Essential Physics II

英語で物理学の

エッセンス II

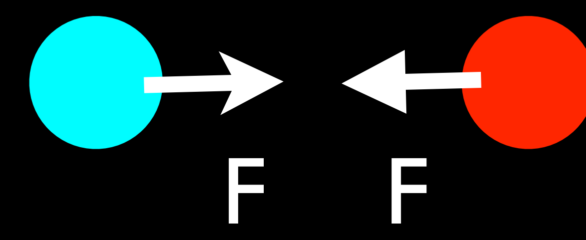
Lecture 6:   02-11-15

Last week....

Force between 2 charges:

$$\bar{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}_{12}$$

Coulomb's law



Electric field: (force per unit charge)

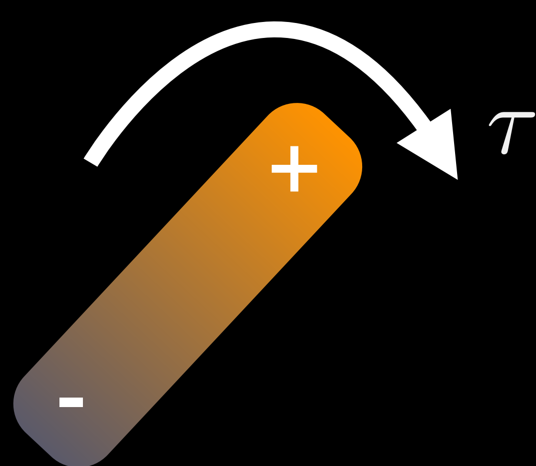
$$\bar{E} = \frac{\bar{F}}{q} = \frac{kq}{r^2}\hat{r}$$

Electric forces + fields can be added: $\bar{E} = \bar{E}_1 + \bar{E}_2 + \ldots = \sum_i \frac{kq_i}{r_i}^{2} \hat{r}_i$

If charge is continuous: integrate $= \int \frac{kdq}{r^2}\hat{r}$

A dipole feels a torque in an E - field

and a force if E - field is not uniform.

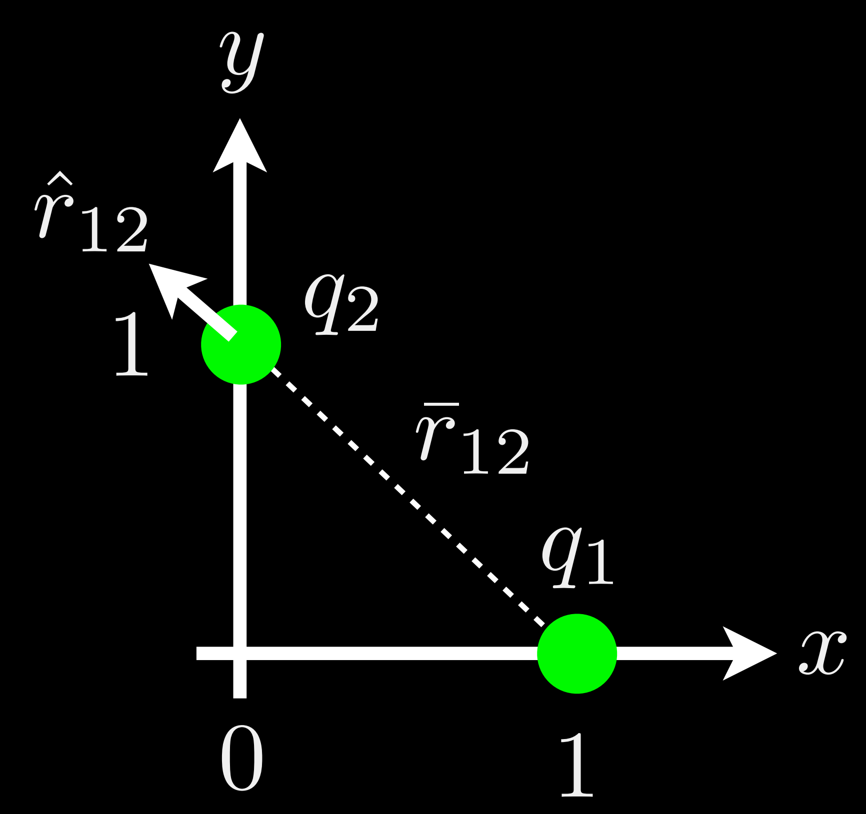


Last week.... Quiz

A charge q_1 is at (x,y) = (1, 0) m.

A charge q_2 is at (x,y) = (0, 1) m.

What is the unit vector, $\hat{r}_{12}$?



(a) $$-\frac{\sqrt{2}}{2}\hat{i} - \frac{\sqrt{2}}{2}\hat{j}$$

(b) $$\frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j}$$

(c) $$\frac{\sqrt{2}}{2}\hat{i} - \frac{\sqrt{2}}{2}\hat{j}$$

(d) $$-\frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j}$$

$$\bar{r}_{12} = (0\hat{i} + 1\hat{j}) - (1\hat{i} + 0\hat{j}) = -\hat{i} + \hat{j}$$

$$|\bar{r}_{12}| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$$

$$\hat{r}_{12} = \frac{\bar{r}_{12}}{|\bar{r}_{12}|} = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

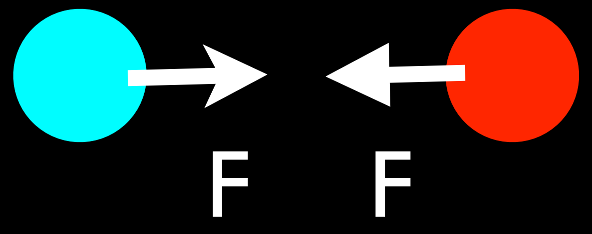
$$= -\frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j}$$

Last week.... Quiz

2 charged particles attract each other with a force, F.

If the charges of both particles are doubled (x 2) and the separation is also doubled (x 2)

What is the force?



(a) F/8

(b) F/2

(c) F

(d) 2F

(e) 8F

Coulomb's law:

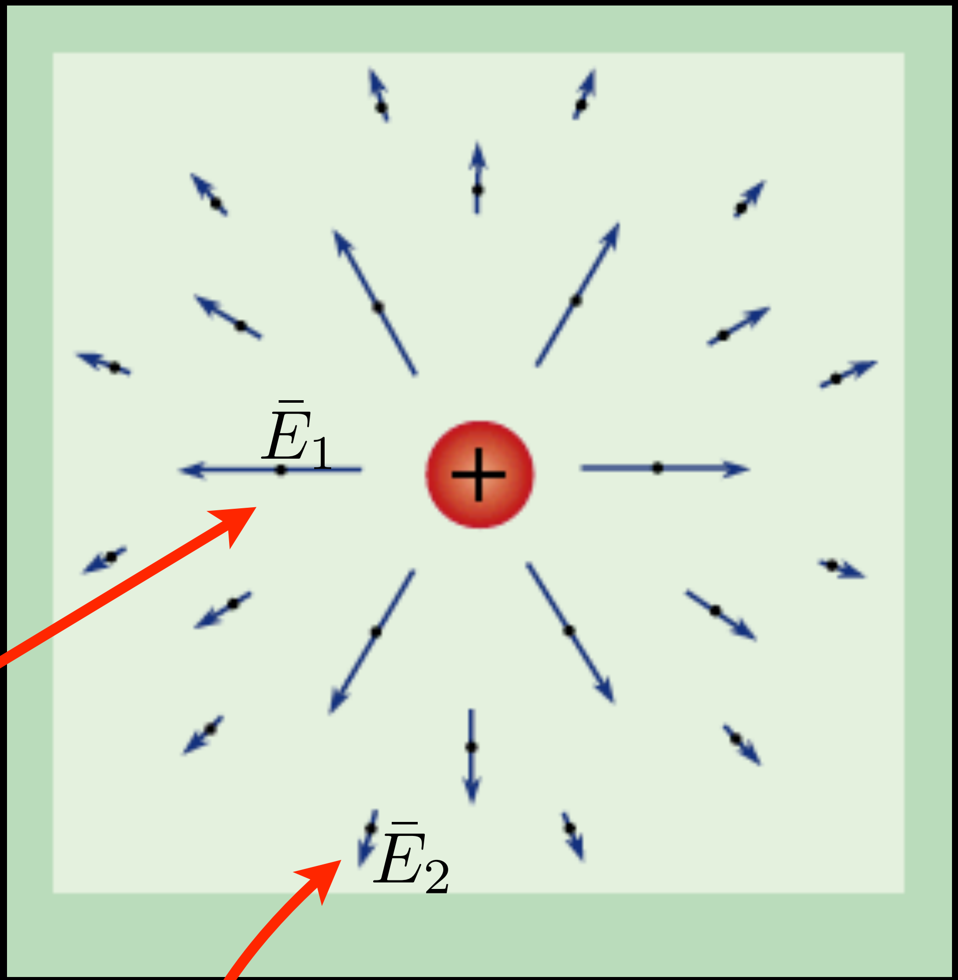
$$\bar{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}_{12} \quad \rightarrow \frac{k(2q_1)(2q_2)}{(2r)^2}\hat{r}_{12}$$

$$\rightarrow \frac{4}{4}\frac{kq_1q_2}{r^2}\hat{r}_{12} \quad = \bar{F}_{12}$$

Last week....

The electric field: $$\bar{E} = \frac{\bar{F}}{q}$$

For a point charge: $$\bar{E} = \frac{kq}{r^2}\hat{r}$$



Close to the charge, the point charge q, would feel $\bar{F} = q\bar{E}_1$

Further from the charge, point charge q would feel a weaker force, $\bar{F} = q\bar{E}_2$ in a different direction.

Electric field lines

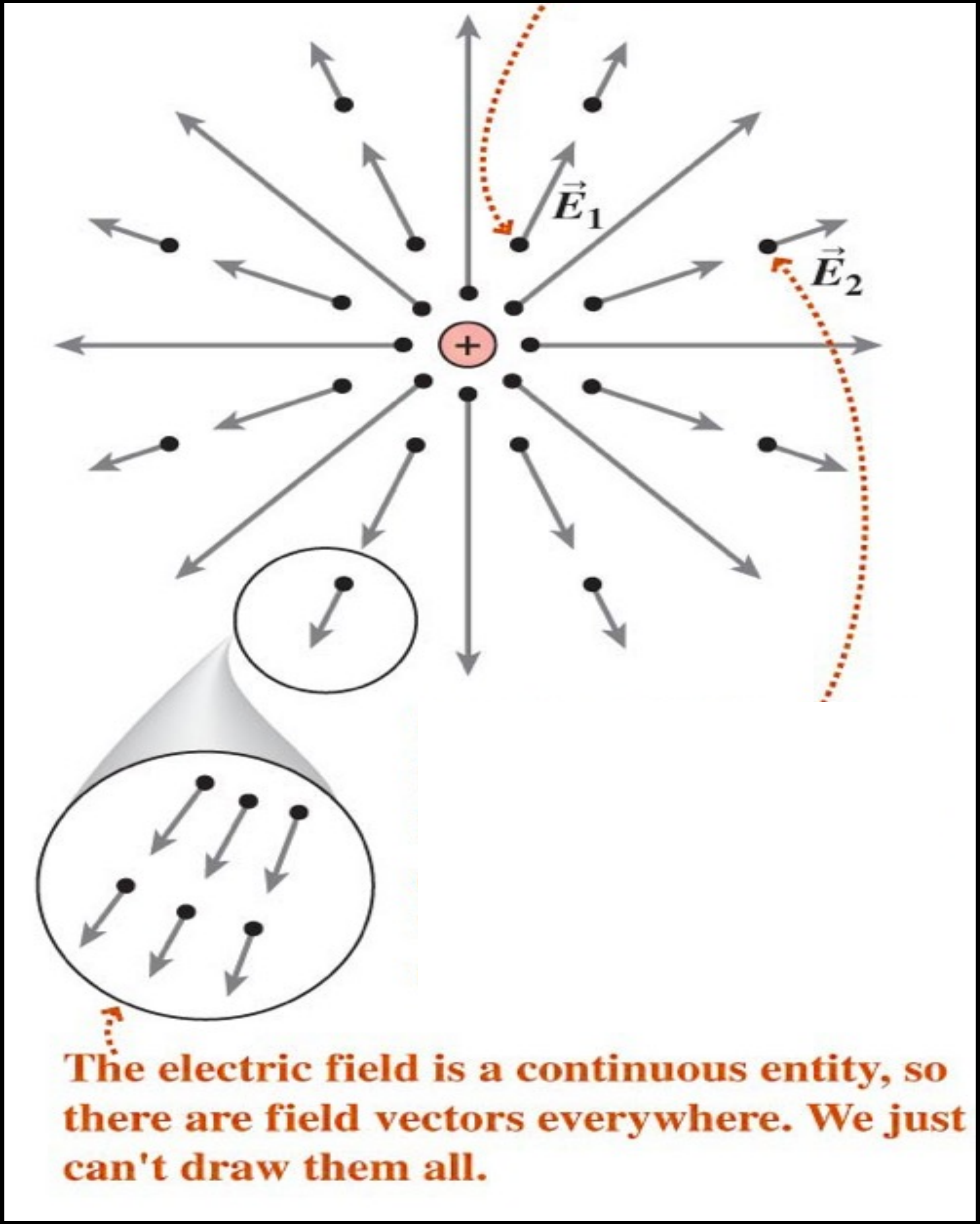
The lines we draw represent the field.

They point in the direction of the field at that location.

But the field is actually everywhere....

.... so each charge really has an infinite number of lines.

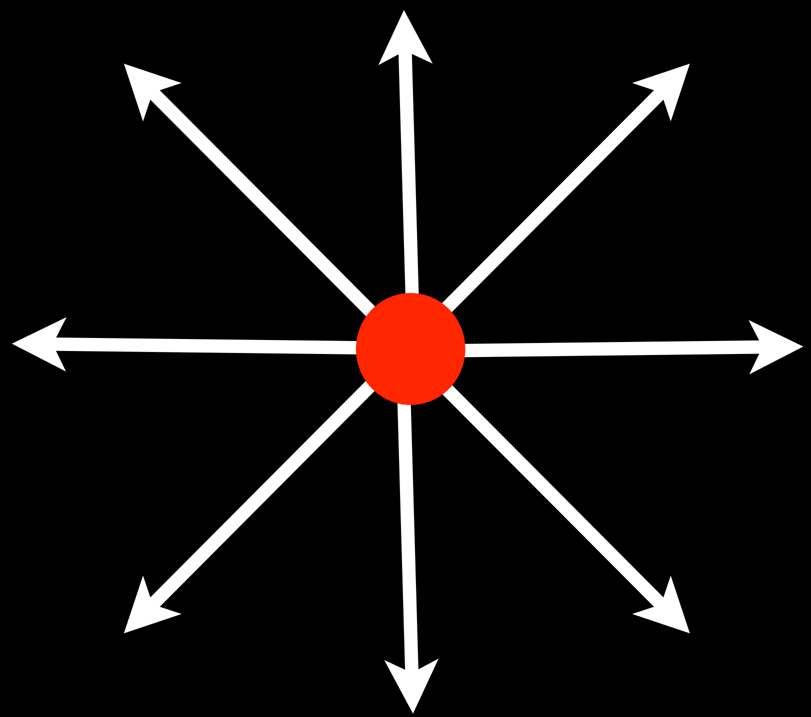
So, does it matter HOW we draw the field?



Electric field lines

A system for drawing field lines can describe the field more accurately.

Rule 1: field lines start on positive charges

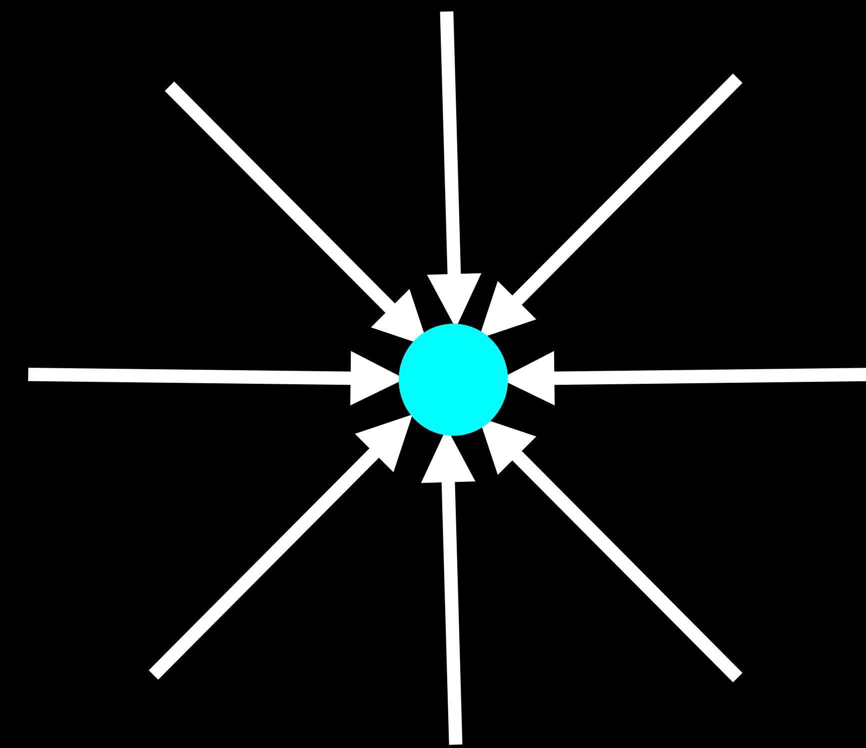


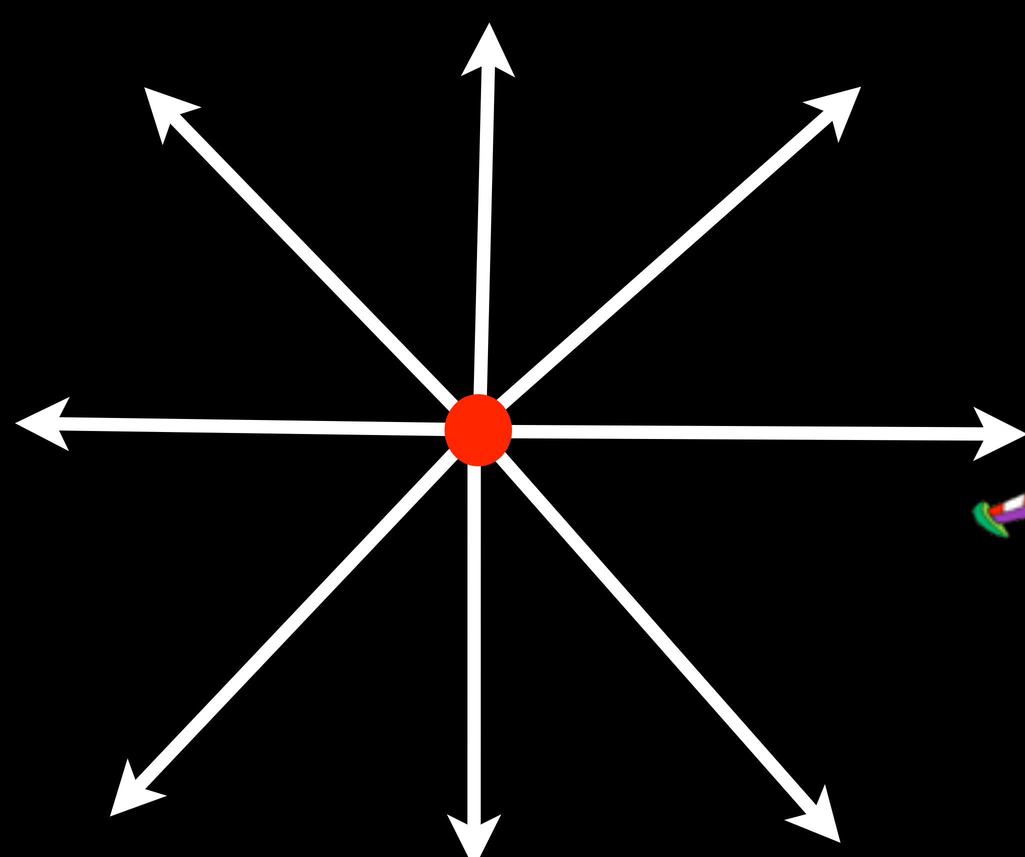
Electric field lines

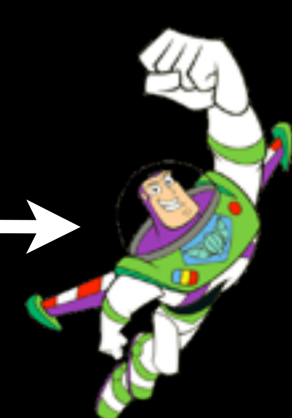
A system for drawing field lines can describe the field more accurately.

Rule 1: field lines start on positive charges

... and end on negative charges







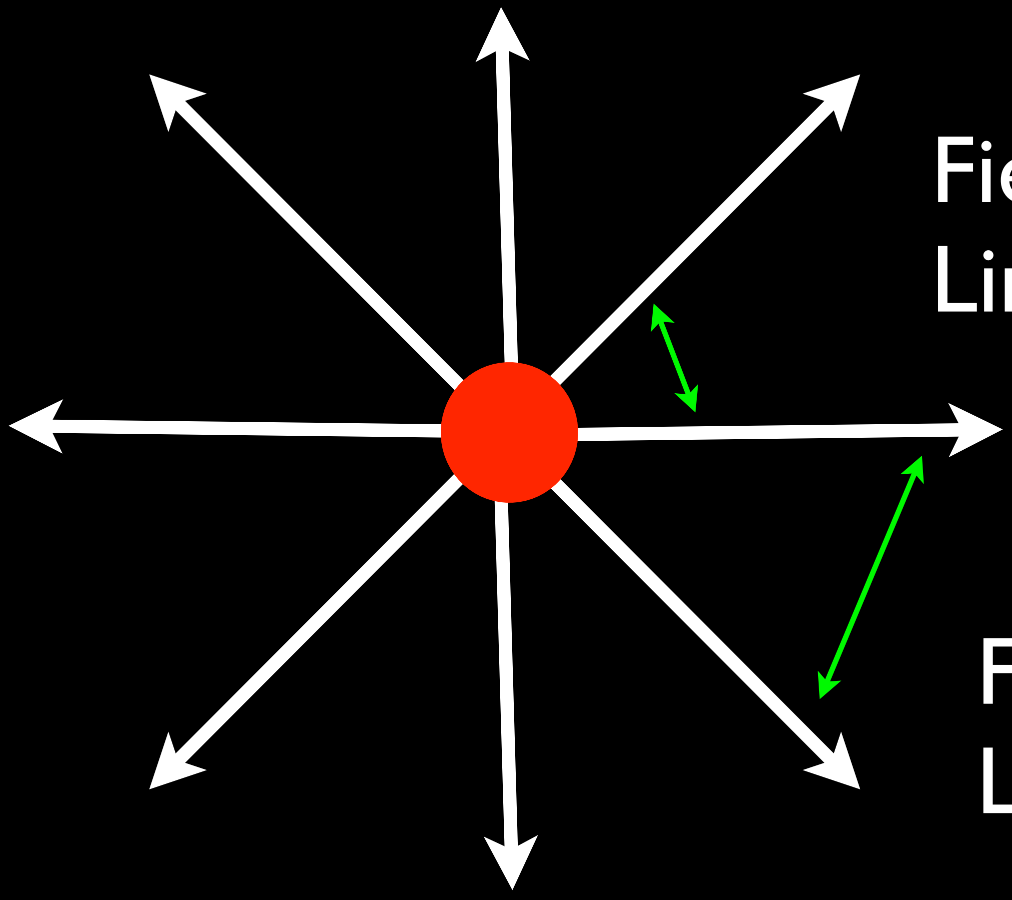
Lines to/from each charge extend to infinity

Draw symmetrically leaving/entering each charge

Electric field lines

A system for drawing field lines can describe the field more accurately.

Rule 2: The closer the field lines, the stronger the field



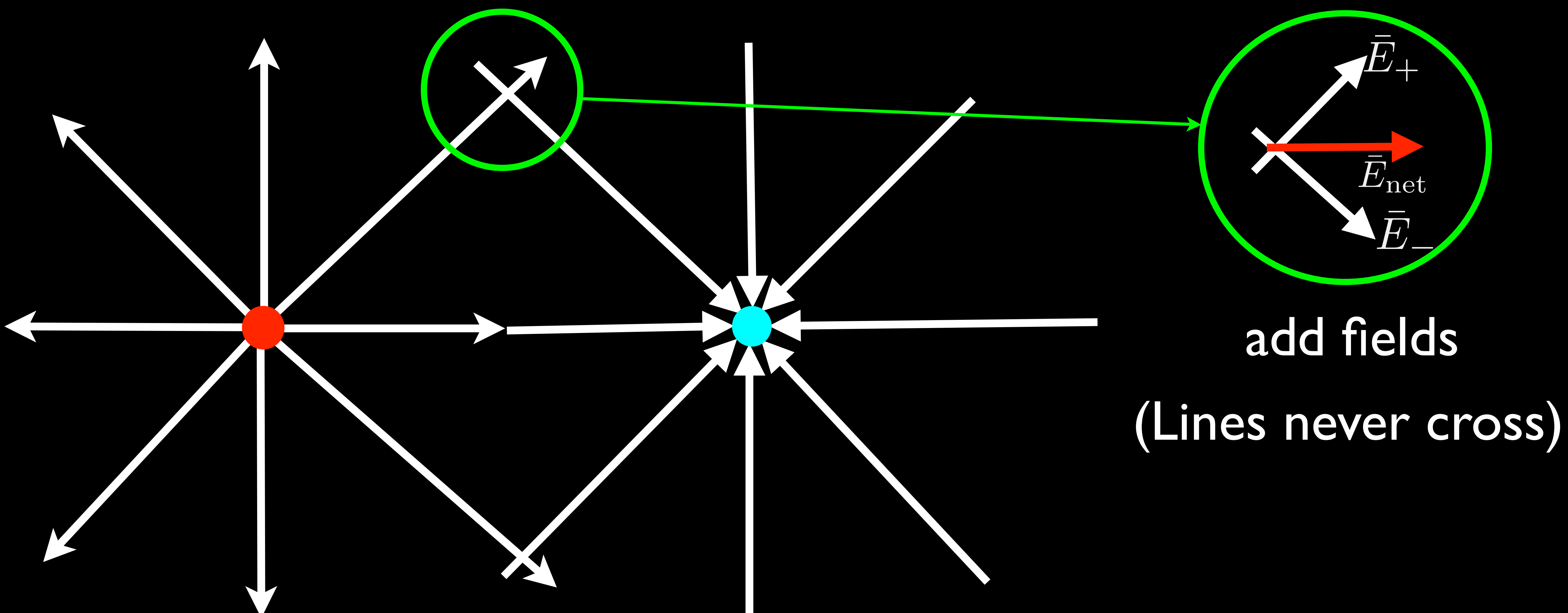
Field is stronger closer to the source.
Lines are closer.

Field is weaker further away.
Lines are further apart.

Electric field lines

A system for drawing field lines can describe the field more accurately.

