Essential Physics II

英語で物理学の

エッセンス II

Lecture 7:   09-11-15

News

Schedule change!!

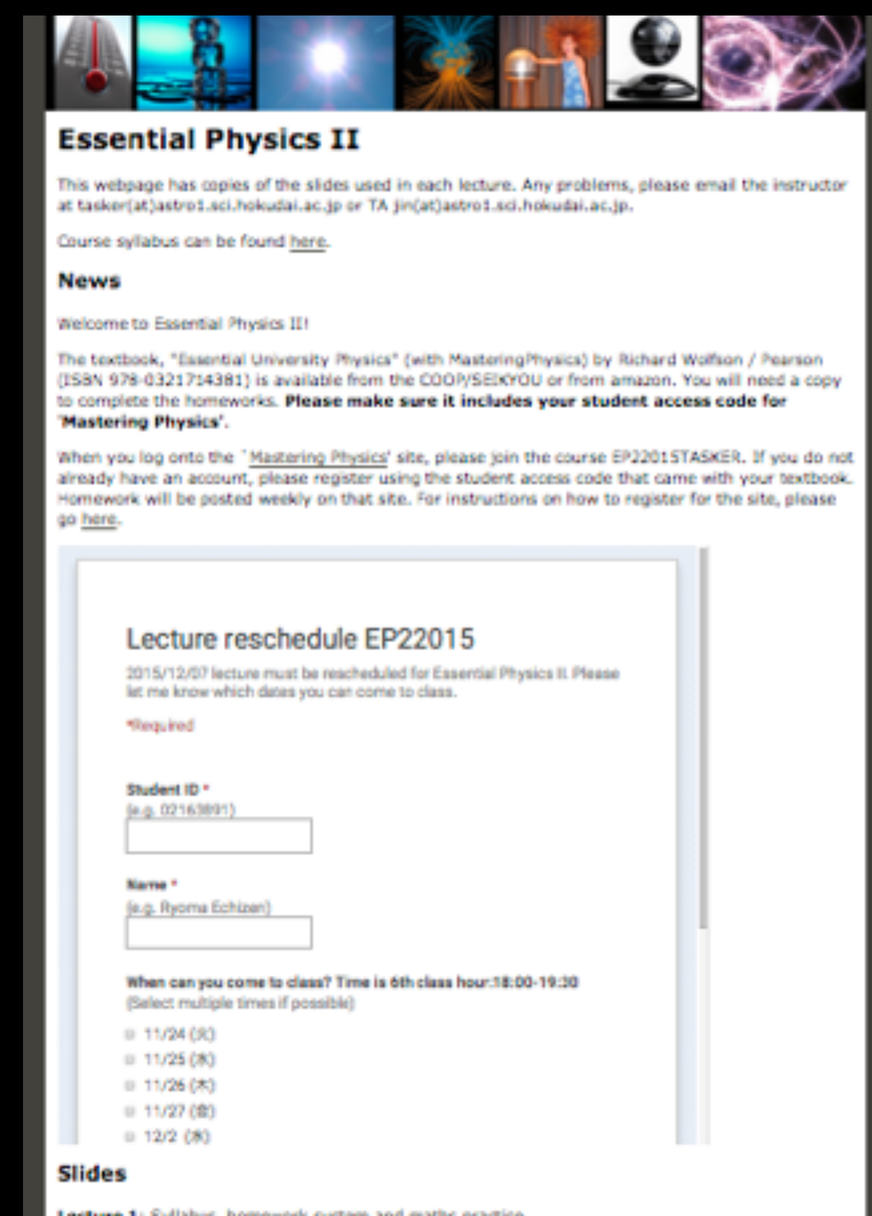
~~Monday 7th December (１２月７日)~~ NO CLASS!

Lecture must be RESCHEDULED

Go to course website

http://astro3.sci.hokudai.ac.jp/~tasker/teaching/ep2/

Choose available dates



Essential Physics II

This webpage has copies of the slides used in each lecture. Any problems, please email the instructor at tasker(at)astro1.sci.hokudai.ac.jp or TA jin(at)astro1.sci.hokudai.ac.jp.

Course syllabus can be found here.

News

Welcome to Essential Physics II!

The textbook, "Essential University Physics" (with MasteringPhysics) by Richard Wolfson / Pearson (ISBN 978-0321714381) is available from the COOP/SEIKYOU or from amazon. You will need a copy to complete the homeworks. **Please make sure it includes your student access code for 'Mastering Physics'.**

When you log onto the 'Mastering Physics' site, please join the course EP2201STASKER. If you do not already have an account, please register using the student access code that came with your textbook. Homework will be posted weekly on that site. For instructions on how to register for the site, please go here.

Lecture reschedule EP22015

2015/12/07 lecture must be rescheduled for Essential Physics II. Please let me know which dates you can come to class.

*Required

Student ID *
(e.g. 02163891)

Name *
(e.g. Ryoma Echizen)

When can you come to class? Time is 6th class hour:18:00-19:30
(Select multiple times if possible)

- 11/24 (火)
- 11/25 (水)
- 11/26 (木)
- 11/27 (金)
- 12/2 (水)

Slides

Quiz

What happens next?

(A) child is angry that kite is caught in power lines

(B) child is electrocuted

Quiz

Kite hitting power line

What happens next?

(A) child is angry that kite is caught in power lines

(B) child is electrocuted

What happens next?

(A) parasailer is angry he landed on a power line.

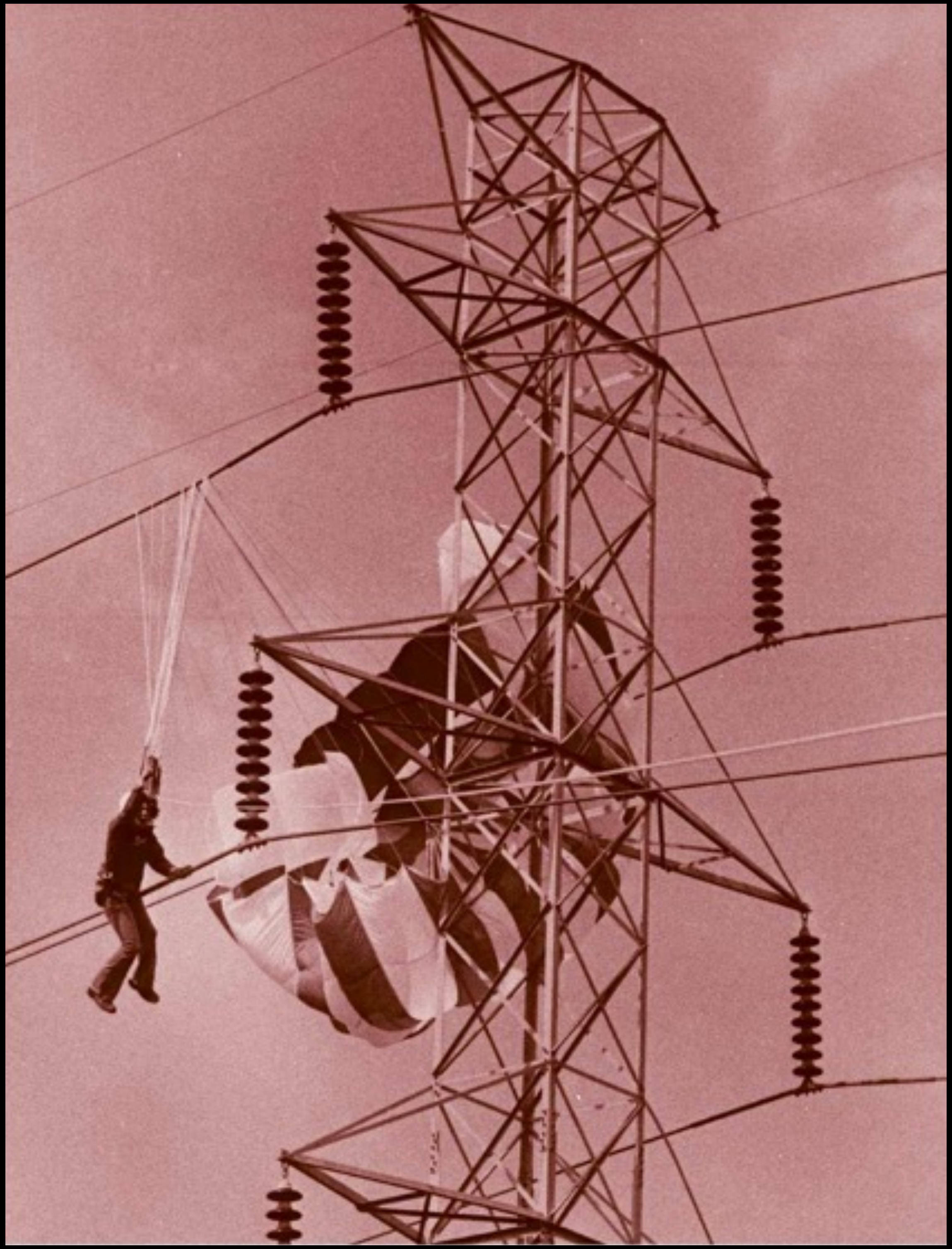
(B) parasailer is electrocuted

Quiz

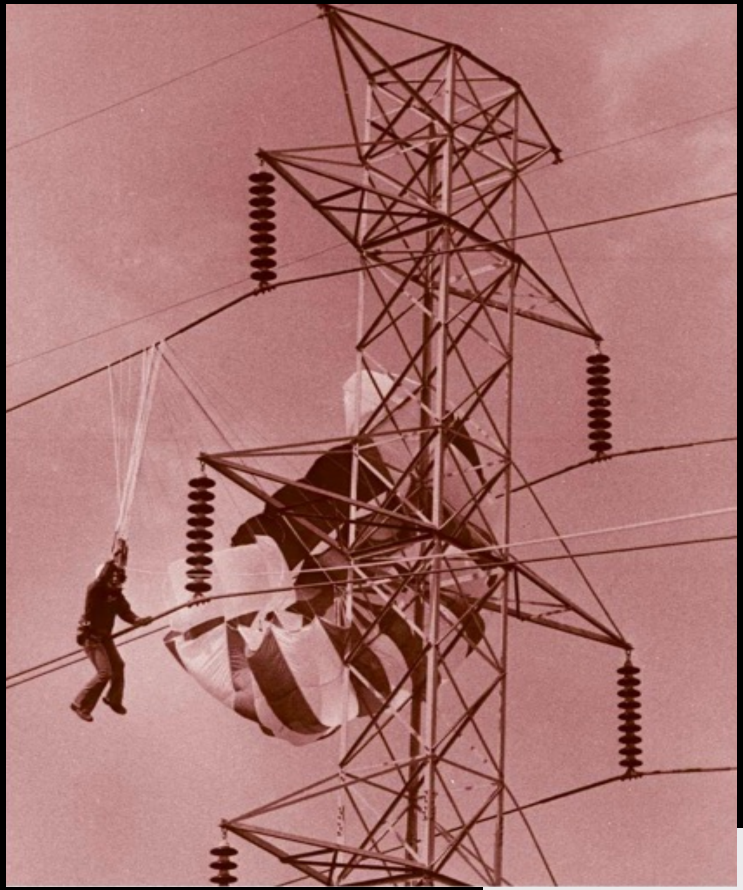
What happens next?

(A) parasailer is angry he landed on a power line.

(B) parasailer is electrocuted

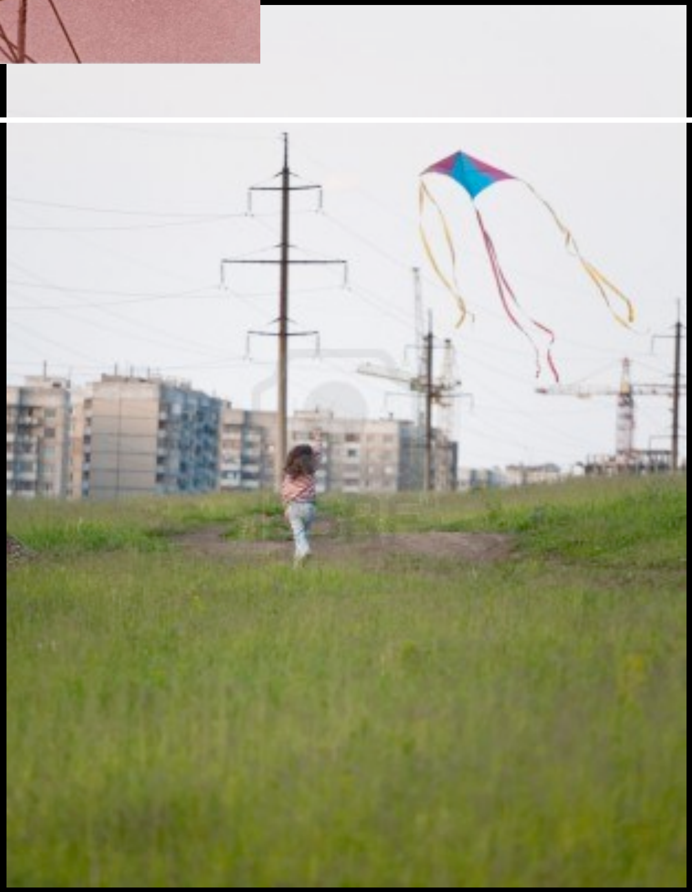


Why the difference?



Chapter 22:

Electric potential



Last semester...

$$\Delta U_{\mathrm{AB}} = U_{\mathrm{B}} - U_{\mathrm{A}} \qquad = -W_{\mathrm{AB}} \quad = -\int_{\mathrm{A}}^{\mathrm{B}} \bar{F} \cdot d\hat{r}$$

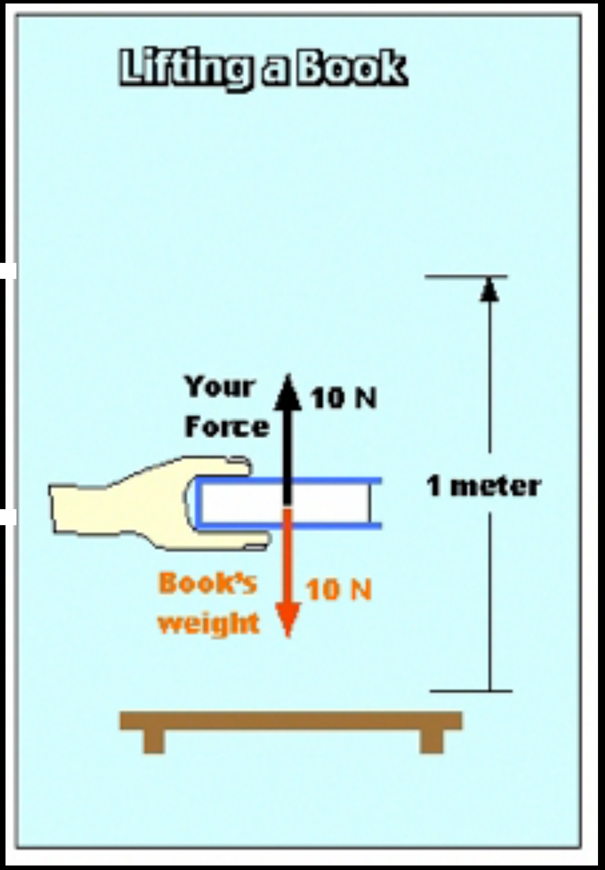
Potential energy difference

Work done
by a conservative force

on an object moved from A to B

B

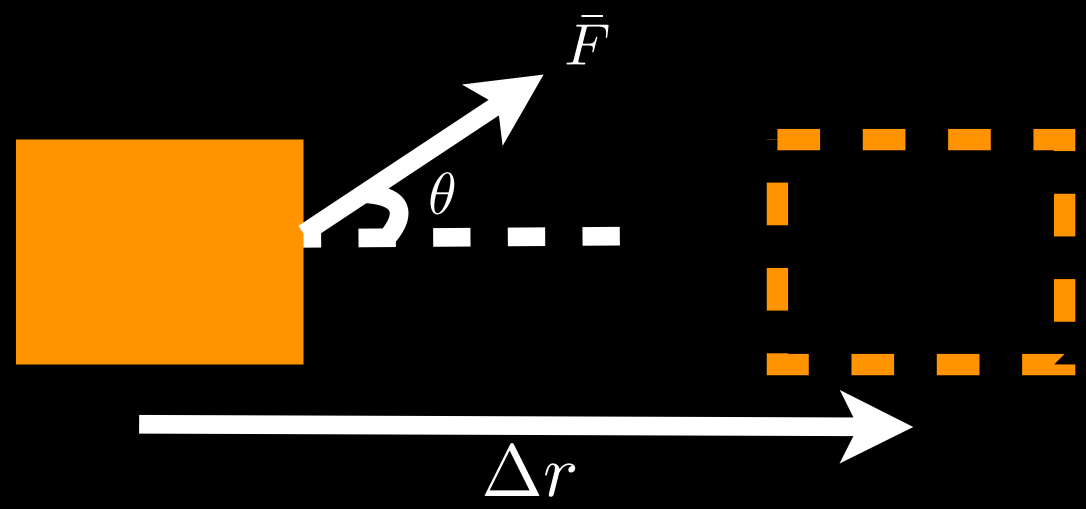
A



Last semester...

