

Essential Physics II

英語で物理学の
エッセンス II

Lecture 5: 26-10-15

Homework: 18.28

What should be the approximate specific-heat ratio of:
50% NO_2 ($\gamma = 1.29$),
30% O_2 ($\gamma = 1.4$)
20% Ar ($\gamma = 1.67$)?

Total Energy: $U = U_{\text{NO}_2} + U_{\text{O}_2} + U_{\text{Ar}}$

$$\frac{1}{n} \frac{dU}{dT} = C_v = \frac{1}{n} \frac{dU_{\text{NO}_2}}{dT} + \frac{1}{n} \frac{dU_{\text{O}_2}}{dT} + \frac{1}{n} \frac{dU_{\text{Ar}}}{dT}$$

but: $n_{\text{NO}_2} = 0.5 \times n$

$$n_{\text{O}_2} = 0.3 \times n$$

$$n_{\text{Ar}} = 0.2 \times n$$

Homework: 18.28

$$\frac{1}{n} \frac{dU}{dT} = C_v = 0.5 \frac{1}{n_{\text{NO}_2}} \frac{dU_{\text{NO}_2}}{dT} + 0.3 \frac{1}{n_{\text{O}_2}} \frac{dU_{\text{O}_2}}{dT} + 0.2 \frac{1}{n_{\text{Ar}}} \frac{dU_{\text{Ar}}}{dT}$$

C_{v,NO_2} C_{v,O_2} $C_{v,\text{Ar}}$

$$\frac{1}{n} \frac{dU}{dT} = 0.5 C_{v,\text{NO}_2} + 0.3 C_{v,\text{O}_2} + 0.2 C_{v,\text{Ar}}$$

Since: $\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v}$

$$C_v = \frac{R}{\gamma - 1}$$

Therefore: $\frac{R}{\gamma - 1} = 0.5 \frac{R}{\gamma_{\text{NO}_2} - 1} + 0.3 \frac{R}{\gamma_{\text{O}_2} - 1} + 0.2 \frac{R}{\gamma_{\text{Ar}} - 1}$

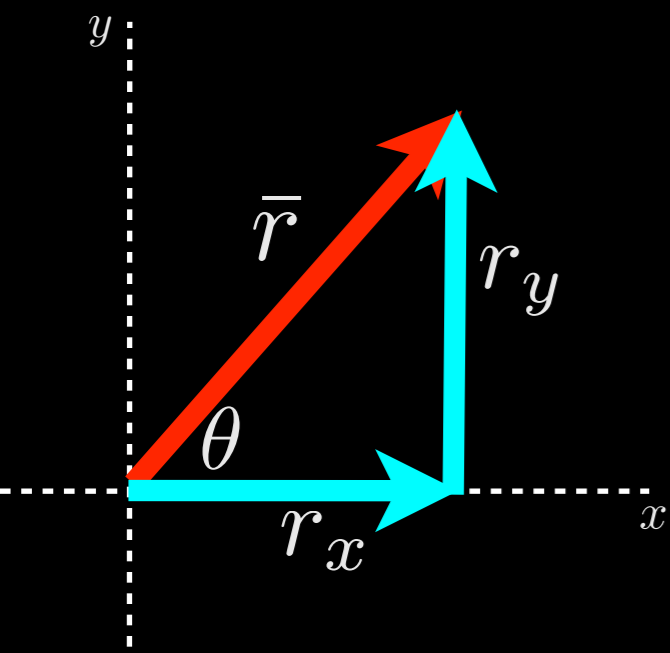
$$\gamma = 1.36$$

Electricity & Magnetism



Reminder: vectors

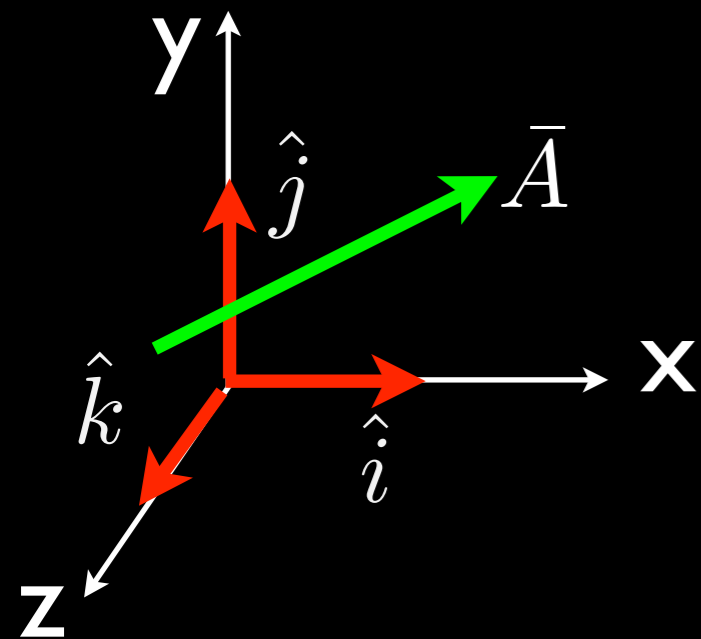
Vector components:



$$r_x = |\vec{r}| \cos \theta$$

$$r_y = |\vec{r}| \sin \theta$$

$$\vec{r} = r_x \hat{i} + r_y \hat{j} \quad |\vec{r}| = \sqrt{r_x^2 + r_y^2}$$



$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\text{if : } A_x = 3 \text{ m} \quad A_y = -4 \text{ m} \quad A_z = 2 \text{ m}$$

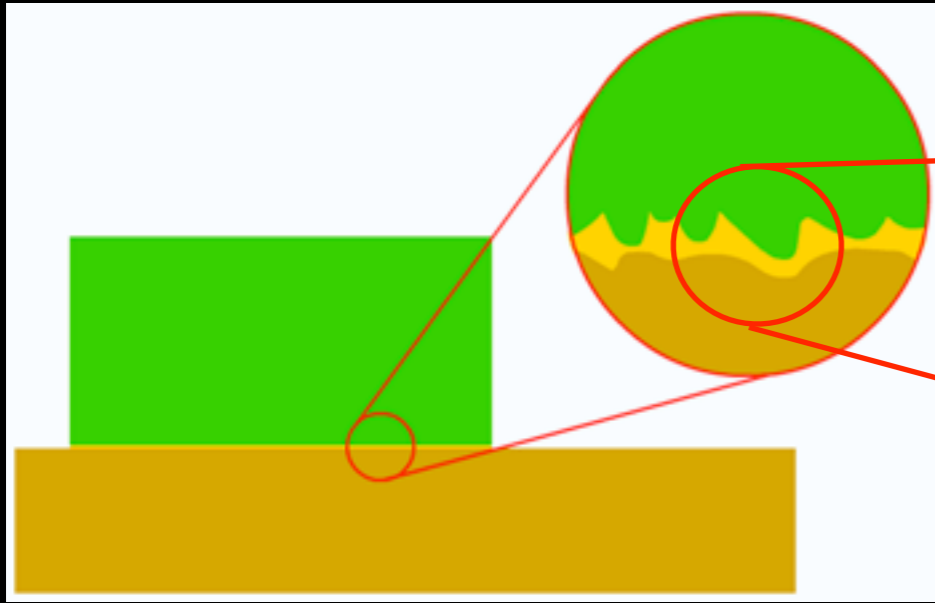
$$\vec{A} = (3\hat{i} - 4\hat{j} + 2\hat{k}) \text{ m}$$

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

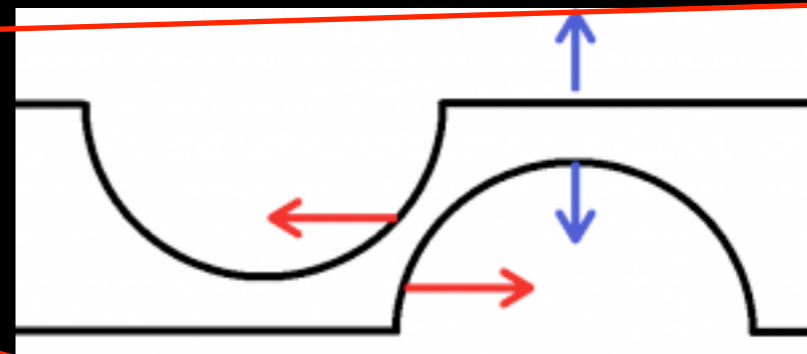
$$|\vec{A}| = \sqrt{3^2 + (-4)^2 + 2^2} = 5.4 \text{ m}$$

The electric force

Friction:



Close in, the smooth surface is irregular



At the atomic level, like charges are being forced together which repel.

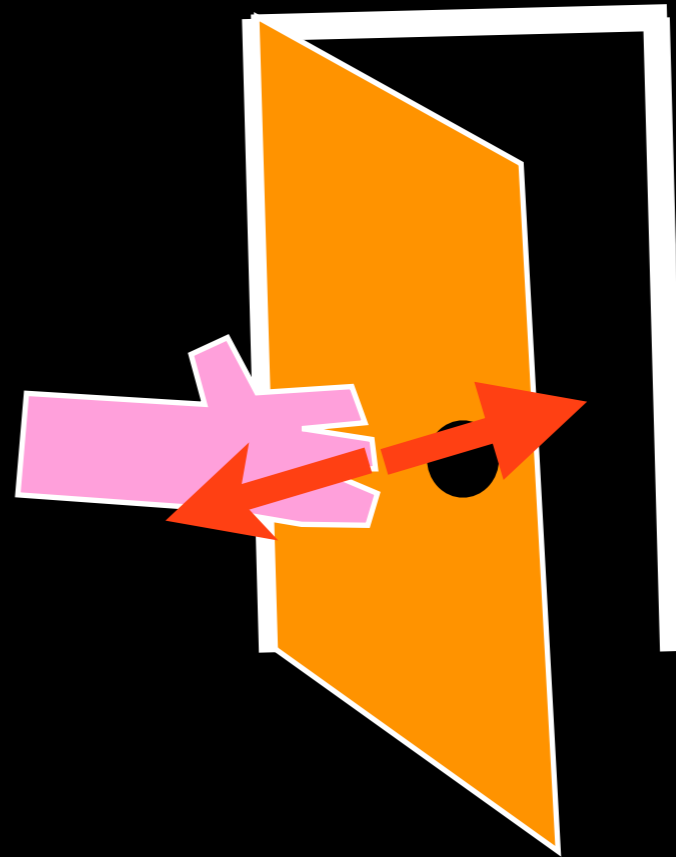


Friction is really an electric force

The electric force

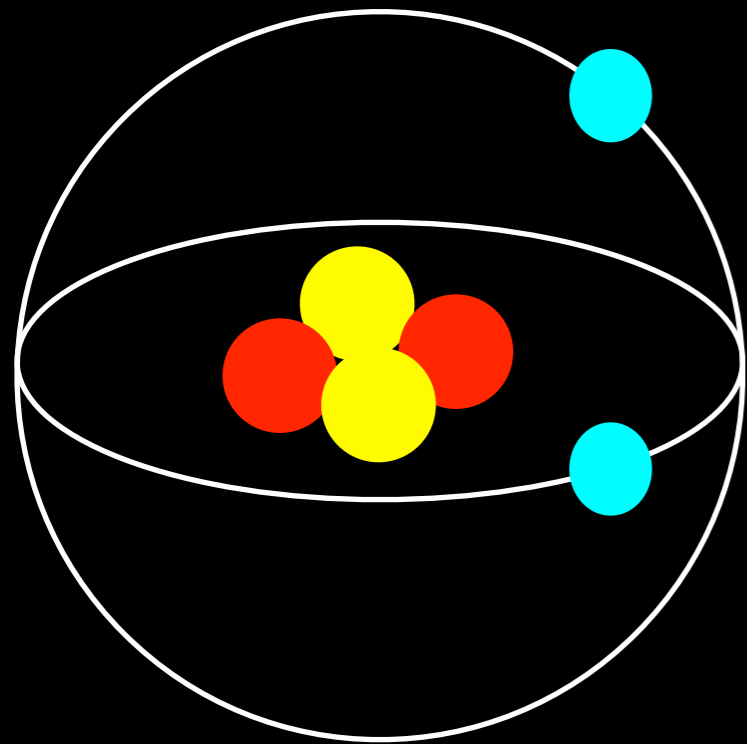
In fact, all forces we have met (except gravity) are electric forces.