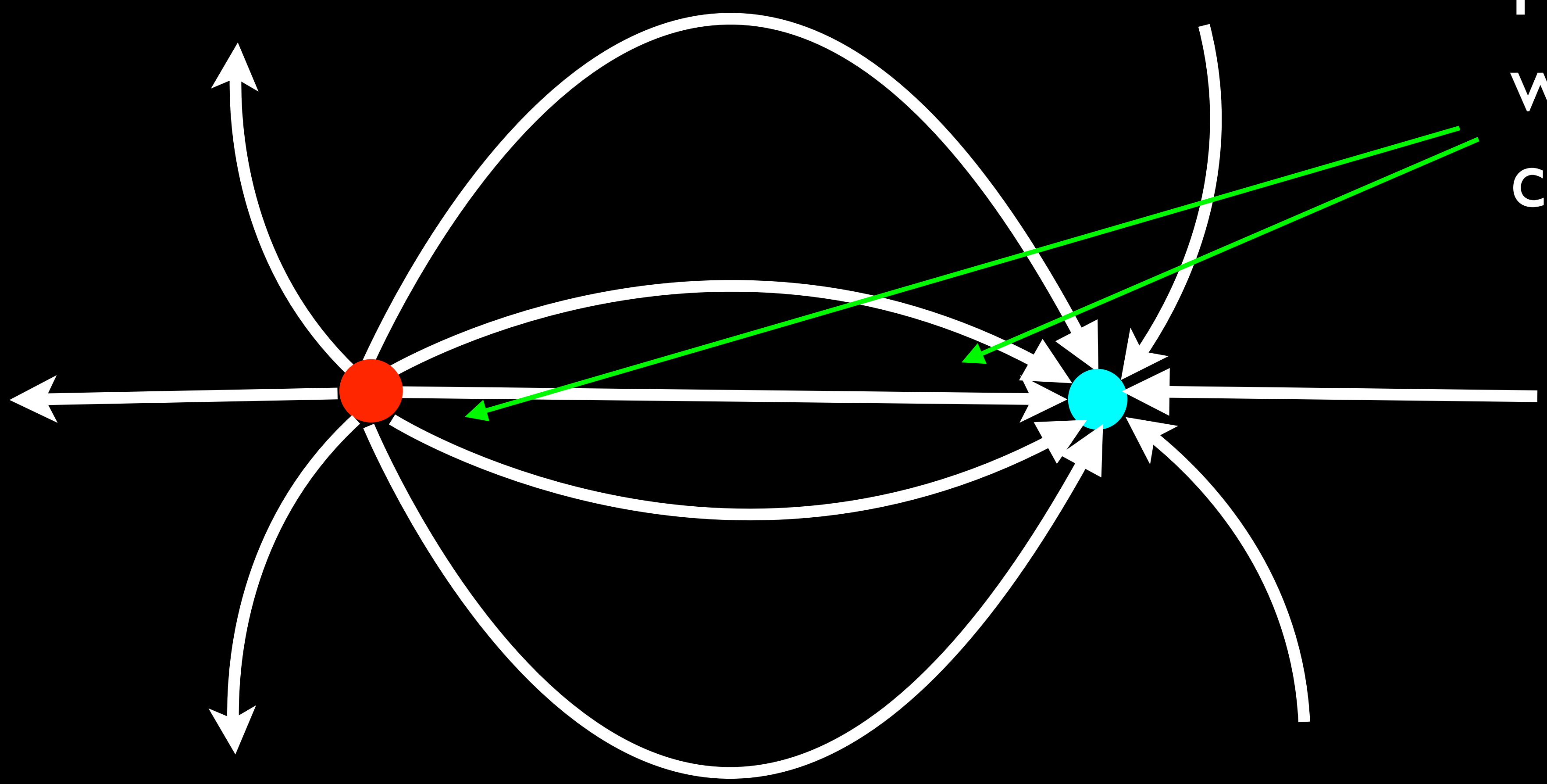
Rule 2: The closer the field lines, the stronger the field



Electric field lines

A system for drawing field lines can describe the field more accurately.

Rule 2: The closer the field lines, the stronger the field

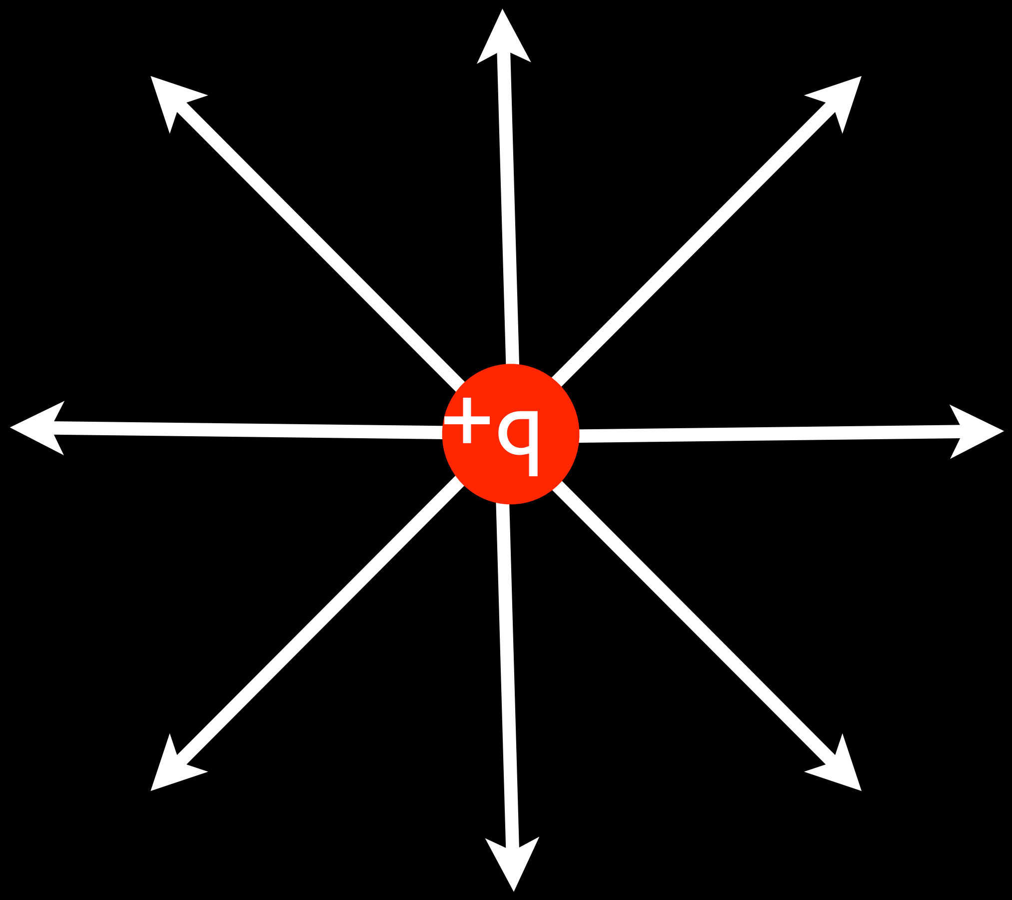


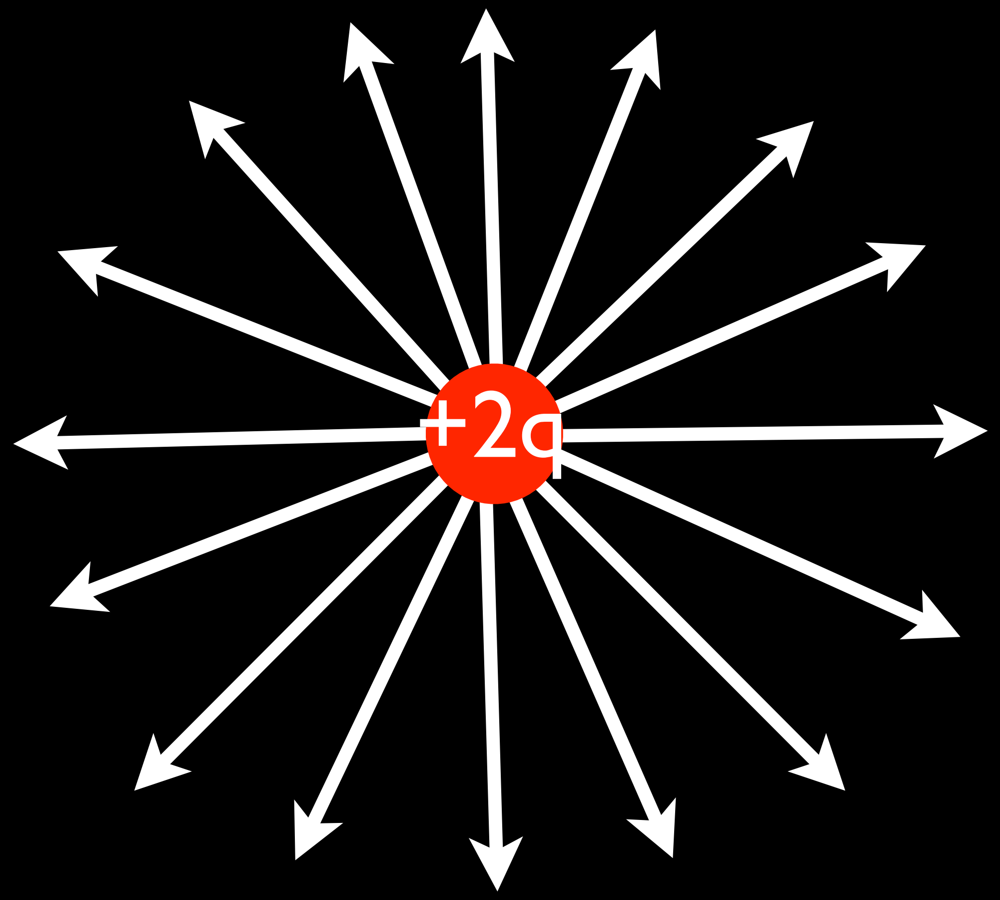
Field is strongest where lines are closest.

Electric field lines

A system for drawing field lines can describe the field more accurately.

Rule 3: The number of lines entering/leaving a charge is proportional to the size of the charge.





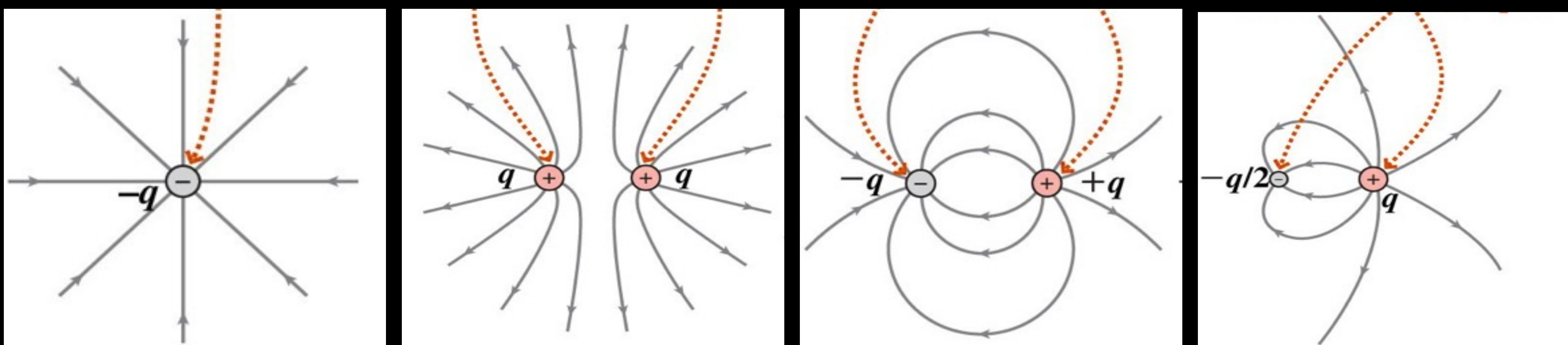
e.g. 8 lines begin on +q

16 lines begin on +2q

Electric field lines

A system for drawing field lines can describe the field more accurately.

Rule 3: The number of lines entering/leaving a charge is proportional to the size of the charge.



8 lines end on -q

8 lines begin on each +q

8 lines begin on +q
8 lines end on -q

8 lines begin on +q

4 lines go to infinity

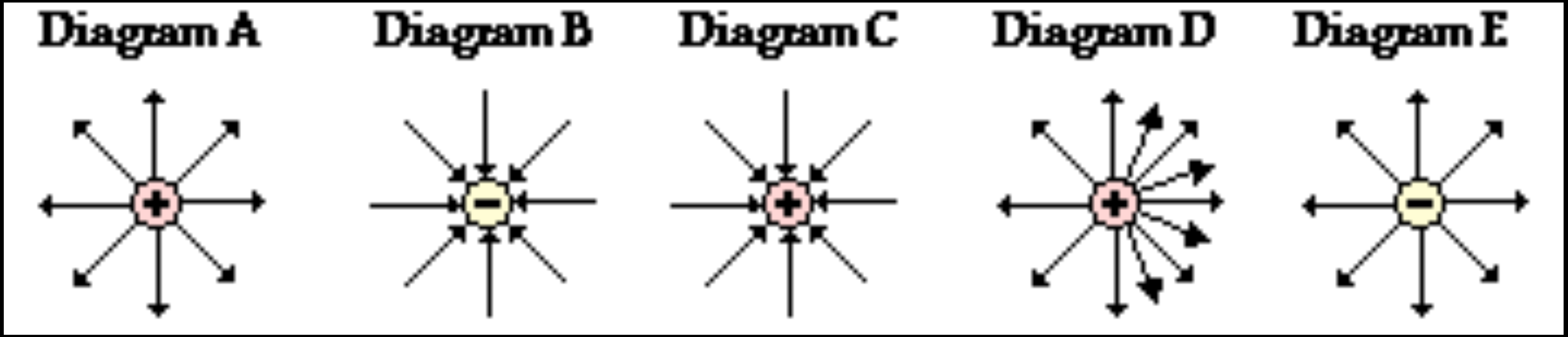
4 lines end on -q

Lines always begin/end on a charge or extend to infinity

Electric field lines

Quiz

Which diagrams are incorrect?



(1) A, B, D

(2) E, C, D

(3) B, D, E

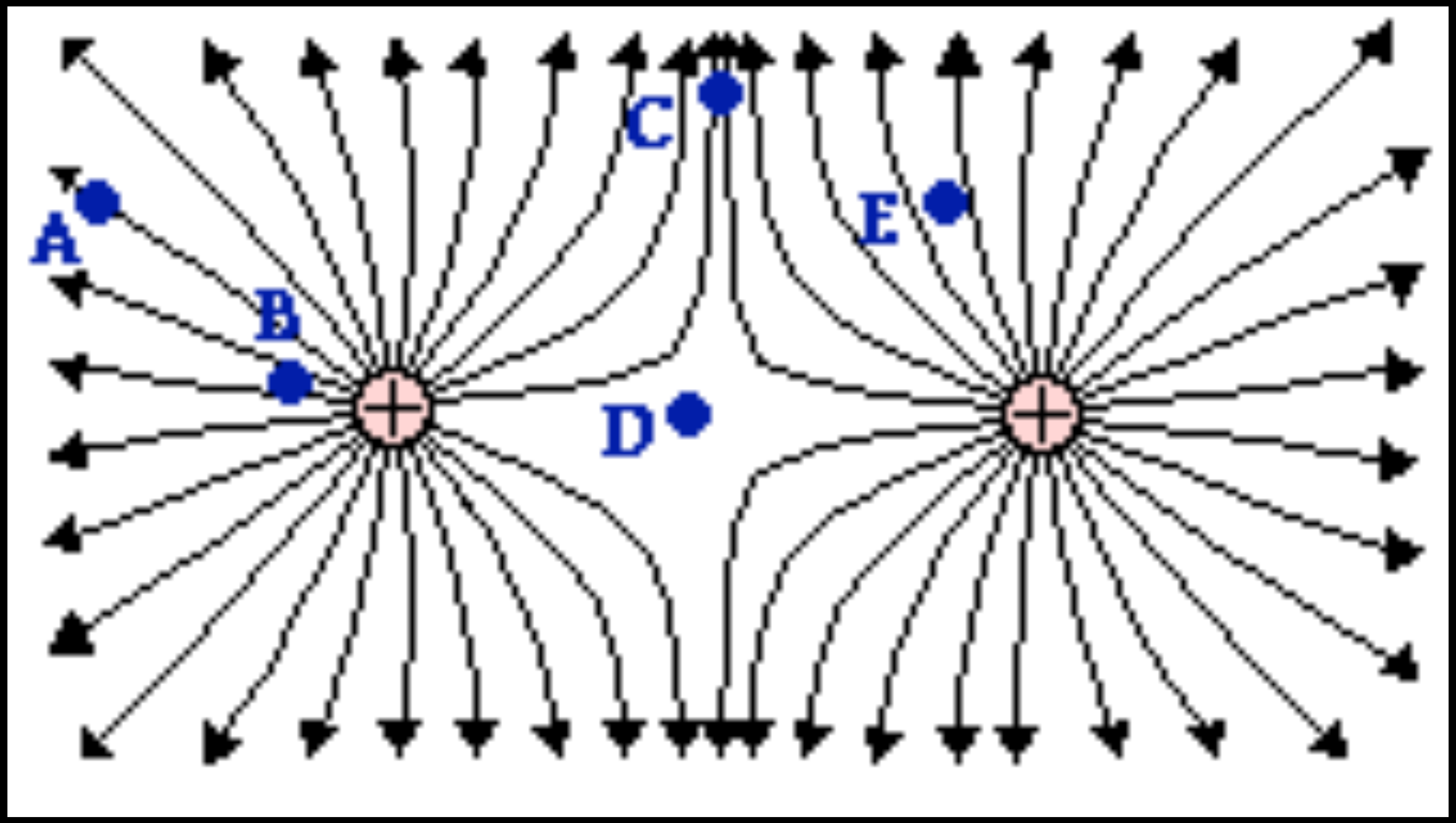
(4) C, D, A

(5) B, A, E

Electric field lines

Quiz

Rank locations in order of electric field strength: smallest to largest



(A) A, B, C, D, E

(B) B, E, C, D, A

(C) D, A, E, C, B

(D) D, C, A, E, B

(E) D, A, C, E, B

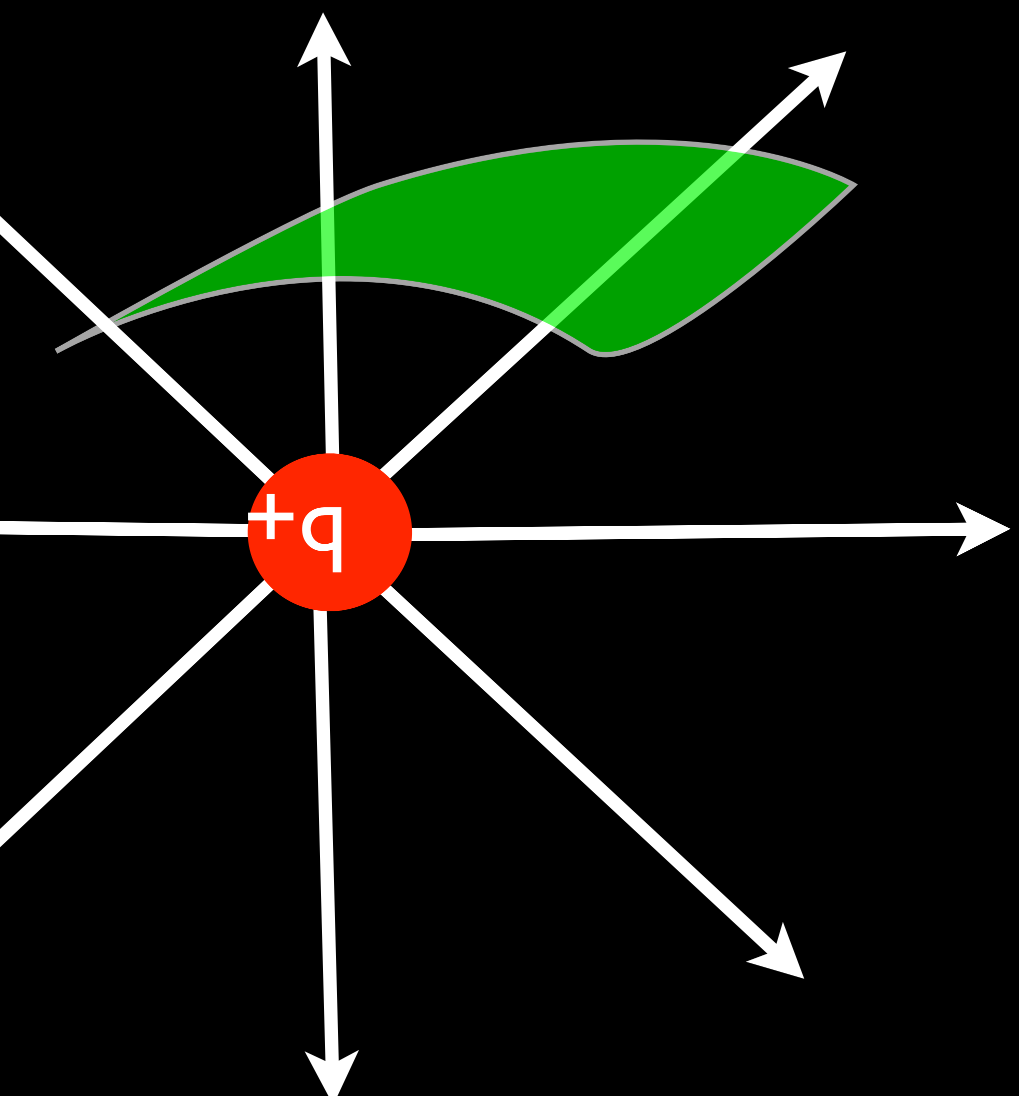
(F) B, C, E, D, A

Electric field strength is greatest where the lines are closest together and weakest where lines are furthest apart.