$$\Delta U_{\mathrm{AB}} = U_{\mathrm{B}} - U_{\mathrm{A}} \qquad = -W_{\mathrm{AB}} \quad = -\int_{\mathrm{A}}^{\mathrm{B}} \bar{F} \cdot d\hat{r}$$

where work:



$$W_{\mathrm{AB}} = \int_{A}^{B} \bar{F} \cdot d\hat{r}$$

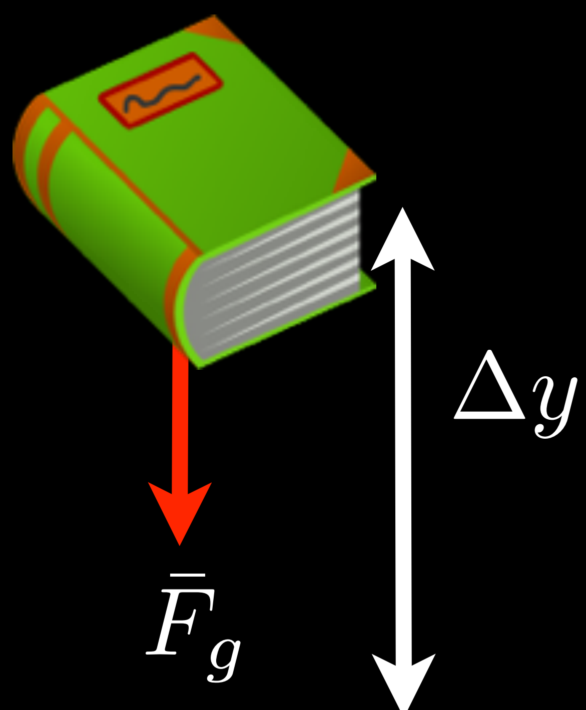
$$= \bar{F} \cdot \Delta\bar{r} \quad = F \Delta r \cos\theta$$

if $\bar{F}$ is constant

Last semester...

$$\Delta U_{\mathrm{AB}} = U_{\mathrm{B}} - U_{\mathrm{A}} \qquad = -W_{\mathrm{AB}} \quad = -\int_{\mathrm{A}}^{\mathrm{B}} \bar{F} \cdot d\hat{r}$$

e.g. when lifting a book:



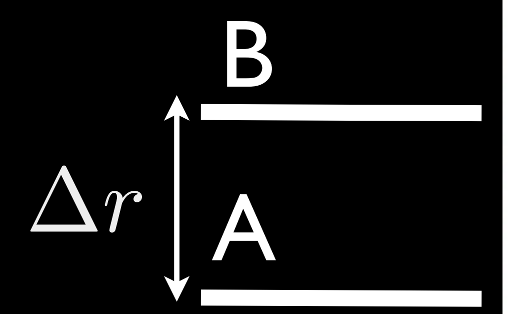
$$- W_{\mathrm{grav}} = -\bar{F} \cdot \Delta \bar{r} \qquad = \quad \Delta U$$
$$= -mg\Delta y$$

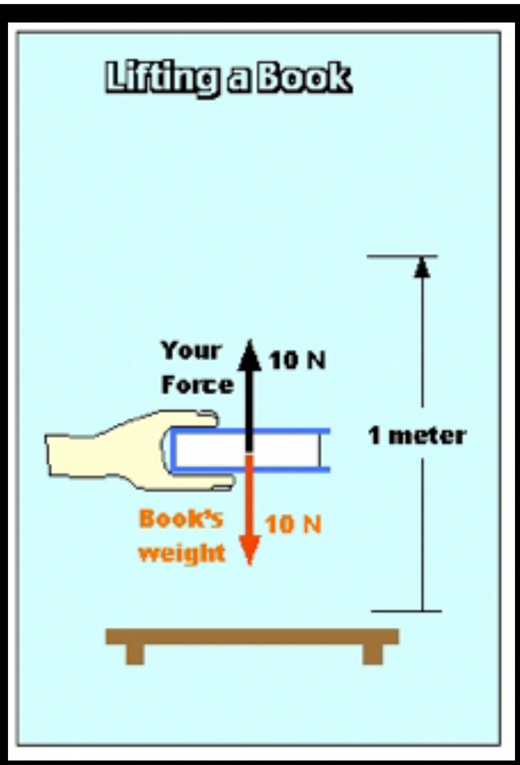
Gravitational potential energy: $\Delta U = -mg\Delta y$

Last semester...

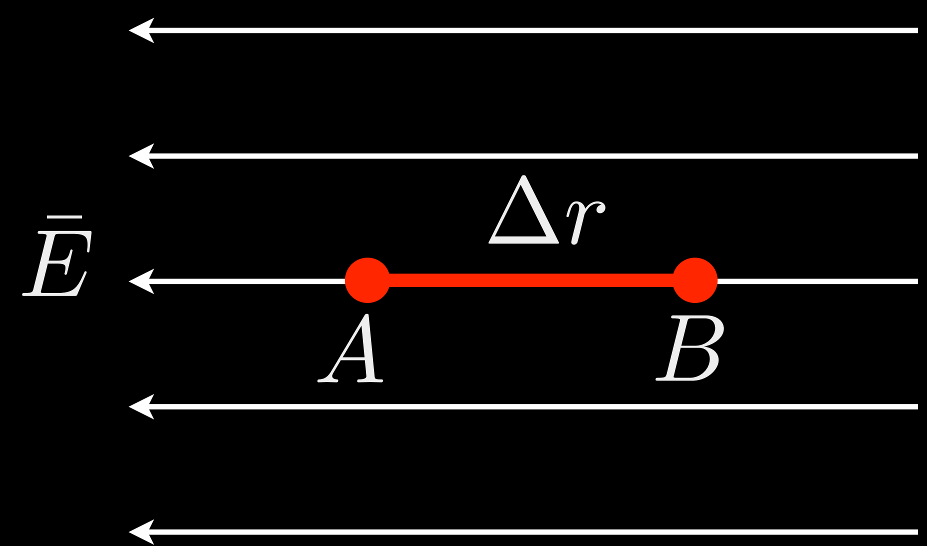
$$\Delta U_{\mathrm{AB}} = U_{\mathrm{B}} - U_{\mathrm{A}} \qquad = -W_{\mathrm{AB}} \quad = -\int_{\mathrm{A}}^{\mathrm{B}} \bar{F} \cdot d\hat{r}$$

Here, the conservative force is gravity.





The electric force is also a conservative force.

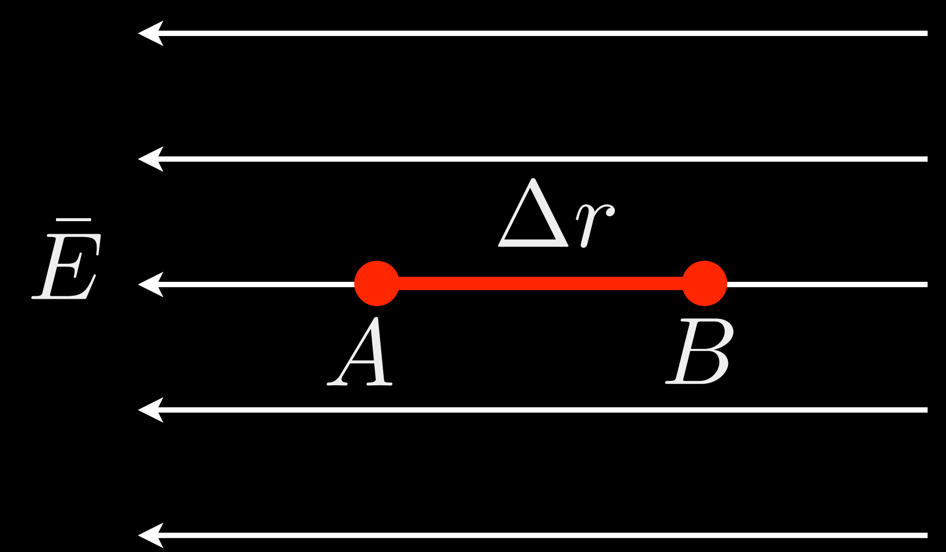


Electric Potential Difference

Positive charge, q

move from A to B

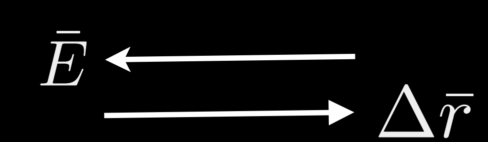
in uniform electric field, $\bar{E}$



Constant electric force: $\bar{F} = q\bar{E}$

$$\begin{aligned}
\Delta U_{\mathrm{AB}} &= -W_{\mathrm{AB}} = -\bar{F} \cdot \Delta\bar{r} \\
&= -q\bar{E} \cdot \Delta\bar{r} \\
&= -qE\Delta r \cos 180^\circ \\
&= qE\Delta r
\end{aligned}$$

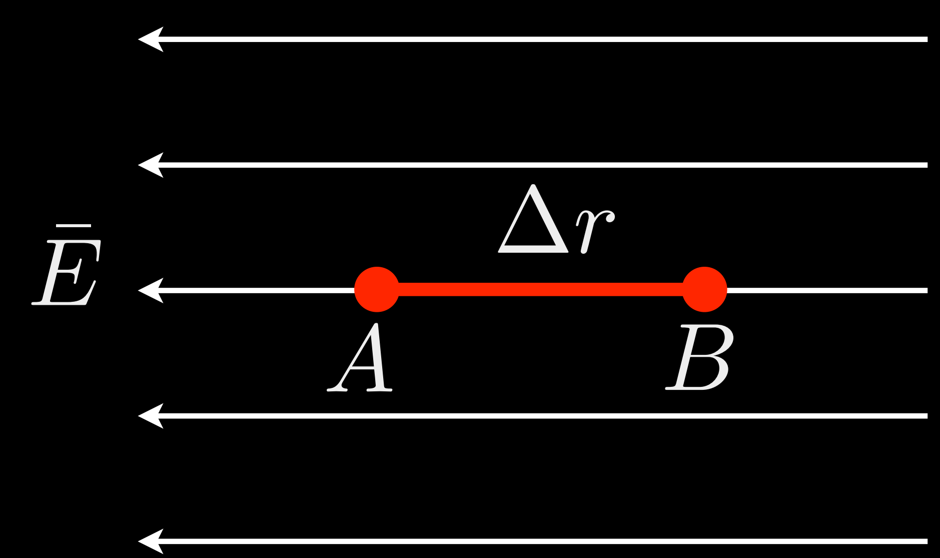
$\bar{E}$ and $\Delta\bar{r}$ point in opposite directions



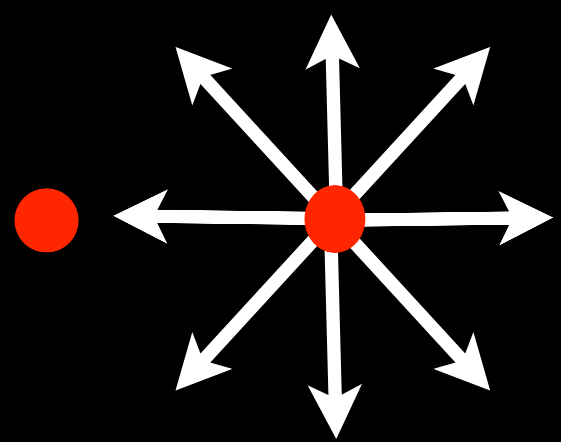
$\cos 180^\circ = -1$

Electric Potential Difference

$\Delta U_{\mathrm{AB}} = qE\Delta r$ - does this make sense?



Pushing a positive charge against the electric field



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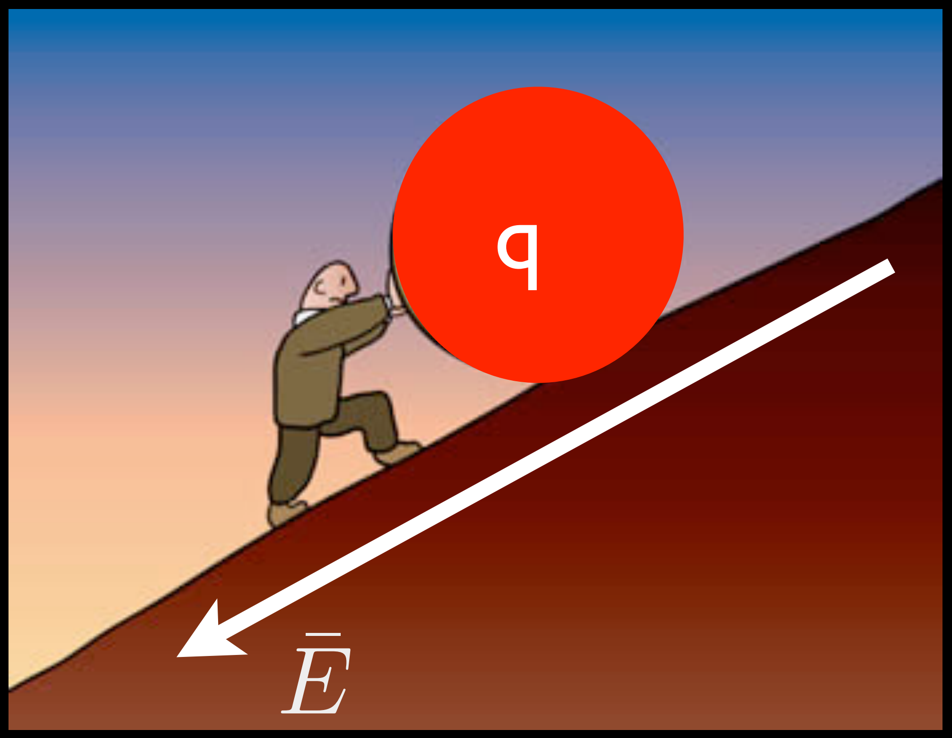
Pushing a car uphill

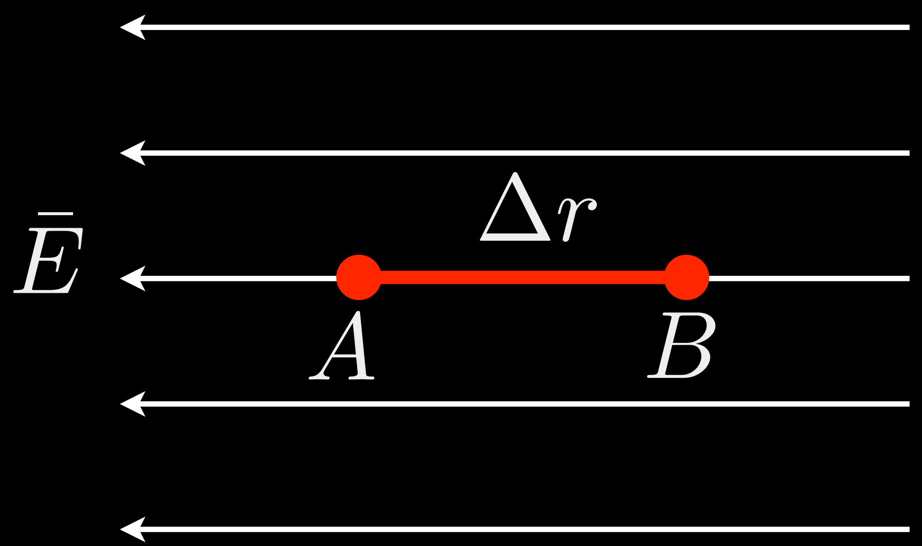


Potential energy increases.

Electric Potential Difference

$$\Delta U_{\mathrm{AB}} = q E \Delta r$$

Potential energy depends on charge, q



Better measure:

$$\Delta V_{\mathrm{AB}} = \frac{\Delta U_{\mathrm{AB}}}{q}$$

electric potential difference

$$= E \Delta r$$

More general:

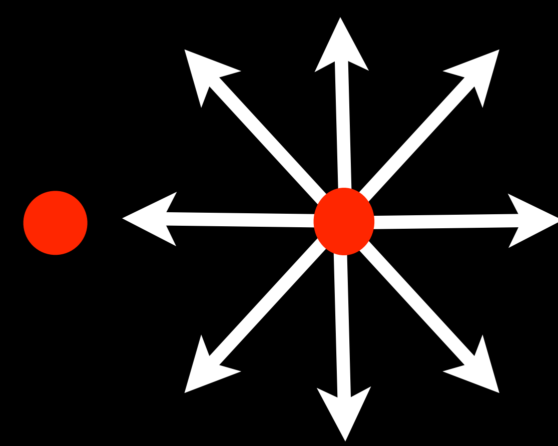
$$\boxed{\Delta V_{\mathrm{AB}} = -\int_A^B \bar{E} \cdot d\bar{r}} \quad = -\bar{E} \cdot \Delta \bar{r}$$