Touch a door and the atoms in your hand and those in the door repel to produce a force



Electric charge

Electric charge is a fundamental property of matter (like mass)



Atom

Normal matter is made from:

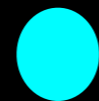


neutrons with no electric charge



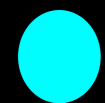
protons with a positive charge

+



electrons with a negative charge

-



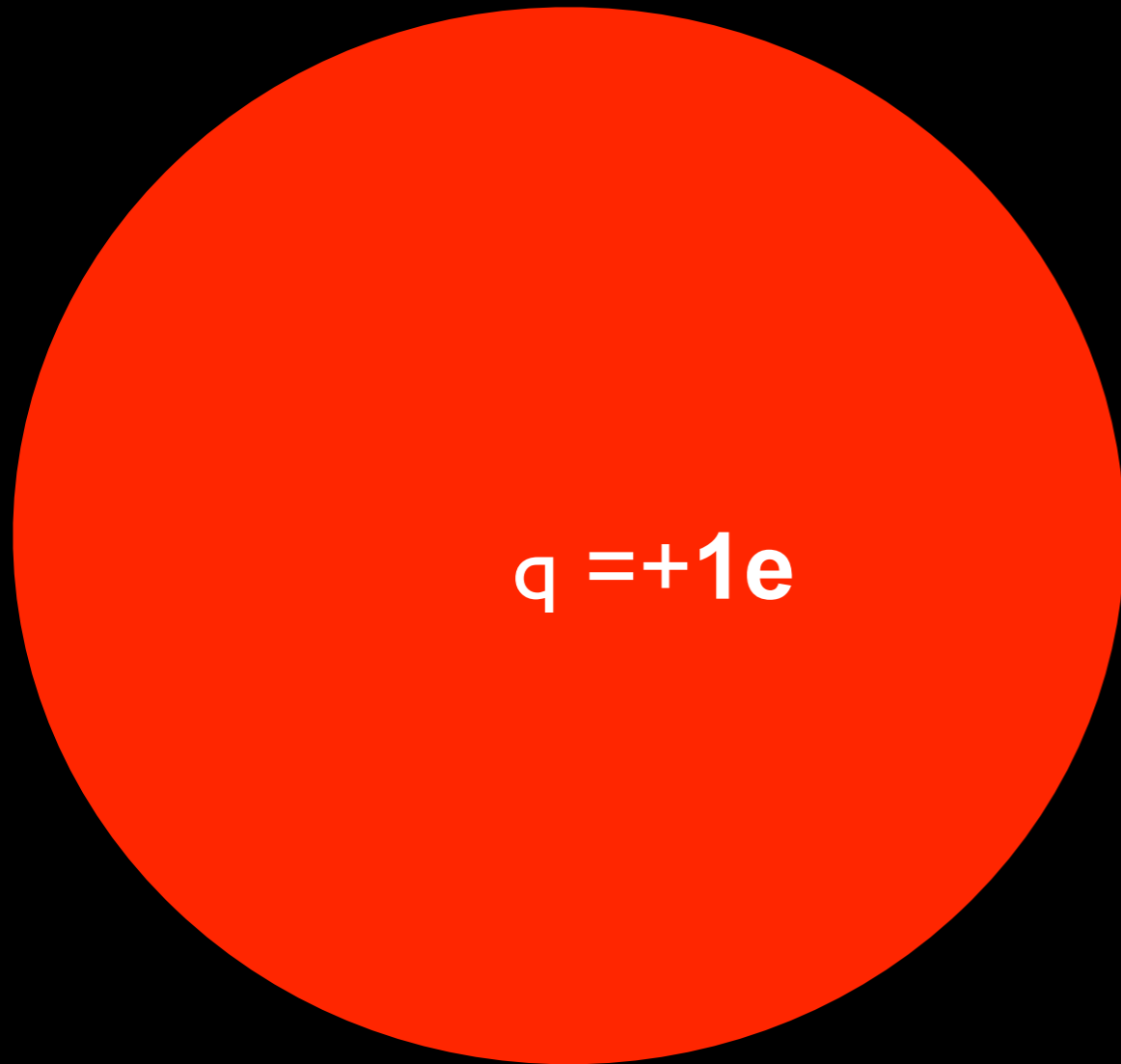
all electrons have exactly the same electronic charge



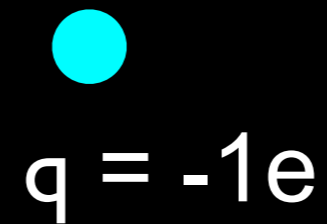
all protons have exactly the same electronic charge

Electric charge

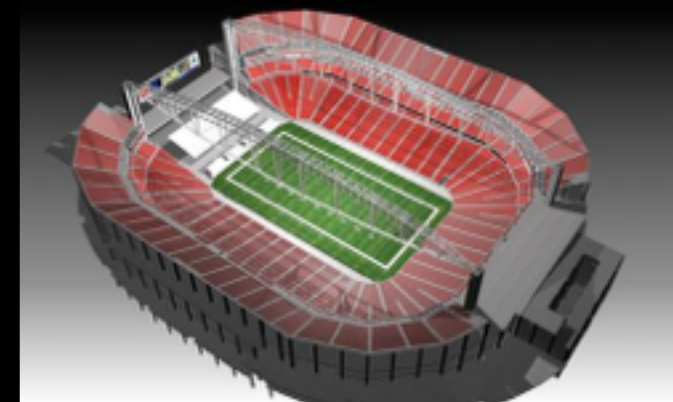
The electric charge of the proton is exactly equal and opposite of that of the electron.



... even though their mass ratio is about 1836 : 1



If a proton was the size of a football and in the centre of a football pitch, the electron would be the size of a pea in the stands.



Electric charge

Electric charge is quantized:

● - 1e $e = 1.6 \times 10^{-19} \text{C}$

●● - 2e

●●● - 3e

●●●● - 4e

SI unit: Coulomb: $1\text{C} = 6.25 \times 10^{18}$ charges

~~●●●●● - 2.5e~~

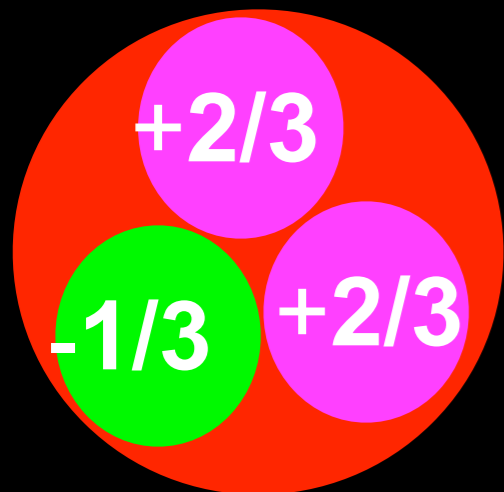
Huge! Normally we see μC to nC

found by experiment

Particle physics theory:

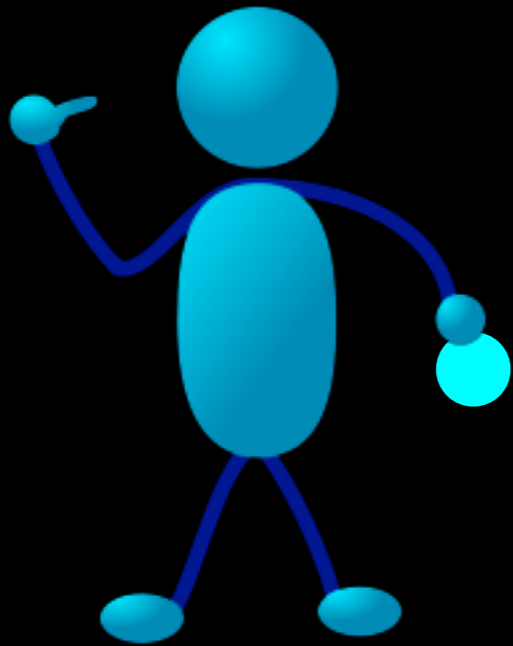
Fundamental charge is actually $\frac{1}{3}e$

But quarks are never found in isolation

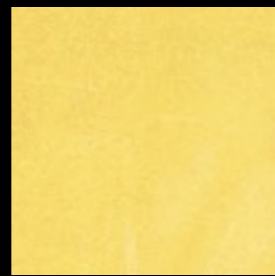


Electric charge

Electric charge is conserved:

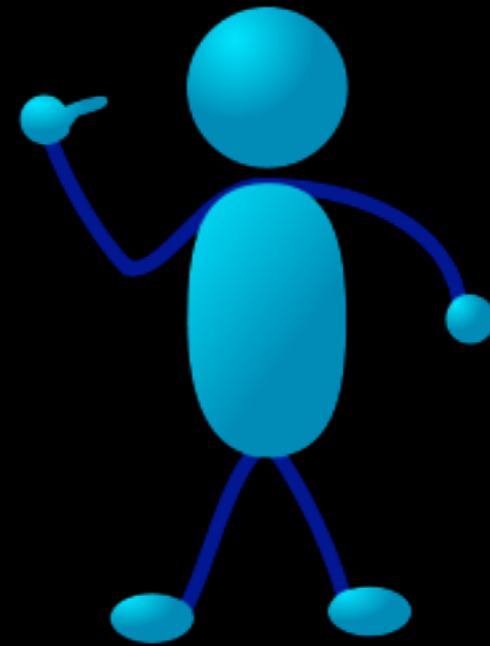


charge,
 $q = 0$

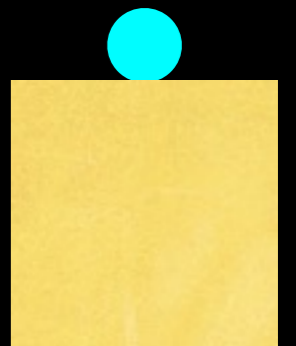


charge:
 $q = 0$

net charge: 0



charge:
 $q = +1$



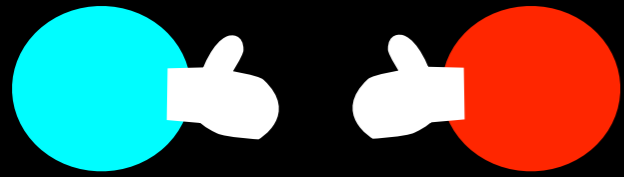
charge:
 $q = -1$

net charge: 0

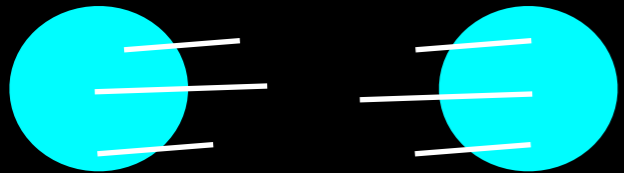
Electric charge summary

- Electric charge is a fundamental property of matter
- Charge comes in two types, positive & negative
- Protons carry a positive (+) charge, electrons an equal negative (-) charge
- Many particles (made from protons & electrons) carry a net electric charge
- Charge is conserved: net charge in a system is constant
(True even if particles are created or destroyed)
- SI unit of charge is the coulomb (C)
1 coulomb: $6.25 \times 10^{18} e$ $e = 1.6 \times 10^{-19} \text{C}$

The electric force



Unlike (different) charges attract.



Like (same) charges repel.

q_1

q_2

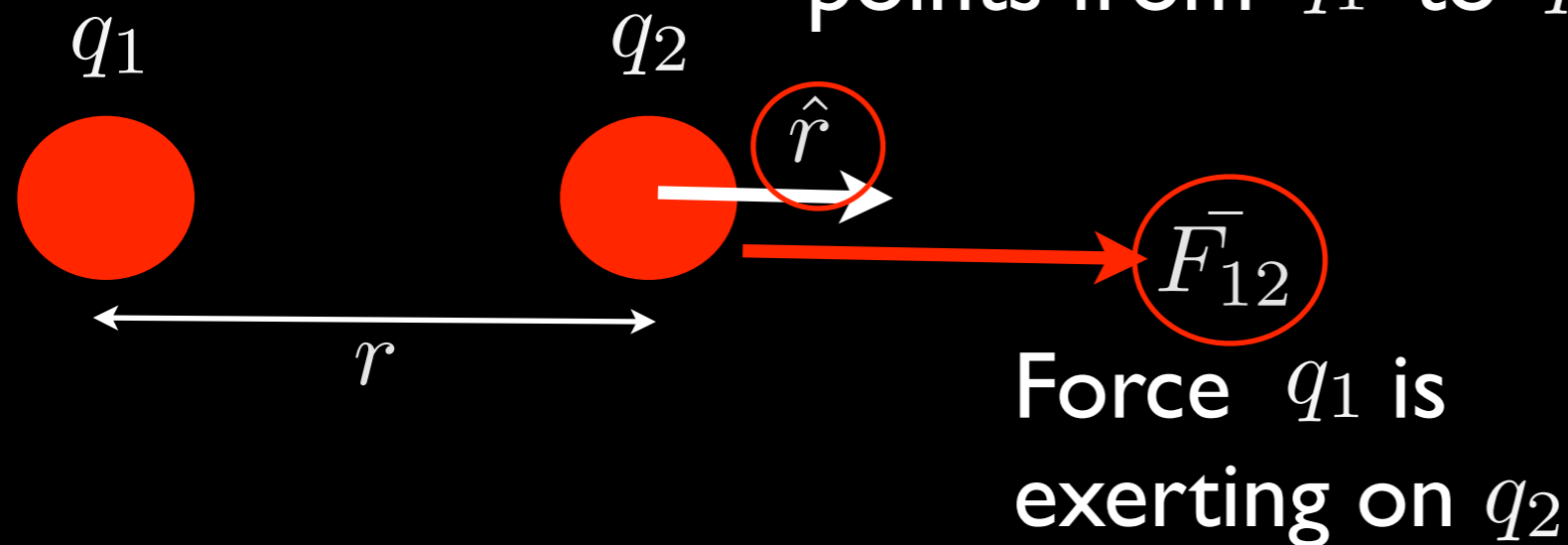
The strength of that force depends on the **magnitude of the charges** and the **distance** between charges.

The electric force

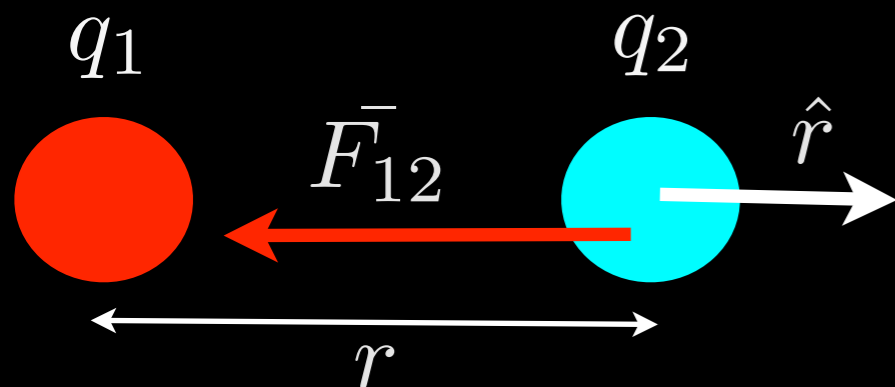
Coulomb's law

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2} \hat{r}_{12} \quad k = 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2$$

unit vector: \hat{r}_{12}
points from q_1 to q_2



Force is away from q_1
when charges are like



Force is towards q_1
when charges are unlike

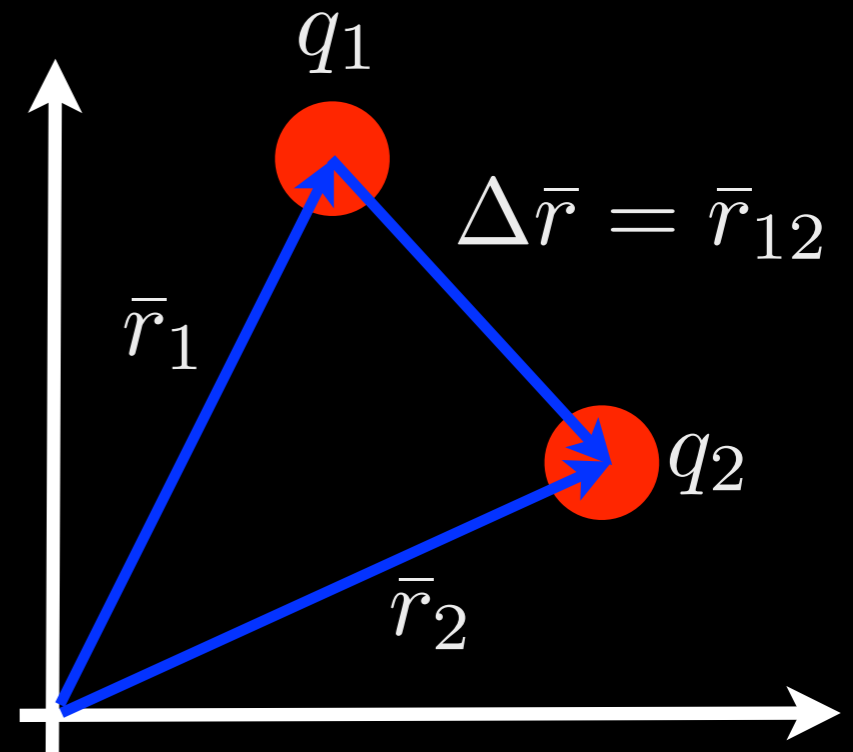
The electric force

The unit vector, \hat{r}_{12}

Unit vectors have a magnitude of 1

\hat{r}_{12} lies on a line through the two charges **from** q_1 **to** q_2

$$\hat{r}_{12} = \frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|} = \frac{\bar{r}_{12}}{|\bar{r}_{12}|}$$



The direction of \hat{r}_{12} is important because force, \bar{F}_{12} is a vector.

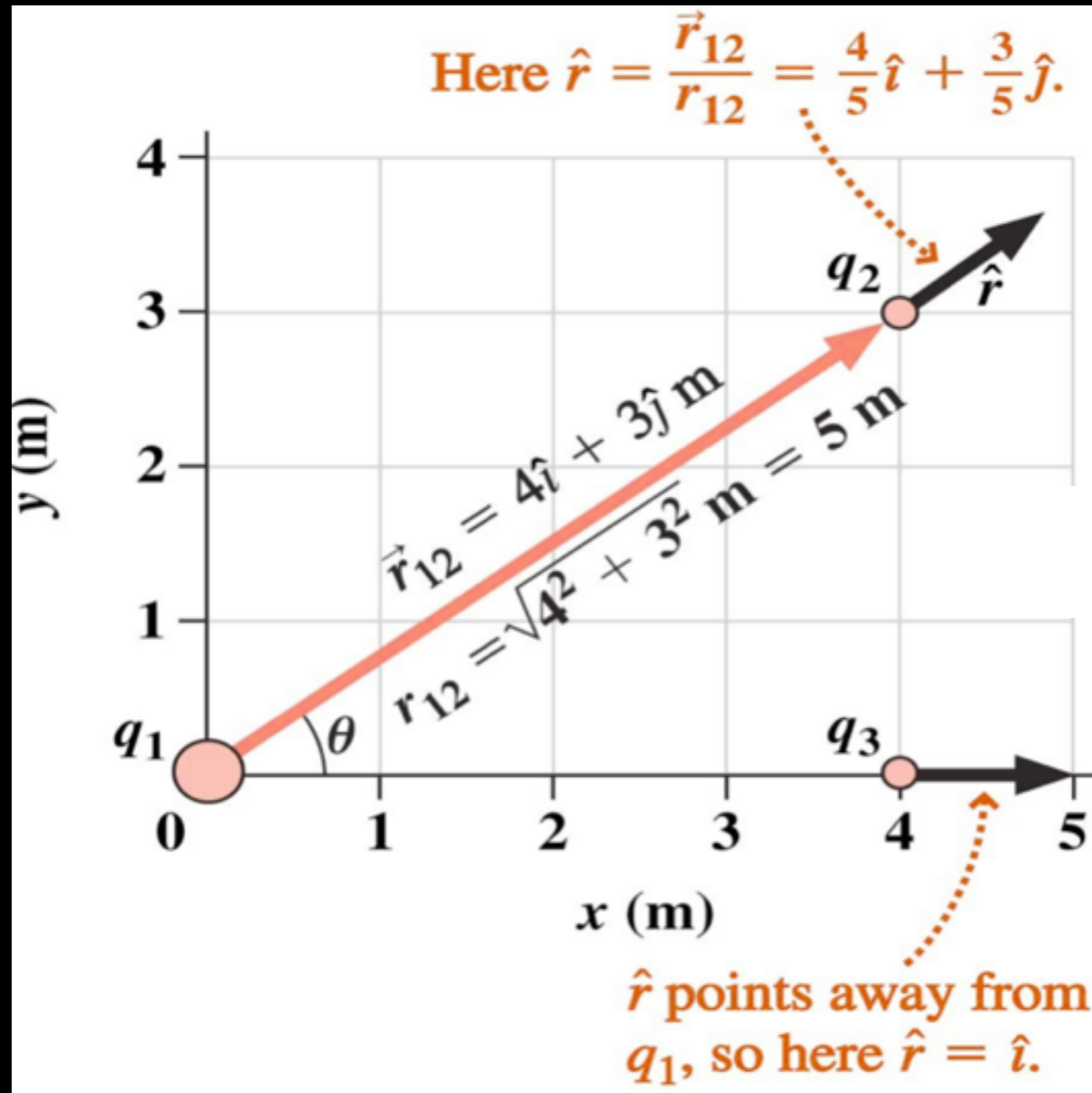
Its direction is given by \hat{r}_{12} :