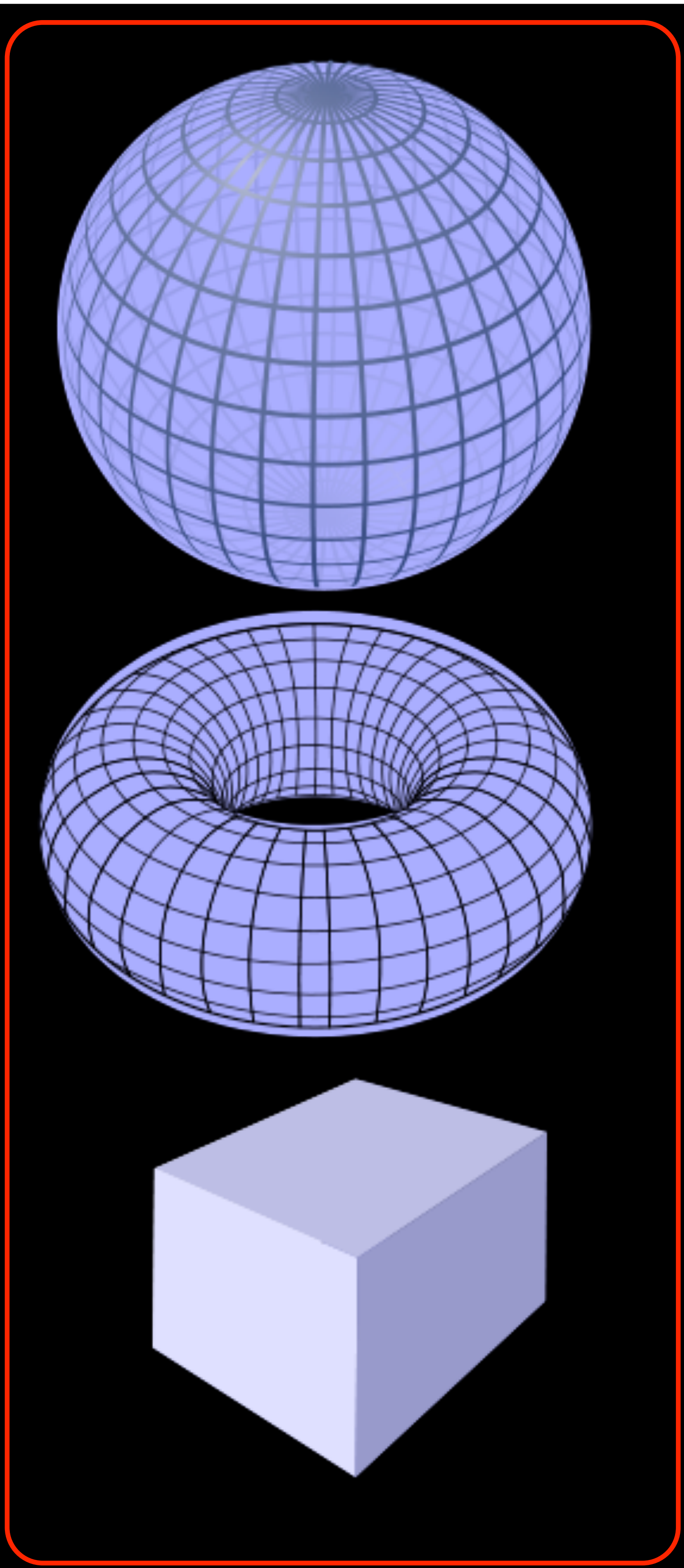
Electric flux

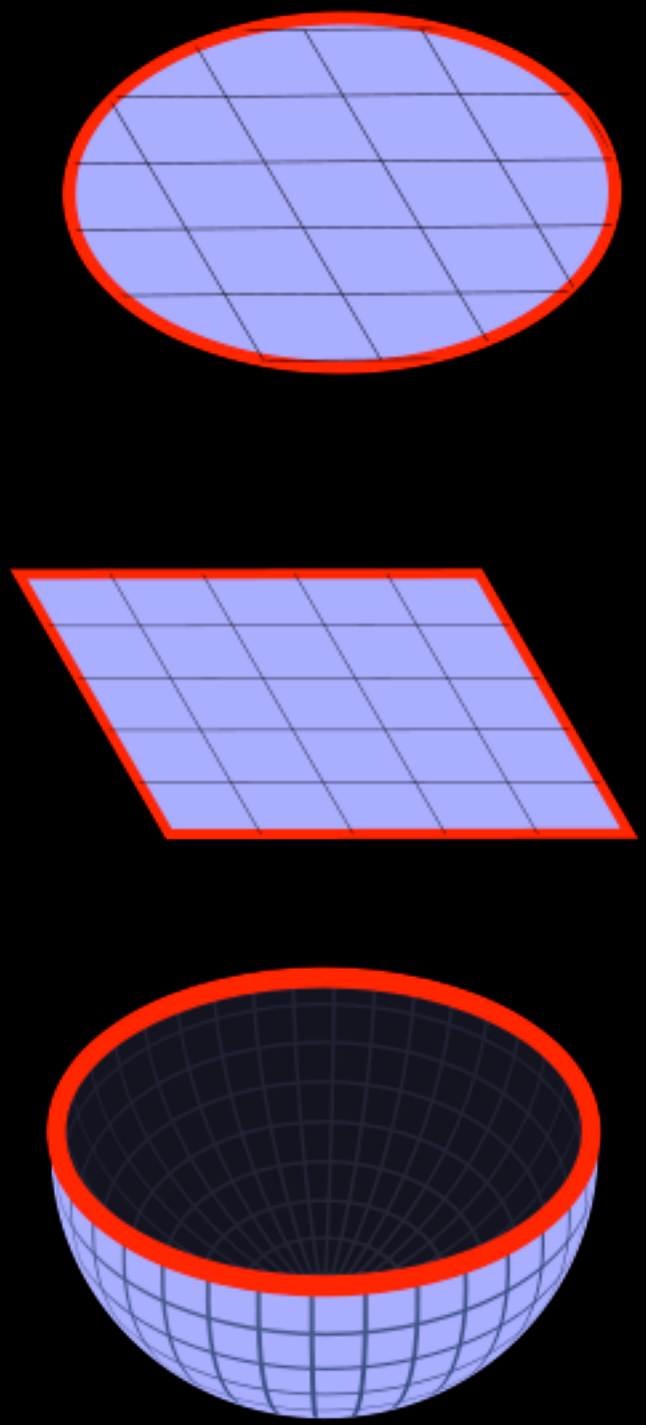
Electric flux

Electric flux counts the number of electric field lines crossing a surface



Electric flux

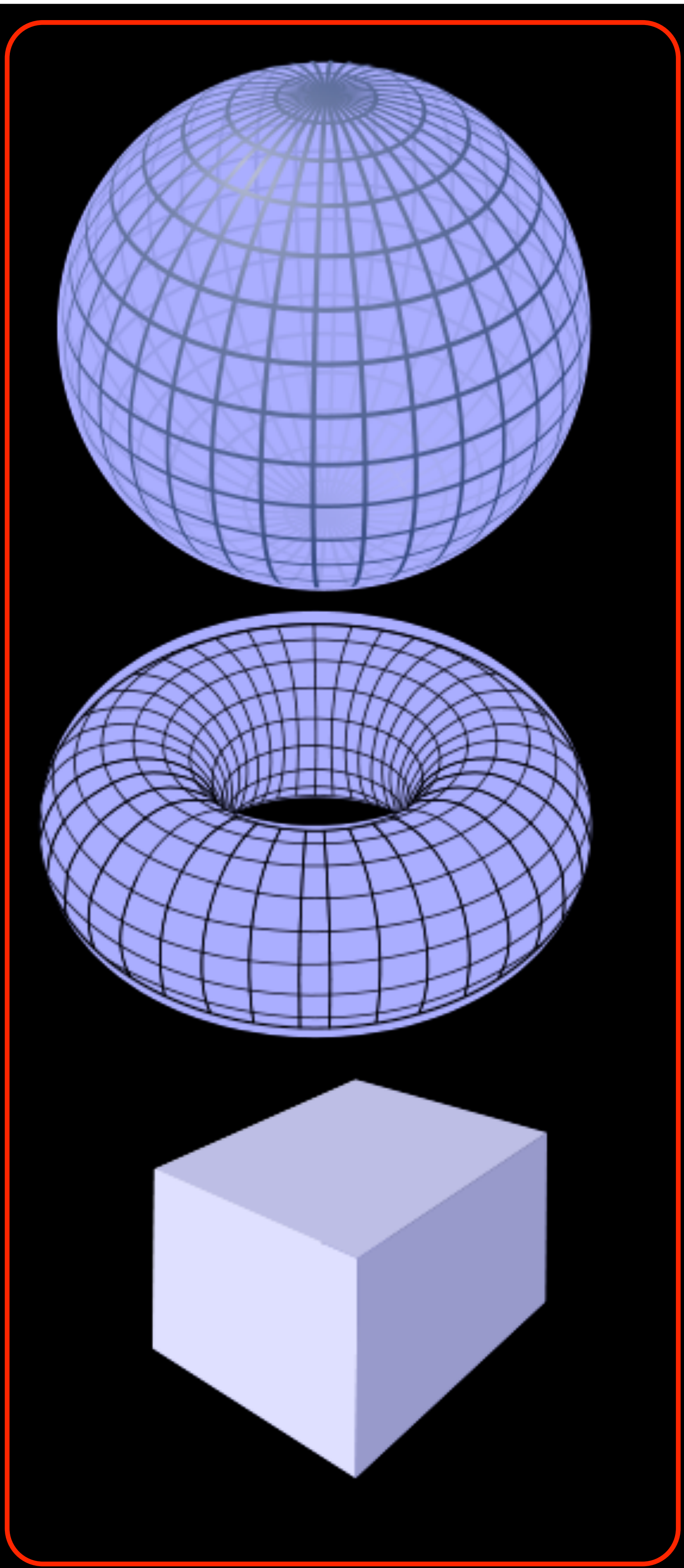


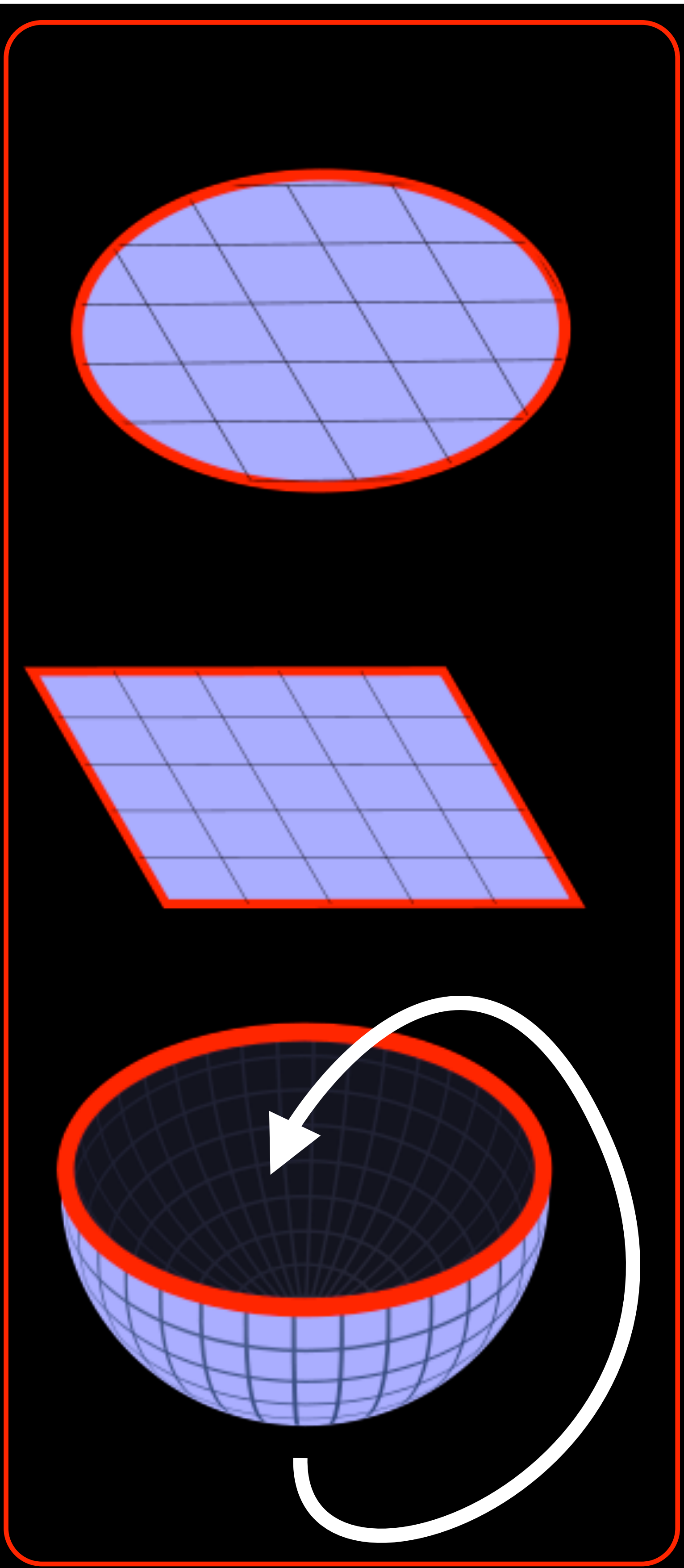


A closed surface:

Impossible to get from the inside to the outside without crossing the surface.

Electric flux



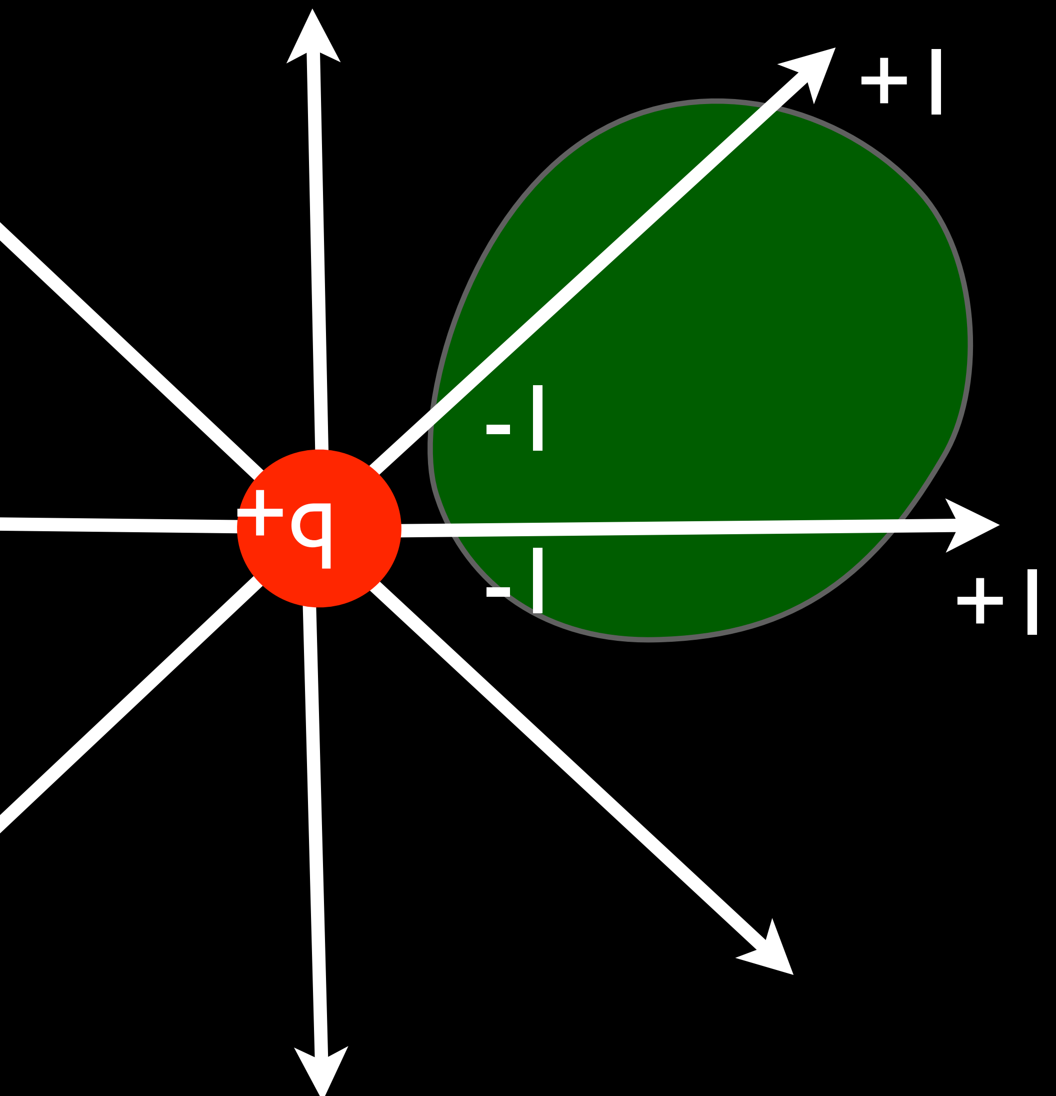


A closed surface:

Impossible to get from the inside to the outside without crossing the surface.

A open surface:

Electric flux



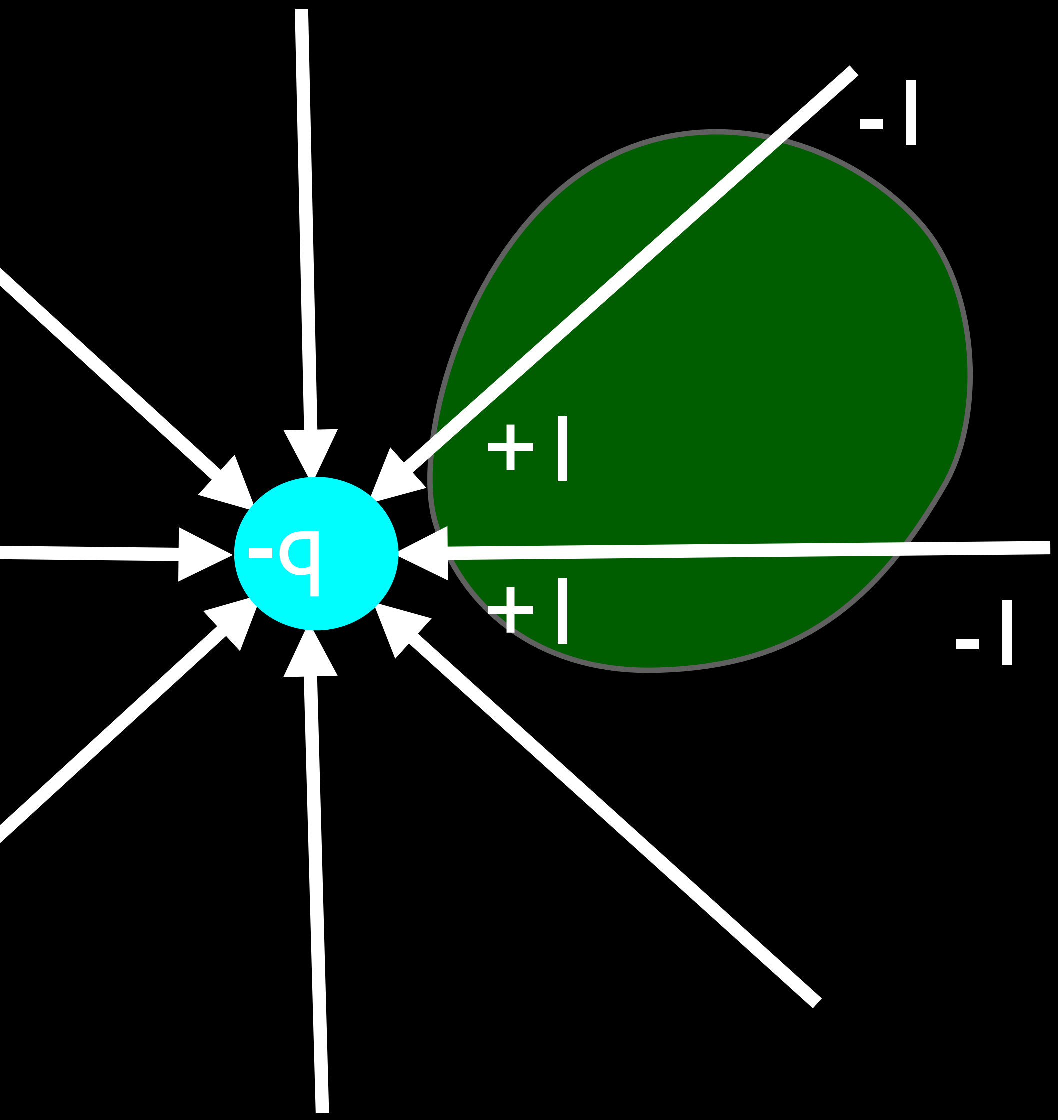
Inward crossing lines: -2 in

Outward crossing lines: +2 out

Net number of field lines crossing surface: 0

Flux: 0

Electric flux



Inward crossing lines: -2 in

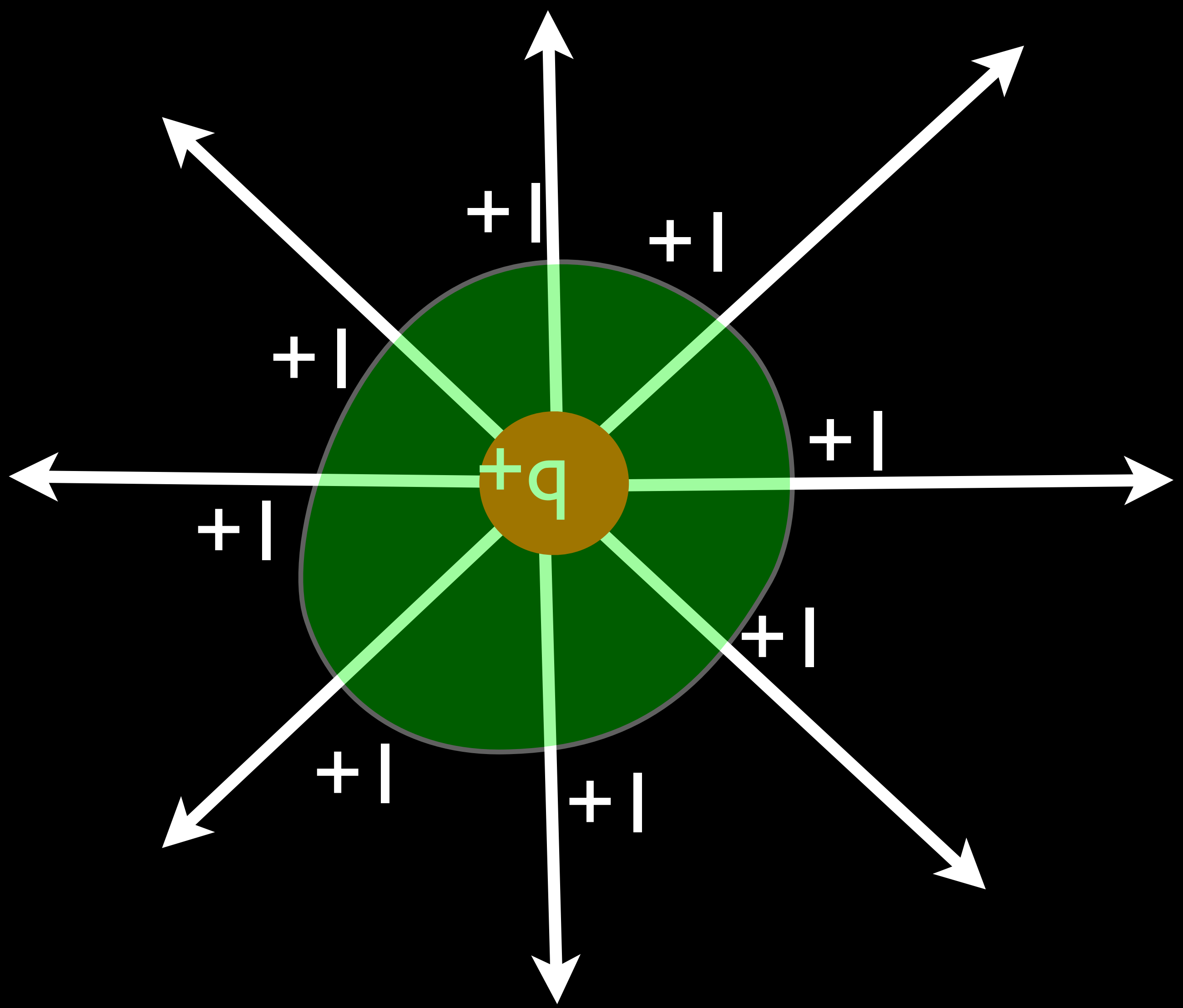
Outward crossing lines: +2 out

Net number of field lines crossing surface: 0

Flux: 0

Electric flux

What if the surface contains a charge?