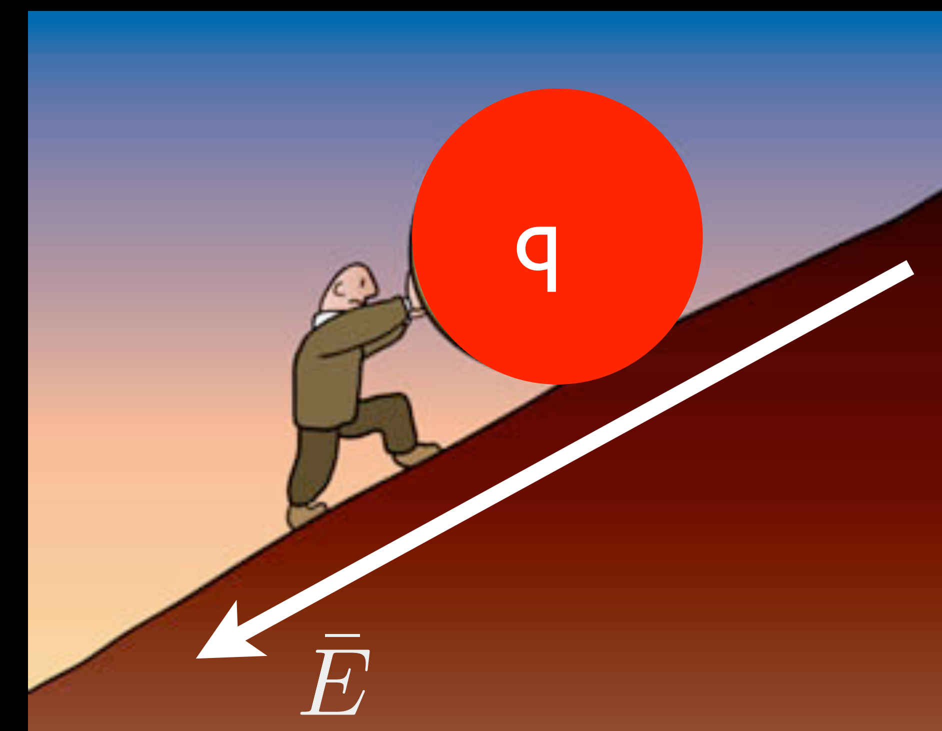
uniform field

Electric Potential Difference

Moving a positive charge through a positive potential difference

Going uphill

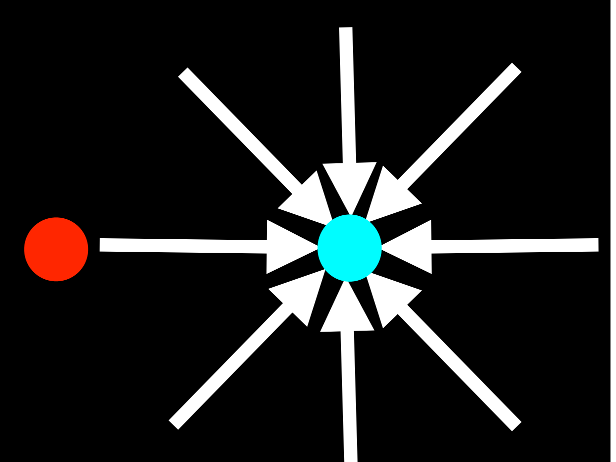


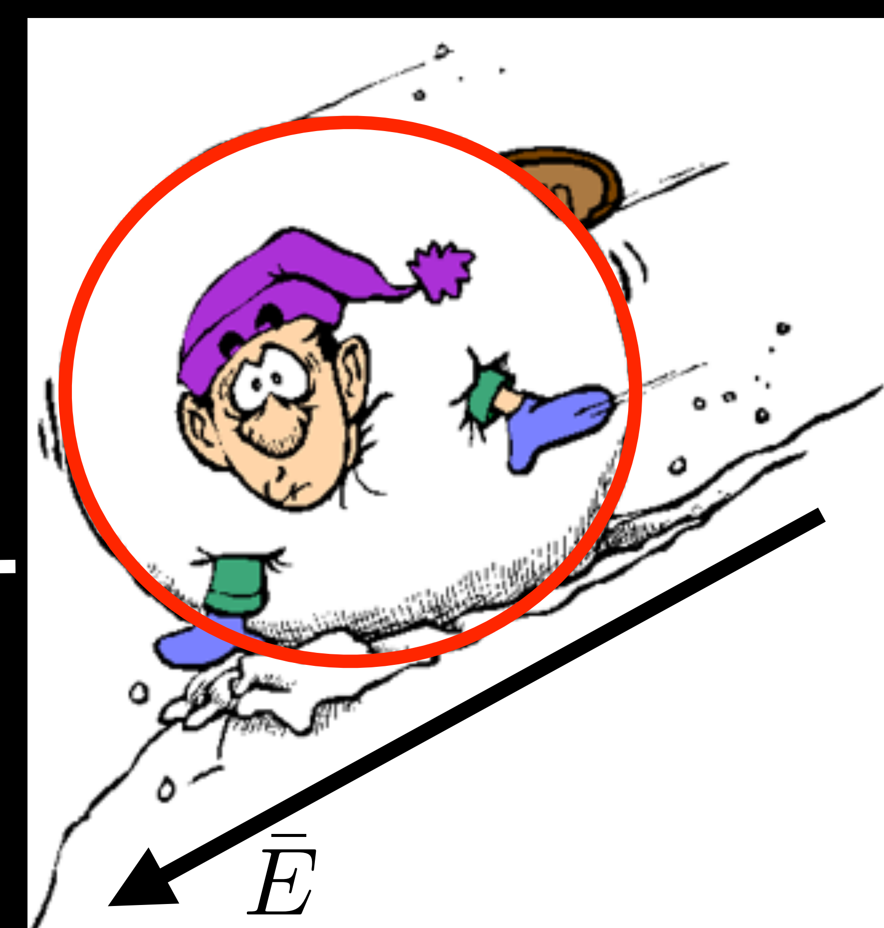


Potential energy increases

Moving a positive charge through a negative potential difference

Going downhill

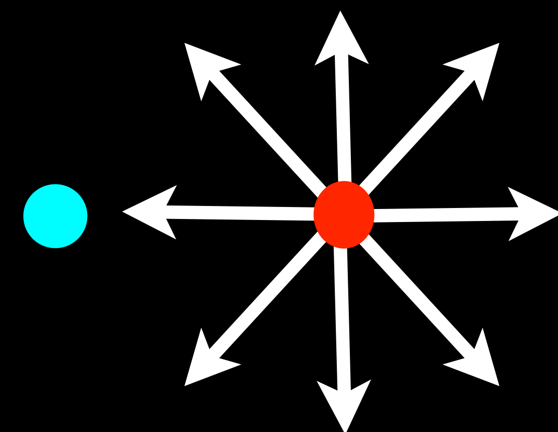




Potential energy decreases

Electric Potential Difference

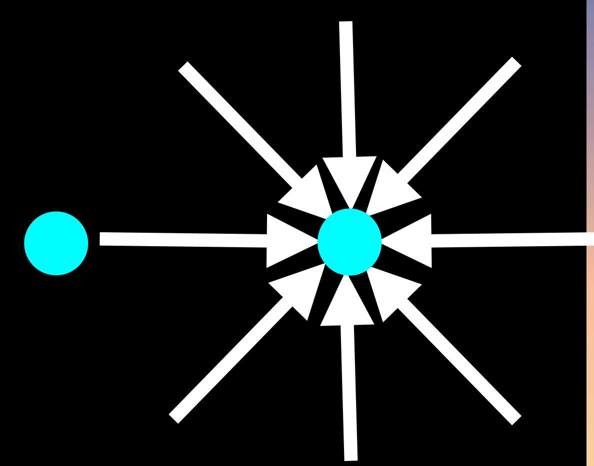
Moving a negative charge through a positive potential difference

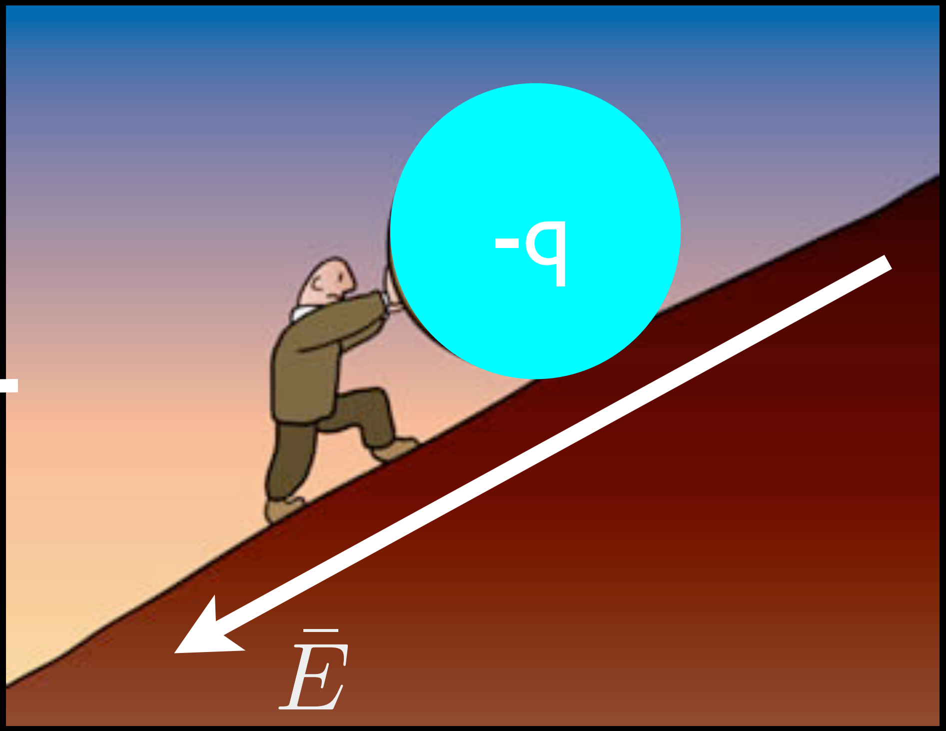


Going downhill

Potential energy decreases

Moving a negative charge through a negative potential difference



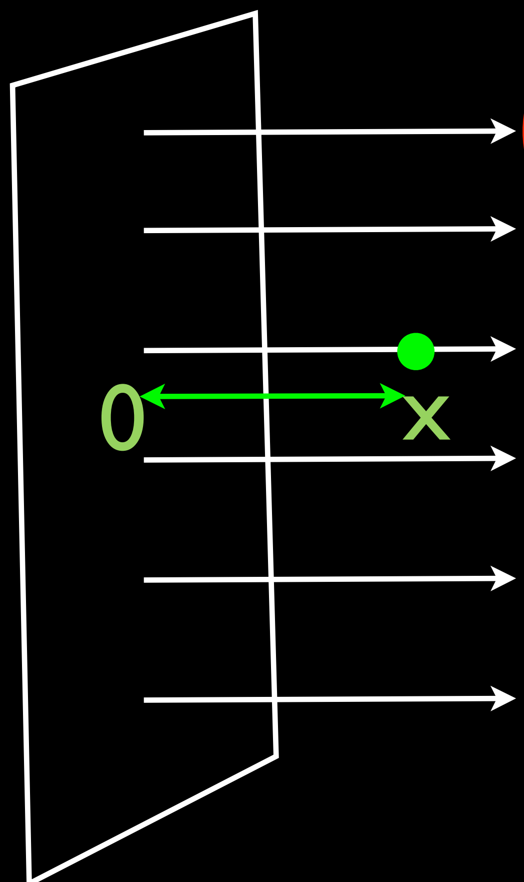


Going uphill

Potential energy increases

Potential of a charged sheet

What is the potential difference from an infinitely charged sheet at x?



$$E = \frac{\sigma}{2\epsilon_0}$$

from last lecture

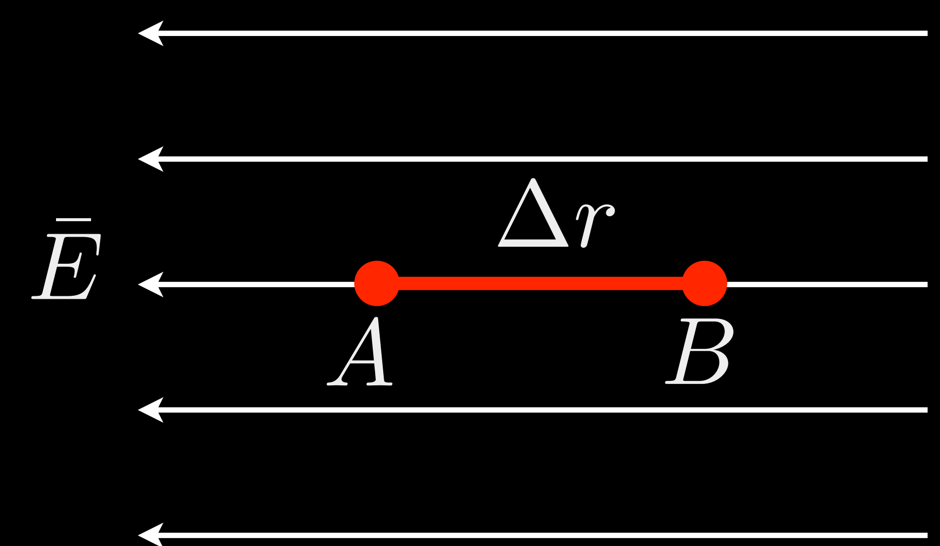
σ charge surface density

Uniform field: $$\Delta V_{0x} = -\bar{E} \cdot \Delta \bar{r}$$

$$= -Ex = -\frac{\sigma x}{2\epsilon_0}$$

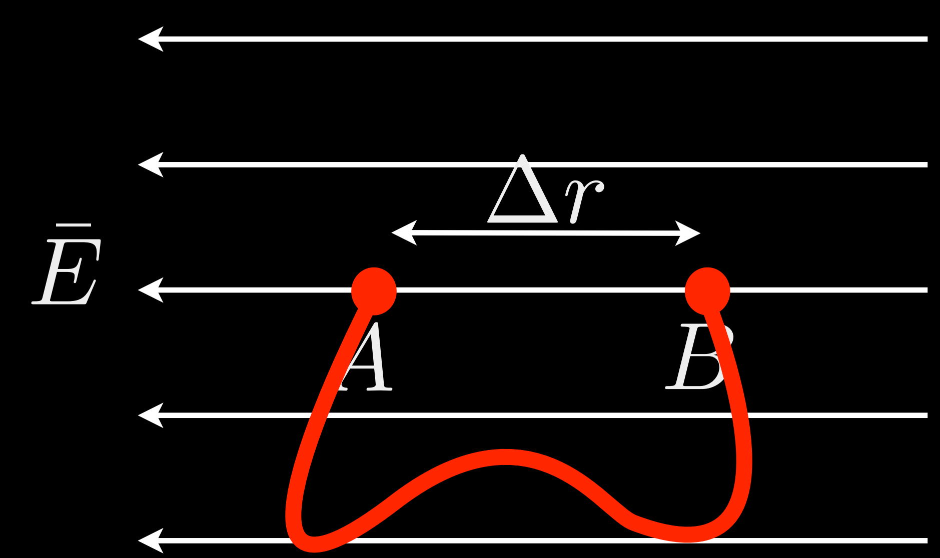
Electric Potential Difference

Potential energy depends on two points (A and B)



Not on the path taken between A and B.

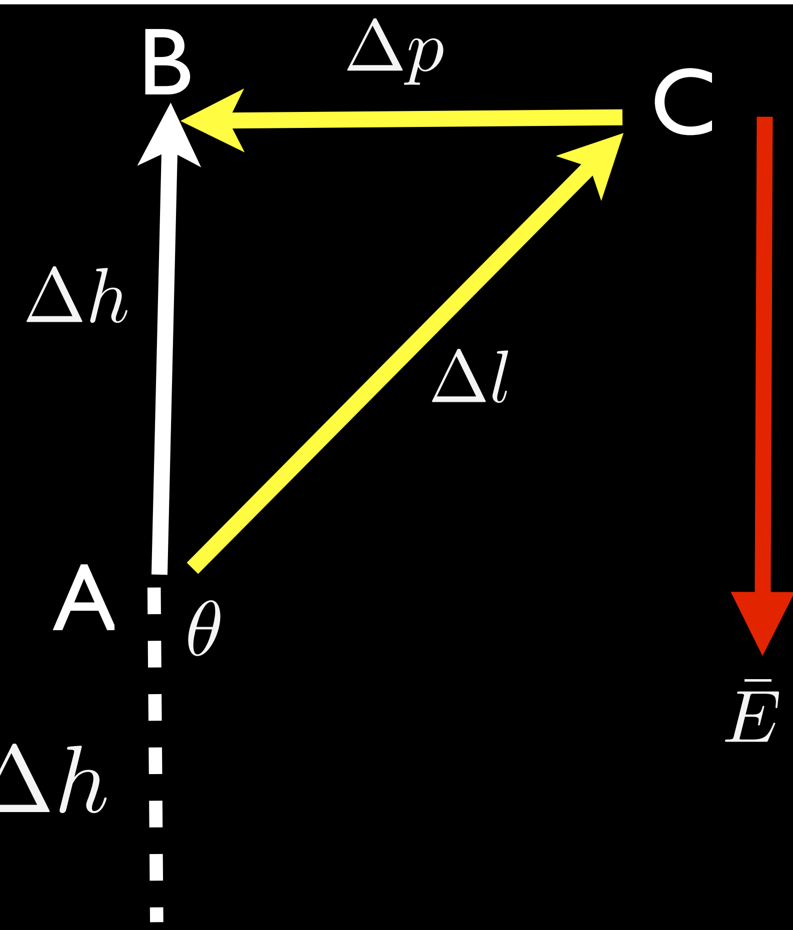
Potential difference is the same for both diagrams.



$$\Delta V_{\mathrm{AB}} = -\bar{E} \cdot \Delta \bar{r}$$

Electric Potential Difference

Is ΔV really independent of path?



$$\Delta V_{\mathrm{AB}} = V_B - V_A = -\int_A^B \bar{E}\cdot d\bar{r}$$

Path 1: A $\longrightarrow$ B

$$\Delta V_{\mathrm{AB}} = -\bar{E}\cdot\Delta\bar{r} = -E\Delta h\cos 180^\circ = E\Delta h$$

uniform field

Path 2: A $\longrightarrow$ C $\longrightarrow$ B

$$\Delta V_{\mathrm{AB}} = -\int_A^C \bar{E}\cdot d\bar{r} - \int_C^B \bar{E}\cdot d\bar{r}$$

$$= -E\Delta l\cos\theta \; -E\Delta p\cos 90^\circ \quad = E\Delta h - 0$$

Electric Potential Difference

Quiz

What would happen to the potential difference between points A and B if the distance Δr were doubled?