Like charges: \bar{F} is in same direction
Unlike, \bar{F} is in opposite direction

Two diagrams illustrating the direction of the force vector \bar{F} relative to the unit vector \hat{r}_{12} . In the top diagram, labeled 'Like charges', a black arrow representing \hat{r}_{12} points to the right, and a red arrow representing \bar{F} also points to the right. In the bottom diagram, labeled 'Unlike', a black arrow representing \hat{r}_{12} points to the right, and a red arrow representing \bar{F} points to the left.

The electric force

The unit vector, \hat{r}_{12}



The electric force

Quiz

A charge q is at $x = 1\text{m}$, $y = 0\text{m}$.

What is the direction of the Coulomb force (\hat{r}_{12}) on a 2nd charge at $x = 2\text{m}$, $y = 3\text{m}$?

(A) $1.0\hat{i} + 3.0\hat{j}$ m

(B) $0.75\hat{i} + 0.66\hat{j}$ m

(C) $0.316\hat{i} + 0.949\hat{j}$ m

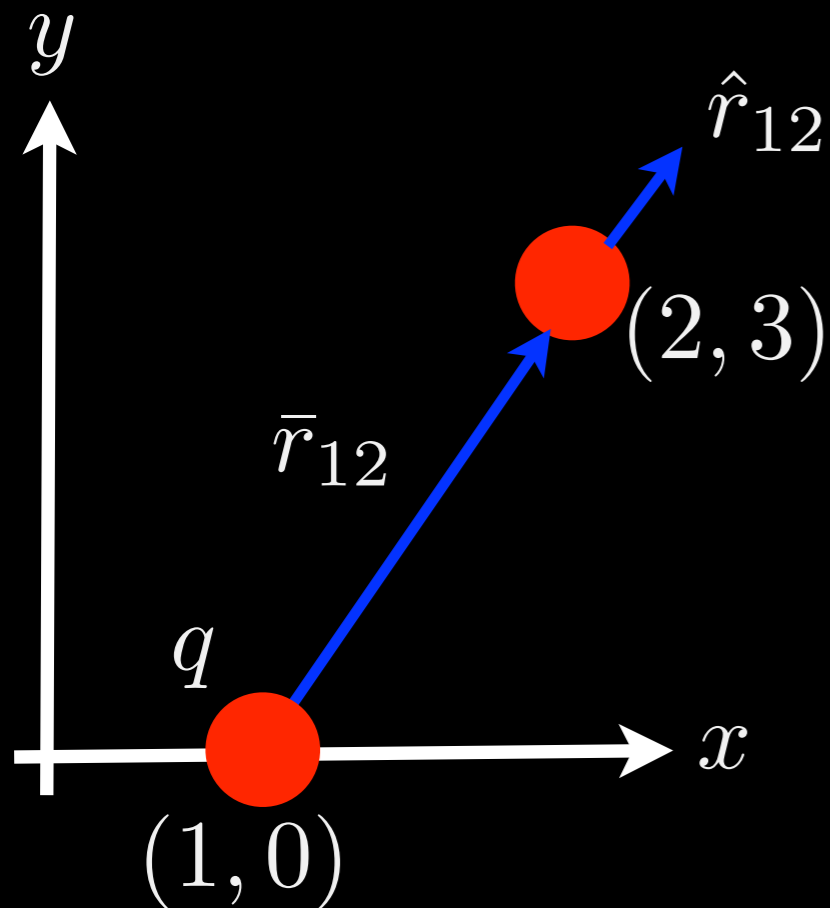
(D) $0.632\hat{i} + 0.949\hat{j}$ m

The electric force

Quiz

A charge q is at $x = 1\text{m}$, $y = 0\text{m}$.

What is the direction of the Coulomb force (\hat{r}_{12}) on a 2nd charge at $x = 2\text{m}$, $y = 3\text{m}$?



$$\hat{r}_{12} = \frac{\bar{r}_{12}}{|\bar{r}_{12}|}$$

$$\begin{aligned}\bar{r}_{12} &= (2\bar{i} + 3\bar{j}) - (1\bar{i}) \\ &= 1\bar{i} + 3\bar{j}\end{aligned}$$

$$|\bar{r}_{12}| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$\hat{r}_{12} = \frac{1\hat{i} + 3\hat{j}}{\sqrt{10}} = (0.316\hat{i} + 0.949\hat{j}) \text{ m}$$

The electric force

Quiz

Remember I said 1C was big?

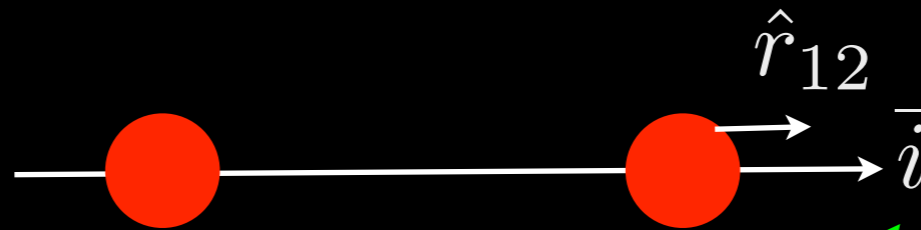
2 point charges each of 1C are 1m apart. What is the magnitude of the force between them?

(A) $2 \times 10^{12} \text{N}$

(B) $9 \times 10^9 \text{N}$

(C) 30N

(D) 5000N



Not given co-ordinates, so assume charges are along x-axis

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2} r \hat{12}$$

$$\vec{F}_{12} = \frac{(9.0 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2)(1.0\text{C})(1.0\text{C})}{(1.0\text{m})^2} \hat{i}$$

$$= 9.0 \times 10^9 \text{N}$$



Gravitational force of ~1000 trains

The electric force

Quiz

The electron and proton in a hydrogen atom are 52.9 pm apart.

Find the electric force between them.

(A) $8.2 \times 10^{20} \text{ N}$

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2} \hat{r}_{12}$$

(B) $-8.2 \times 10^{-8} \text{ N}$

$$= \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})(-1.6 \times 10^{-19} \text{ C})}{(52.9 \times 10^{-12} \text{ m})^2} \hat{i}$$

(C) $-4.3 \times 10^{-18} \text{ N}$

$$= -8.2 \times 10^{-8} \text{ N}$$

(D) $8.2 \times 10^{-8} \text{ N}$

The electric force

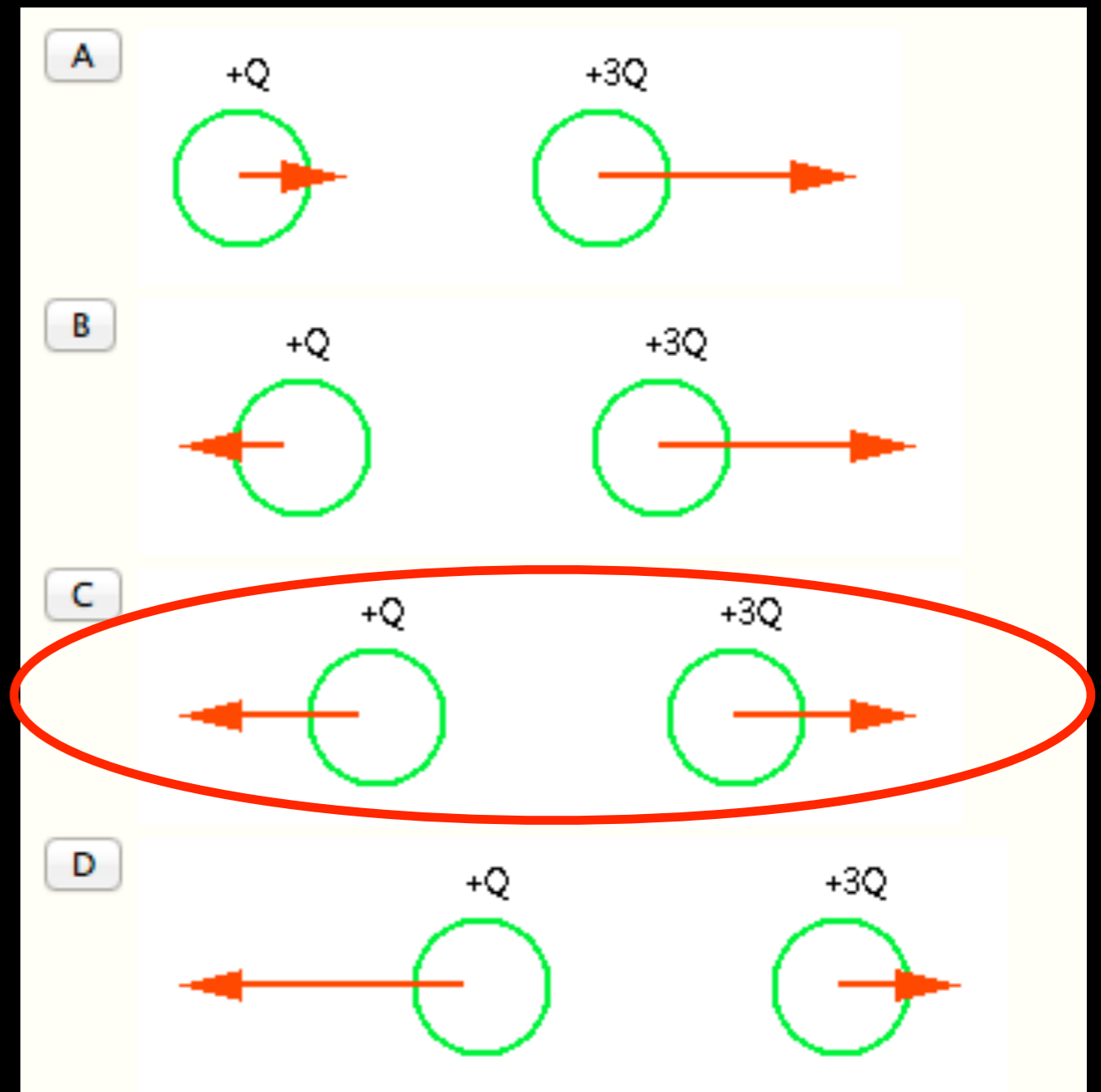
Quiz

2 balls rest on a frictionless surface. 1 ball is given a charge of $+Q$, the other ball is given a charge of $+3Q$.