Outward crossing lines: +8

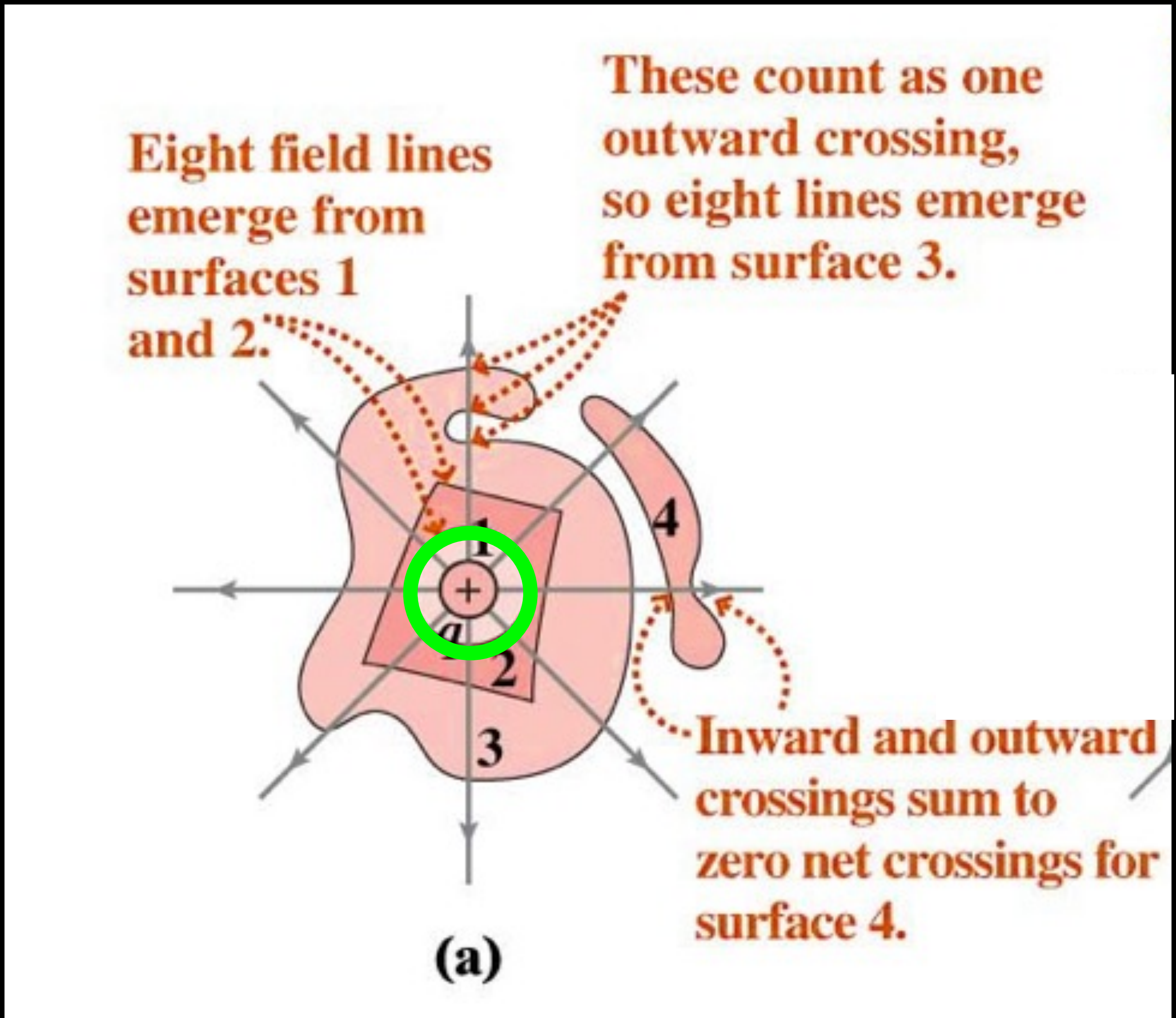
Inward crossing lines: 0

Net number of field lines crossing surface: +8

Flux: $\neq 0$

Electric flux

Four different surfaces

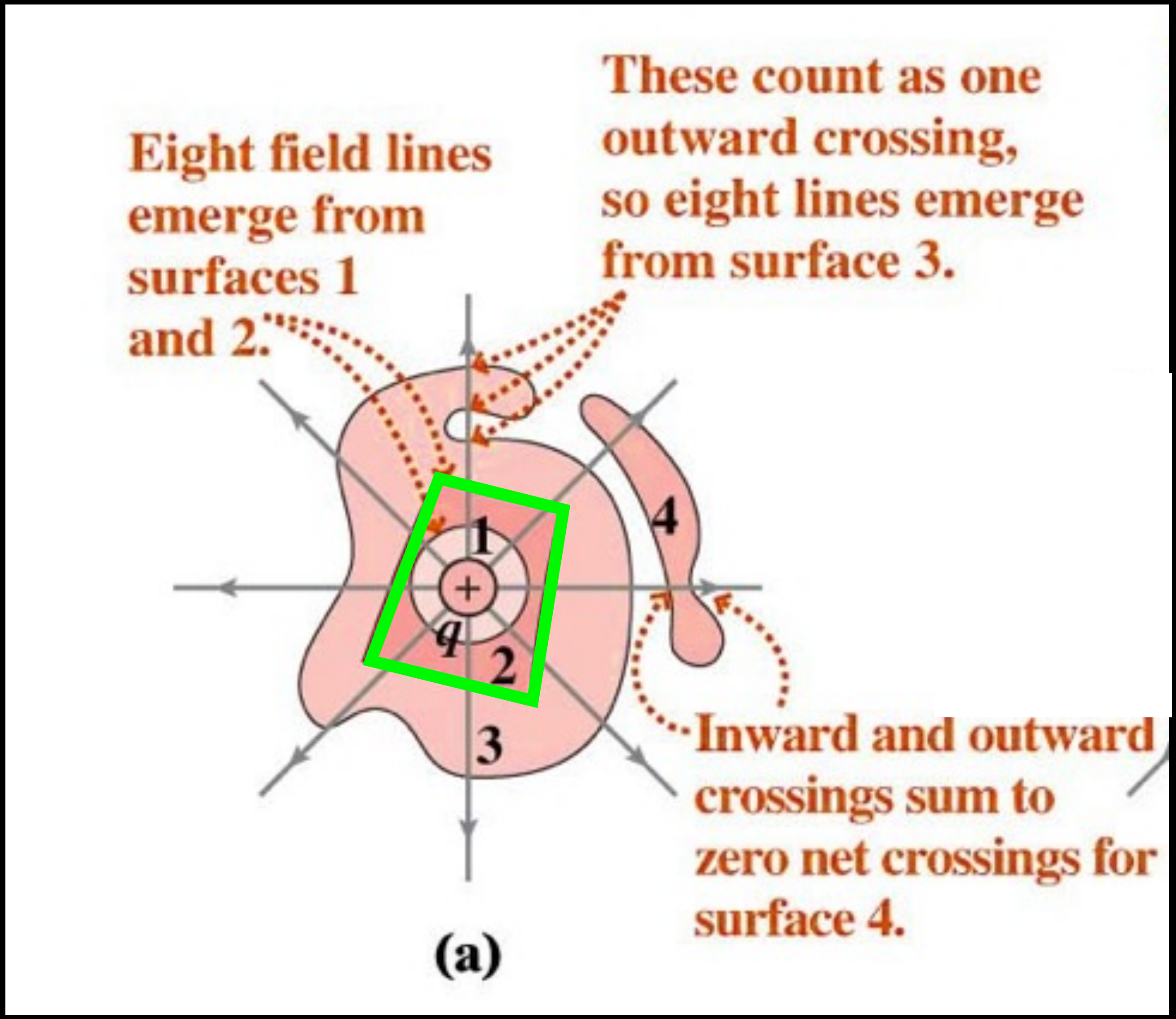


Surface 1

Outward crossing lines: +8

Electric flux

Four different surfaces



Surface 1

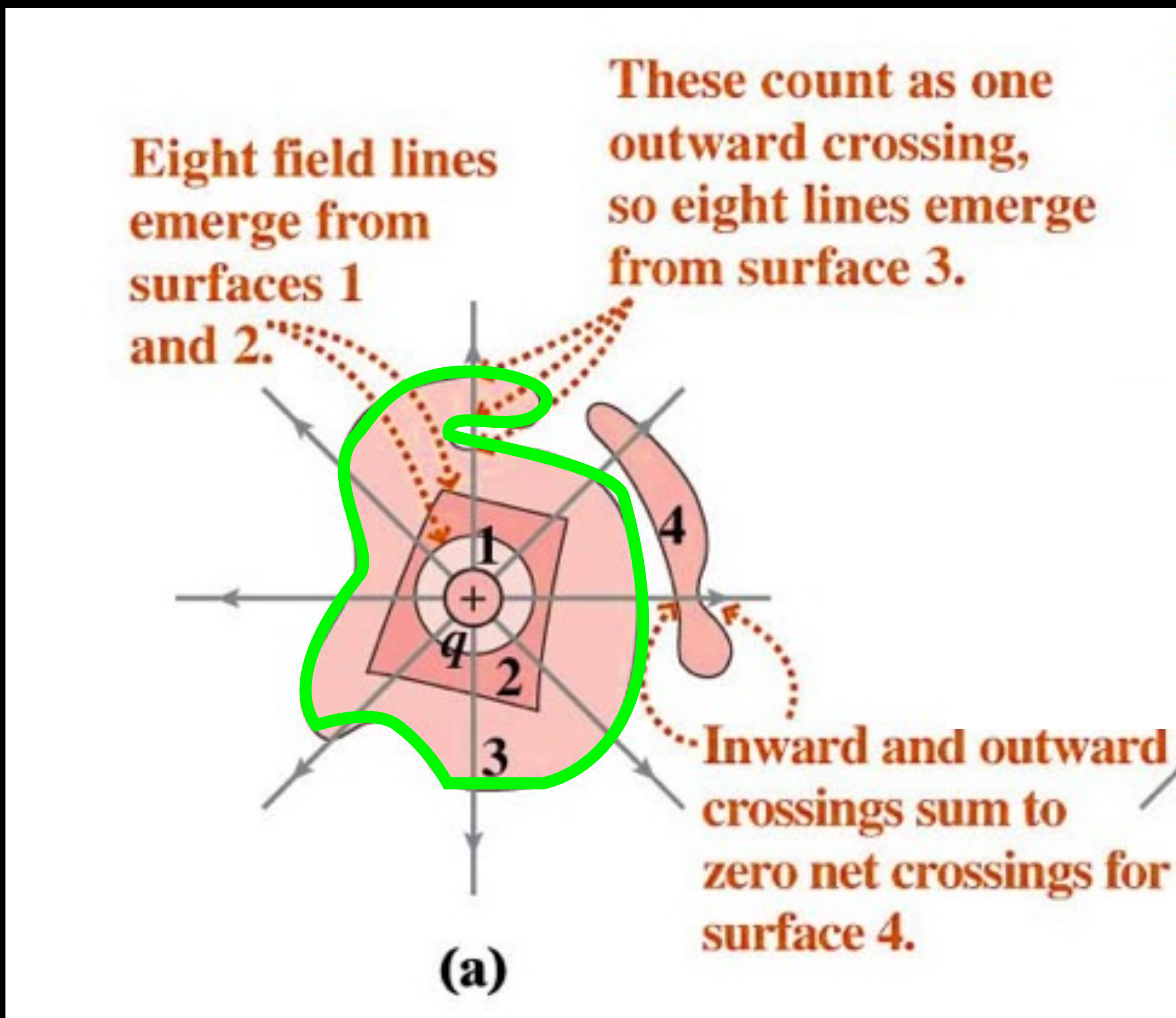
Outward crossing lines: +8

Surface 2

Outward crossing lines: +8

Electric flux

Four different surfaces



Surface 1

Outward crossing lines: +8

Surface 2

Outward crossing lines: +8

Surface 3

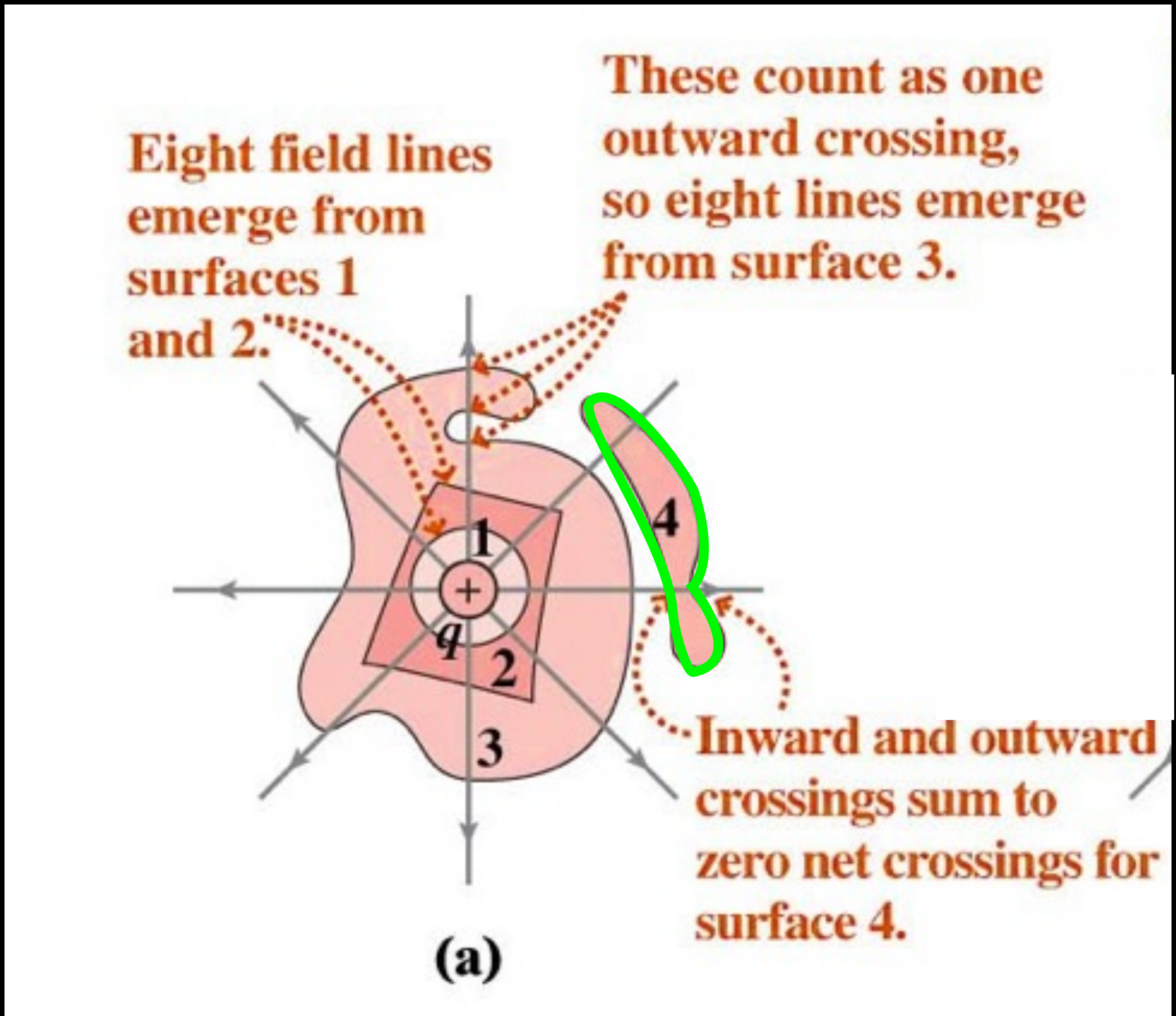
Outward crossing lines: +8

Inward crossing lines: -1

Outward crossing lines: +1

Electric flux

Four different surfaces



Surface 1

Outward crossing lines: +8

Surface 2

Outward crossing lines: +8

Surface 3

Outward crossing lines: +8

Inward crossing lines: -1

Outward crossing lines: +1

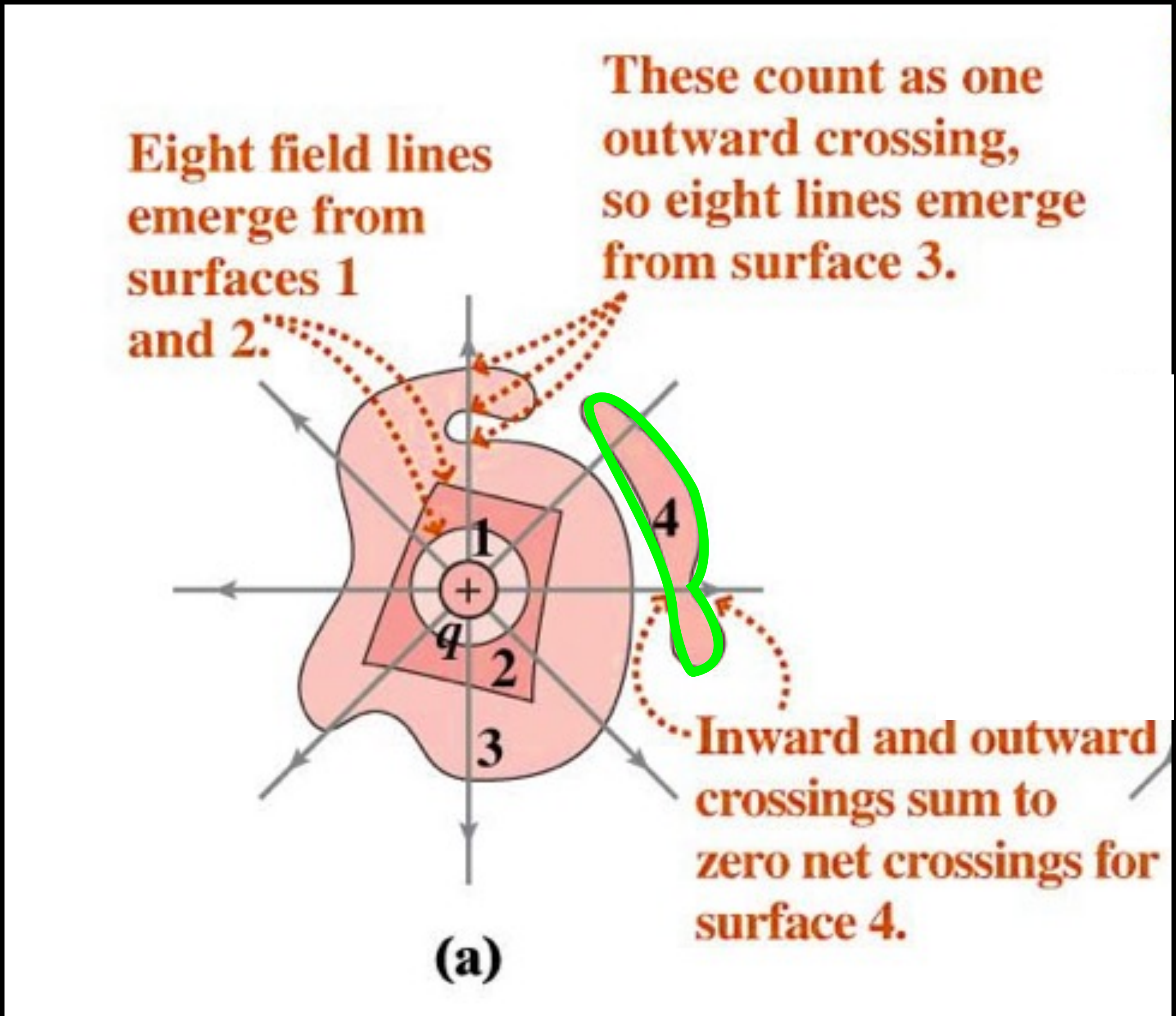
Surface 4

Inward crossing lines: -2

Outward crossing lines: +2

Electric flux

Four different surfaces



Surface 1

Outward crossing lines: +8

Surface 2

Outward crossing lines: +8

Surface 3

Outward crossing lines: +8

Inward crossing lines: -1

Outward crossing lines: +1

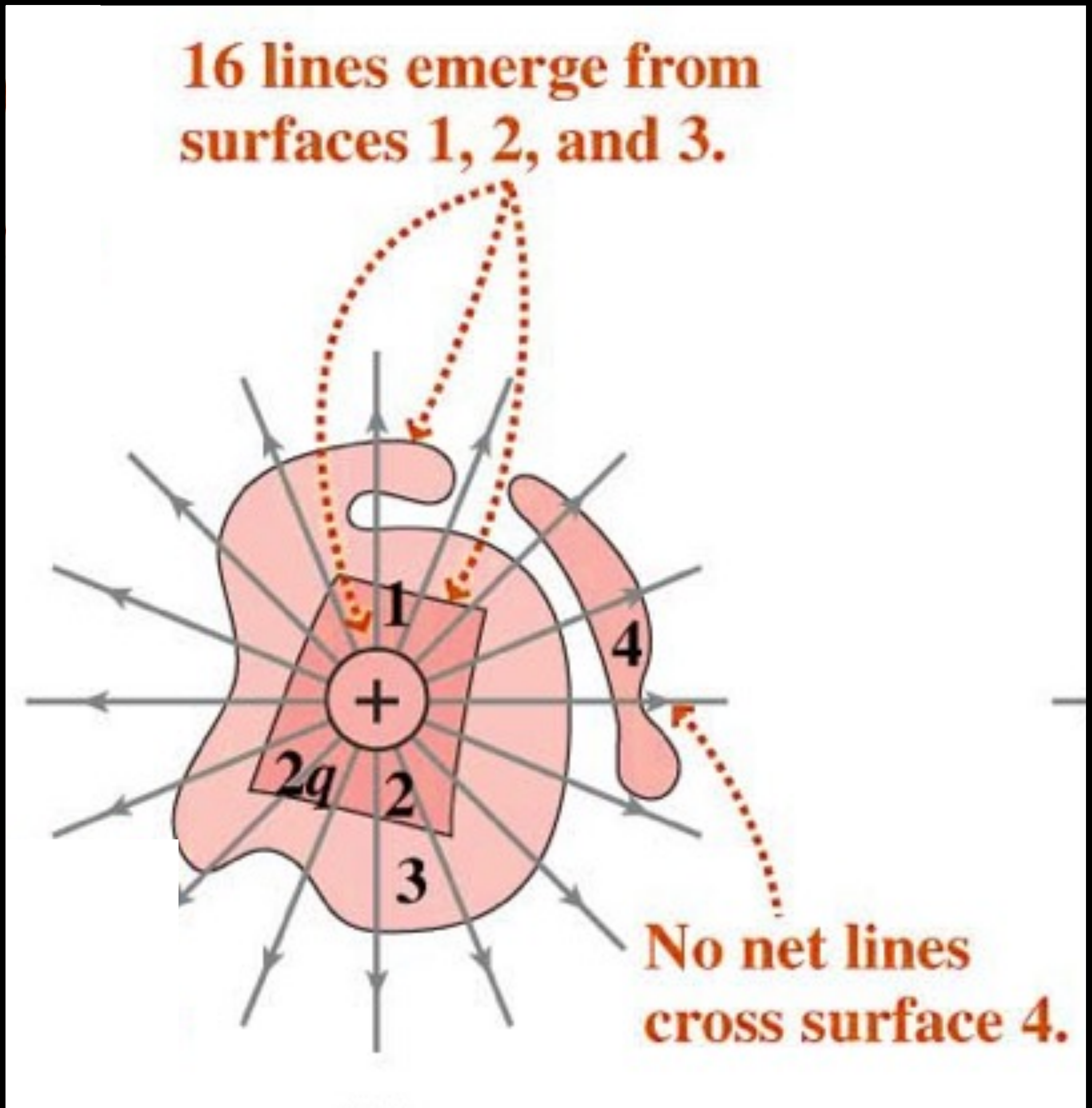
Surface 4

Inward crossing lines: -2

Outward crossing lines: +2

Electric flux

Four different surfaces



(b)

Surface 1

Outward crossing lines: +16

Surface 2

Outward crossing lines: +16

Surface 3

Outward crossing lines: +16

Inward crossing lines: -1

Outward crossing lines: +1

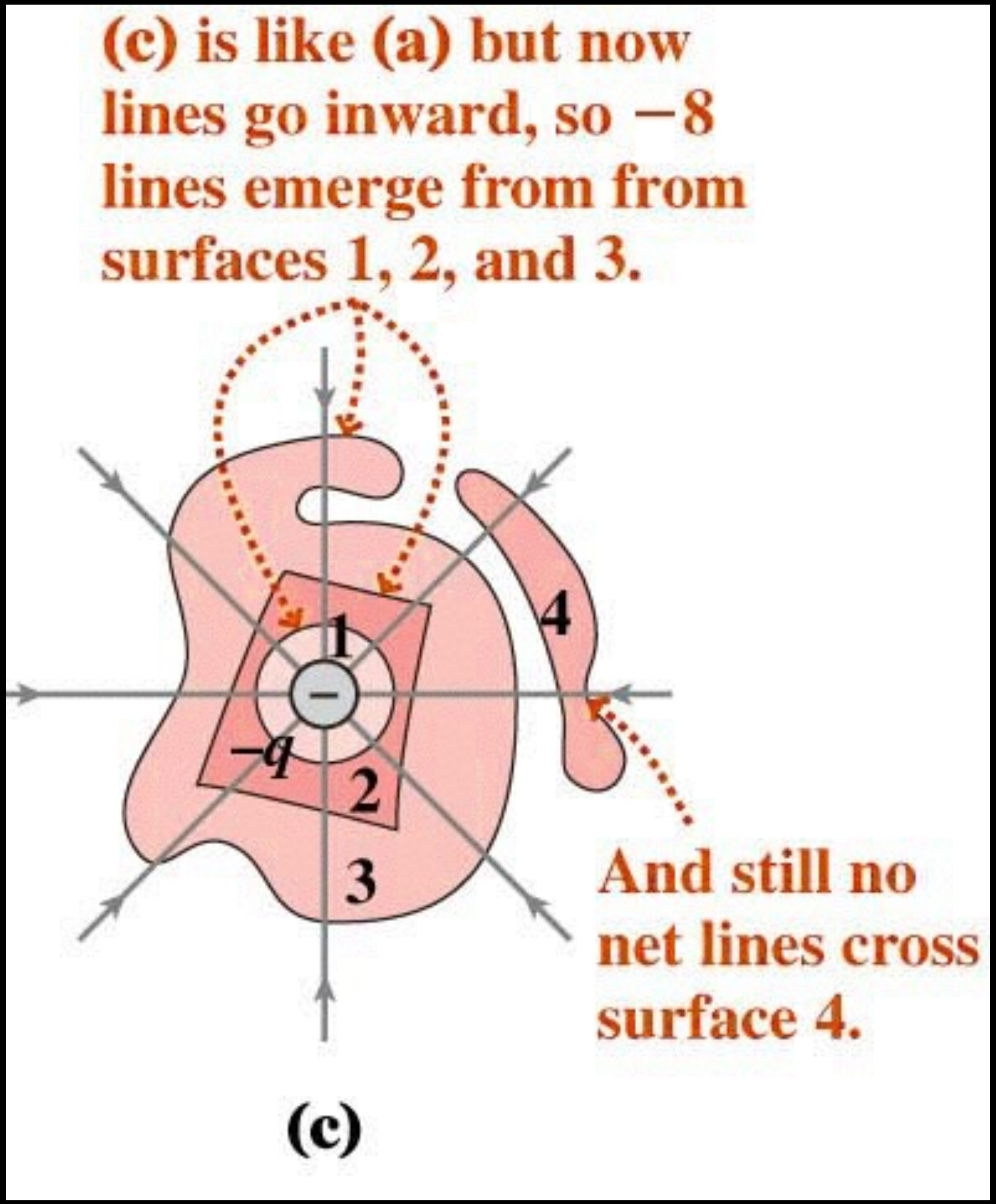
Surface 4

Inward crossing lines: -3

Outward crossing lines: +3

Electric flux

Four different surfaces



Surface 1

Inward crossing lines: -8

Surface 2

Inward crossing lines: -8

Surface 3

Inward crossing lines: -8

Outward crossing lines: +1

Inward crossing lines: -1

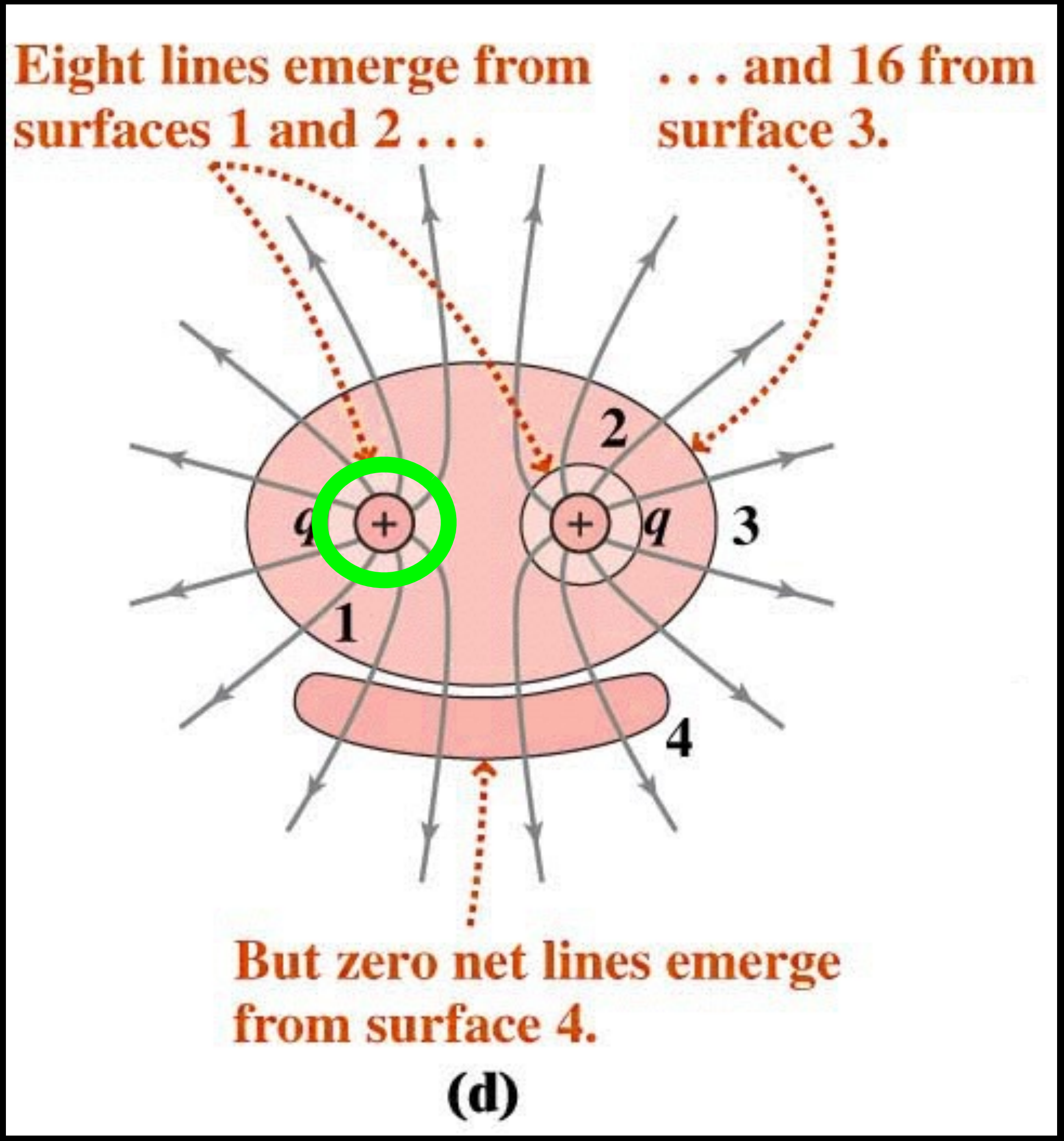
Surface 4

Inward crossing lines: -2

Outward crossing lines: +2

Electric flux

Four different surfaces

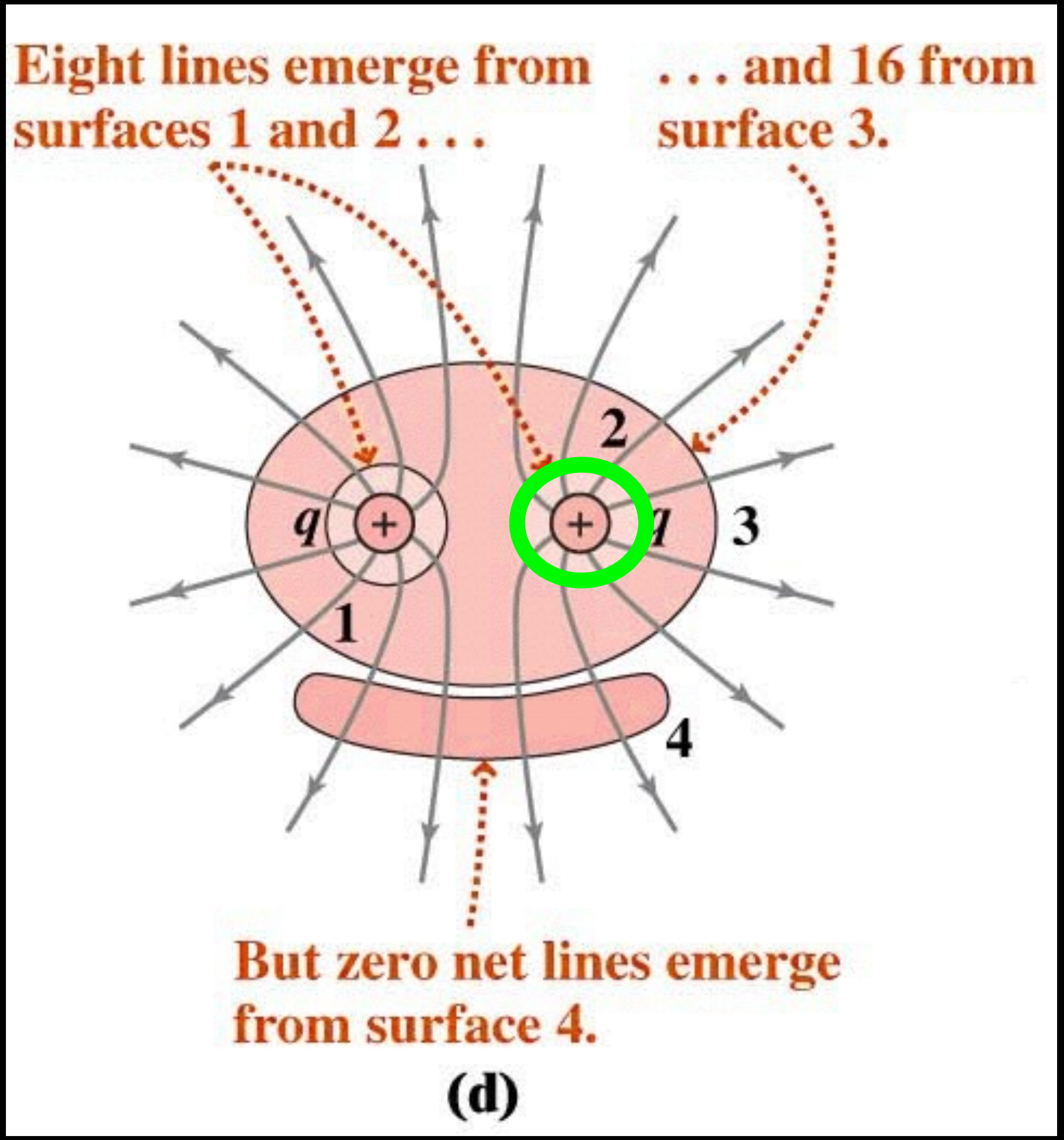


Surface 1

Outward crossing lines: +8

Electric flux

Four different surfaces



Surface 1

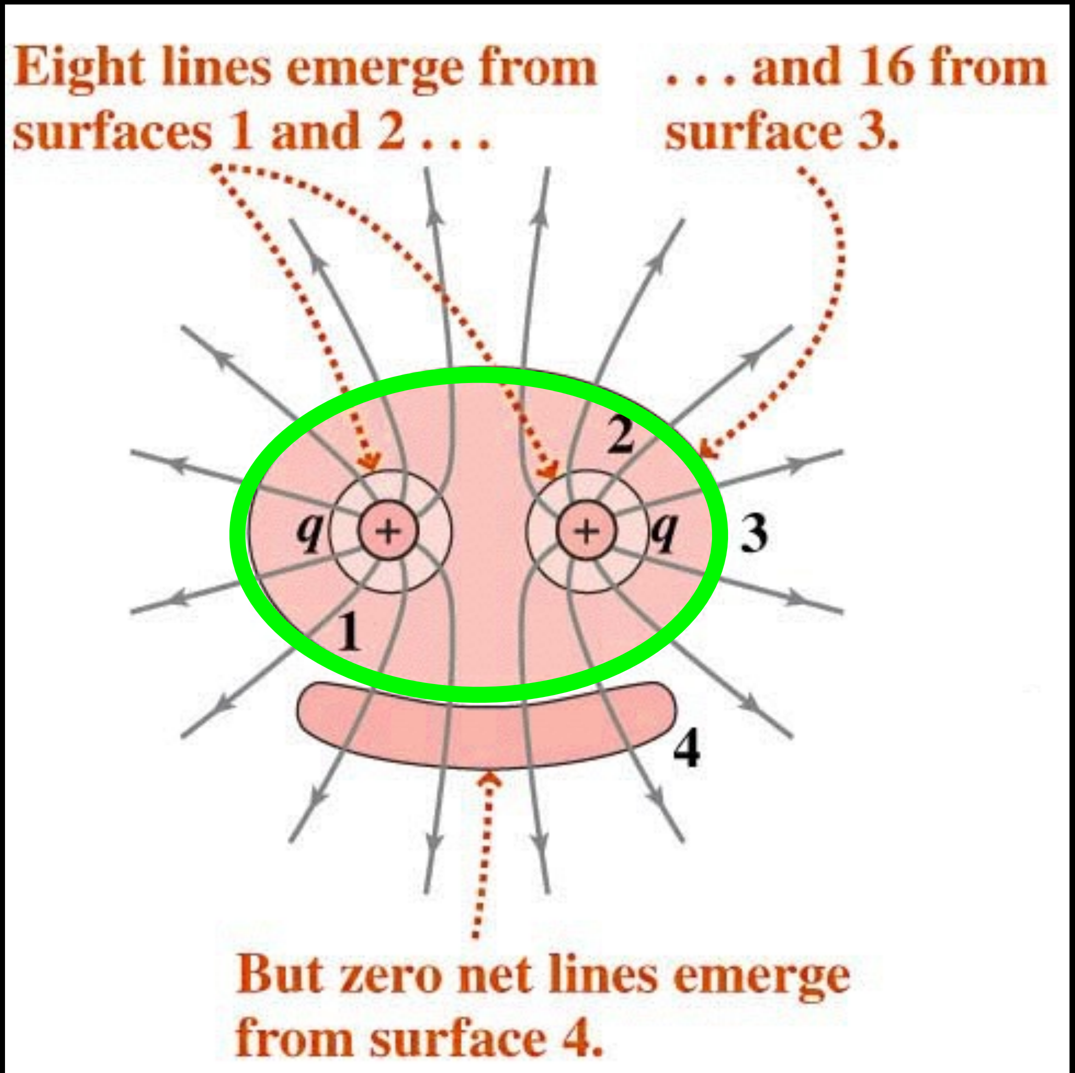
Outward crossing lines: +8

Surface 2

Outward crossing lines: +8

Electric flux

Four different surfaces



(d)