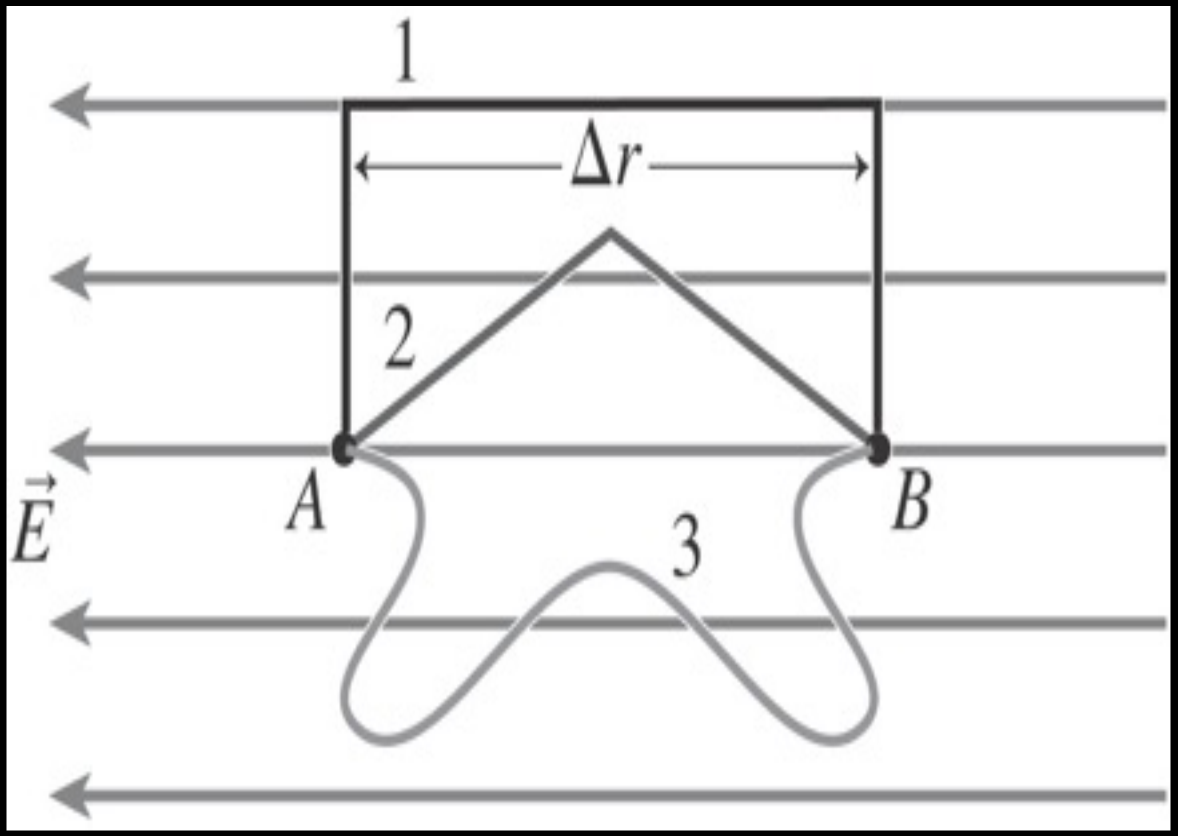
(A) ΔV doubled

$\Delta V \propto \Delta r$

(B) ΔV halved

(C) ΔV quadrupled (x4)

(D) ΔV quartered (x 1/4)

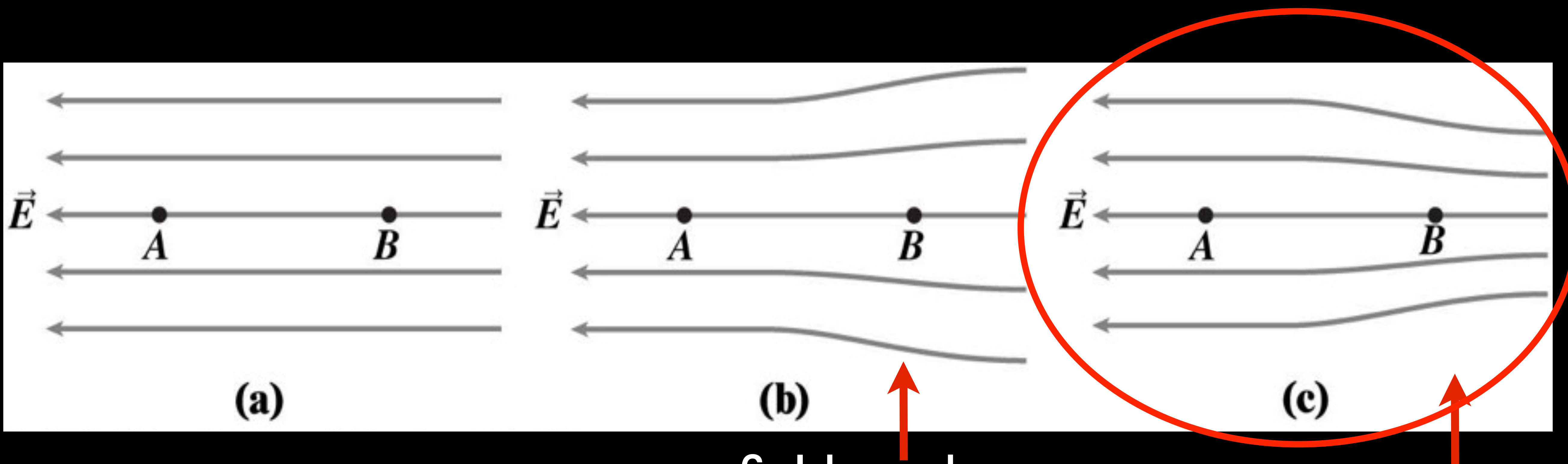


Electric Potential Difference

Quiz

3 straight paths A to B of the same length, each in a different electric field.

Which one of the three has the largest potential difference, ΔV_{AB} , between the two points?



field constant

field weaker

"hill" gradient
less steep

field stronger

"hill" gradient
more steep

The volt and the electron volt

Potential difference, ΔV, is energy per unit charge.

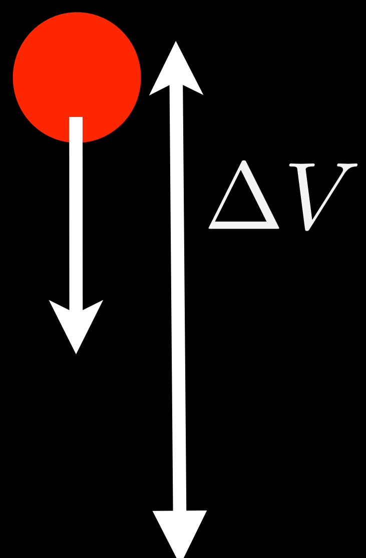
$$\Delta V_{\mathrm{AB}} = \frac{\Delta U_{\mathrm{AB}}}{q} = -\int_A^B \bar{E} \cdot d\bar{r}$$

Unit is the volt (V).

1 volt (V) = 1 joule per coulomb (J/C)

Related energy unit: electronvolt (eV)

Energy gained by q = e falling through $\Delta V = 1\mathrm{V}$



since: $\mathrm{e} = 1.6 \times 10^{-19}\mathrm{C}$

$$1\mathrm{eV} = 1.6 \times 10^{-19}\mathrm{J}$$

The volt and the electron volt

Quiz

An alpha particle (charge 2e) moves through a 10-V potential difference. How much work, expressed in eV, is done on the alpha particle?

(A) 5 eV

(B) 10 eV

(C) 20 eV

(D) 40 eV

$$\Delta U = q\Delta V = -W$$

work done by electric field

Work energy gained by particle:

$$W = q\Delta V = (2e)(10\,\mathrm{V})\,\mathrm{J}$$

$$= (2)(1.6 \times 10^{19}\,\mathrm{C})(10\,\mathrm{V})\,\mathrm{J}$$

$$= \frac{(2)(1.6 \times^{-19}\,\mathrm{C})(10\,\mathrm{V})}{1.6 \times 10^{19}\,\mathrm{J}}\,\mathrm{eV}$$

$$= 20\,\mathrm{eV}$$

Potential of a point charge

$\bar{E}$ from a point charge varies with radius:

$$\bar{E} = \frac{kq}{r^2}\hat{r}$$

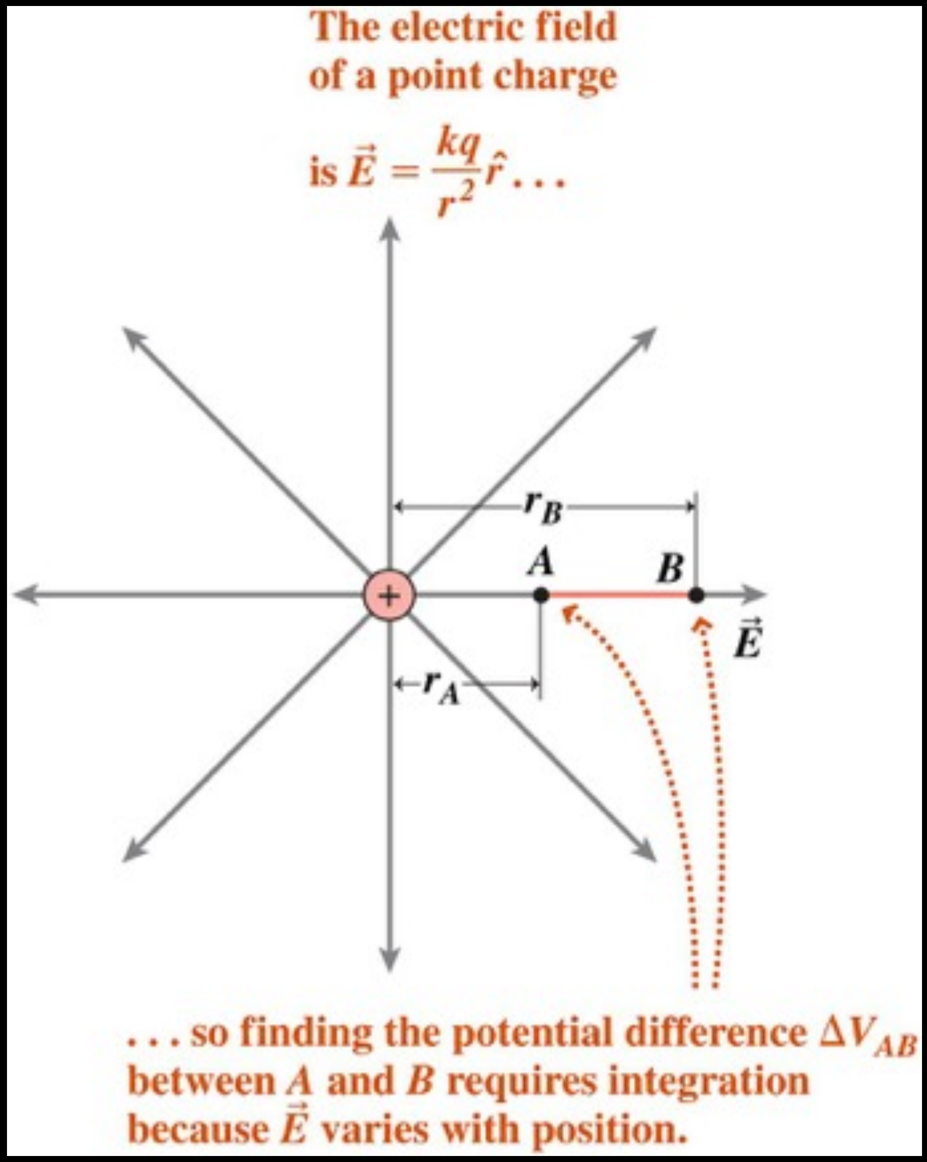
Use integral to get potential difference:

$$\Delta V_{\mathrm{AB}} = -\int_{r_A}^{r_B} \bar{E}\cdot d\bar{r} \;= -\int_{r_A}^{r_B} \frac{kq}{r^2}\hat{r}\cdot d\bar{r}$$

$$d\bar{r} = dr\hat{r}$$
$$\hat{r}\cdot\hat{r} = 1$$

$$= -kq\int_{r_A}^{r_B} r^{-2}dr$$

$$= kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$



Potential of a point charge

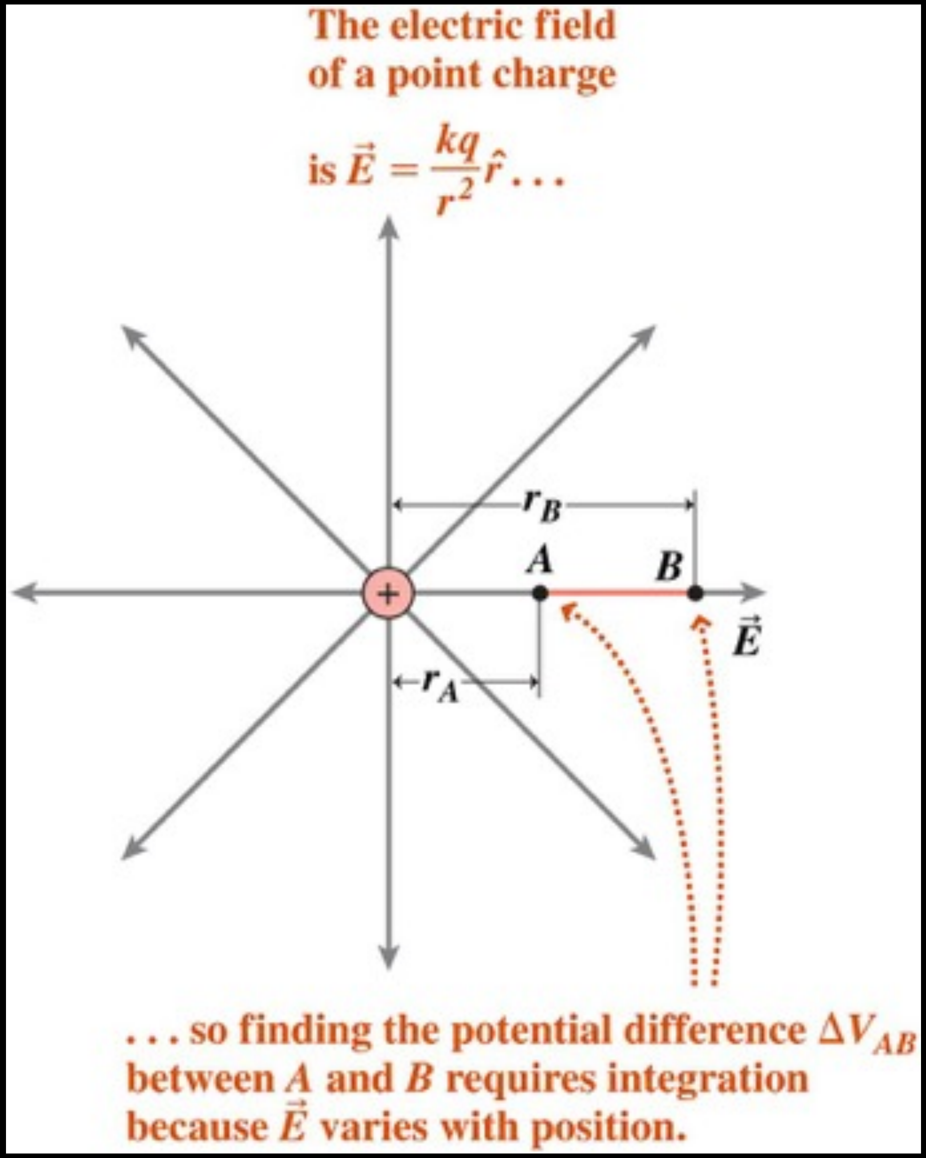
Where is the potential zero? Actually...

Only potential difference (ΔU and ΔV) have physical meaning.

But it is useful to define 'zero point'.

For an isolated point charge, choose:
'zero potential' at infinity

$$U = 0 \qquad\qquad r = \infty$$



Then, $r_A \rightarrow \infty$ and $\frac{1}{r_A} \rightarrow 0$ so:

$$\boxed{V_{\infty}r = V(r) = \frac{kq}{r}}$$

point charge potential

(actually, potential difference between infinity and r)

Potential of a point charge

Quiz

You measure $\Delta V = 50V$ between 2 points 10 cm apart parallel to the field produced by a point charge. Suppose you move closer to the point charge.

How will the potential difference change?

(A) The potential difference will remain the same.

(B) The potential difference will increase. $\Delta V \propto \frac{1}{r}$

(C) The potential difference will decrease.

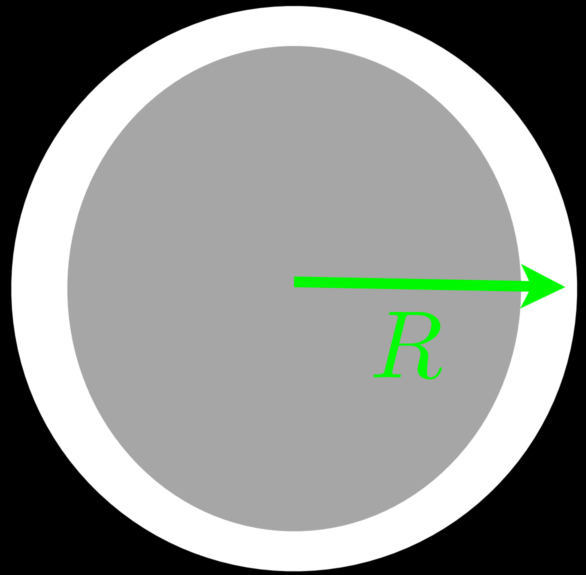
(D) We cannot find this without knowing how much closer we are.

Potential of a charged spherical shell

$\bar{E}$ field (Last lecture: Gauss's Law)

$$r > R: \quad E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2}$$

$$R = 2.3m$$
$$Q = 640\mu C$$



same as point charge!