Which picture best represents the force vectors on the balls?

Equal and opposite forces are felt!

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2} \hat{r}_{12}$$



The electric force

Strange but true

The **gravitational force** between 2 masses is: $F_g = \frac{Gm_1m_2}{r^2}$

Which is very similar to the **electrical force**: $F_E = \frac{kq_1q_2}{r^2}$

But they have very different magnitudes. For the force between a proton and electron:

$$\frac{F_E}{F_g} = \frac{ke^2}{Gm_em_p} = 2.3 \times 10^{39}$$

The electrical force is 39 orders of magnitude stronger!!

And yet gravity is the more obvious force in normal life. **Why?**

The electric force

Quiz

Gravity is more obvious because...

- (A) On large scales, electric forces often cancel
- (B) The electric force decreases faster with distance than gravity
- (C) We recognise gravity more easily
- (D) Gravity is actually an electric force

The electric force

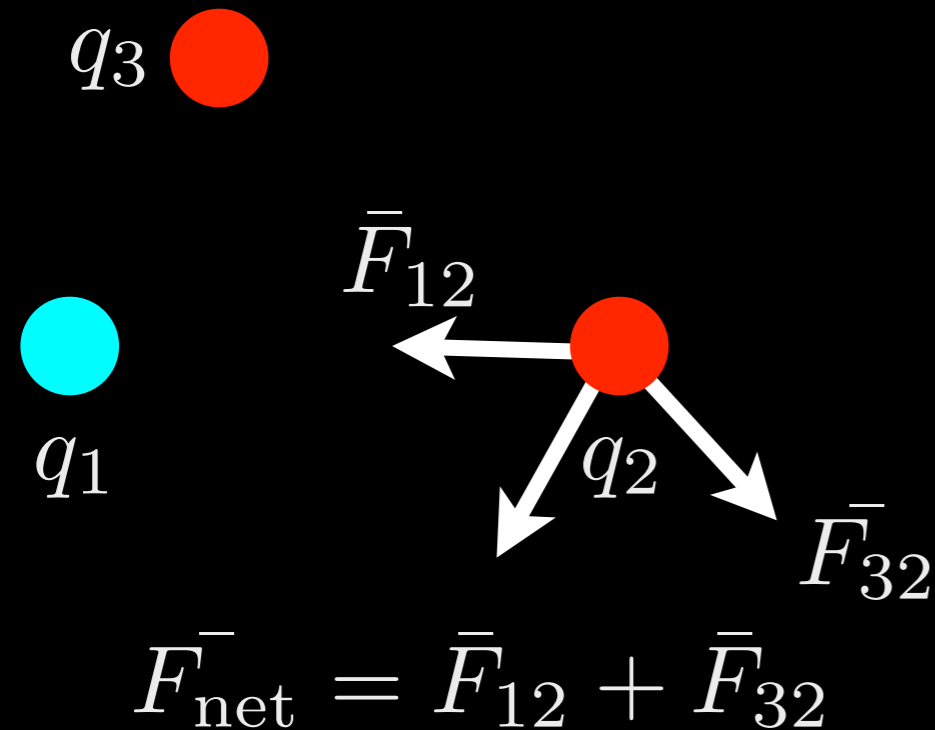
	Electric force	Gravitational force
Equation	$F_E = \frac{kq_1q_2}{r^2}$	$F_g = \frac{Gm_1m_2}{r^2}$
Source	charge	mass
Type	Positive (+ve) or negative (-ve)	one type only
Direction	Can attract or repel	Only attracts

The strength of the electric force actually makes it less obvious:

Opposite charges bind and cancel, so most matter is neutral on scales we see.

Superposition of point charges

Forces from point charges can be added:



Point charge: size is negligible (~ 0)

Remember: vector addition

If $\vec{F}_{12} = -3\hat{i}$ and $\vec{F}_{32} = 2\hat{i} - 3\hat{j}$

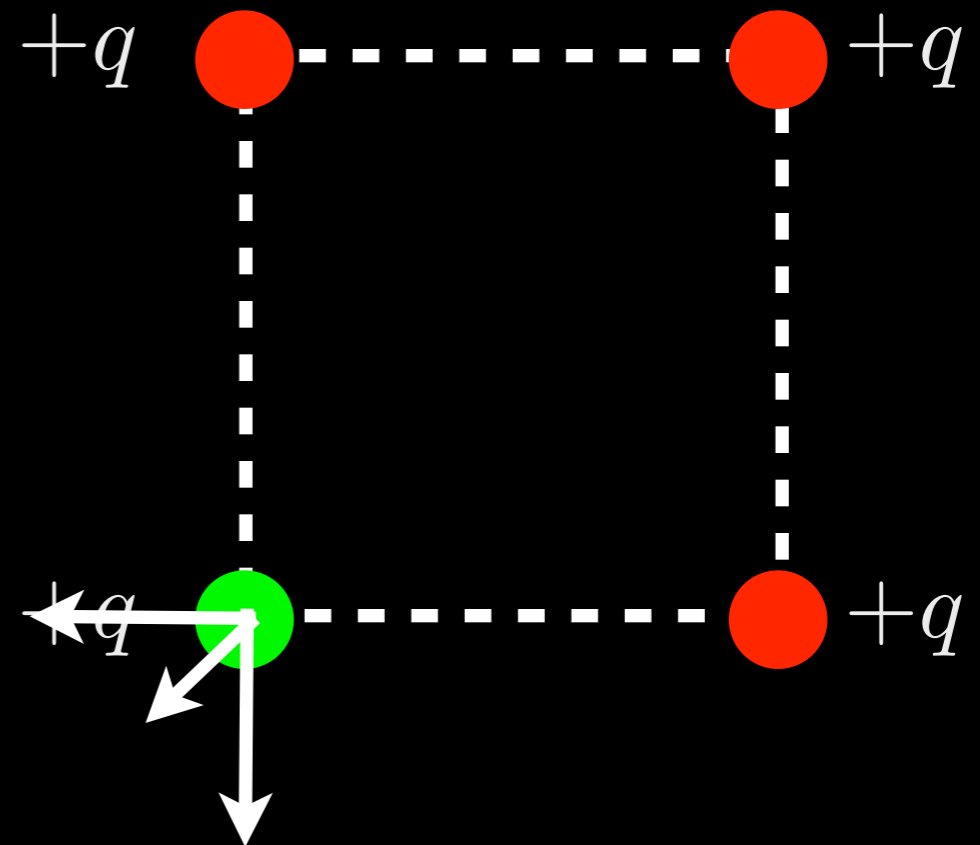
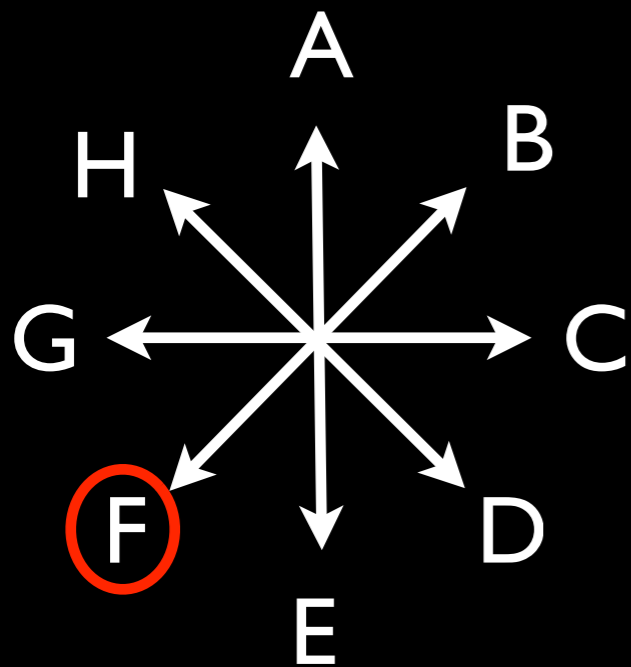
then $\vec{F}_{\text{net}} = -\hat{i} - 3\hat{j}$

Superposition of point charges

Quiz

4 equal charges are fixed to the corners of a square.

What is the direction of the net force on the green charge?

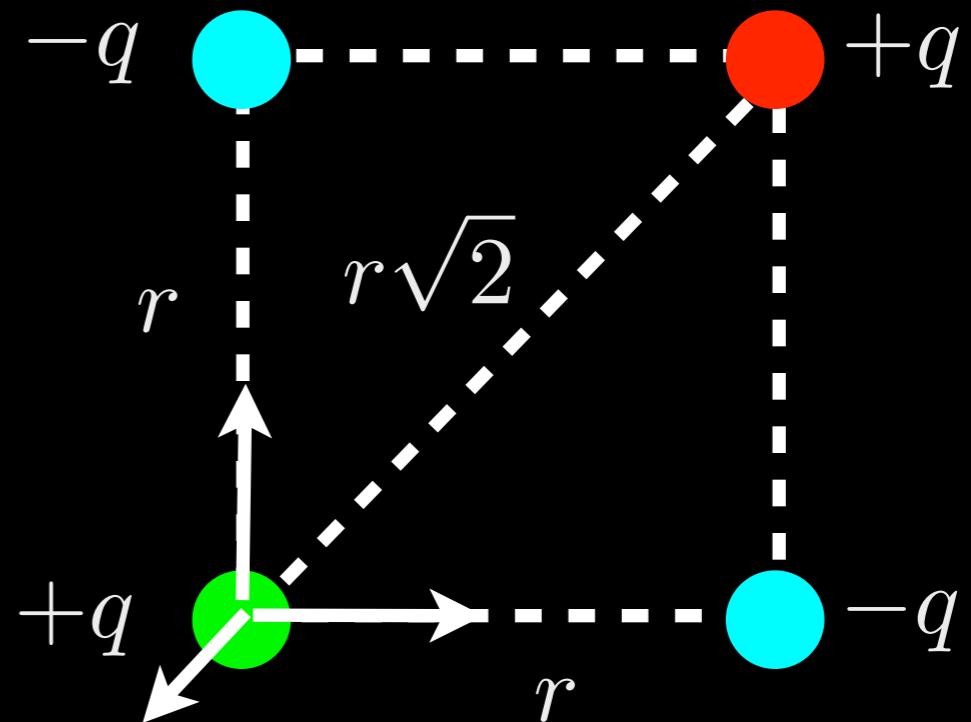
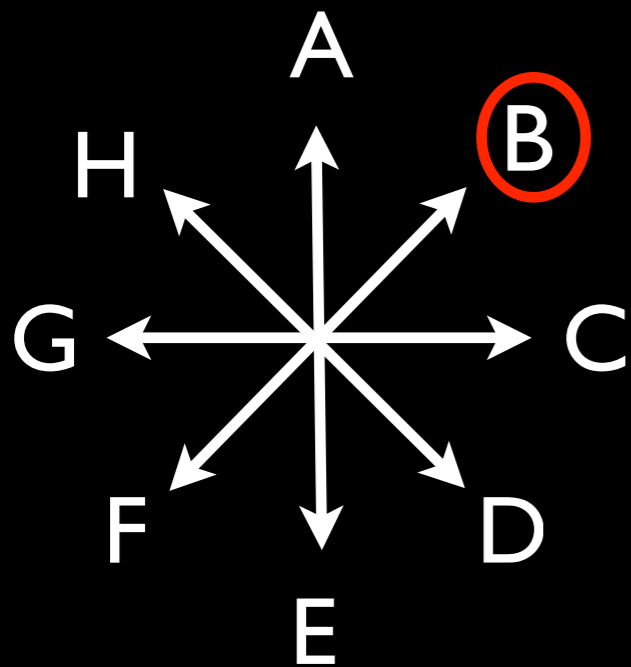


Superposition of point charges

Quiz

4 equal charges are fixed to the corners of a square.