Surface 1

Outward crossing lines: +8

Surface 2

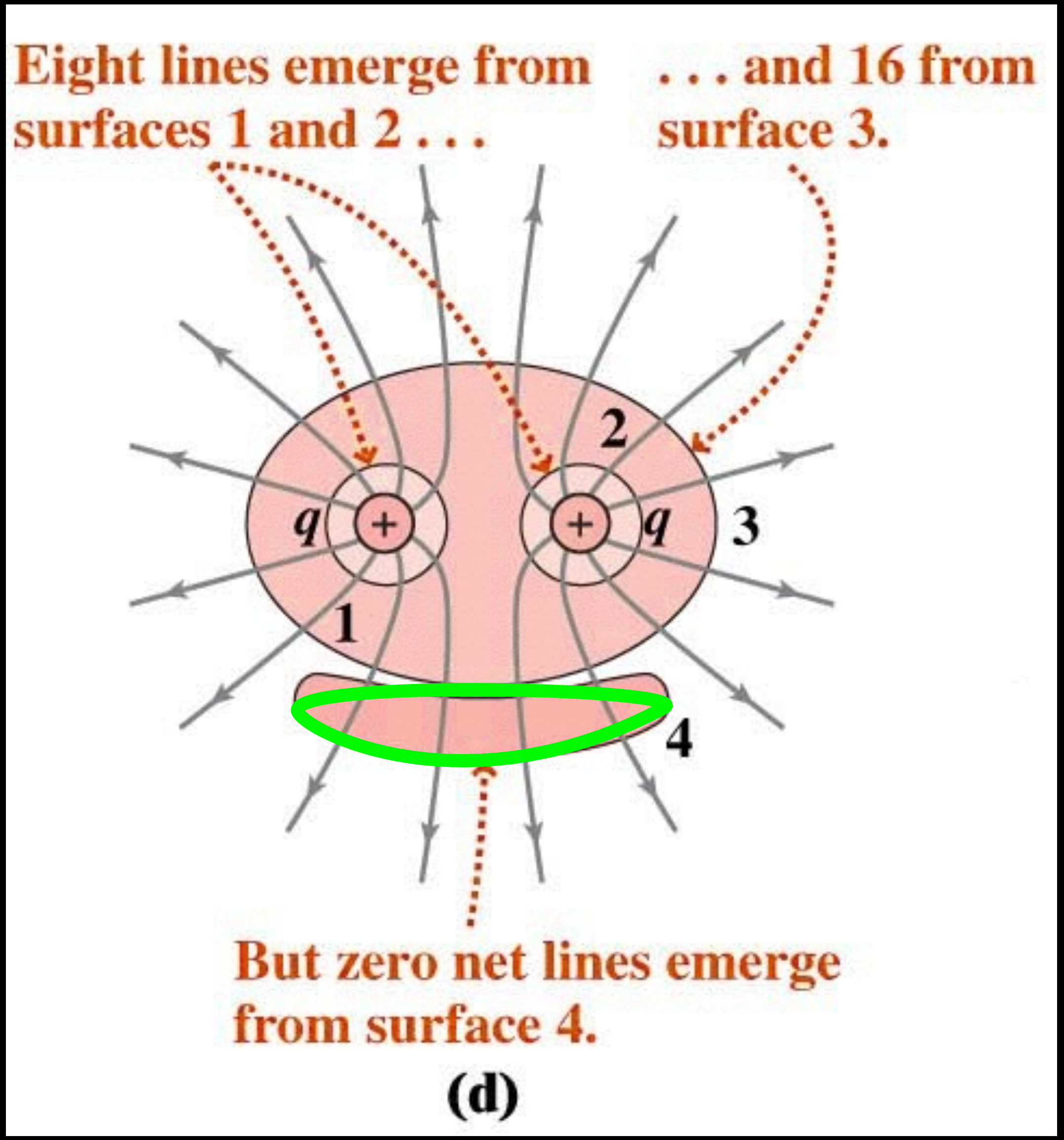
Outward crossing lines: +8

Surface 3

Outward crossing lines: +16

Electric flux

Four different surfaces



Surface 1

Outward crossing lines: +8

Surface 2

Outward crossing lines: +8

Surface 3

Outward crossing lines: +16

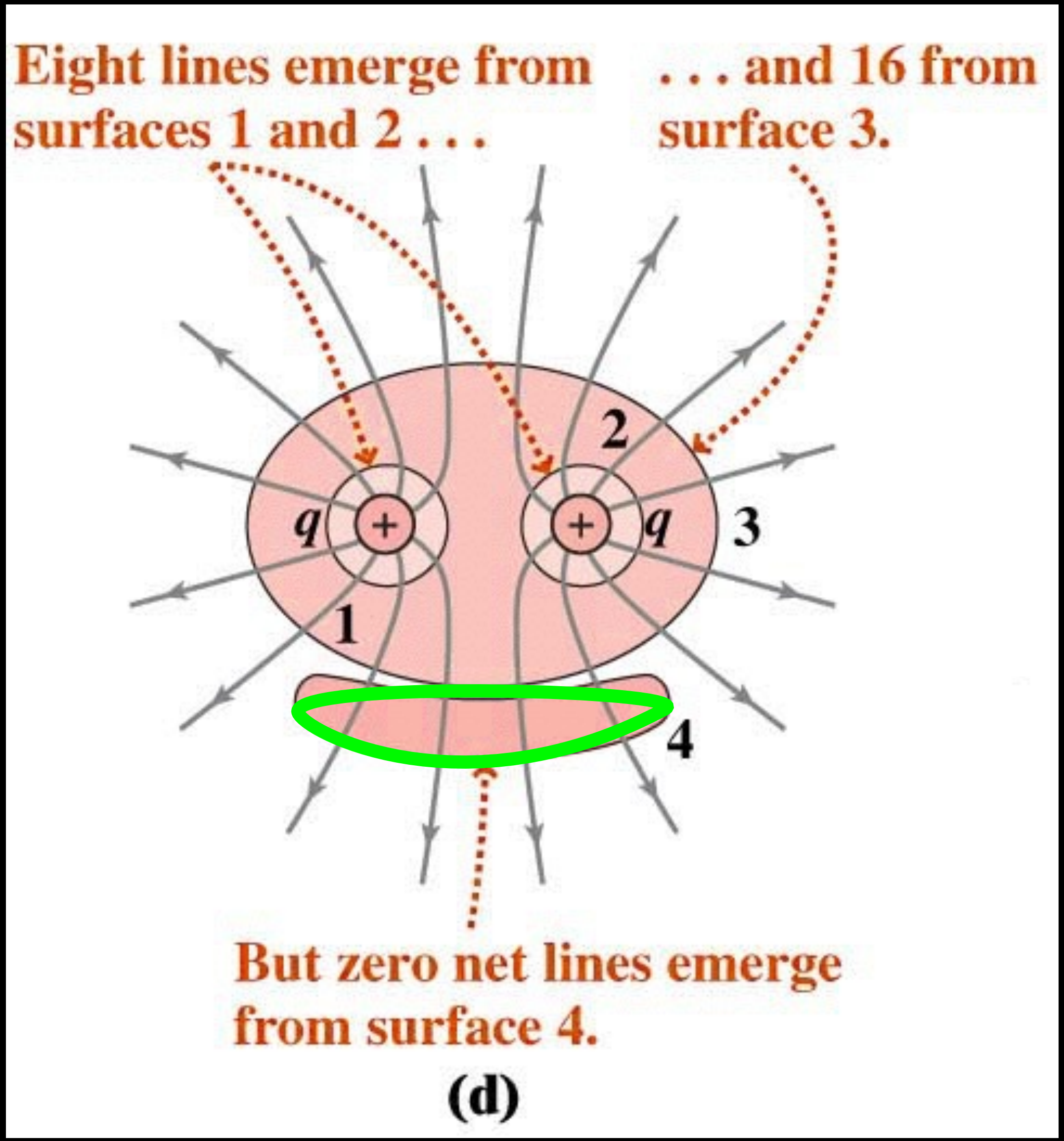
Surface 4

Inward crossing lines: -4

Outward crossing lines: +4

Electric flux

Four different surfaces



Surface 1

Outward crossing lines: +8

Surface 2

Outward crossing lines: +8

Surface 3

Outward crossing lines: +16

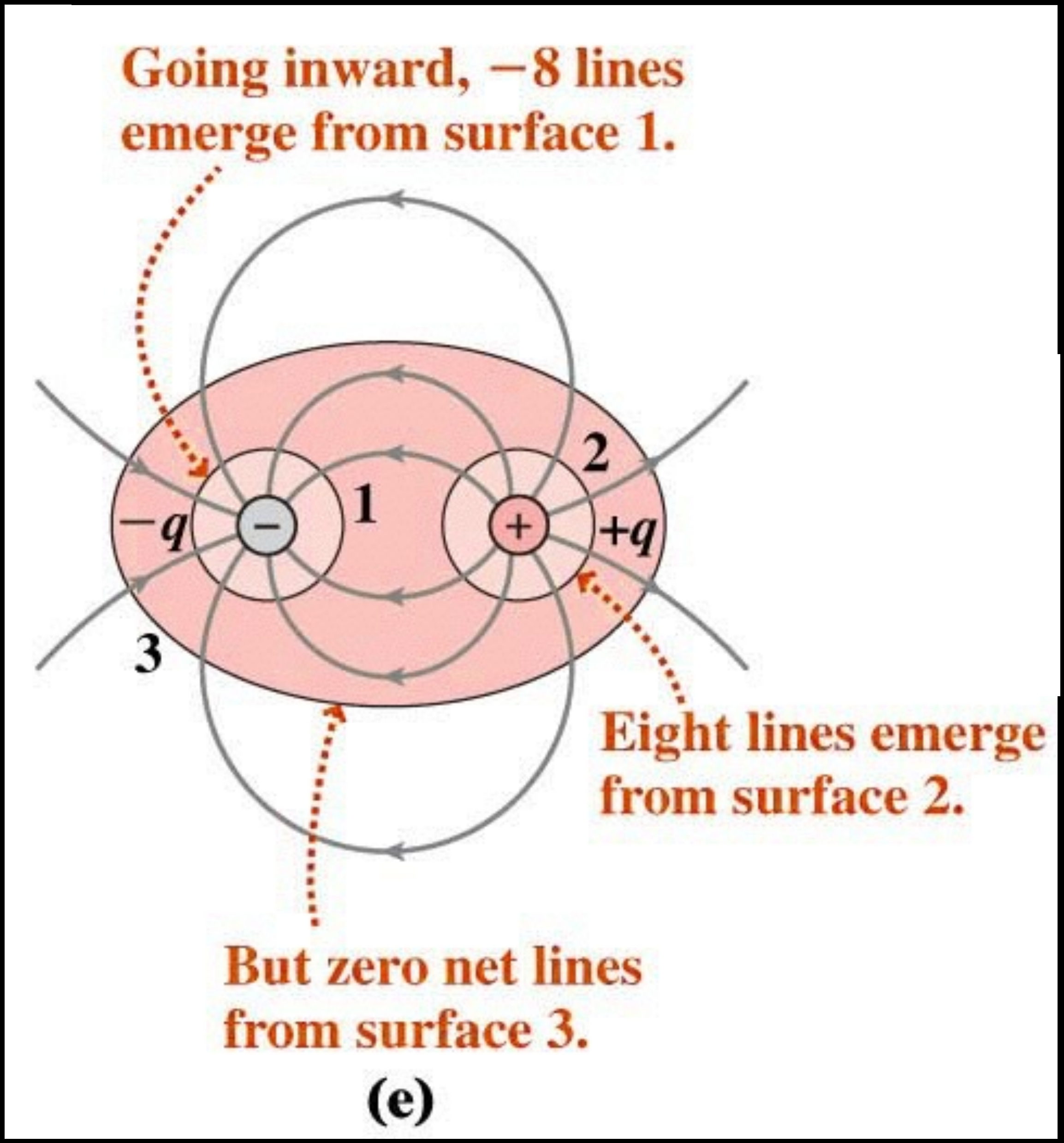
Surface 4

Inward crossing lines: -4

Outward crossing lines: +4

Electric flux

Three different surfaces



Surface 1

Inward crossing lines: -8

Surface 2

Outward crossing lines: +8

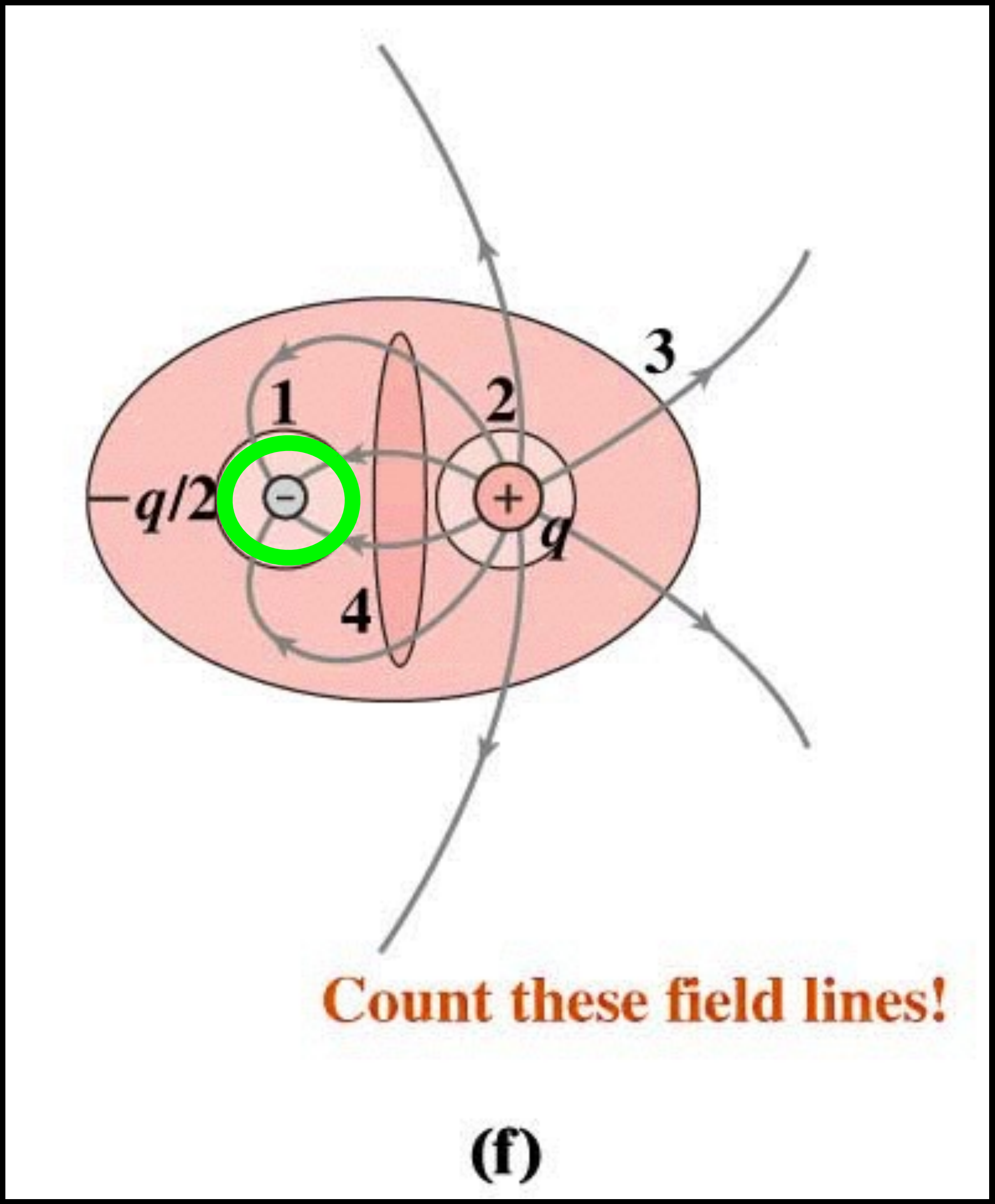
Surface 3

Outward crossing lines: +4

Inward crossing lines: -4

Electric flux

Surface 1: net number of lines?



(A) +4

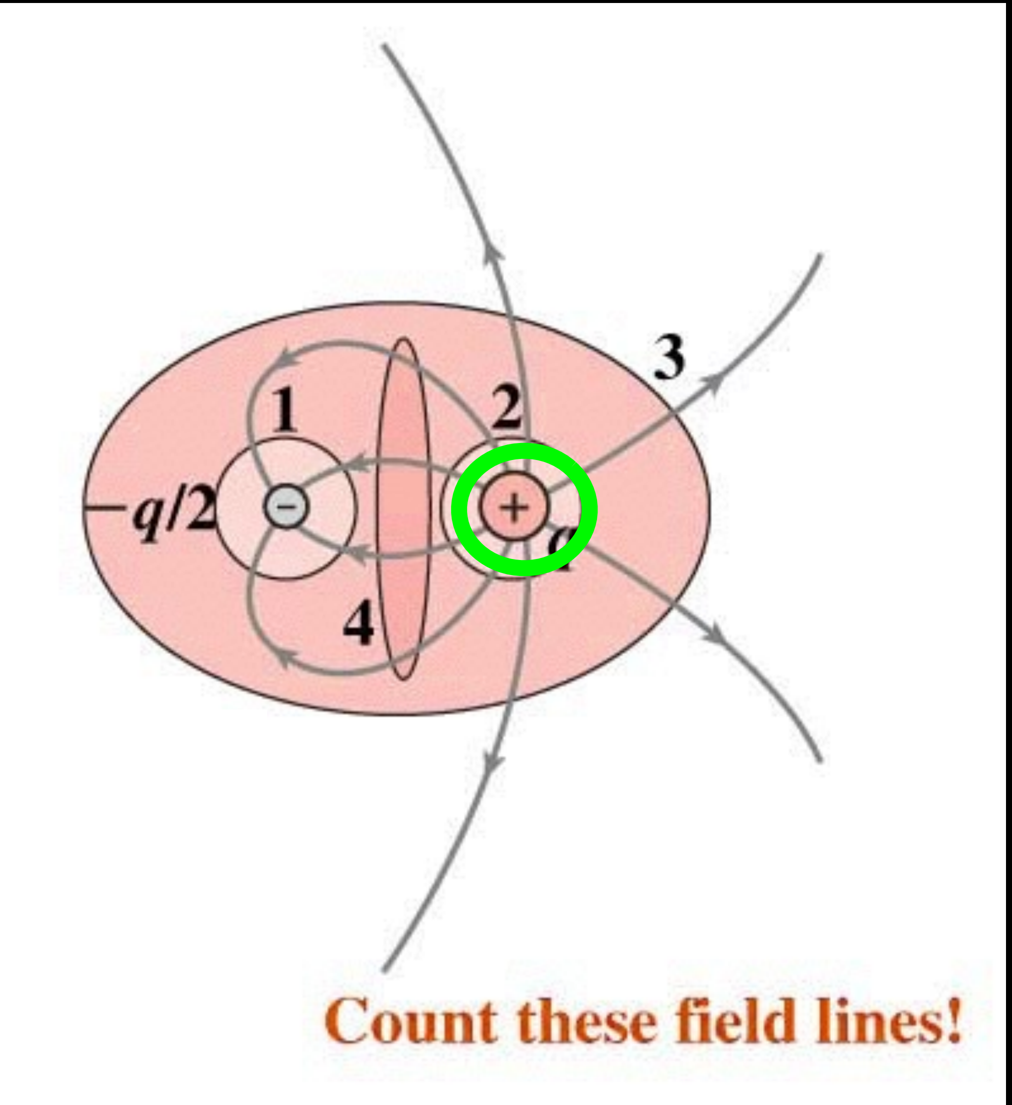
(B) -4

(C) -8

(D) +8

Electric flux

Surface 2: net number of lines?



(f)

(A) +4

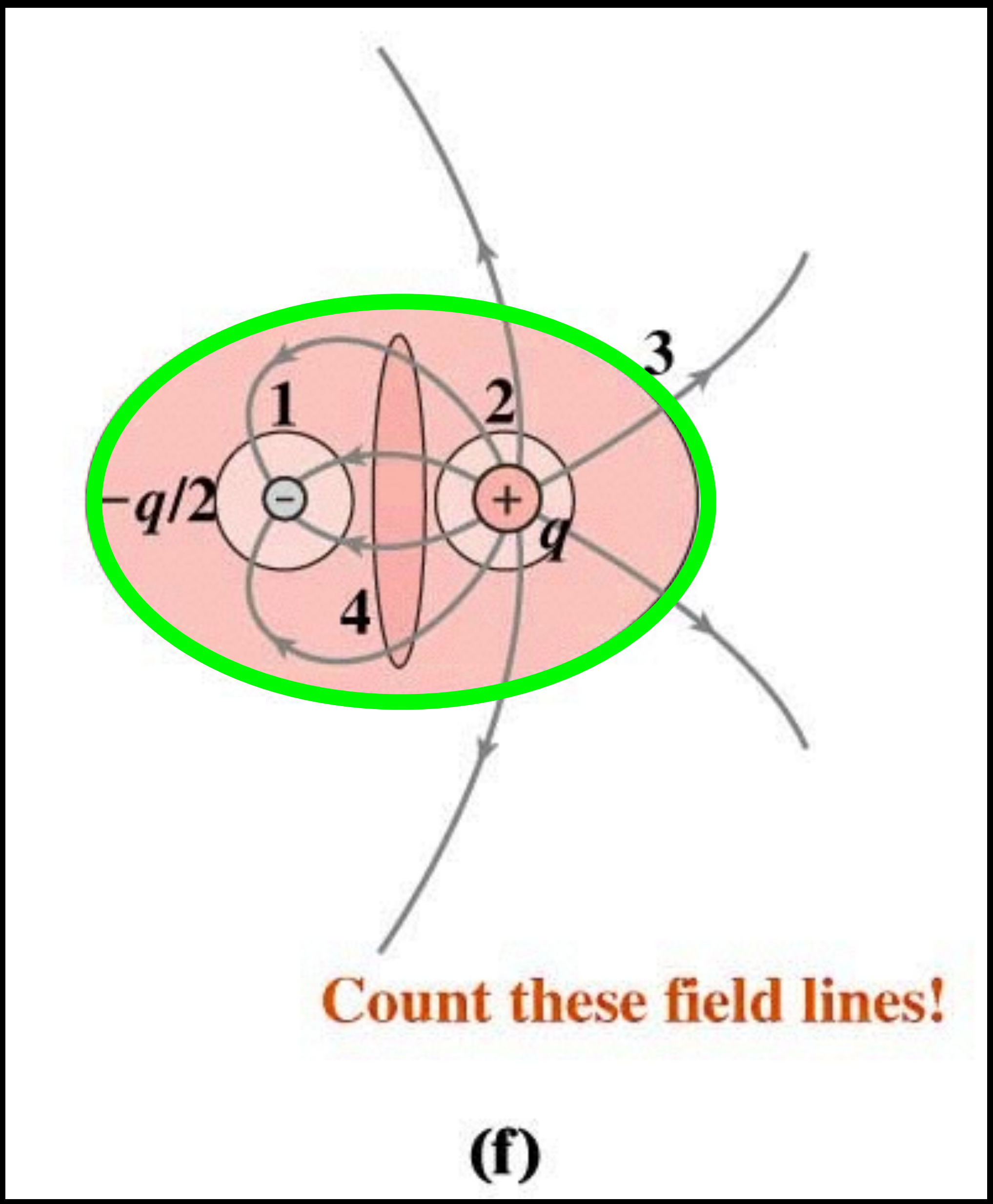
(B) -4

(C) -8

(D) +8

Electric flux

Surface 3: net number of lines?