Potential at surface: (ΔV between zero point (infinity) and surface)

$$V(R) = \frac{kq}{r} = \frac{kQ}{R} = 2.5\,\mathrm{MV}$$

Work need to bring proton (q = e) from infinity to sphere surface:

$$2.5MeV = 4.0 \times 10^{-13} J$$

ΔV between sphere surface and point 2R from centre:

$$\Delta V_{\mathrm{R2R}} = V(2R) - V(R) = \frac{kq}{2R} - \frac{kq}{R} = -\frac{1}{2}\left(\frac{kq}{R}\right) = -1.25\,\mathrm{MV}$$

Potential of a charged spherical shell Quiz

A charged hollow sphere has r = 5 cm. The electric potential at a point 15 cm from the sphere's centre is 30 Volts.

What is the potential on the surface of the sphere?

(A) 90 V

(B) 100 V

(C) 30 V

(D) 15 V

(E) Can not tell; need to know the charge on the sphere

Outside sphere, potential = point charge: $V(R) \propto \frac{1}{r}$

$$V(R = 15\,\mathrm{cm}) = 30\,\mathrm{V} \propto \frac{1}{0.15\,\mathrm{m}} = \frac{C}{0.15\,\mathrm{m}}$$

$$V(R = 5\,\mathrm{cm}) = X\,\mathrm{V} \propto \frac{1}{0.05\,\mathrm{m}} = \frac{C}{0.05\,\mathrm{m}}$$

$$\frac{X}{30} = \frac{0.15}{0.05} \qquad X = 90\,\mathrm{V}$$

constant

Calculating potential

Quiz

If an E field in the x- direction has form: $E = aqx^2$

What is $V(x)?$ (Assume V = 0 at x = 0)

ΔV between zero point (x = 0) and x

(A) $V(x) = 2aqx$

(B) $V(x) = -2aqx$

$$\Delta V_{0x} = -\int_0^x \bar{E} \cdot d\bar{r} = -\int_0^x aqx^2 dx$$

(C) $V(x) = aqx^3/3$

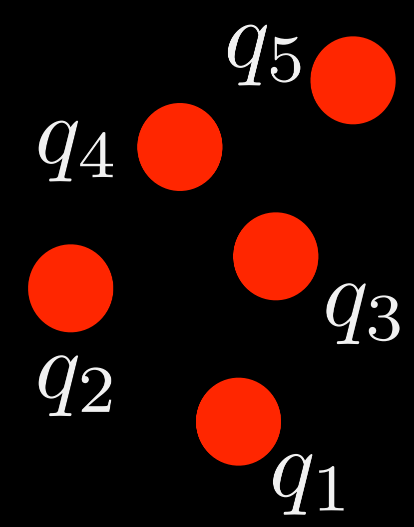
(D) $V(x) = -aqx^3/3$

(E) Can't tell; need to know charge

Potential of a charge distribution

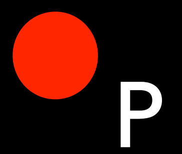
If we know $\bar{E}$ from a charge distribution, we can integrate to find ΔV.

$$\Delta V_{\mathrm{AB}} = -\int_A^B \bar{E} \cdot d\bar{r}$$



For a distribution of discrete point charges, we can sum:

$$V(P) = \sum_i \frac{kq_i}{r_i}$$



Where $V(P)$ is the potential difference between infinity and a point P.

For continuous charge distribution:

$$V(P) = \int \frac{kdq}{r}$$

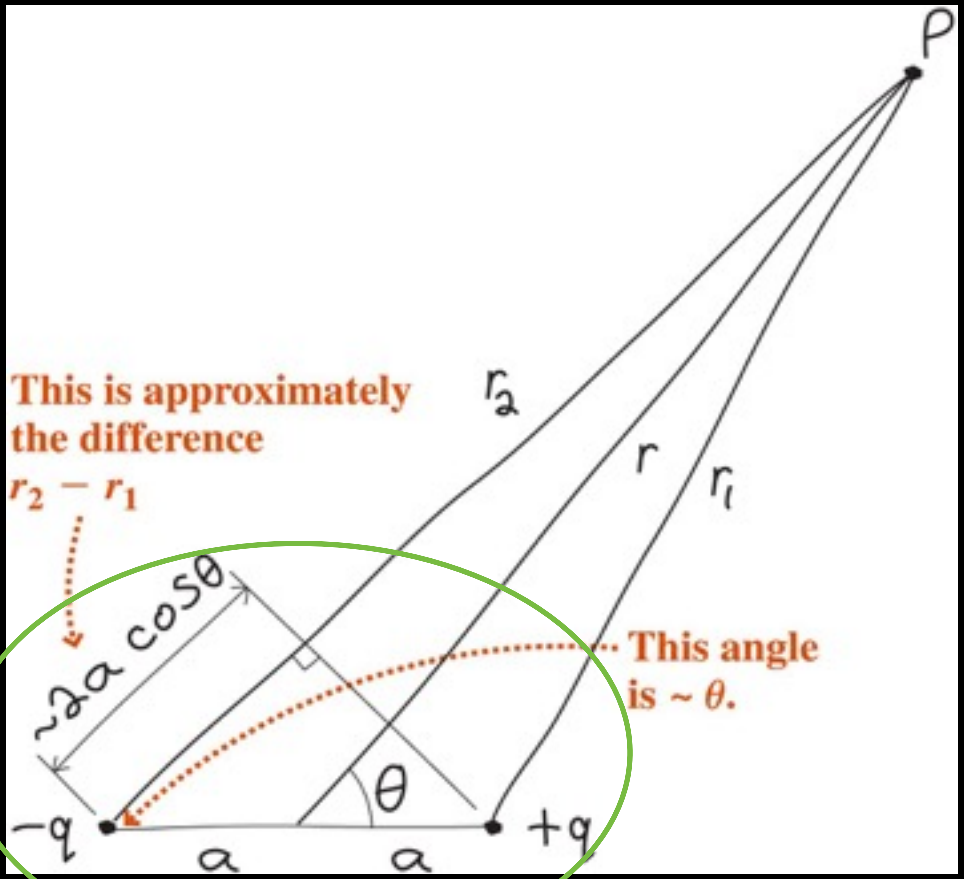
Potential of a charge distribution

Potential of electric dipole at point P.

= distribution of discrete point charges.

$$V(P) = \sum_i \frac{kq_i}{r_i} = \frac{kq}{r_1} + \frac{k(-q)}{r_2}$$

$$= \frac{kq(r_2 - r_1)}{r_1 r_2}$$



When r is large compared to dipole spacing, 2a:

$$V(r, \theta) = \frac{k(2aq)\cos\theta}{r^2}$$

$$\boxed{r_2 - r_1 = 2a\cos\theta}$$

$$\boxed{r_1 \simeq r_2}$$

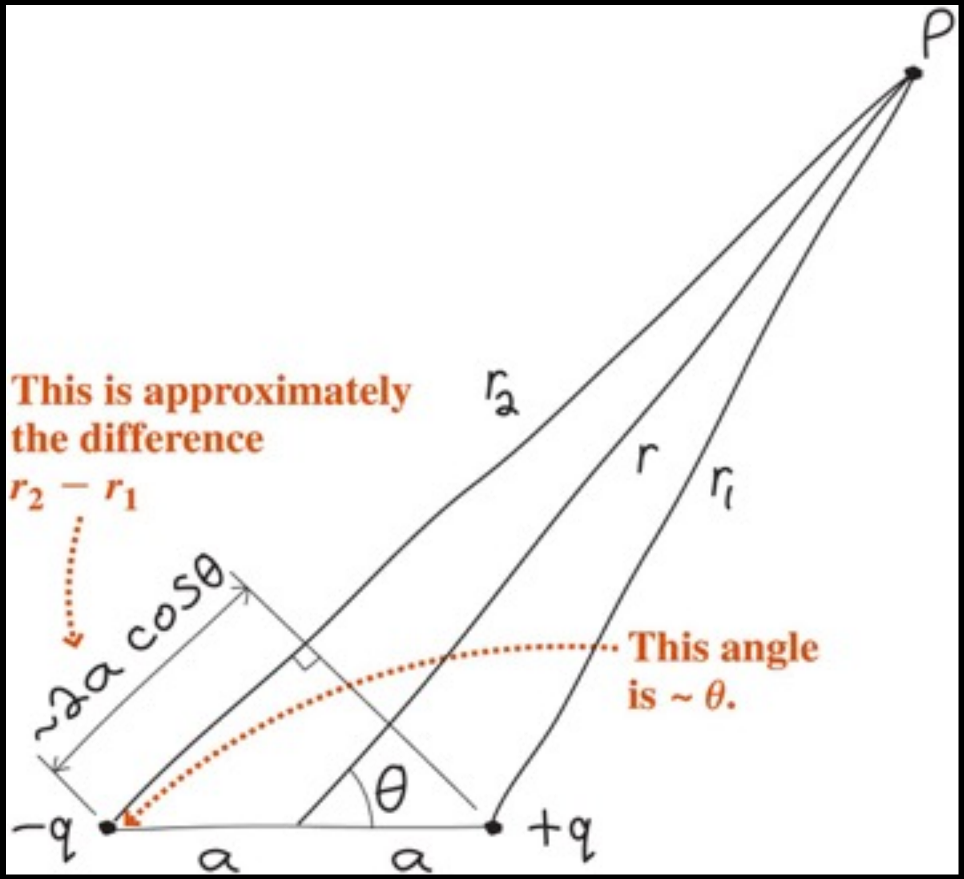
Potential of a charge distribution

Potential of electric dipole at point P.

= distribution of discrete point charges.

$$V(P) = \sum_i \frac{kq_i}{r_i} = \frac{kq}{r_1} + \frac{k(-q)}{r_2}$$

$$= \frac{kq(r_2 - r_1)}{r_1 r_2}$$



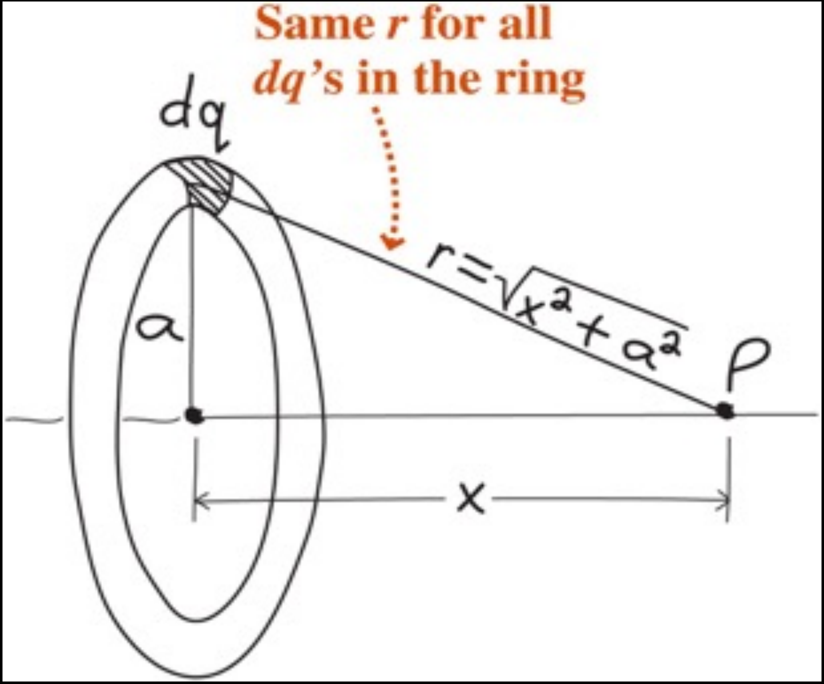
When r is large compared to dipole spacing, 2a:

$$V(r,\theta) = \frac{k(2aq)\cos\theta}{r^2}$$

since dipole moment, p = 2aq: $$V(r,\theta) = \frac{kp\cos\theta}{r^2}$$

Potential of a charge distribution

Potential of uniformly charged ring at point P.



= continuous charge distribution

$$V(P) = \int \frac{k\,dq}{r} = \frac{k}{r} \int dq$$

radius is constant

$$r = \sqrt{x^2 + a^2}$$

Q = total charge

$$\int dq = Q$$

$$= \frac{kQ}{\sqrt{x^2 + a^2}}$$

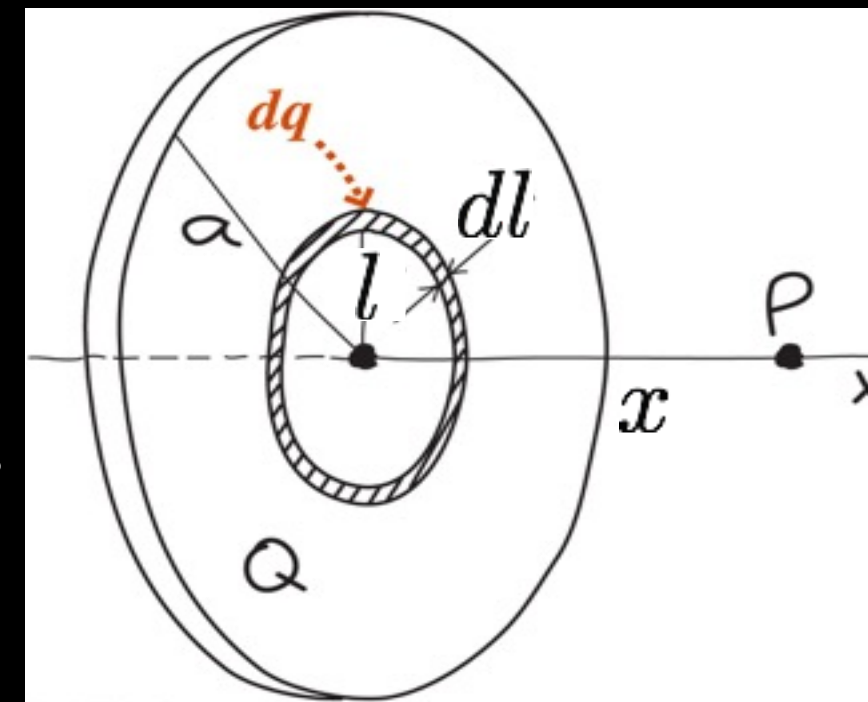
Potential of a charge distribution

Potential of uniformly charged disc at point P.

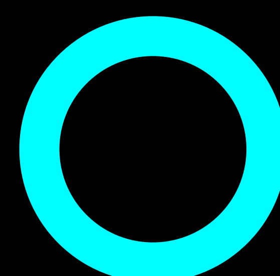
= continuous charge distribution

$$V(P) = \int \frac{kdq}{r}$$

but distance from P varies (radius not constant)



Divide the disc into rings:



dq

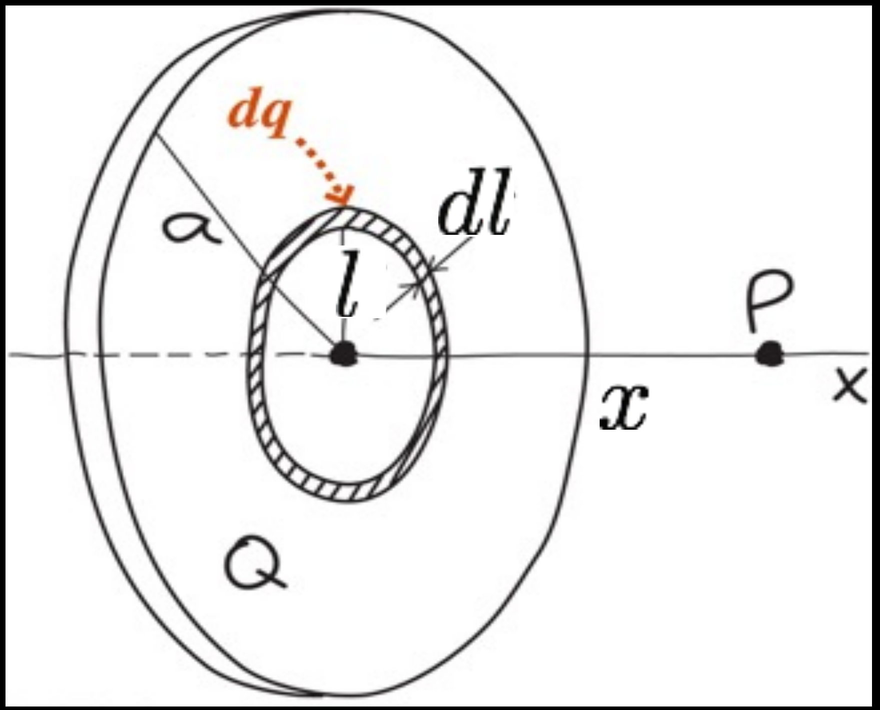
$$dV = \frac{kdq}{\sqrt{x^2 + l^2}}$$

constant radius from p

Potential of a charge distribution

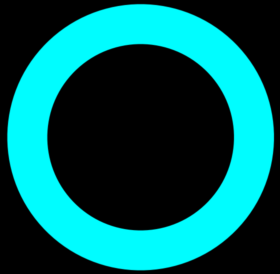
Potential of uniformly charged disc at point P.