What is the direction of the net force on the green charge?



Superposition of point charges

Quiz

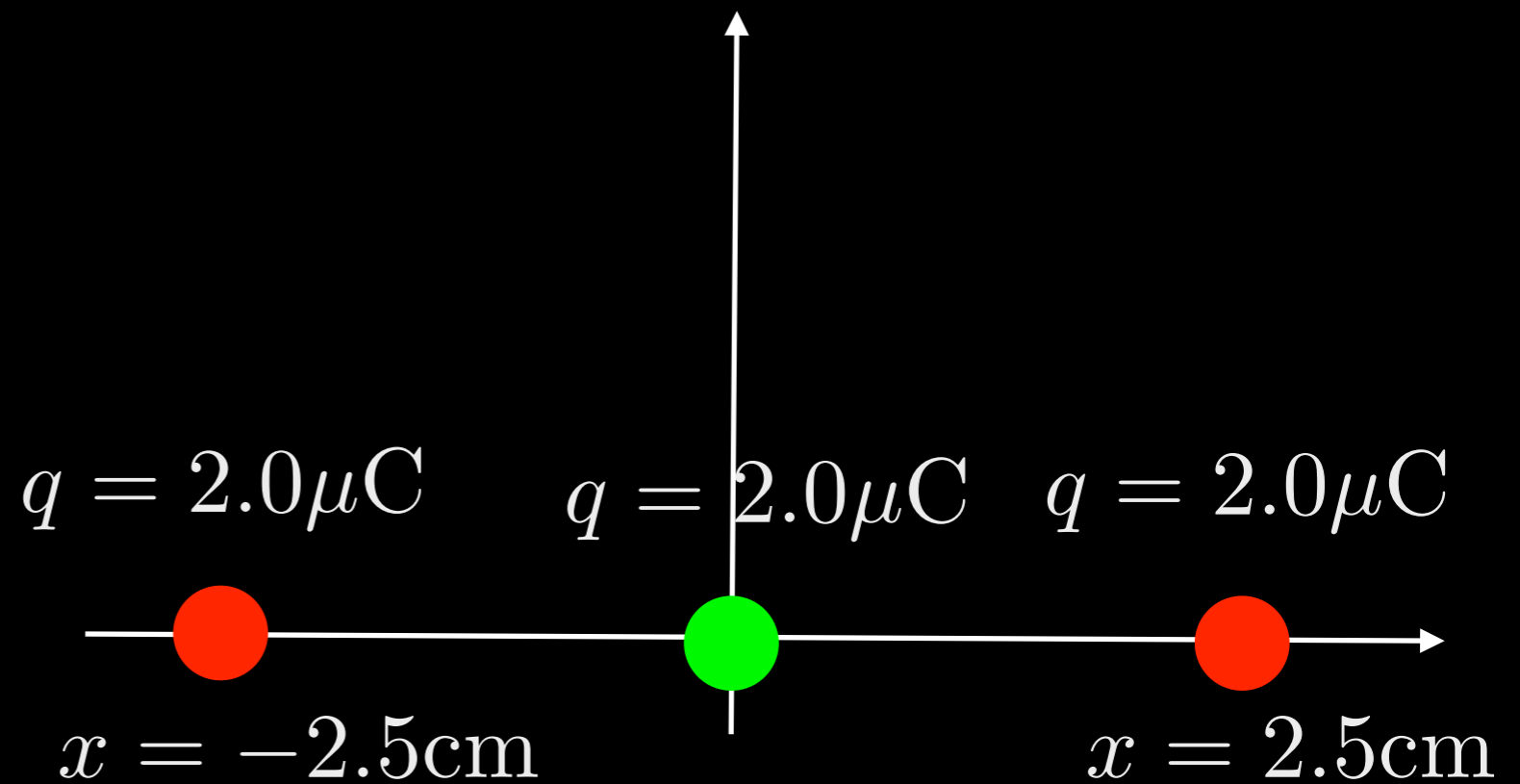
What is the force on the green charge? $k = 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2$

(A) 0N

(B) 57.6N

(C) 115.2N

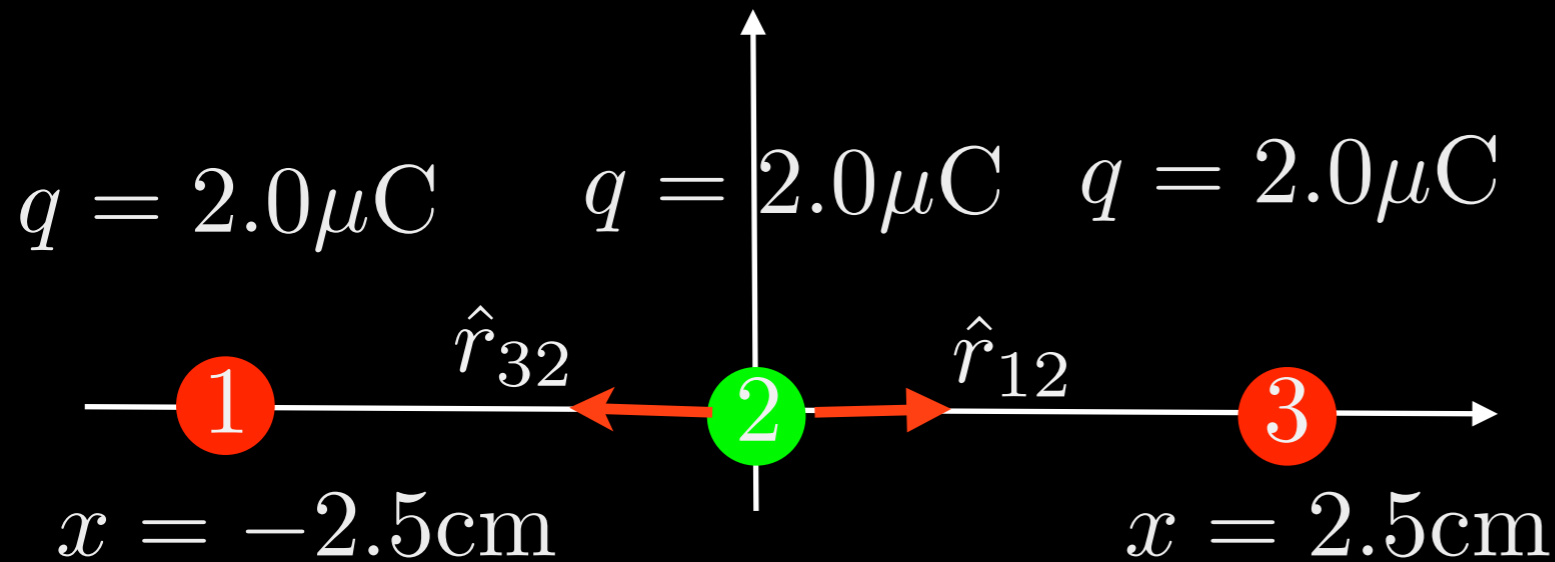
(D) -57.6N



Superposition of point charges

Quiz

What is the force on the green charge?



$$\vec{F}_{12} = \frac{kq_1q_2}{r^2} \hat{r}_{12} = \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(2 \times 10^{-6} \text{ C})^2}{(2.5 \times 10^{-2} \text{ m})^2} \hat{i}$$

$$\vec{F}_{32} = \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(2 \times 10^{-6} \text{ C})^2}{(2.5 \times 10^{-2} \text{ m})^2} (-\hat{i}) = -\vec{F}_{12}$$