(A) +4

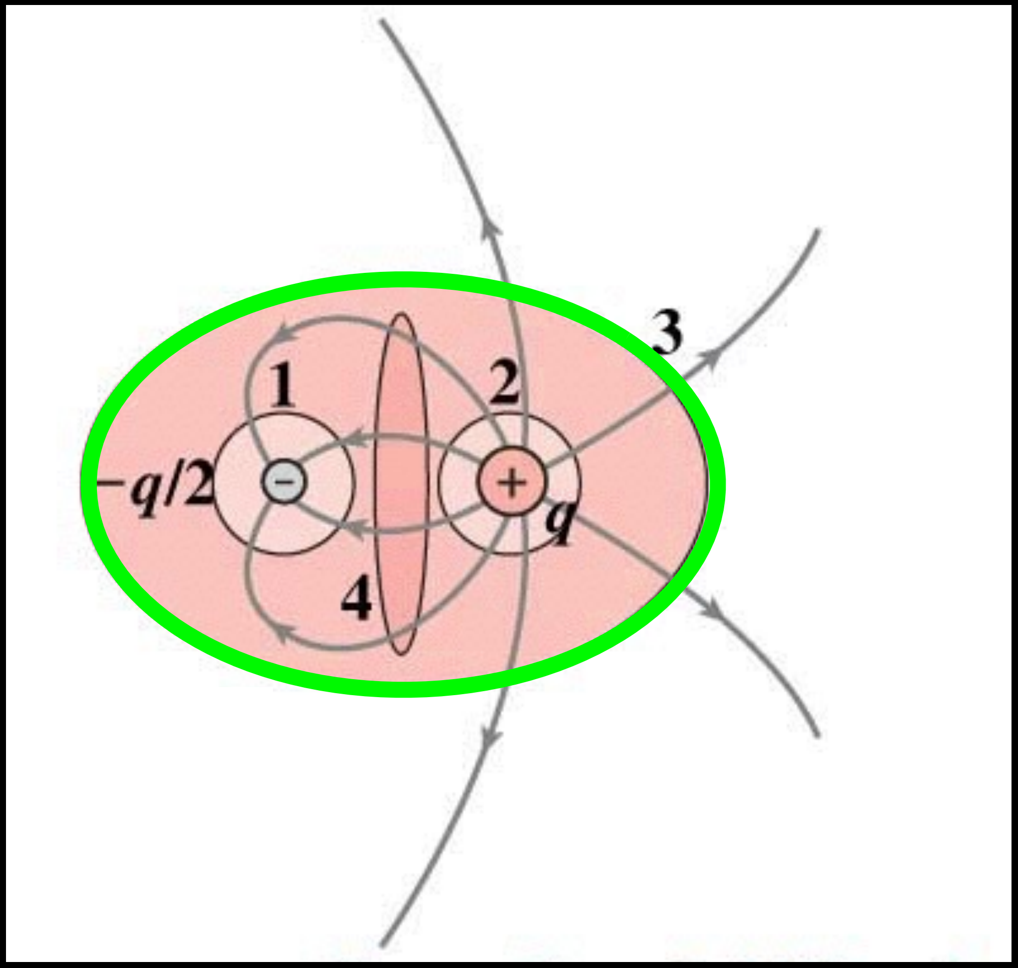
(B) -4

(C) -8

(D) +8

Electric flux

Is there a pattern?



The number of field lines (= flux) emerging from any closed surface is proportional to the net charge enclosed.

Surface 1:

Net charge: $-q/2$

Net lines: -4

Surface 2:

Net charge: q

Net lines: +8

Surface 3:

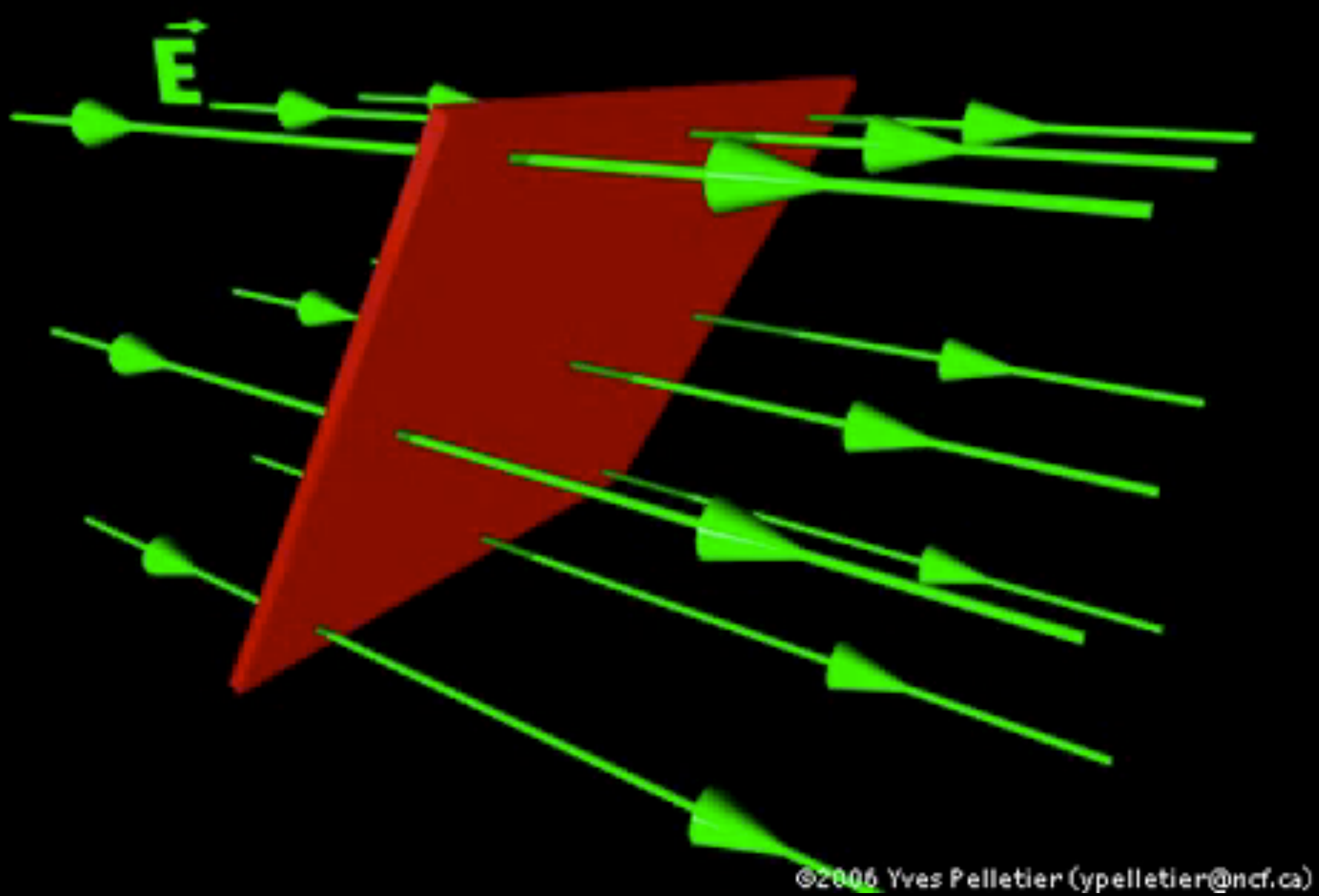
Net charge: $q/2$

Net lines: +4

Electric flux

We've seen how the interior charge affects the number of field lines

What controls how many of those field lines pass through a surface?

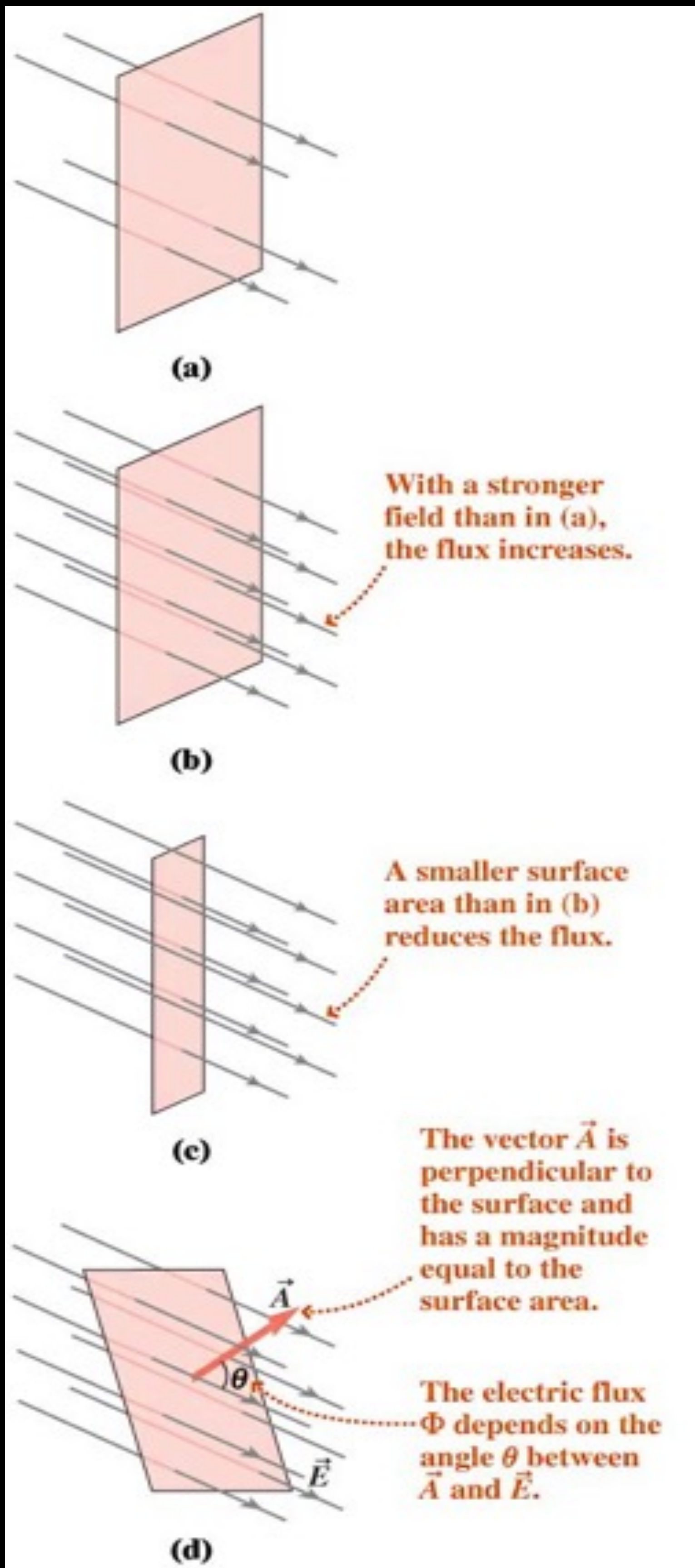


A smaller surface area reduces the flux.

A stronger field increases the flux

The flux depends on the angle between the electric field the surface

Electric flux

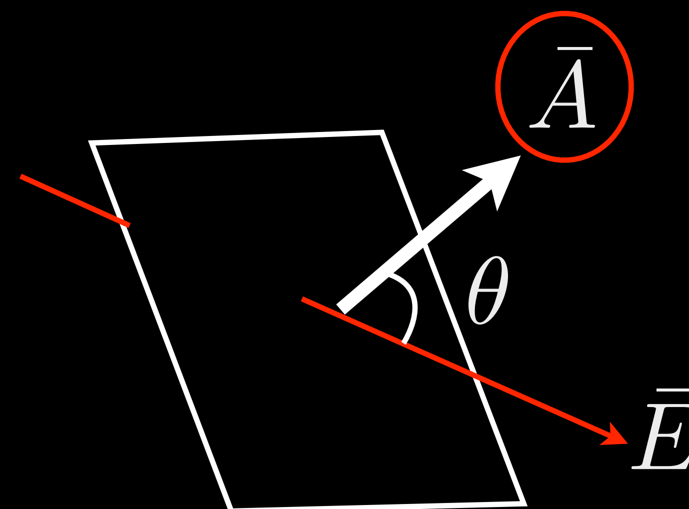


Flux depends on:

$$\Phi = |\bar{E}||\bar{A}|\cos\theta$$

Flux

$\propto E$



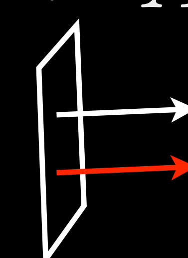
Direction perpendicular to surface

Magnitude = surface area

$\propto A$

Area perpendicular to field: $\bar{A}$ parallel to $\bar{E}$

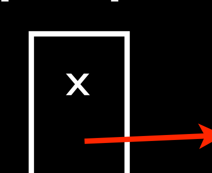
$$\cos\theta = \cos 0 = 1$$



$\propto \theta$

Area parallel to field: $\bar{A}$ perpendicular to $\bar{E}$

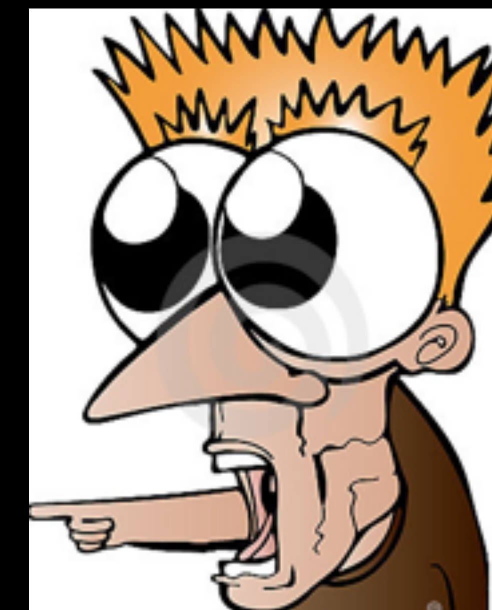
$$\cos\theta = \cos 90 = 0$$



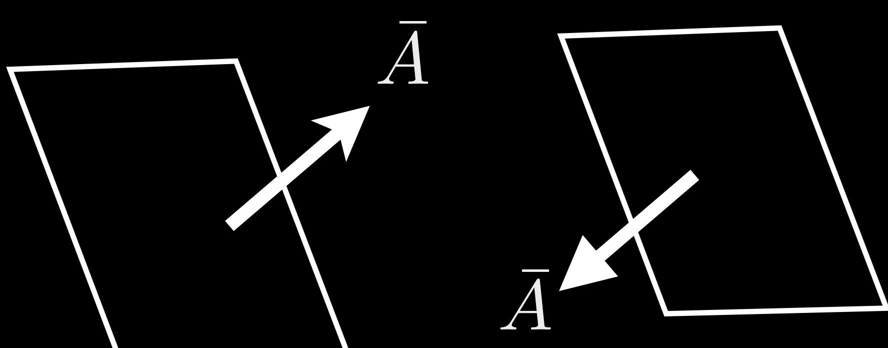
Electric flux

$$\boxed{\begin{aligned}\Phi &= |\bar{E}||\bar{A}|\cos\theta \\ &= \bar{E}\cdot\bar{A}\end{aligned}}$$

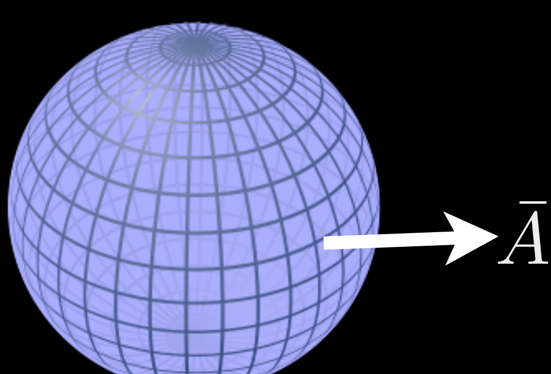
Scalar product!



units: Nm^2/C



with open surfaces, $\bar{A}$ can point in either direction.

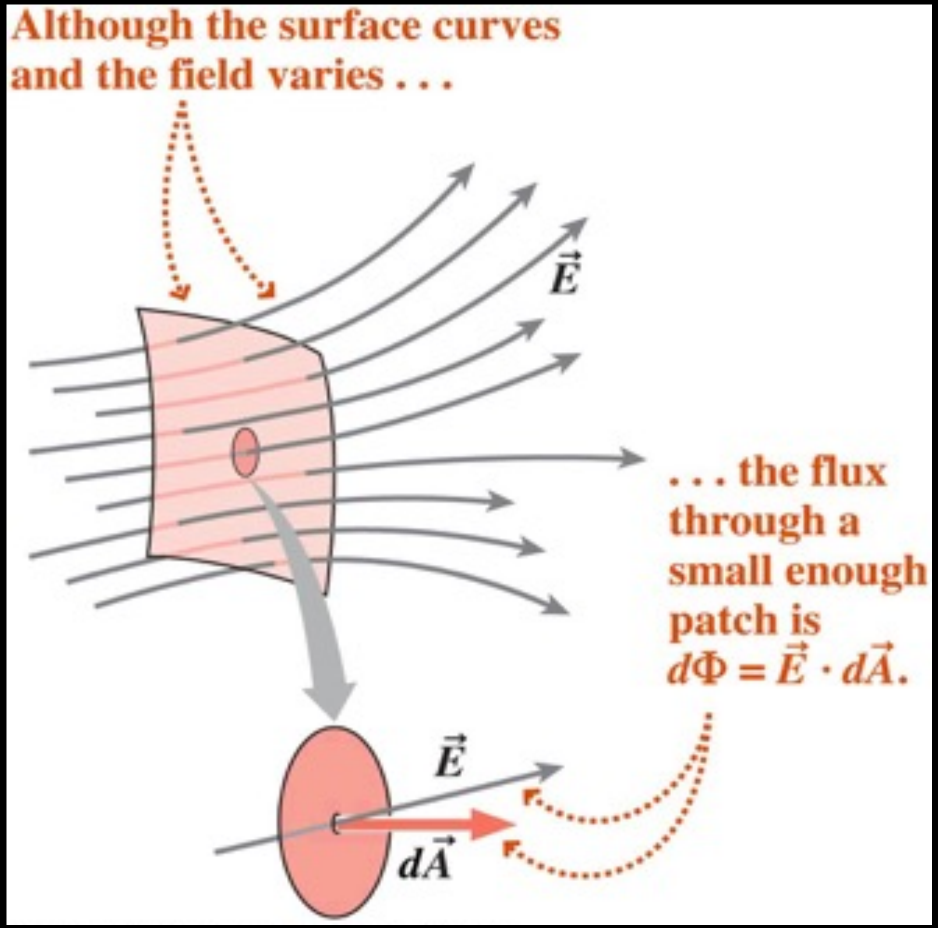


with closed surfaces, $\bar{A}$ points outwards.

Electric flux

What if the surface curves and/or the field varies with position?

Divide surface into small patches, $d\bar{A}$



Each patch ~ flat

Field ~ uniform

$$d\Phi = \bar{E} \cdot d\bar{A}$$

If patch is very small, the total flux becomes:

$$\Phi = \sum_i \bar{E}_i \cdot d\bar{A}_i = \int_{\text{surface}} \bar{E} \cdot d\bar{A}$$

Integral over the whole surface.

Electric flux

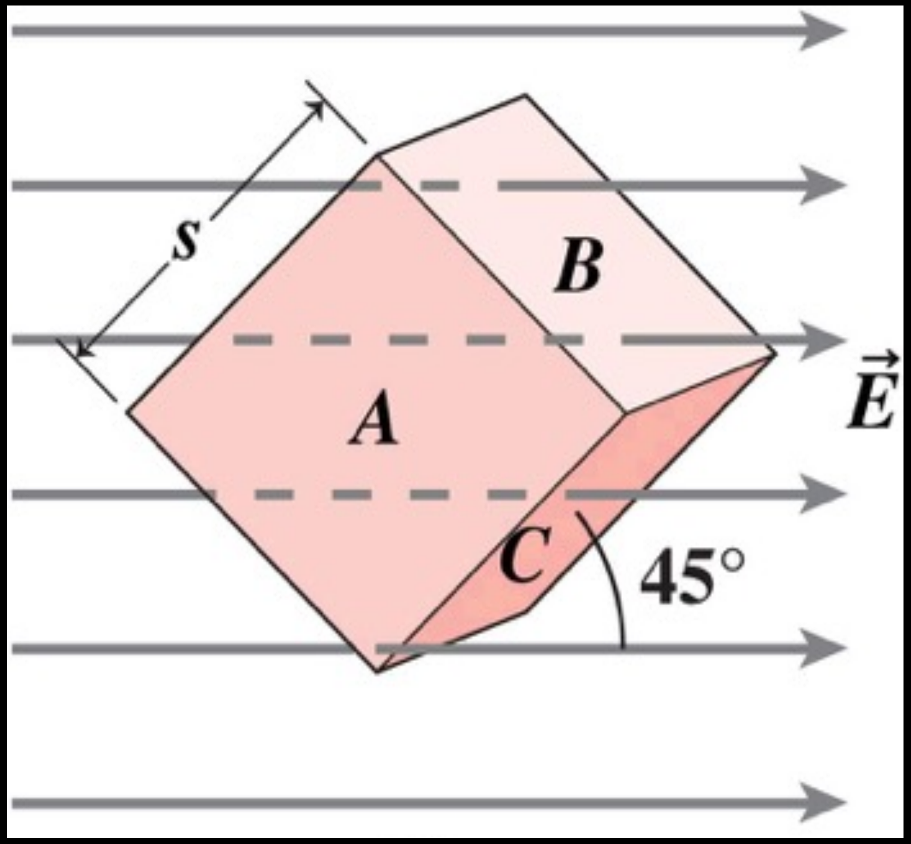
The flux through side B of the cube in the figure is the same as the flux through side C. What is a correct expression for the flux through each of these sides?

(A) $\Phi = s^3 E$

(B) $\Phi = s^2 E$

(C) $\Phi = s^3 E \cos 45°$

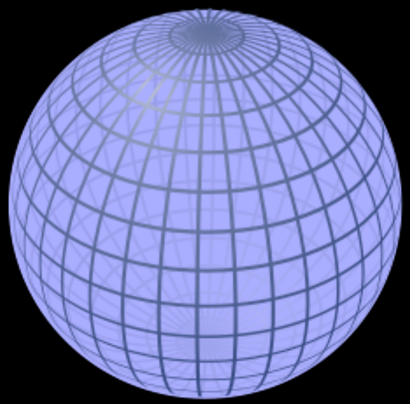
(D) $\Phi = s^2 E \cos 45°$



Gauss's Law

Gauss's Law

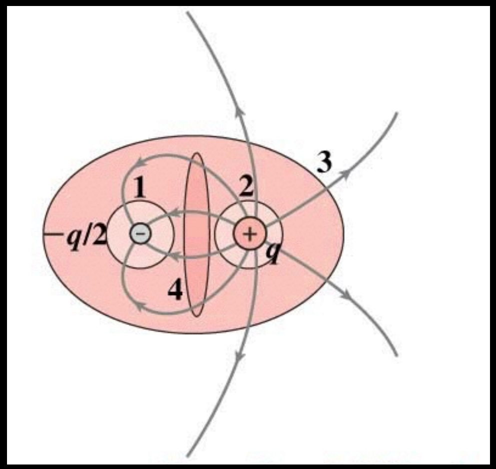
We have seen:



The flux through a surface is: $\Phi = \int_{\text{surface}} \bar{E} \cdot d\bar{A}$

If the surface is closed: $\Phi = \oint \bar{E} \cdot d\bar{A}$

But, for a closed surface, we have seen:



The flux emerging is proportional to the net charge enclosed.

$$\Phi \propto q_{\text{enclosed}}$$

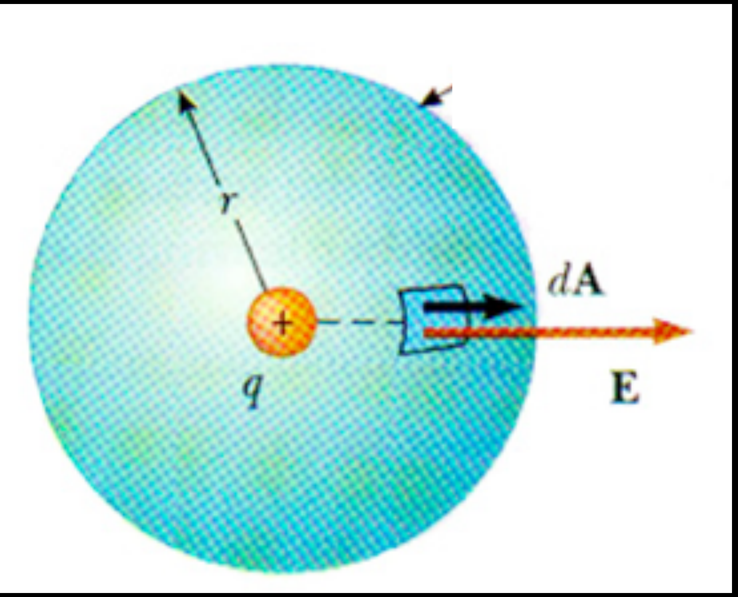
Therefore: $\Phi = \oint \bar{E} \cdot d\bar{A} \propto q_{\text{enclosed}}$

Gauss's Law

$$\Phi = \oint \bar{E} \cdot d\bar{A} \propto q_{\text{enclosed}}$$

What is the actual value?

Consider simple system: Point charge in a sphere.



$$\Phi = \oint \bar{E} \cdot d\bar{A} = \oint E dA \cos\theta$$

$d\bar{A}$ and $\bar{E}$ are parallel on the sphere: $\cos\theta = 1$

$E \propto 1/r^2$: constant on sphere

$$\Phi = \oint_{\text{sphere}} E dA = E \oint_{\text{sphere}} dA = E(4\pi r^2) = \left(\frac{kq}{r^2}\right)(4\pi r^2)$$

area of the sphere

Coulomb's law

$$= 4\pi k q$$

Gauss's Law

Therefore: $$\Phi = \oint \bar{E} \cdot d\bar{A} = 4\pi k q$$

our constant!

$$\epsilon_0 = \frac{1}{4\pi k}$$

k = Coulomb constant

$$\epsilon_0 = 8.85 \times 10^{-12} \mathrm{C}^2/\mathrm{Nm}^2$$

So: $$\boxed{\oint \bar{E} \cdot d\bar{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}}$$

Gauss's law

Valid for all shapes of closed surfaces and all charge distributions!

One of the 4 fundamental laws of electromagnetism.

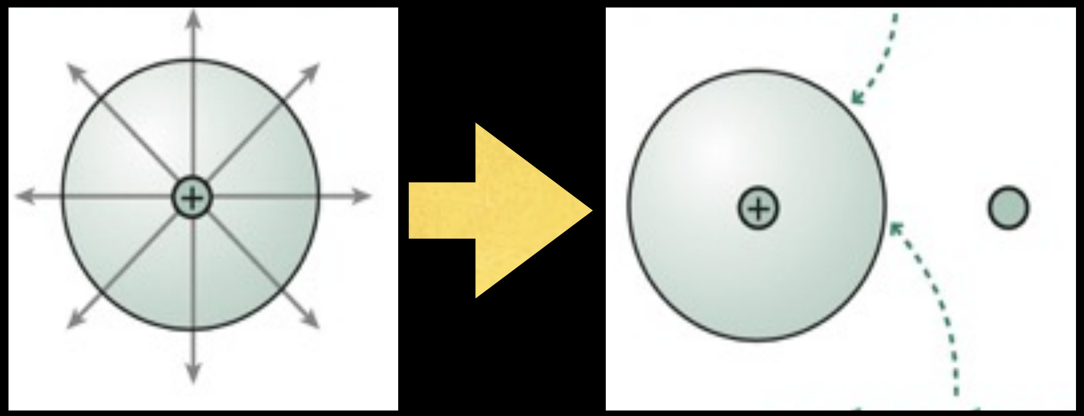
Gauss's Law

Quiz

A spherical surface surrounds an isolated positive charge. We can calculate the electric flux for this surface. If a second charge is placed outside the spherical surface, what happens to the magnitude of the flux?