= continuous charge distribution



$$V(P) = \int \frac{k\,dq}{r}$$

but distance from P varies (radius not constant)

Divide the disc into rings:  dq

$$dV = \frac{k\,dq}{\sqrt{x^2 + l^2}}$$

$$V(P) = \int dV = \int_{l=0}^{l=a} \frac{k\,dq}{\sqrt{x^2 + l^2}}$$

Charge (q) proportional to area:

$$\frac{dq}{Q} = \frac{2\pi l\,dl}{\pi a^2}$$

area of ring

area of disc

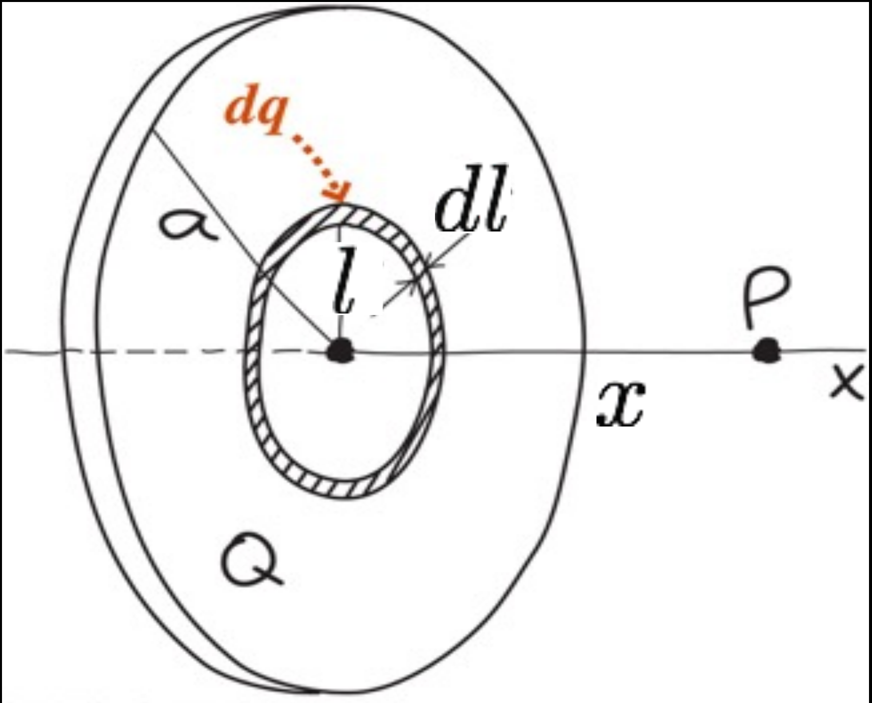
Potential of a charge distribution

$$V(P) = \int_{l=0}^{l=a} \frac{2kQ}{a^2} \frac{l\,dl}{\sqrt{x^2+l^2}}$$

$$= \frac{2kQ}{a^2} \boxed{\int_{l=0}^{l=a} \frac{l\,dl}{\sqrt{x^2+l^2}}}$$

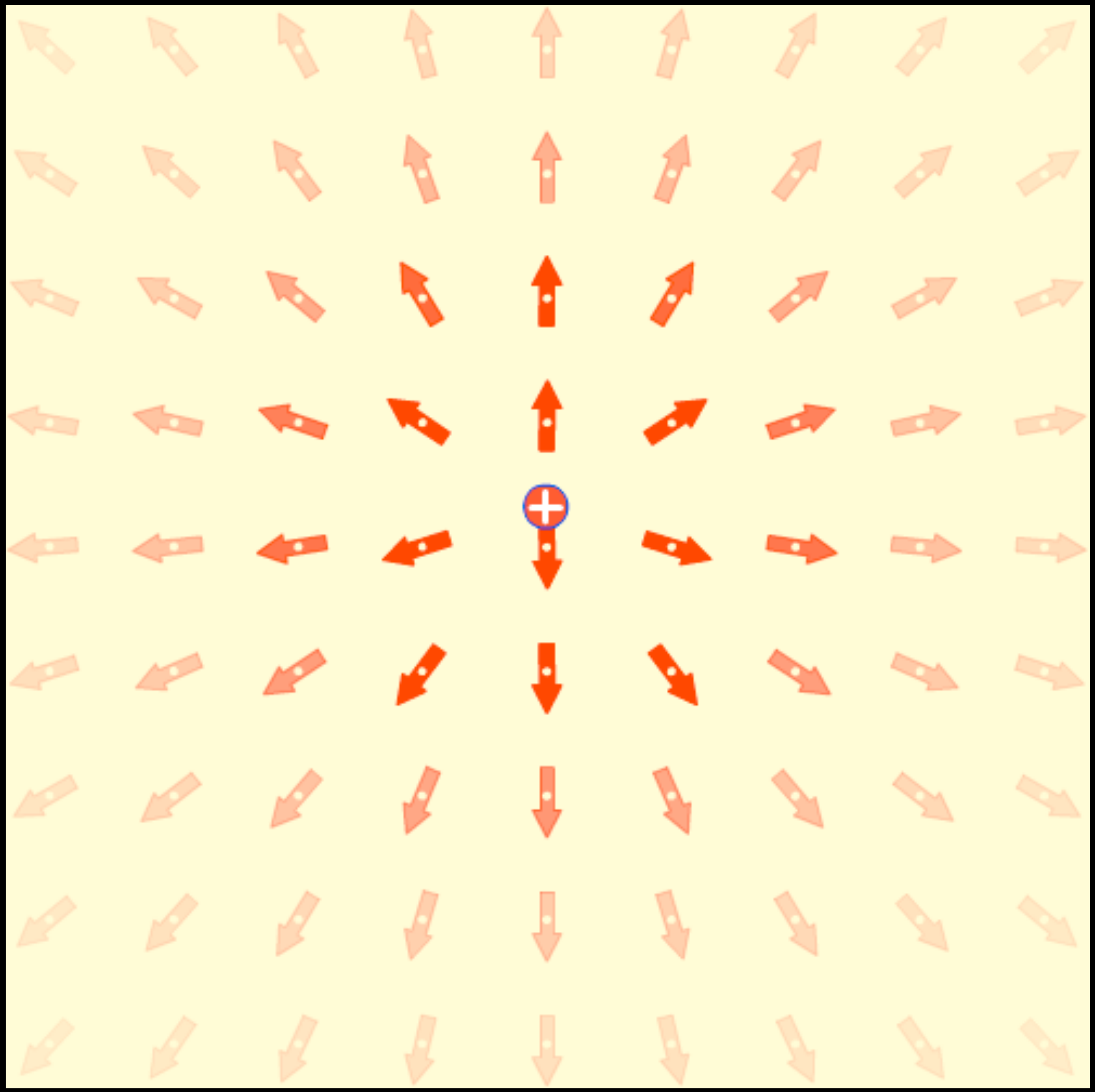
$$\boxed{= \sqrt{x^2+l^2}}$$

$$= \frac{2kQ}{a^2}(\sqrt{x^2+a^2} - |x|)$$



Potential difference and the electric field

Moving perpendicular to the electric field requires no work.



Potential difference and the electric field

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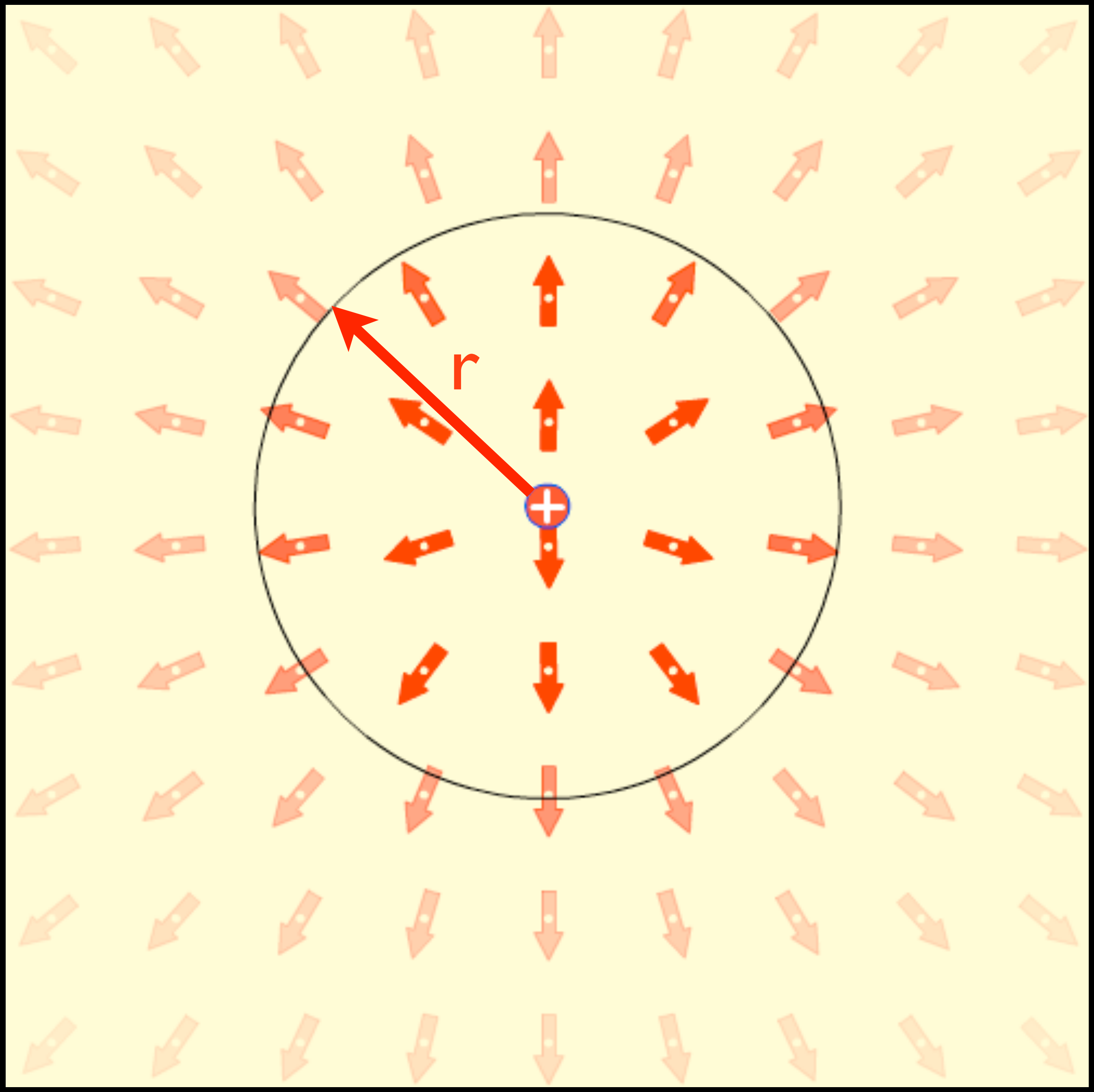


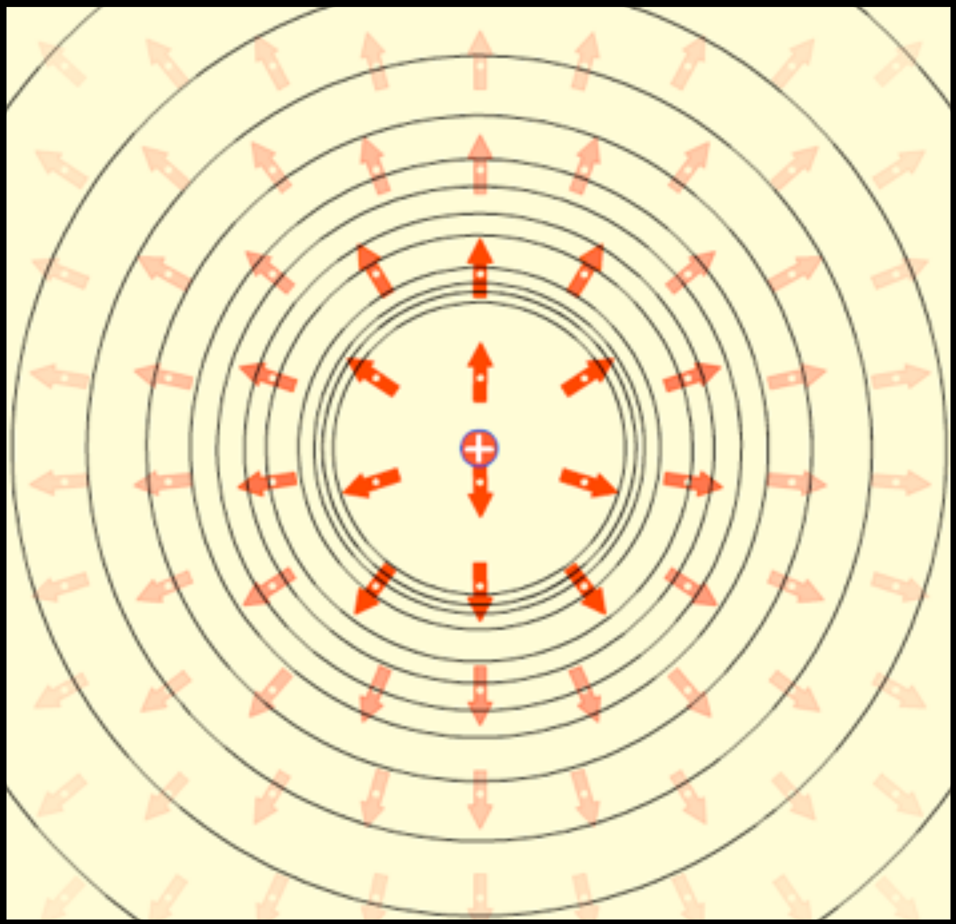
For a point charge, r = constant

Therefore, there is no potential difference along this line.

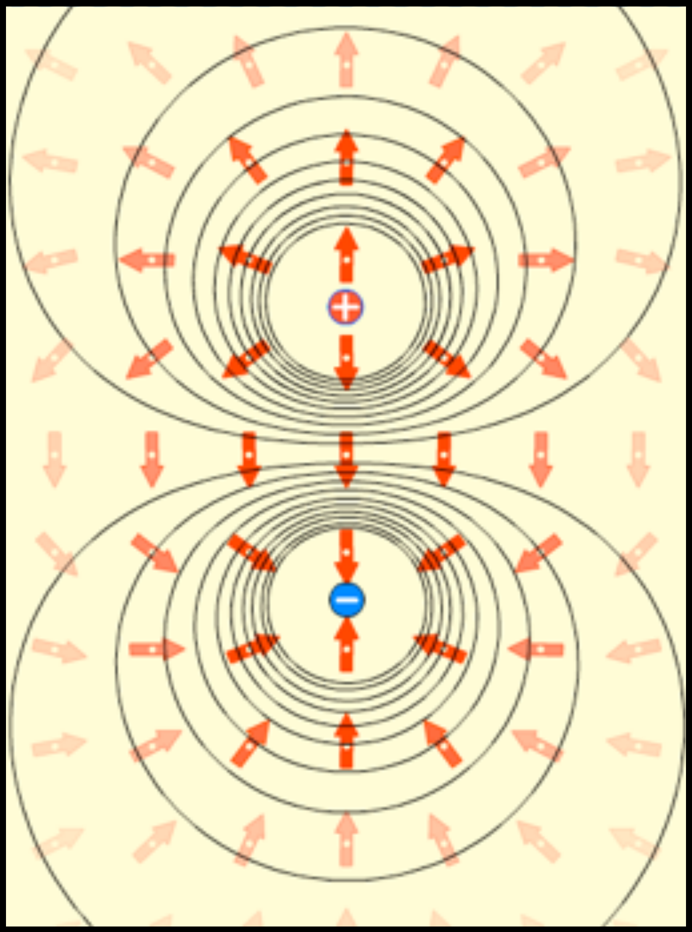
$$\Delta V = 0$$

This is called an equipotential.

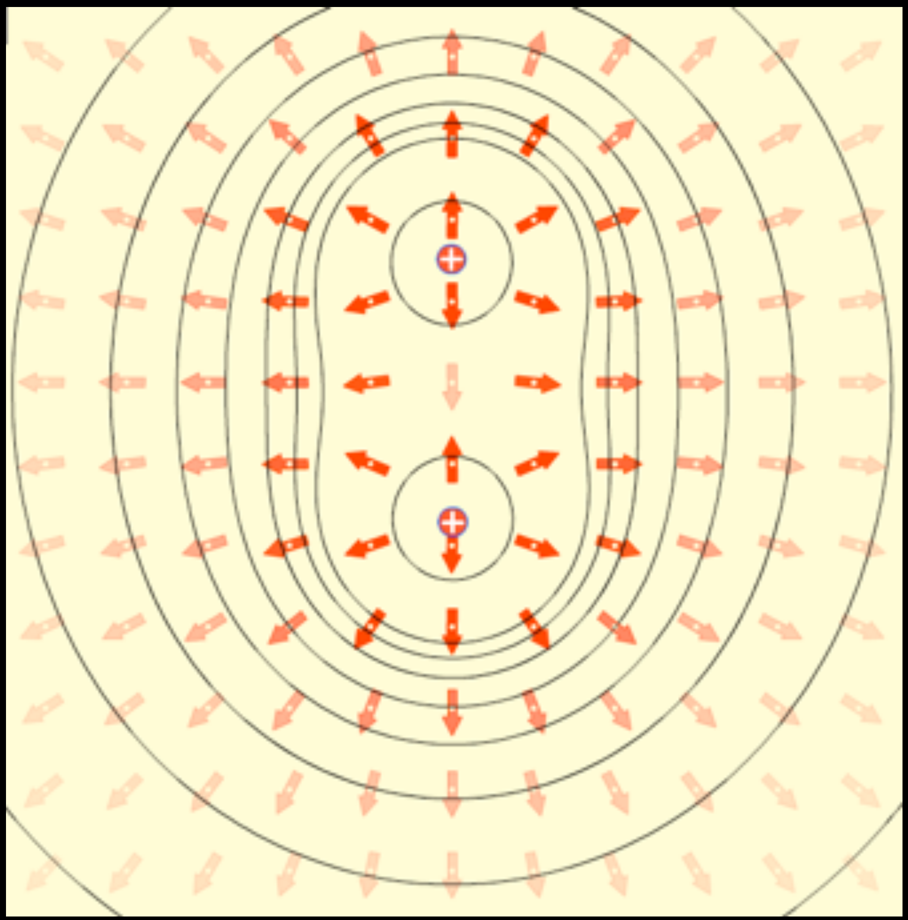
Potential difference and the electric field