$$\vec{F}_{12} + \vec{F}_{32} = 0$$

Superposition of point charges

Quiz

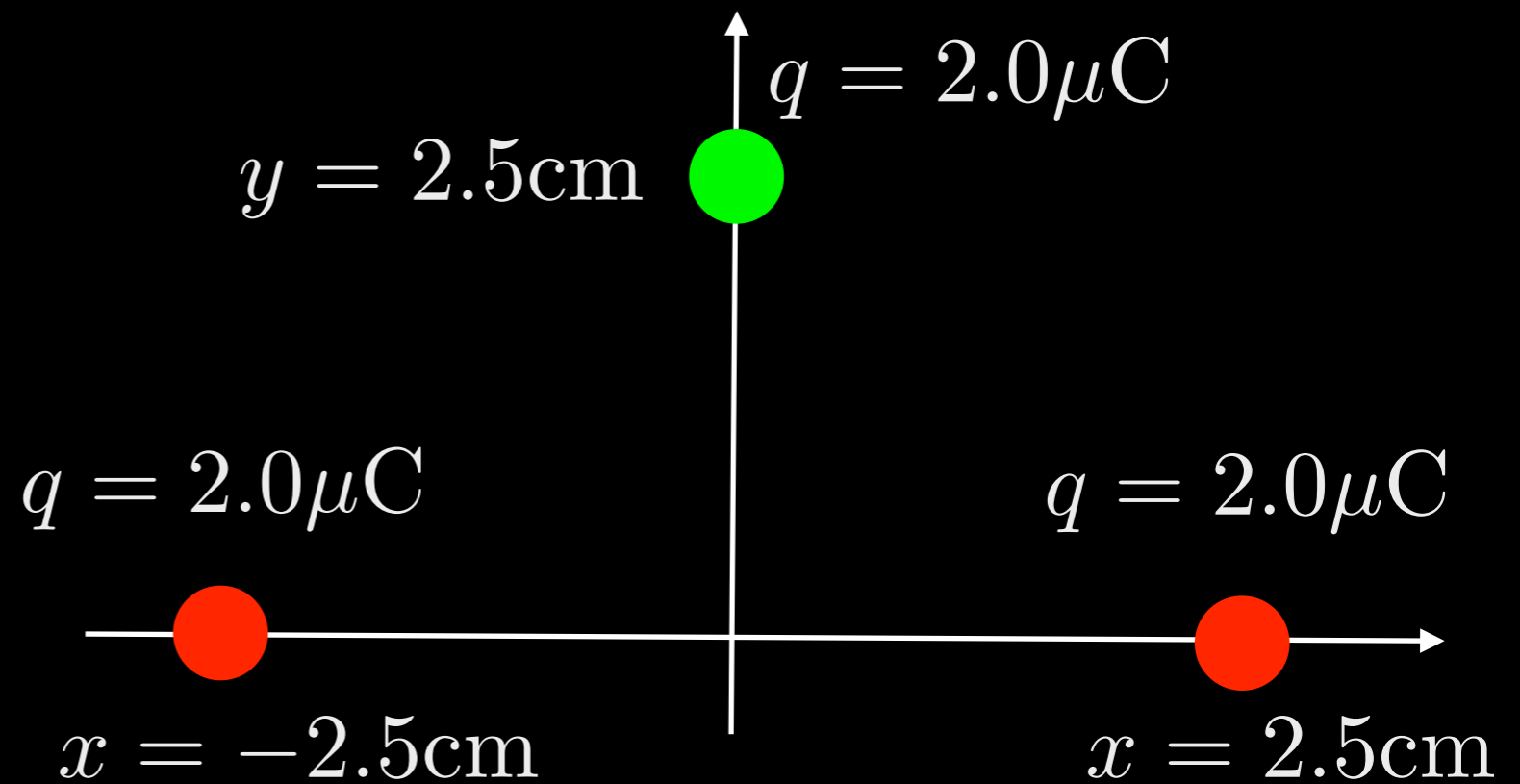
What is the force on the green charge? $k = 9.0 \times 10^9 \text{Nm}^2/\text{C}^2$

(A) 0N

(B) 81.45N

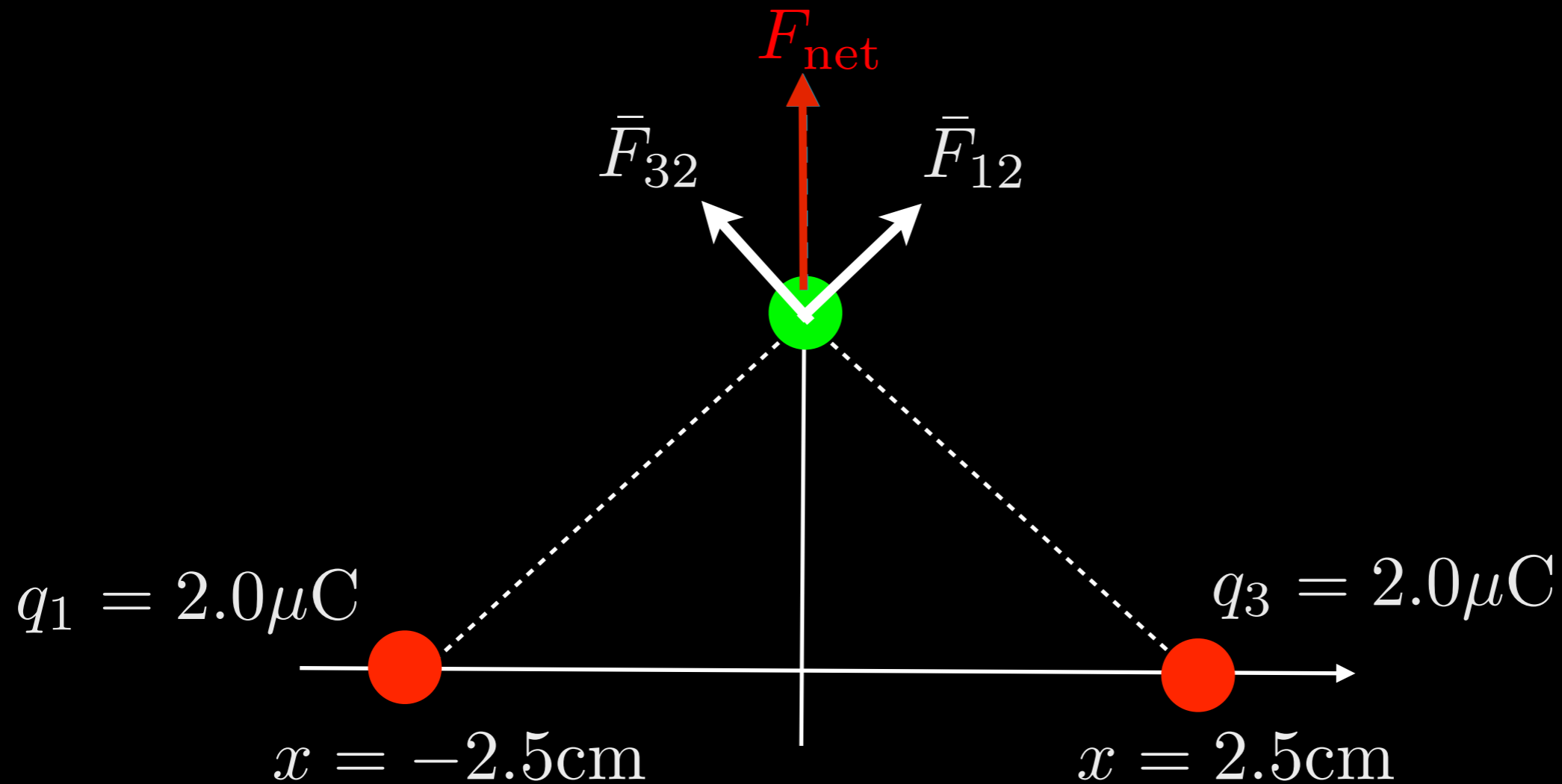
(C) 40.7N

(D) 115.2N



Superposition of point charges

Quiz



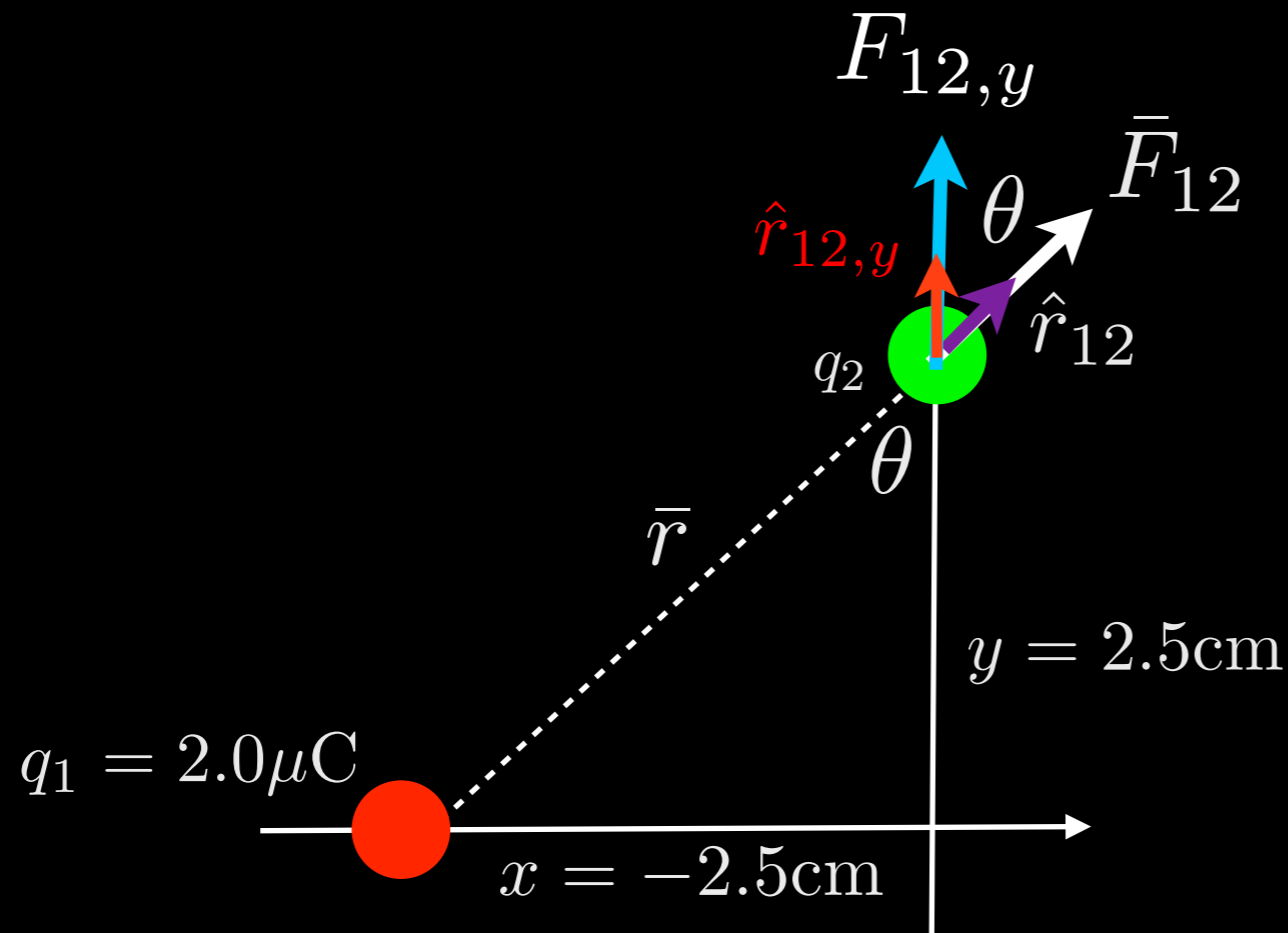
x-axis components of \vec{F}_{12} and \vec{F}_{32} will cancel.