(A) The flux increases proportionally to the magnitude of the second charge.

(B) The flux decreases proportionally to the magnitude of the second charge.



(C) The flux does not change.

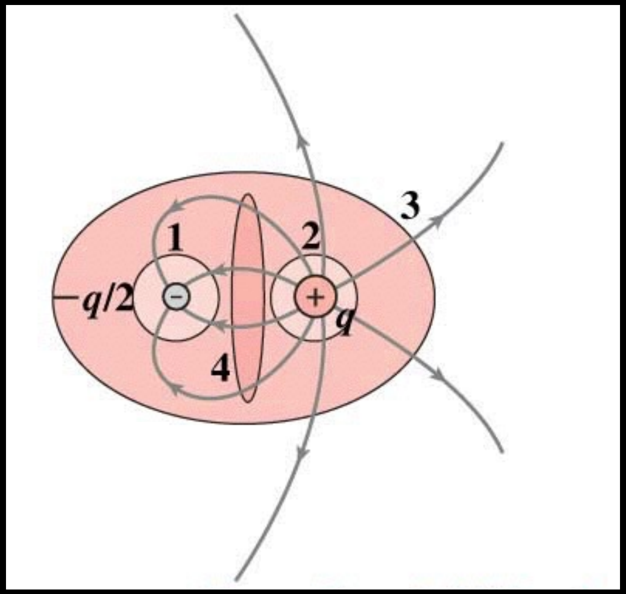
(D) The answer depends on whether the second charge is positive or negative.

Gauss's Law

Gauss's law may look complicated but:

$$\oint \bar{E} \cdot d\bar{A} = \frac{q_{\mathrm{enclosed}}}{\epsilon_0}$$

=



It just tells us the number of field lines emerging from a closed surface is proportional to the net charge

Gauss's law is always true....

.... but it's only useful for calculating the electric field when the system has a lot of symmetry.

Let's look at the main examples.

Gauss's Law

How to find the electric field with Gauss's law:

Step 1: Choose your surface

$\bar{E} \,//\, \bar{A}$ or $\bar{E} \perp \bar{A}$

$\rightrightarrows$ $\qquad$ $\rightarrow \uparrow$

$\bar{E} \cdot \bar{A} = |E||A|$ $\qquad$ $\bar{E} \cdot \bar{A} = 0$

Step 2: Find the enclosed charge

Step 3: Use Gauss's law

$$\oint \bar{E} \cdot d\bar{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

to find $\bar{E}$

Gauss's Law

Field of a uniformly charged sphere

Charge Q is distributed uniformly (equally) throughout a sphere of radius R.

Find $\bar{E}$ at all points (1) outside the sphere (2) inside the sphere.

Step 1: Flux integral

$$\Phi = \oint \bar{E} \cdot d\bar{A}$$

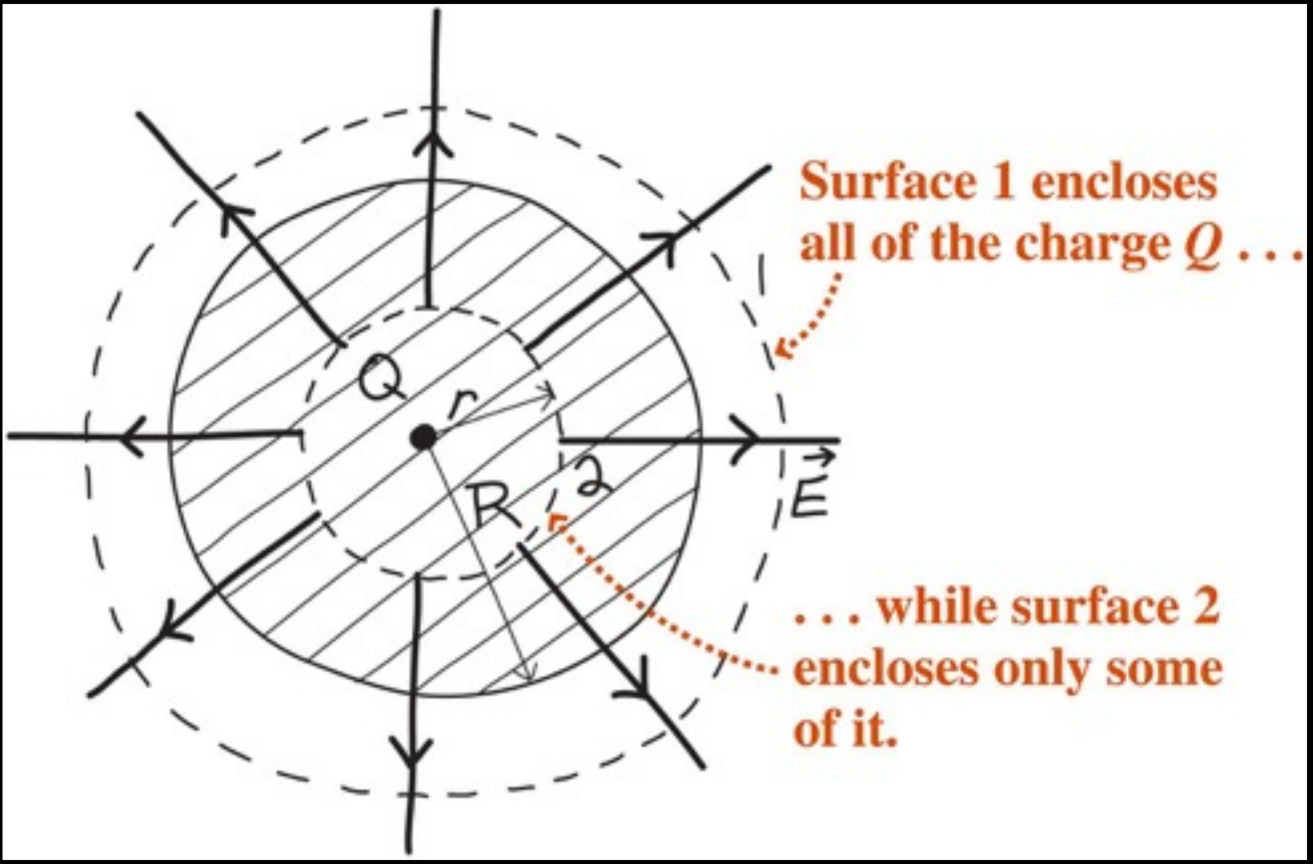
Spherical symmetry:

$d\bar{A}$ and $\bar{E}$ are parallel: $\cos\theta = 1$

$\bar{E}$ constant on sphere (symmetry)

$$\Phi = E \oint dA = 4\pi r^2 E$$

True for both surfaces.



Evaluate Gauss's law on spherical surfaces

Gauss's Law

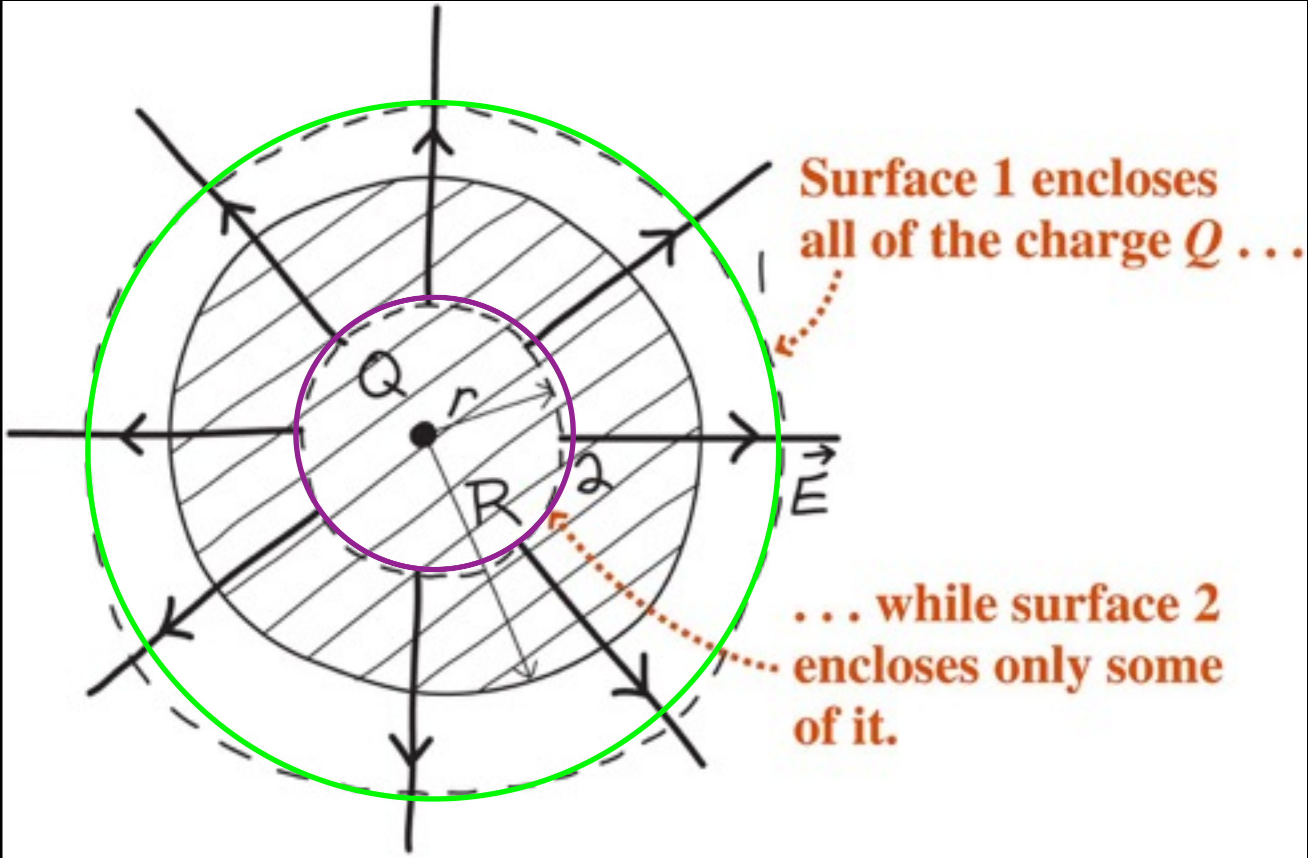
Field of a uniformly charged sphere

Step 2: Enclosed charge. Different for each surface

Surface 1: outside charged sphere

Encloses whole sphere, Q

$$q_{\text{enclosed}} = Q \qquad r > R$$



Surface 2: inside charged sphere

Encloses volume: $\frac{4}{3}\pi r^3$

Whole volume: $\frac{4}{3}\pi R^3$

Fraction of volume enclosed: $\frac{r^3}{R^3}$

charge distributed evenly so:

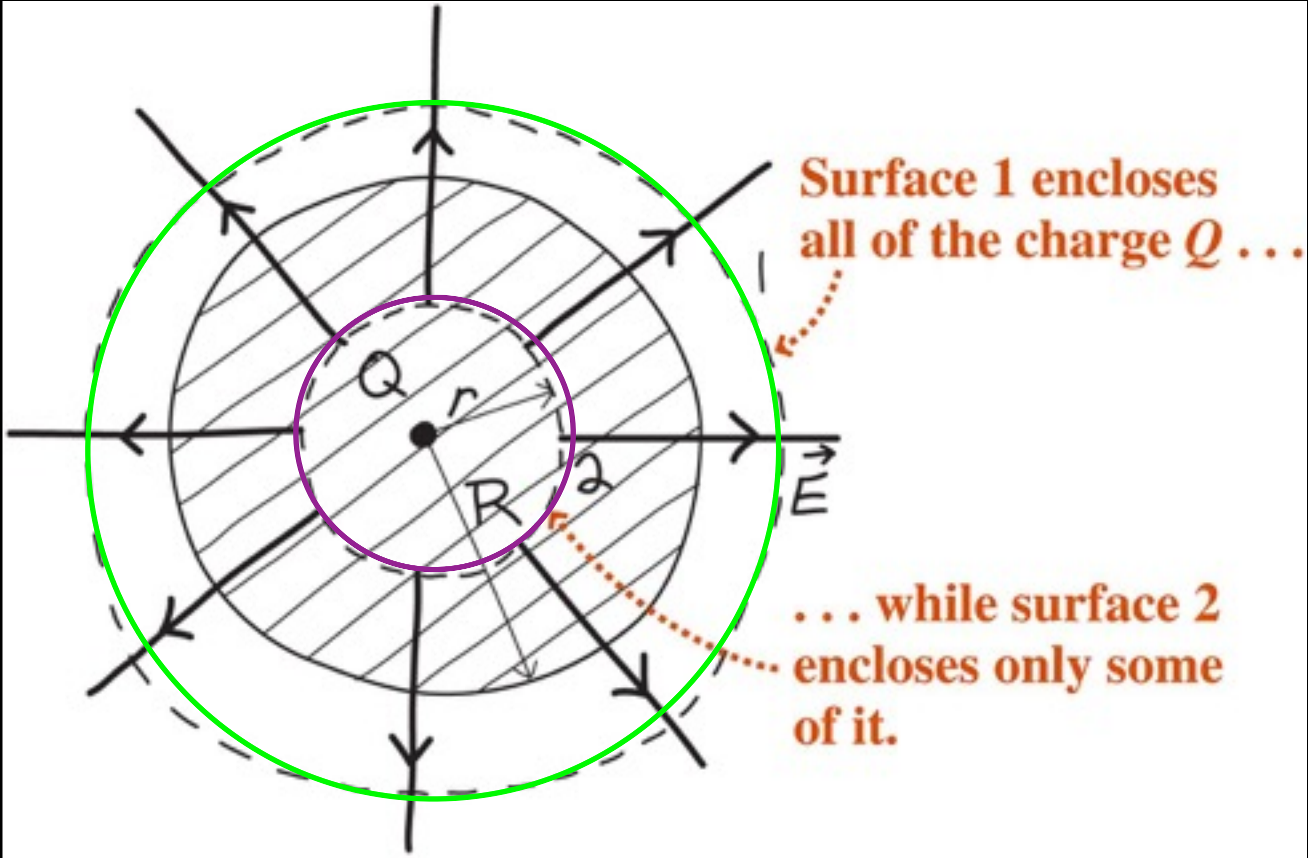
$$q_{\text{enclosed}} = Q\frac{r^3}{R^3}$$

Gauss’s Law

Field of a uniformly charged sphere

Step 3: Apply Gauss’s Law

$$\Phi = 4\pi r^2 E = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



Surface 1: outside charged sphere

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Surface 2: inside charged sphere

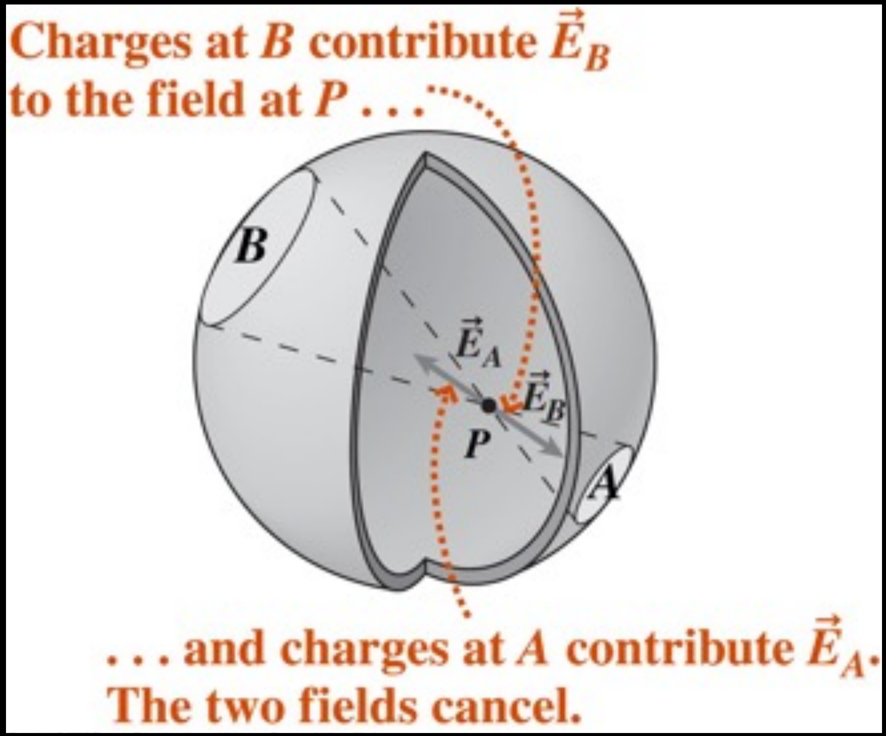
$$E = \frac{Qr}{4\pi\epsilon_0 R^3}$$

Gauss's Law

Field of a hollow spherical shell

Charge Q is distributed uniformly (equally) over shell of radius R.

Find $\bar{E}$ at all points (1) outside the shell (2) inside the shell.



Step 1: Flux integral

$$\Phi = E \oint dA = 4\pi r^2 E$$

Since true for any surface that encloses a spherical charge distribution.

Step 2: Enclosed charge.

Outside the shell, the enclosed charge is Q: $q_{\text{enclosed}} = Q \qquad r > R$

Inside the shell, there is no net charge: $q_{\text{enclosed}} = 0$

Step 3: Apply Gauss's Law

Outside the shell: $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$ Inside the shell: $E = 0$

Gauss's Law

A spherical shell carries charge Q uniformly distributed over its surface. If the charge on the shell doubles, what happens to the electric field strength inside the shell?

(A) The electric field strength is zero.

(B) The electric field strength quadruples (x 4)

(C) The electric field strength is halved (x 0.5)

(D) The electric field strength doubles (x 2)

Gauss's Law

Line symmetry: Field depends only on distance, r, from line.

Find $\bar{E}$ from infinite line charge carrying charge density, λ [C/m]

Step 1: Flux integral

Cylinder: curved section

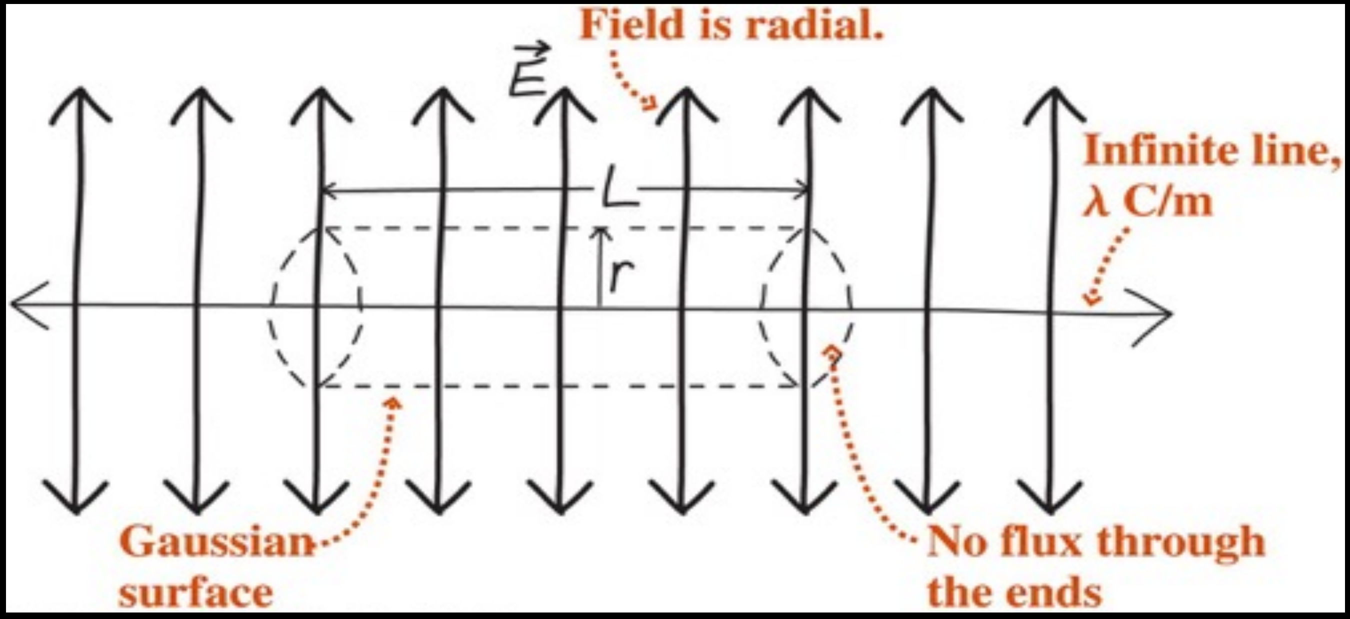
$d\bar{A}$ and $\bar{E}$ are parallel: $\cos\theta = 1$

Cylinder: ends

$d\bar{A}$ and $\bar{E}$ are perpendicular:
$\cos\theta = 0 \rightarrow \Phi = 0$

$\bar{E}$ constant on cylinder (symmetry)

Evaluate Gauss's law on cylindrical surface



$$\Phi = \oint \bar{E} \cdot d\bar{A} = E \int_{\text{curved}} dA \quad = E(2\pi r L)$$

Gauss's Law

<u>Line symmetry:</u> Field depends only on distance, r, from line.

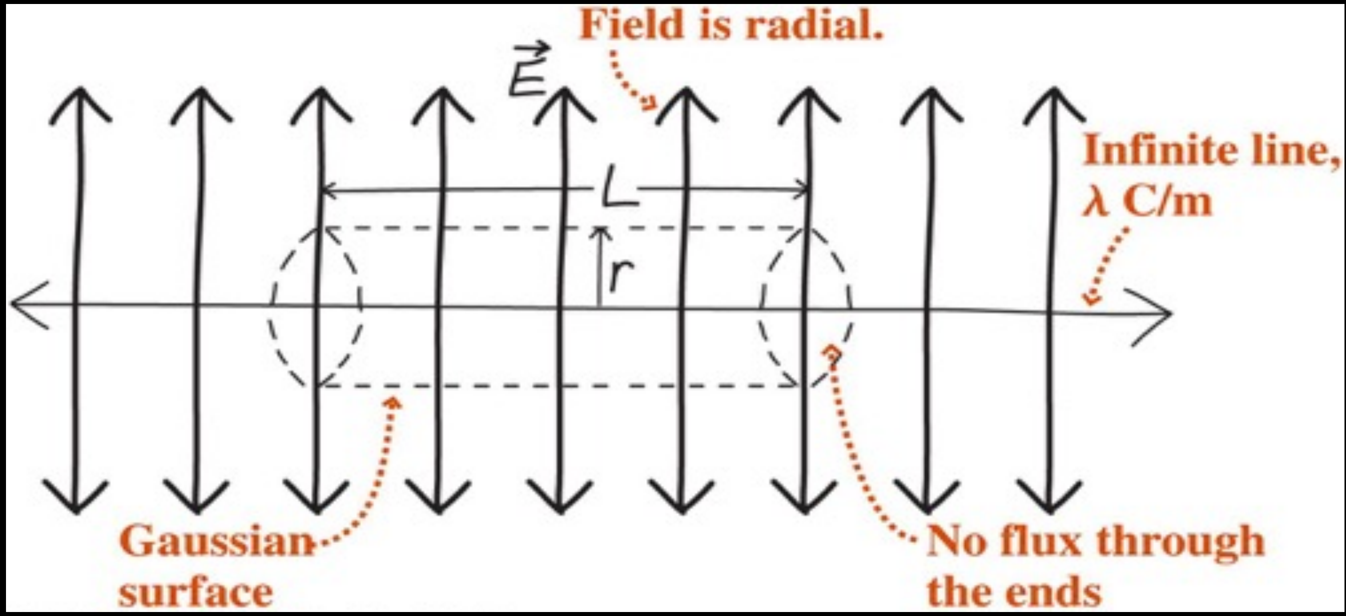
Find $\bar{E}$ from infinite line charge carrying charge density, λ [C/m]

Step 2: Enclosed charge.

charge density is λ [C/m]

$$q_{\text{enclosed}} = \lambda L$$

Evaluate Gauss's law on cylindrical surface



Step 3: Apply Gauss's Law

$$\Phi = 2\pi r L E = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Gauss's Law

Plane symmetry: Field depends only on perpendicular distance, r, from plane.