Point charge



Dipole

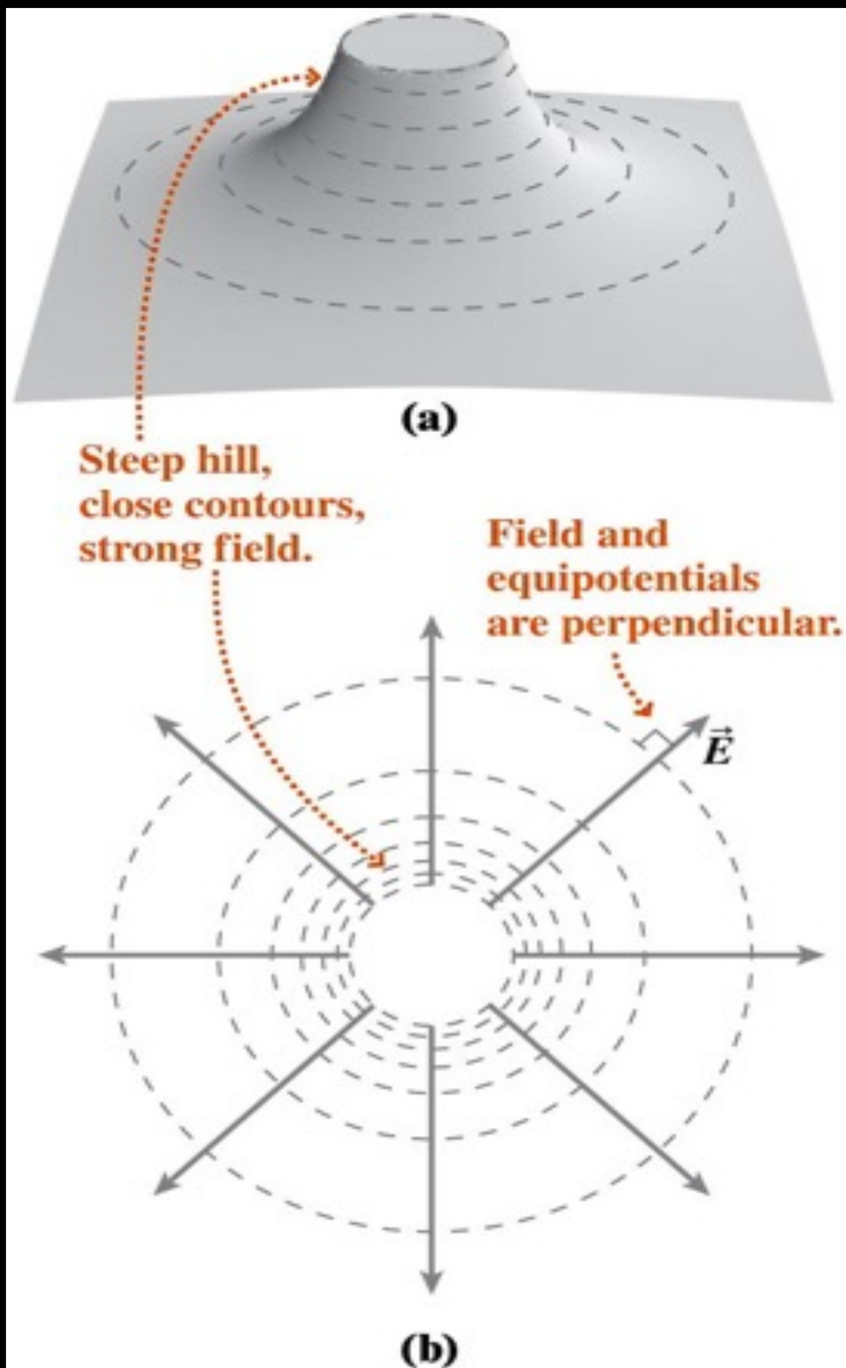


2 like charges

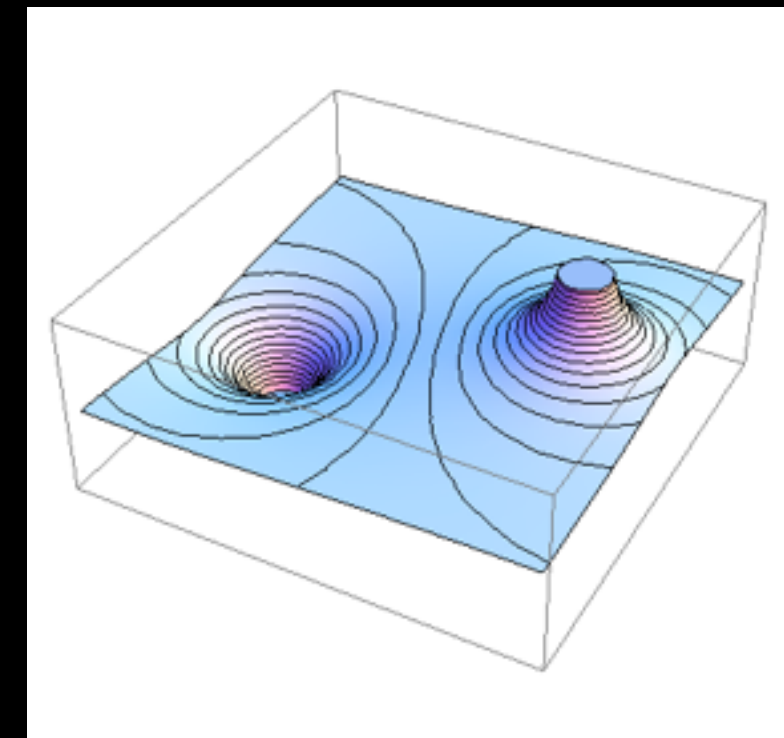
Electric field and equipotential are perpendicular.

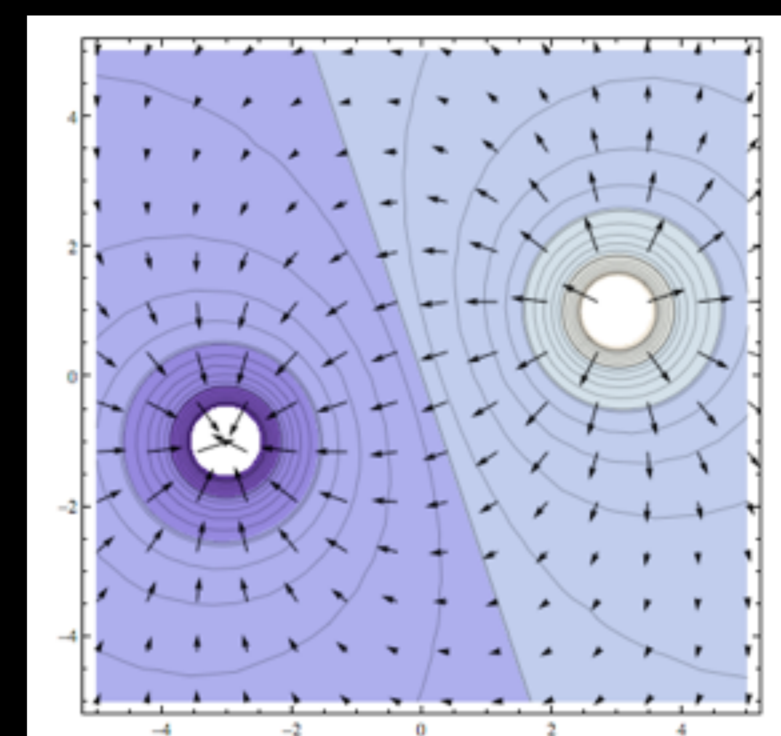
Equipotential is a contour map of the field.

Potential difference and the electric field



Equipotentials can be shown in 3D or 2D





Point charge

Dipole

Potential difference and the electric field

Since: $$\Delta V_{\mathrm{AB}} = -\int_A^B \bar{E} \cdot d\bar{r}$$

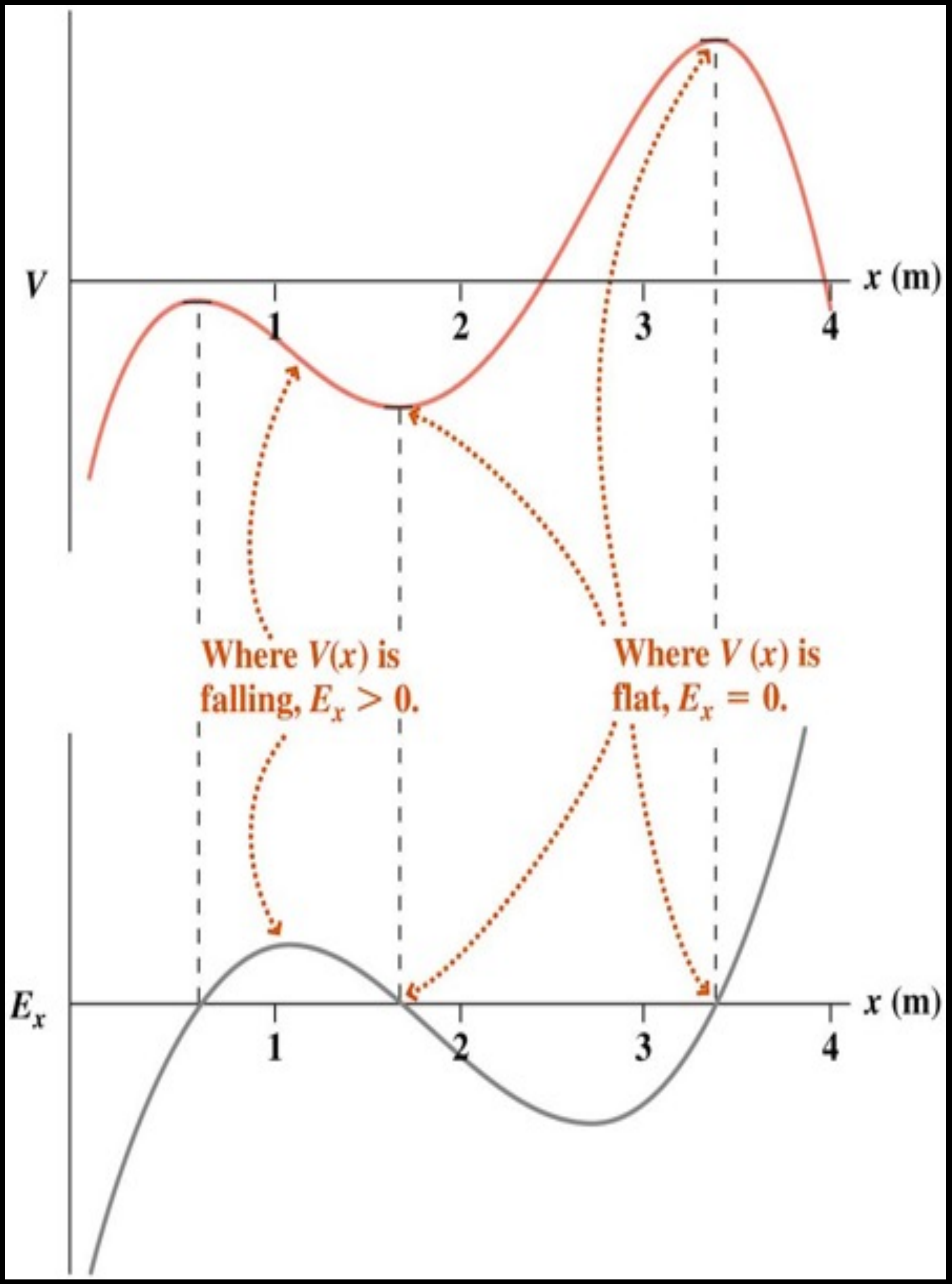
The electric field is the derivative (rate-of-change) of the potential.

But, potential difference is scalar, while the field is a vector with direction.

Electric field in x-direction: $$E_x = -\frac{\partial V}{\partial x}$$

Electric field in y-direction: $$E_y = -\frac{\partial V}{\partial y}$$

Electric field in z-direction: $$E_z = -\frac{\partial V}{\partial z}$$



Potential difference and the electric field

3 components together give:

$$\bar{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

Partial derivative: ∂

Used when a function depends on multiple variables (e.g. x, y and z)

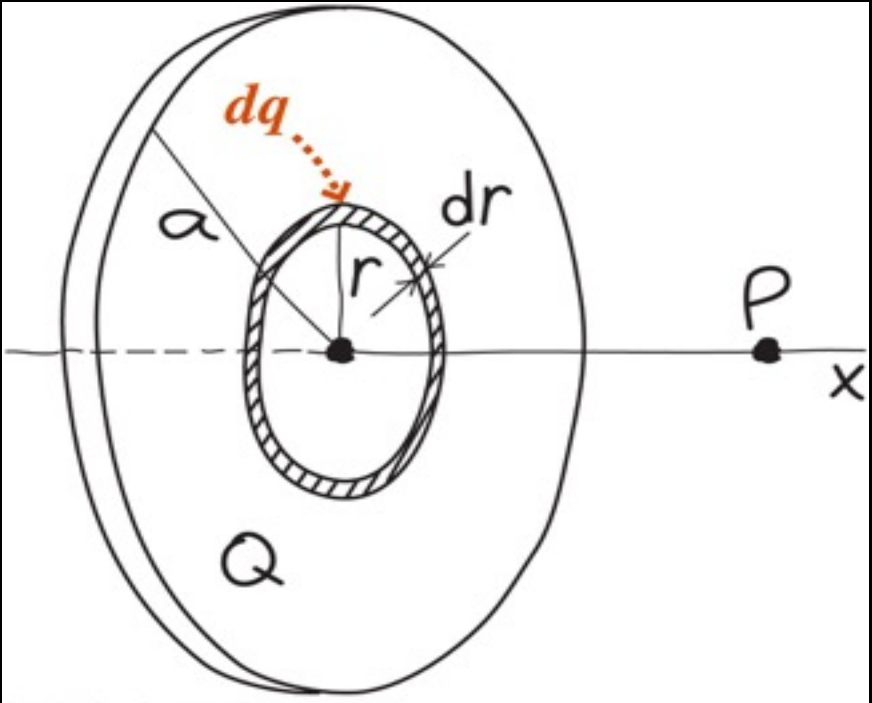
Electric field is strong where potential changes rapidly.

Because V is a scaler, it can be easier to calculate and use it to get E.

Charged disc: revisited

If we have the potential:

$$V(P) = \frac{2kQ}{a^2}(\sqrt{x^2+a^2} - |x|)$$



We can get the electric field easily.

V only depends on x, so: $\frac{\partial V}{\partial y} = 0 \quad \frac{\partial V}{\partial z} = 0$

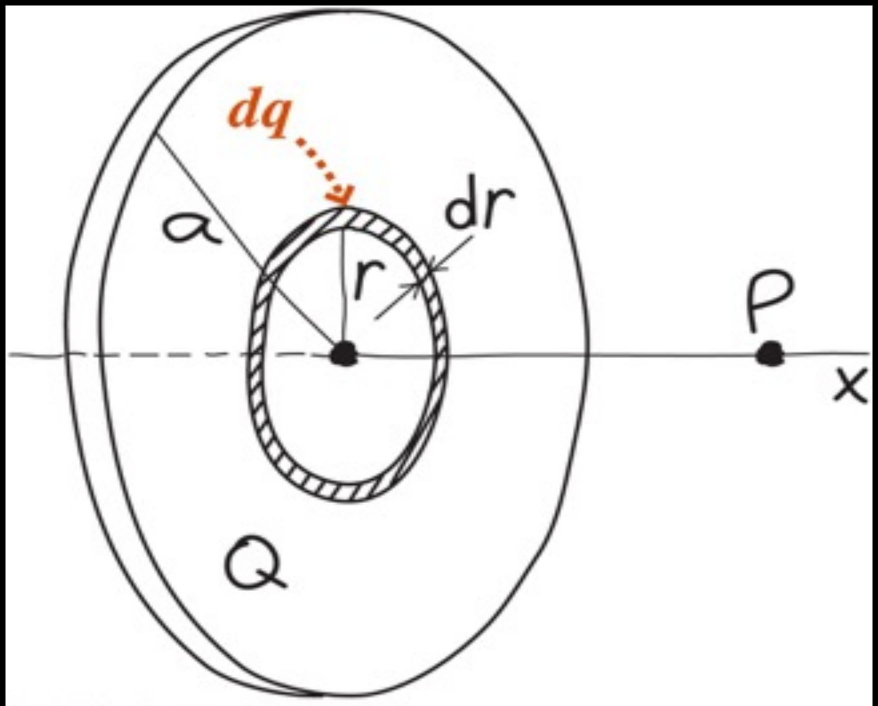
Leaving: $$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left[\frac{2kQ}{a^2}\left(\sqrt{x^2+a^2} - |x|\right)\right]$$

V depends only on x:
full derivative

Charged disc: revisited

If we have the potential:

$$V(P) = \frac{2kQ}{a^2}(\sqrt{x^2+a^2} - |x|)$$



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Leaving: $$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left[\frac{2kQ}{a^2}\left(\sqrt{x^2+a^2} - |x|\right)\right]$$

for each side of disc

$$= \frac{2kQ}{a^2}\left(\pm 1 - \frac{x}{\sqrt{x^2+a^2}}\right)$$

Potential difference and the electric field

Quiz

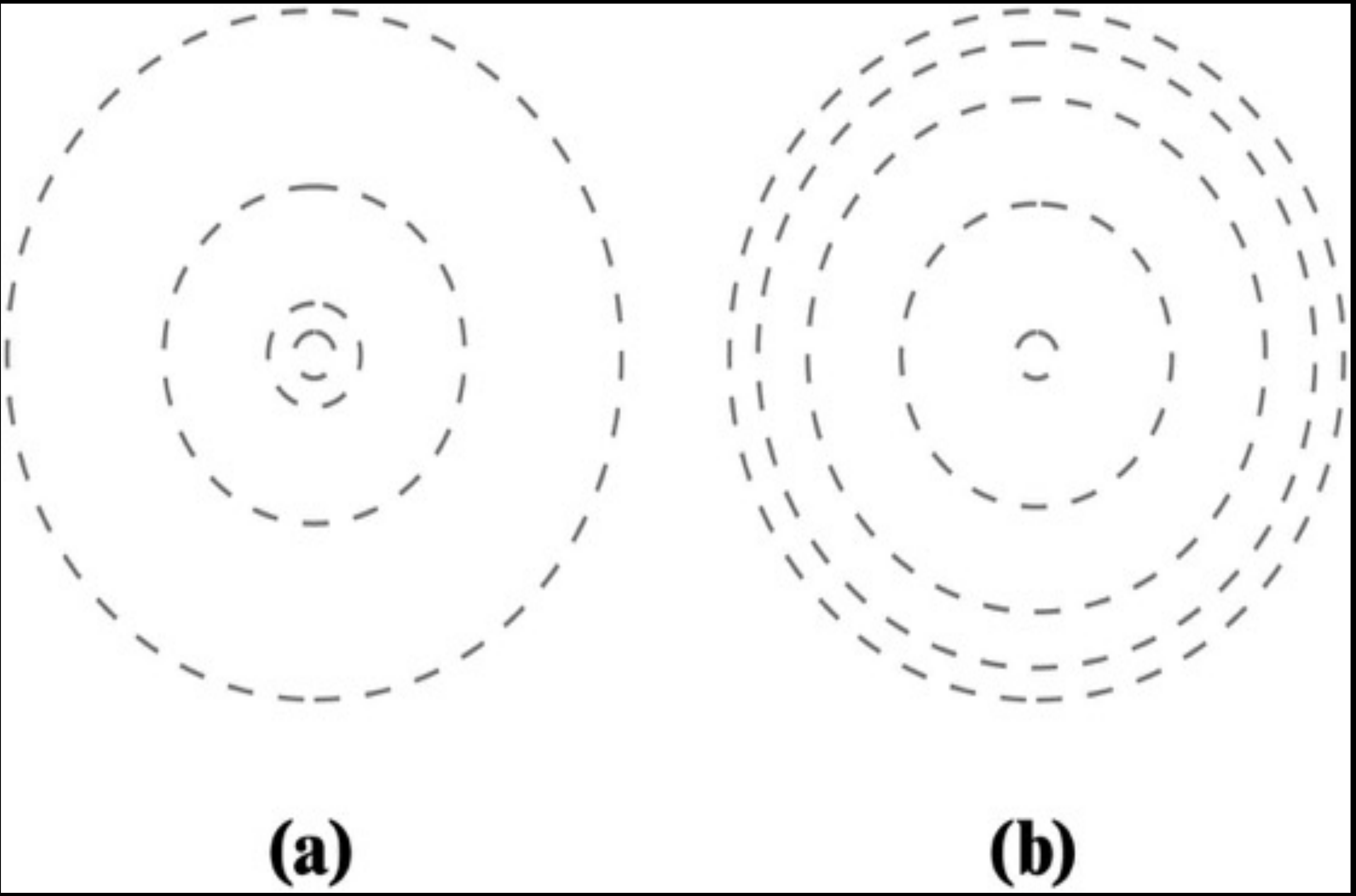
The figure shows cross sections through two equipotential surfaces. In both diagrams the potential difference between adjacent equipotentials is the same. Which of these two could represent the field of a point charge?

(A) a

(B) b

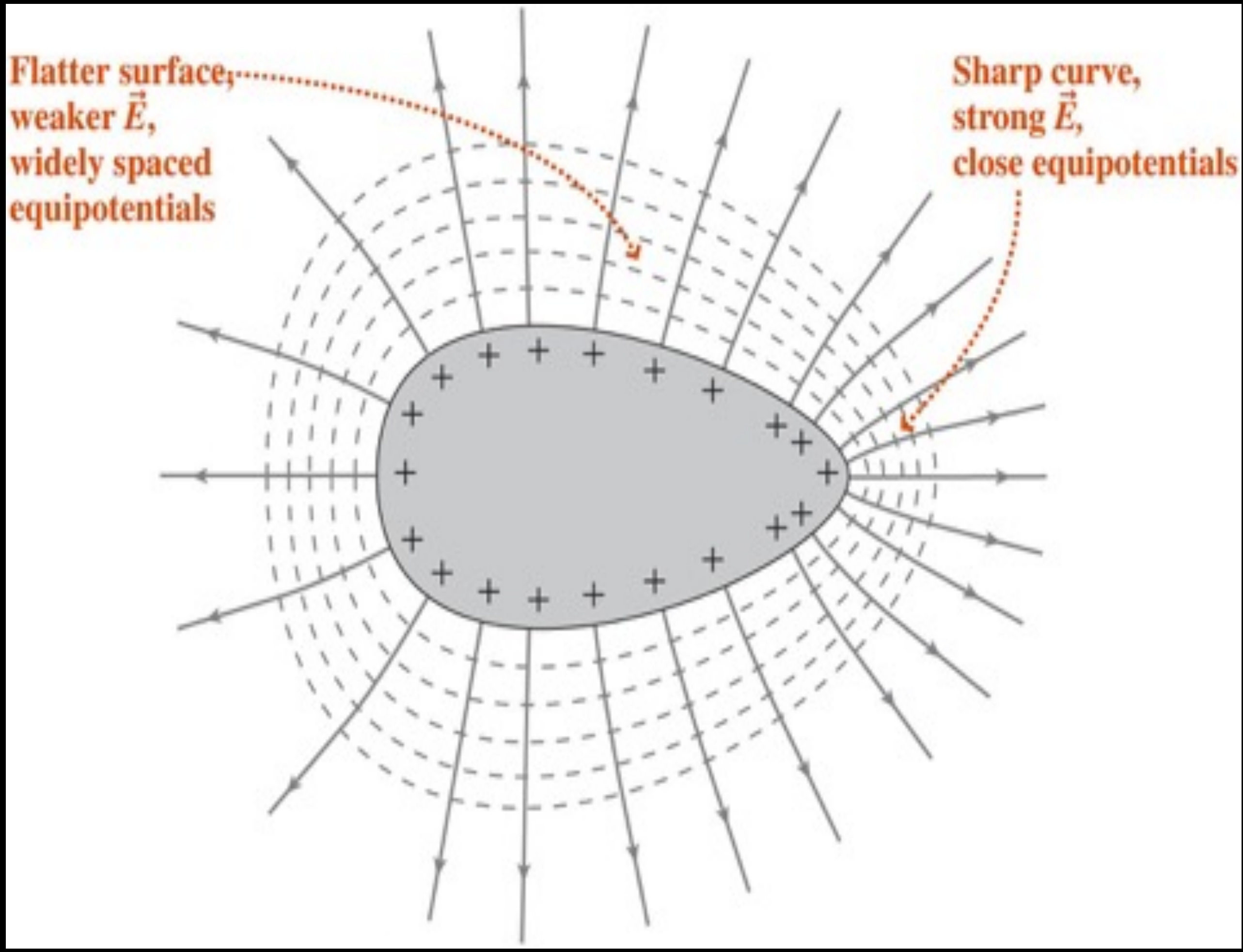
(C) neither a or b

Field gets stronger at higher r in (b)



Charged conductors

Inside a conductor: $\bar{E} = 0$



On the conductor surface,

$\bar{E}$ is perpendicular to the surface.

Therefore, it takes no work to move a test charge on or inside a conductor.

So a conductor (in electrostatic equilibrium) is an equipotential.

This means equipotential surfaces follow the conductor's shape.

Equipotentials are closer at sharp curves:

electric field is stronger on sharp curves.

Summary

Electric potential difference is the work per unit charge done in moving charge between two points in an electric field.

$$\Delta V_{\mathrm{AB}} = -\int_A^B \bar{E} \cdot d\bar{r}$$

The SI unit of electric potential is the volt (V), equal to 1 J/C.

Electric potential always involves 2 points; "the potential at a point" assumes a 2nd point at which the potential is defined to be zero.

Electric potential differences from a point charge: $V_{\infty} r = V(r) = \dfrac{kq}{r}$

(Zero is at infinity)

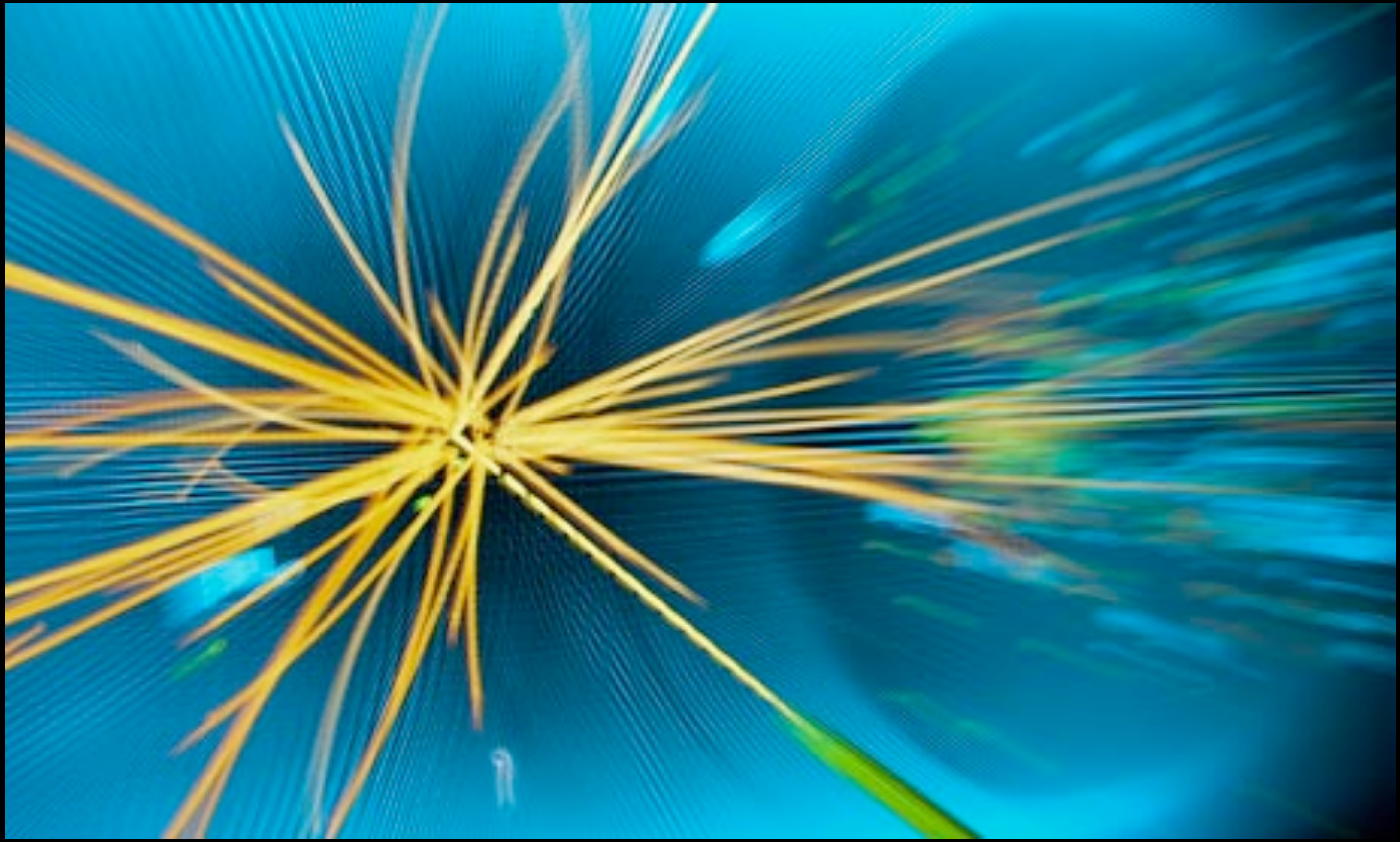
Electric field from potential: $\bar{E} = -\left(\dfrac{\partial V}{\partial x}\hat{i} + \dfrac{\partial V}{\partial y}\hat{j} + \dfrac{\partial V}{\partial z}\hat{k} \right)$

Equipotentials are surfaces of constant potential.

Power lines have same potential!

So no electric field flows through parasailer

Research: the Higgs boson



Research: the Higgs boson

What is the Higgs field?

(A) a force that causes particles to collide

(B) an intrinsic property of particles, like charge

(C) an energy field that fills the entire Universe

(D) the force that causes the Universe to expand

Research: the Higgs boson

Why was the Higgs field proposed?

(A) to explain Universe expansion

(B) to explain the different masses of particles

(C) to explain the existence of quarks

(D) to explain the discovery of a new particle

Research: the Higgs boson

Interacting strongly with the Higgs field means...

(A) you slow down

(B) your mass is small

(C) you collide more frequently with other particles

(D) your mass is large

(E) you speed up

Research: the Higgs boson

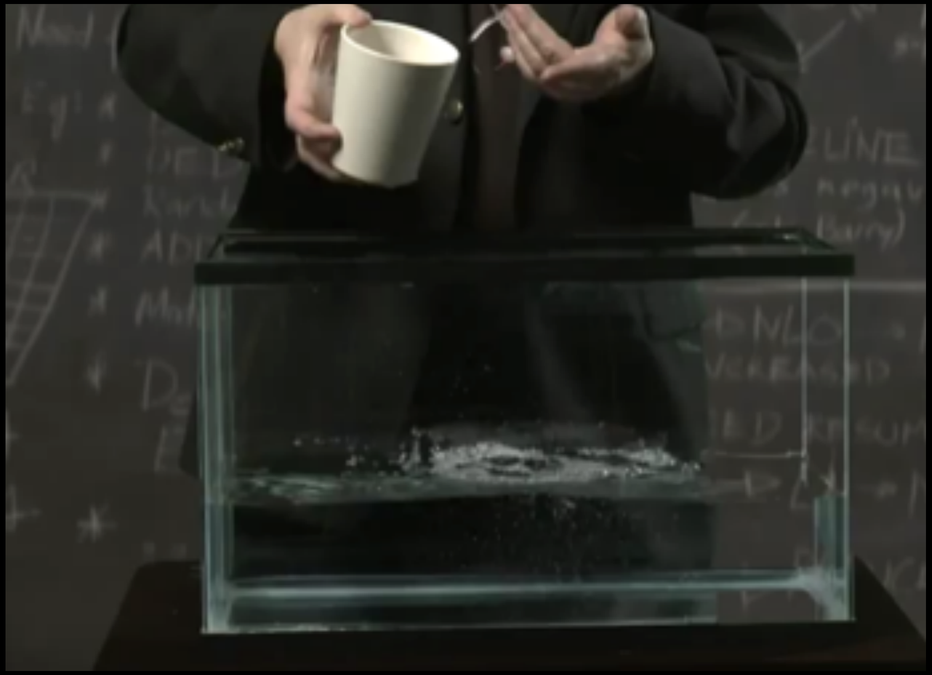
In the analogy (comparison) with water, the water is....

(A) low mass particles

(B) the Higgs field

(C) high mass particles

(D) not relevant



Research: the Higgs boson

In the analogy (comparison) with water, the barracuda (fish) is....

(A) low mass particles

(B) the Higgs field

(C) high mass particles

(D) not relevant

Research: the Higgs boson

In the analogy (comparison) with water, Eddie is....

(A) low mass particles

(B) the Higgs field

(C) high mass particles

(D) not relevant

Research: the Higgs boson

Why is the top quark more massive than the electron?

(A) it interacts more strongly with the Higgs field

(B) it is physically larger

(C) it is physically smaller

(D) it has zero size

Research: the Higgs boson

If the Higgs field did not exist...

(A) we would have more mass

(B) we would have less mass

(C) we would have no mass

(D) the top quark would become larger than the electron

(E) nothing would change

Research: the Higgs boson

How is the Higgs boson related to the Higgs field?

(A) it is the smallest part of the Higgs field

(B) the Higgs field is a property of the Higgs boson

(C) the Higgs boson creates a Higgs field

(D) it is not; only the name is the same

Research: the Higgs boson

In the analogy with water, what is the H_2O molecule?

(A) the Higgs field

(B) the Higgs boson

(C) a massless particle

(D) a massive particle

(E) the analogy does not hold at that level

Research: the Higgs boson

Has the Higgs boson been discovered?

(A) Yes!

(B) No!

(C) .... maybe

The Nobel Prize in Physics 2013
François Englert, Peter Higgs

The Nobel Prize in Physics 2013

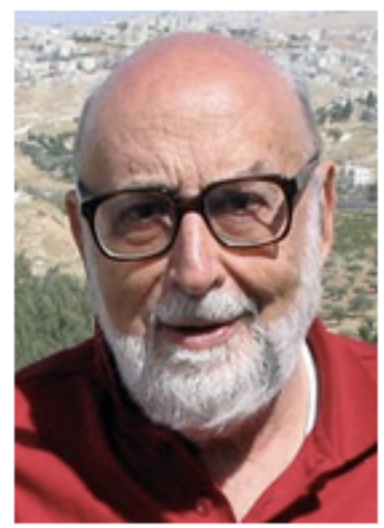


Photo: Pnicolet via Wikimedia Commons

François Englert

Photo: G-M Greuel via Wikimedia Commons

Peter W. Higgs

The Nobel Prize in Physics 2013 was awarded jointly to François Englert and Peter W. Higgs *"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"*

Homework hint:

$$\int \frac{da}{\sqrt{a^2 + b^2}} = \ln\left(\frac{a}{b} + \sqrt{1 + \frac{a^2}{b^2}}\right)$$

For small x:

$$\sqrt{1 + x^2} \approx 1 + \frac{1}{2}x^2$$