Find $\bar{E}$ from infinite sheet carrying charge surface density, σ $[\mathrm{C/m^2}]$

Step 1: Flux integral

Cylinder: curved section

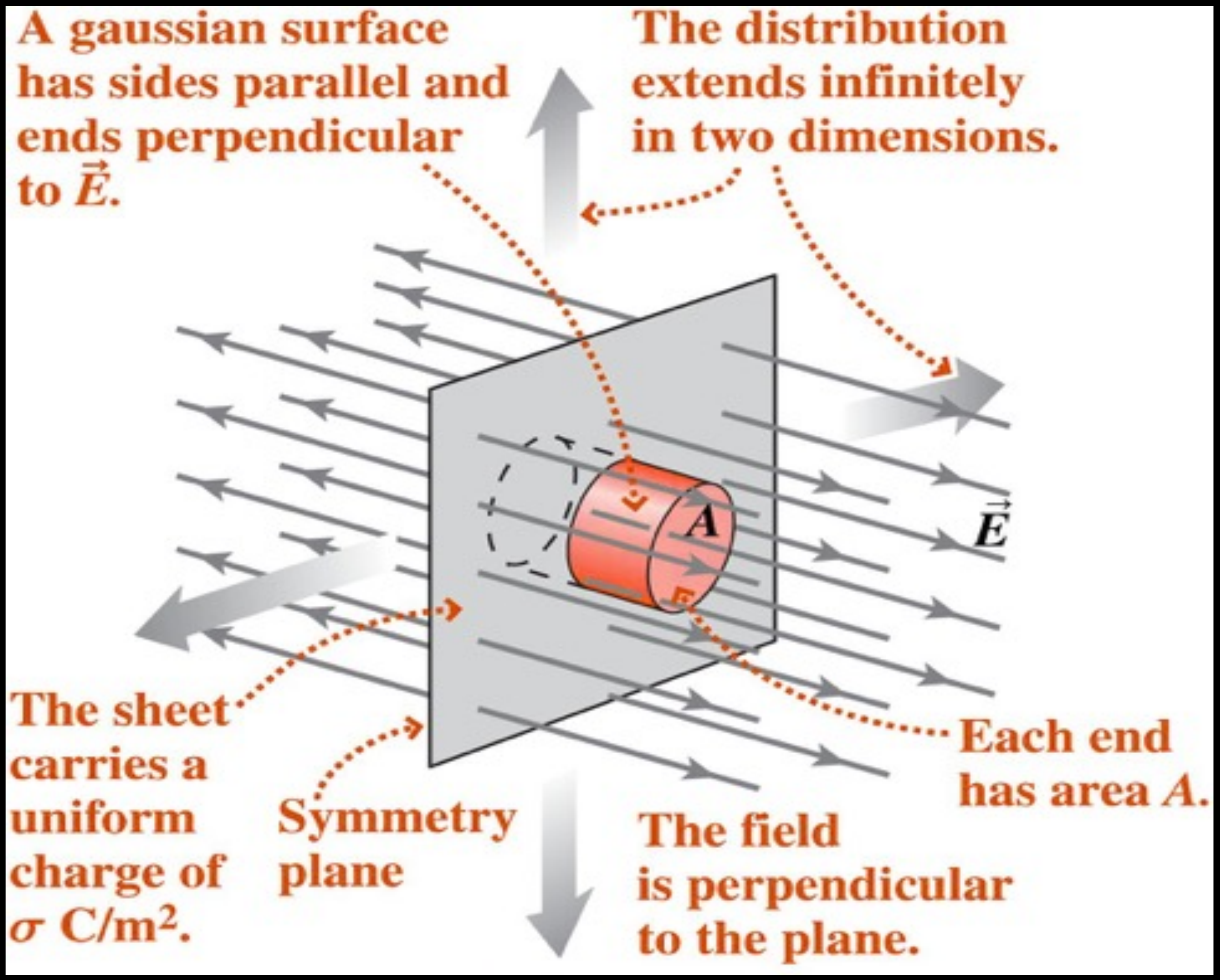
$d\bar{A}$ and $\bar{E}$ are perpendicular:

$\cos\theta = 0 \rightarrow \Phi = 0$

Cylinder: ends

$d\bar{A}$ and $\bar{E}$ are parallel: $\cos\theta = 1$

$\bar{E}$ constant at ends (symmetry)



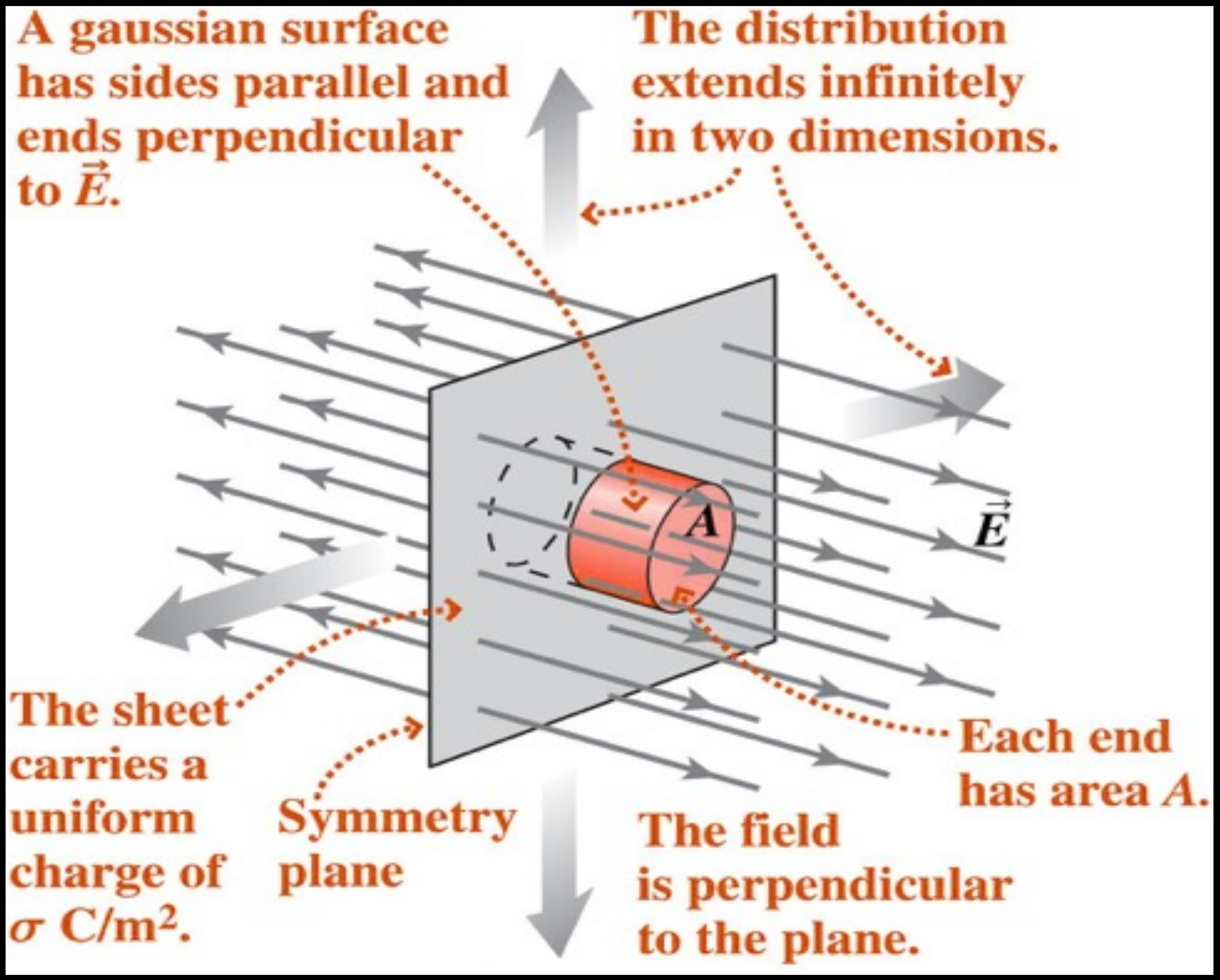
Evaluate Gauss's law on cylindrical surface through sheet.

$$\Phi = \oint \bar{E}\cdot d\bar{A} = E\int_{\text{ends}} dA = 2EA$$

Gauss's Law

Plane symmetry: Field depends only on perpendicular distance, r, from plane.

Find $\bar{E}$ from infinite sheet carrying charge surface density, $\sigma \; [\mathrm{C/m^2}]$



Step 2: Enclosed charge.

charge surface density is $\sigma \; [\mathrm{C/m^2}]$

$$q_{\mathrm{enclosed}} = \sigma A$$

Step 3: Apply Gauss' Law

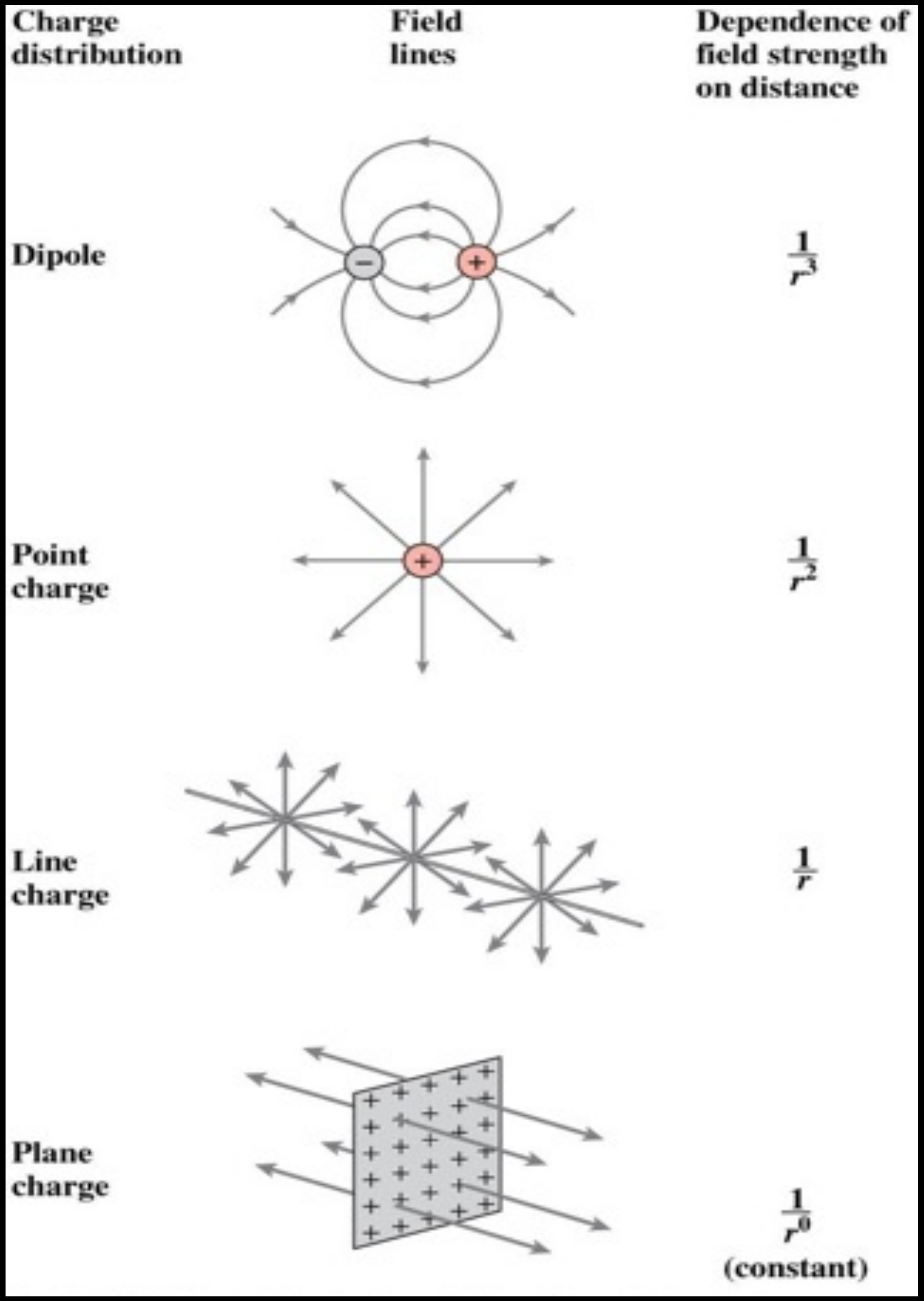
$$\Phi = 2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

Gauss's Law

Gauss's law is always true, but most charge distributions lack the symmetry needed to find the field.

The alternative, Coulomb's law, is hard to use except in the simplest cases.



But in many cases, we can approximate the system with one of our known distributions.

e.g.

far from a finite-size distribution,

the field ~ point source

Near a flat, uniformly charges region,

the field ~ plane charge

Gauss's Law & Conductors

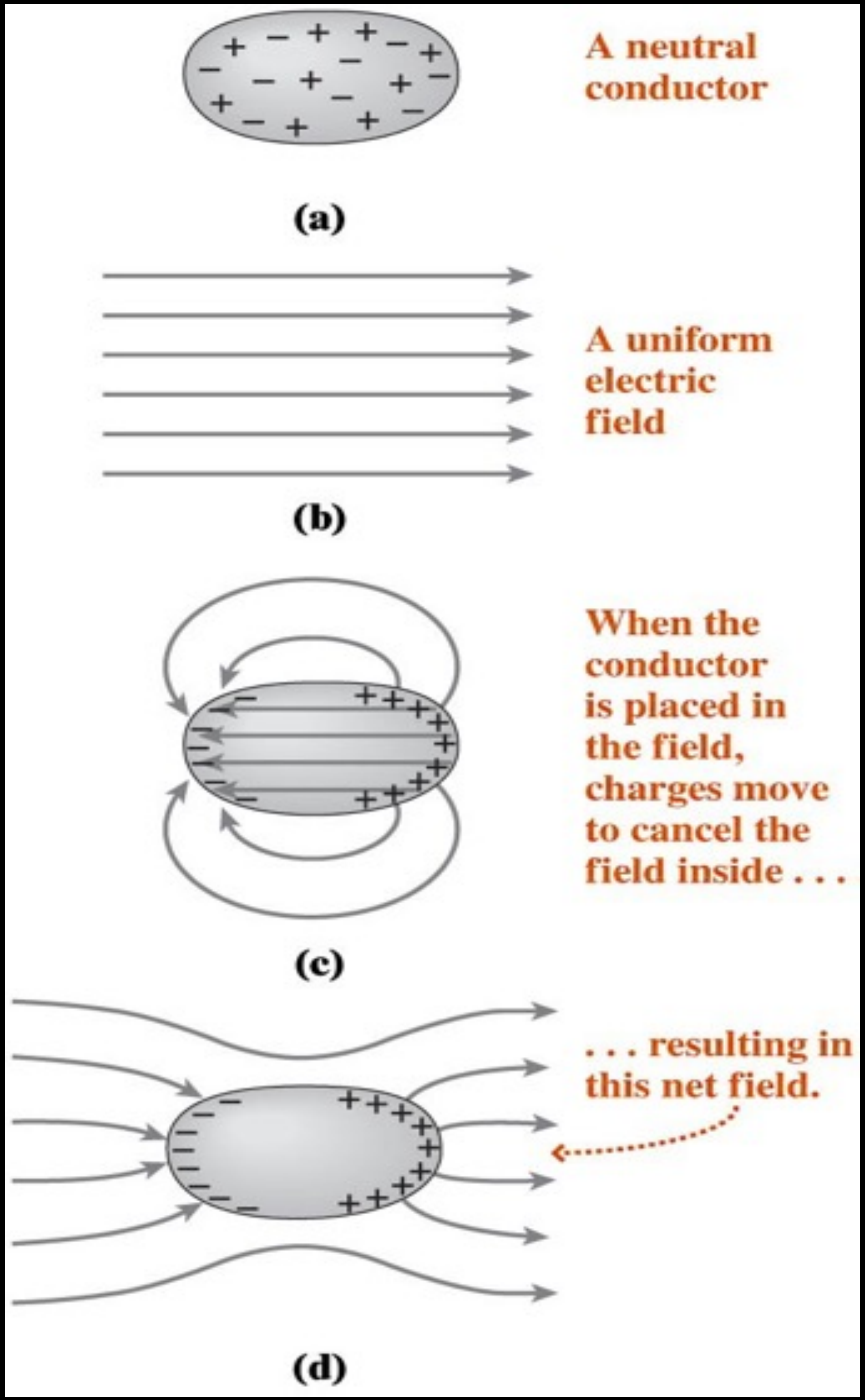
Gauss's Law & Conductors

Charges in conductors can move in response to an electric field.

If the conductor is in electrostatic equilibrium, the charges have moved to cancel the field inside the conductor.

Therefore inside a conductor in electrostatic equilibrium:

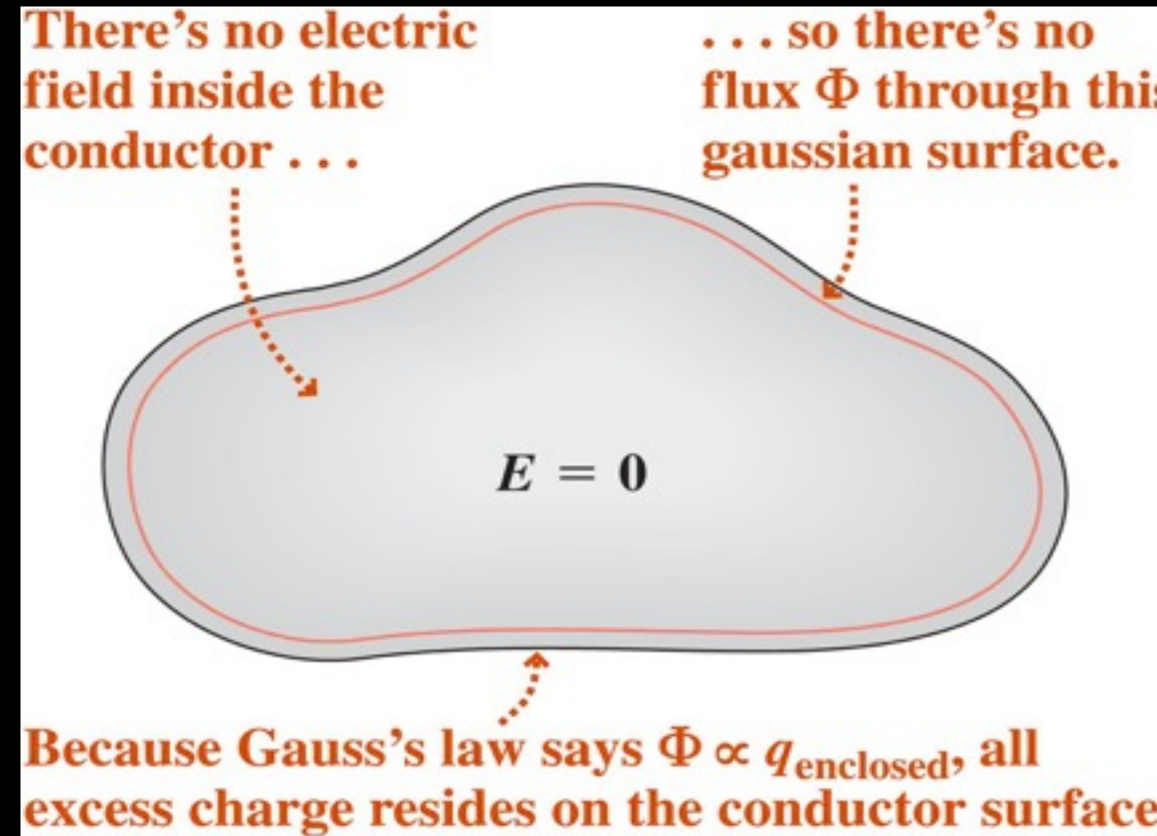
$$\bar{E} = 0$$



Gauss's Law & Conductors

If you add charge to a conductor in equilibrium....

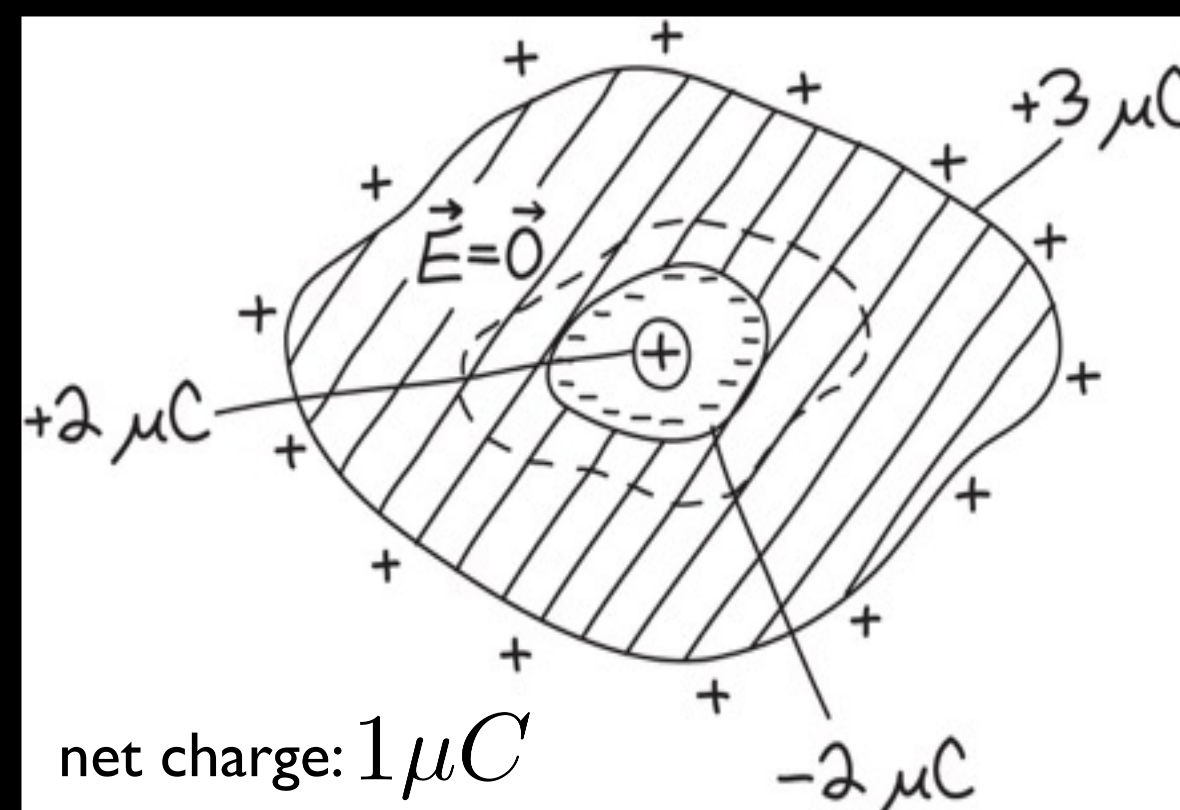
Gauss's law requires that the free charge on a conductor sits on the surface.



If a charge sits in a hollow inside a conductor....

charge in the conductor will move to the inside surface to give a net charge of 0 inside the conductor.

The remains of the charge will sit on the surface.



Gauss's Law & Conductors

What is the field from the charge on the conductor's surface?

Conductor surface ---> Plane surface

Step 1: Flux integral

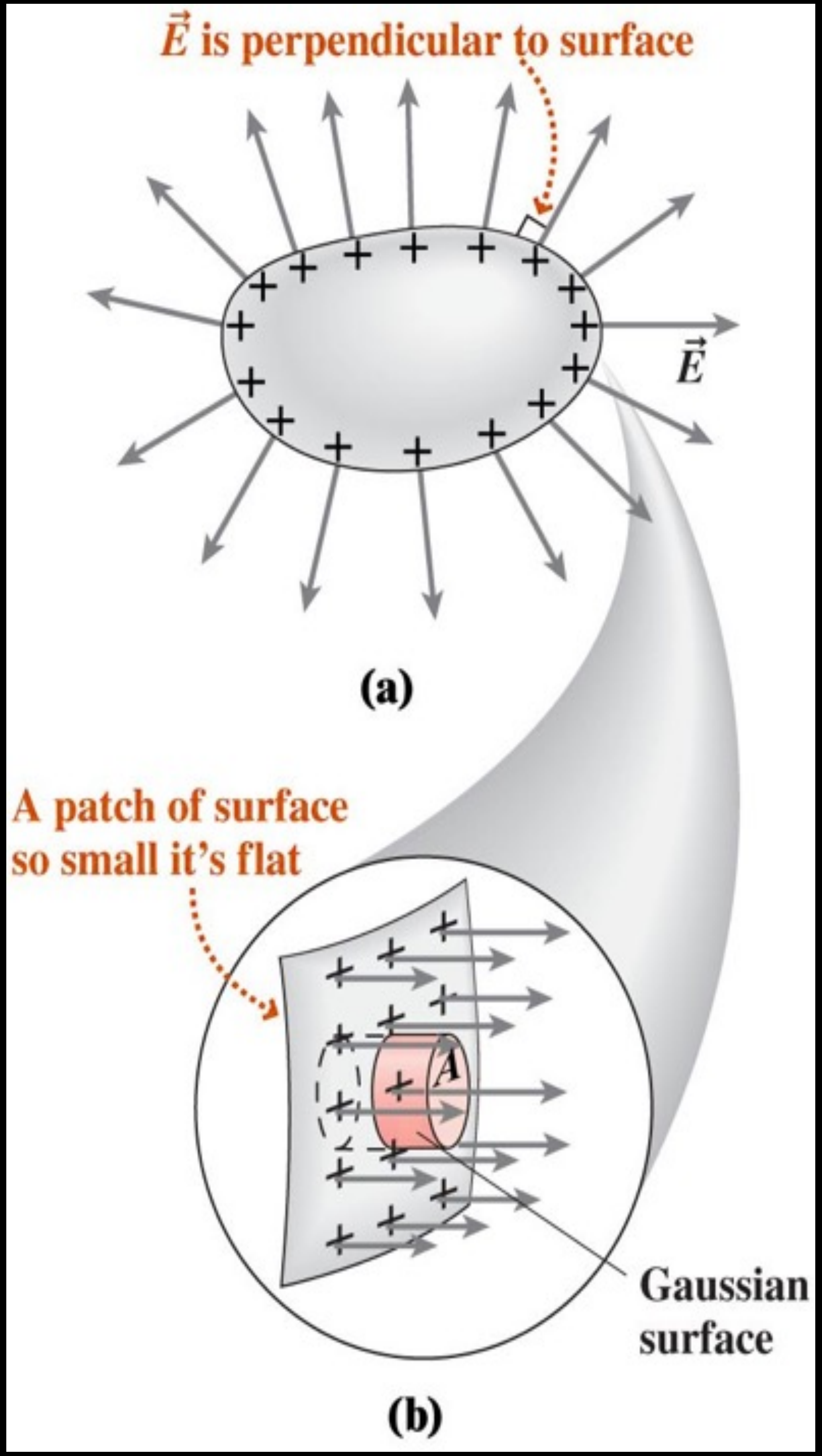
$$\Phi = \oint \bar{E} \cdot d\bar{A} = E \int_{\text{ends}} dA$$

But! Cylinder end inside conductor has no flux.

$$\Phi = EA$$

Step 2: Enclosed charge. $q_{\text{enclosed}} = \sigma A$

Step 3: Apply Gauss' Law $\boxed{E = \dfrac{\sigma}{\epsilon_0}}$

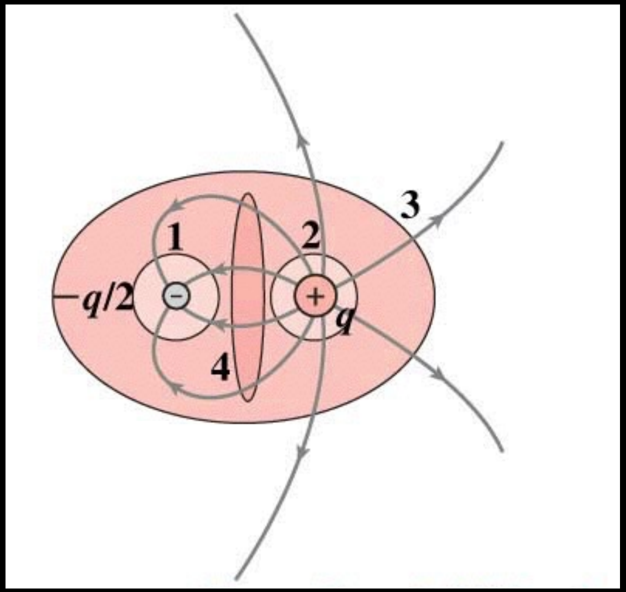


Key point

Gauss's Law: One of the 4 fundamental laws of electromagnetism

$$\oint \bar{E} \cdot d\bar{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

=



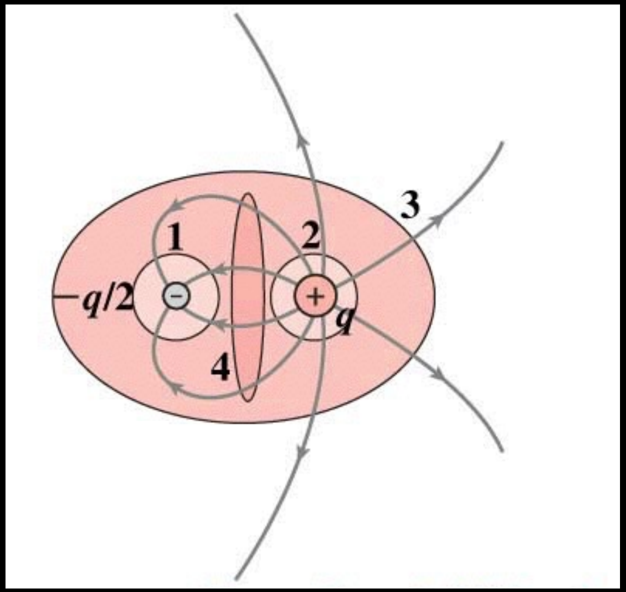
The number of field lines emerging from a closed surface is proportional to the net charge

Key point

Gauss's Law: One of the 4 fundamental laws of electromagnetism

$$\oint \bar{E} \cdot d\bar{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

=



The number of field lines emerging from a closed surface is proportional to the net charge

(The End)