Find y-components $\vec{F}_{12,y}$ and $\vec{F}_{32,y}$

$$\vec{F}_{\text{net}} = \vec{F}_{12,y} + \vec{F}_{32,y}$$

Superposition of point charges

Quiz



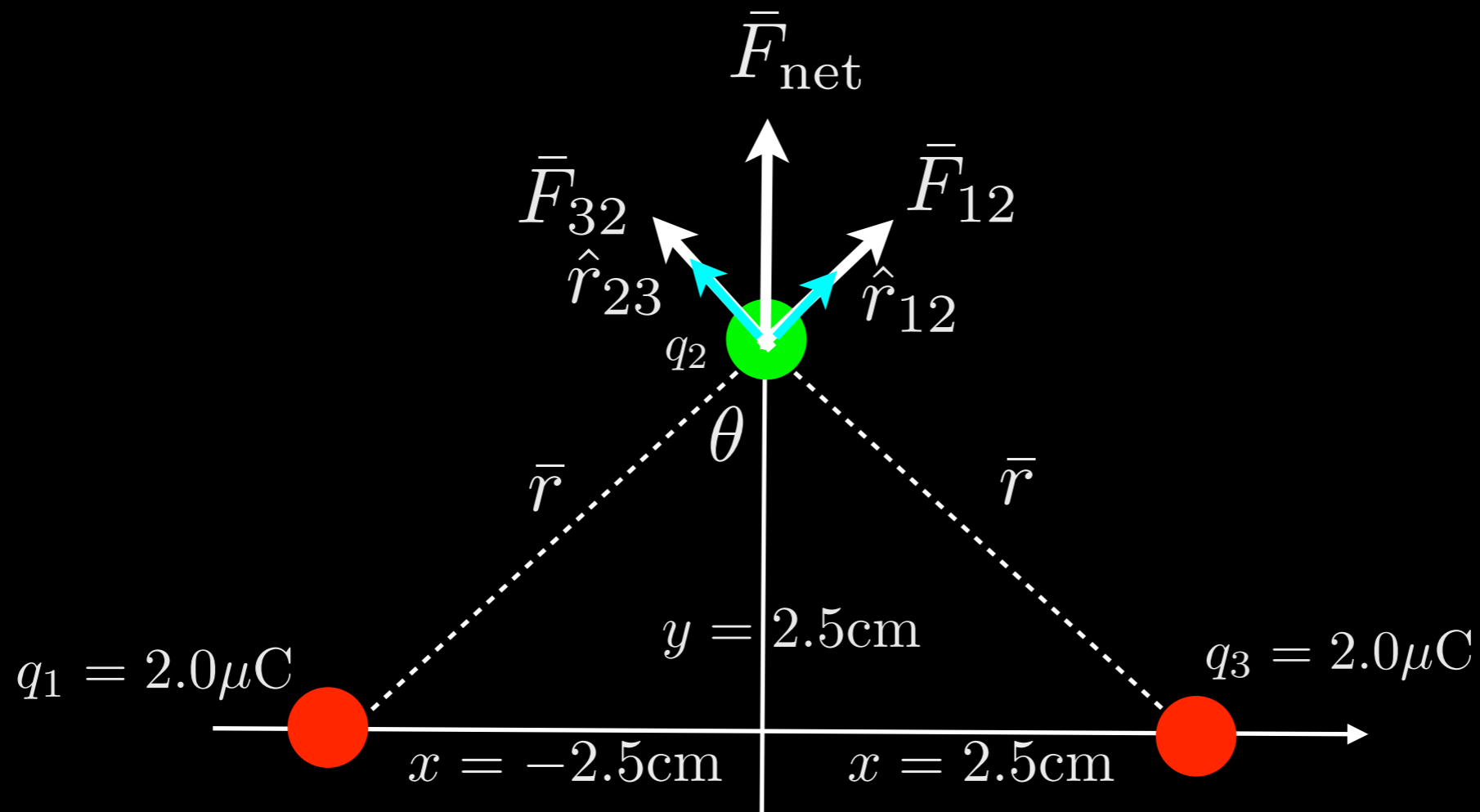
$$\vec{F}_{12,y} = \frac{kq_1q_2}{r^2} \hat{r}_{12,y} = \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(2 \times 10^{-6})^2}{(\sqrt{0.025^2 + 0.025^2})^2} \hat{r}_{12,y}$$

$$\hat{r}_{12,y} = \hat{r}_{12} \cdot \hat{j} = \cos \theta = \left(\frac{0.025}{\sqrt{0.025^2 + 0.025^2}} \right)$$

unit vectors, magnitude 1

Superposition of point charges

Quiz



$$\vec{F}_{12,y} = 20.4$$

$$\vec{F}_{32,y} = \vec{F}_{12,y}$$

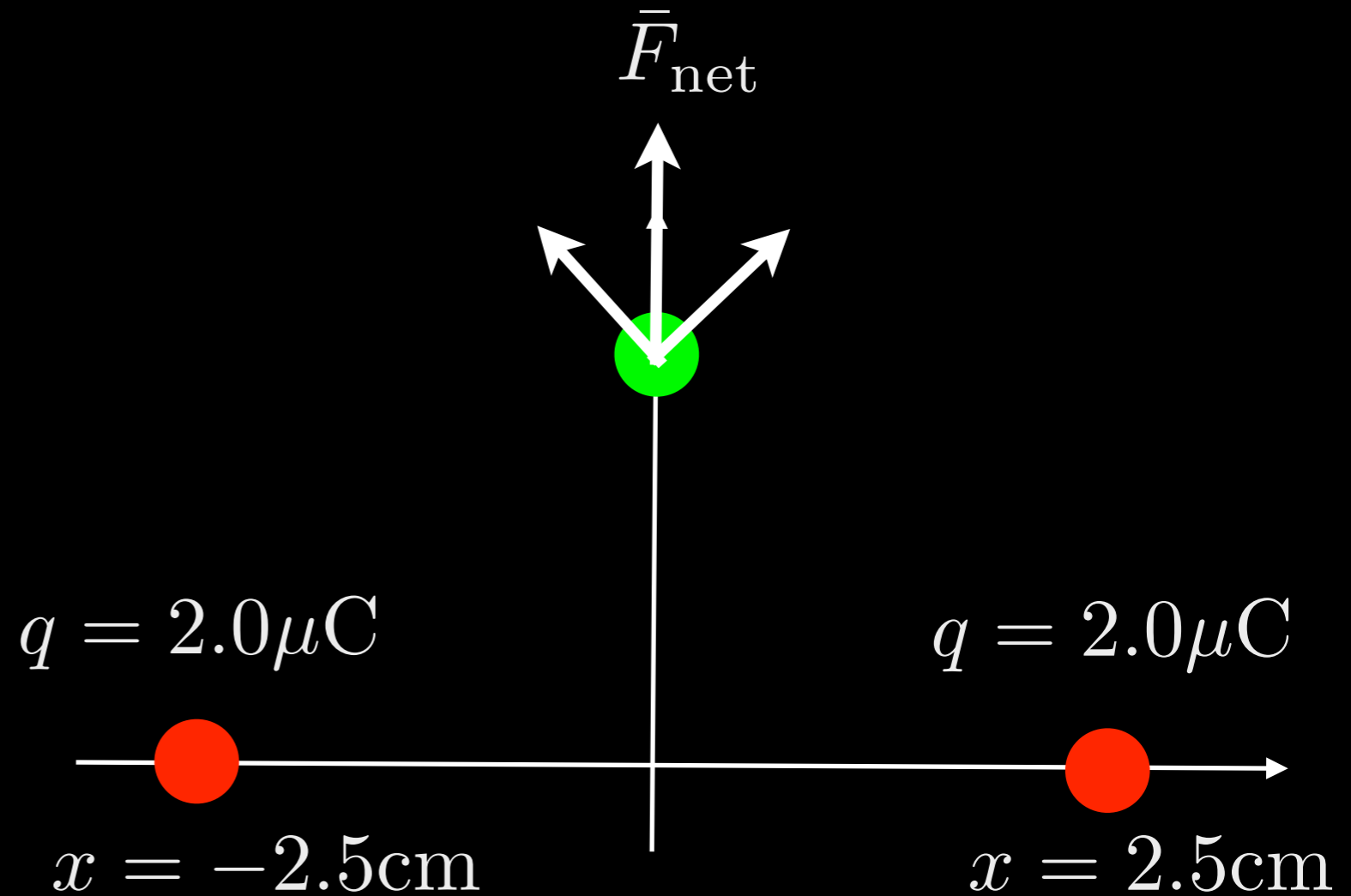
$$\vec{F}_{\text{net}} = 40.8$$

Superposition of point charges

Quiz

What is the force on the green charge?

- (A) 0N
- (B) 81.45N
- (C) 40.7N**
- (D) 115.2N

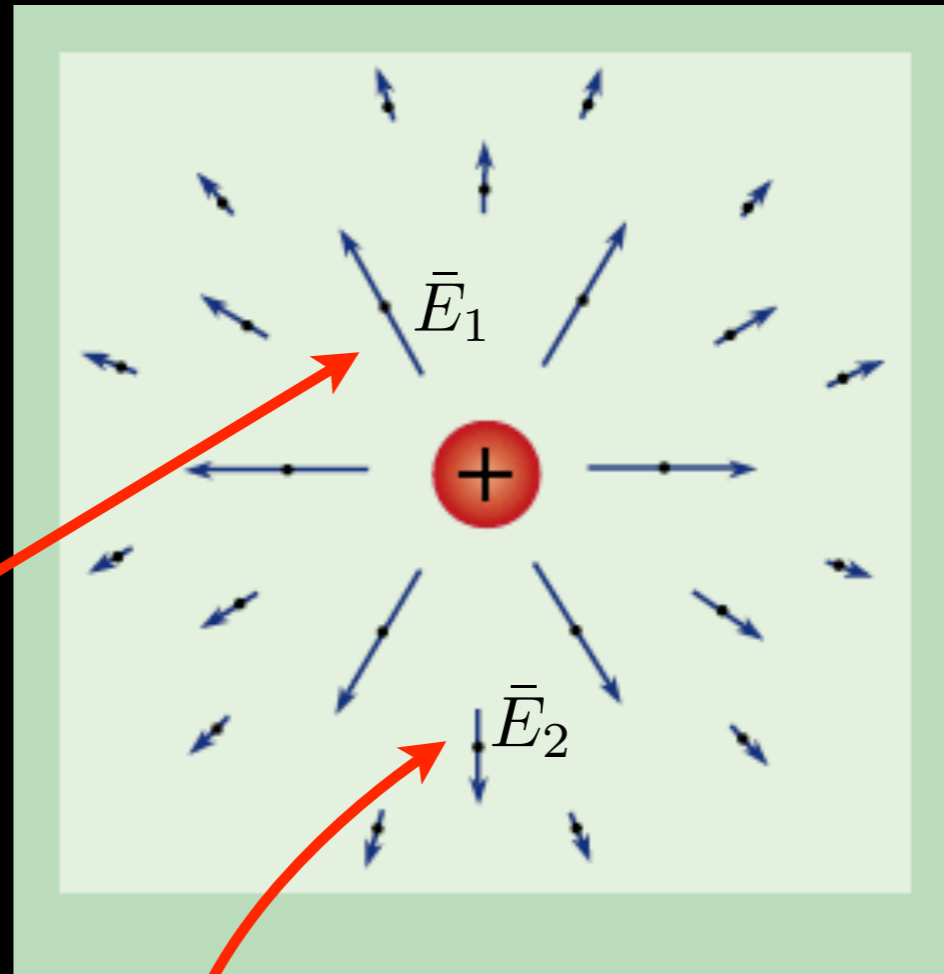


Here, the electric force **increases** as you initially move away.

The electric field

The **electric field** is the force per unit charge that an object would experience in a location.

$$\vec{E} = \frac{\vec{F}}{q}$$



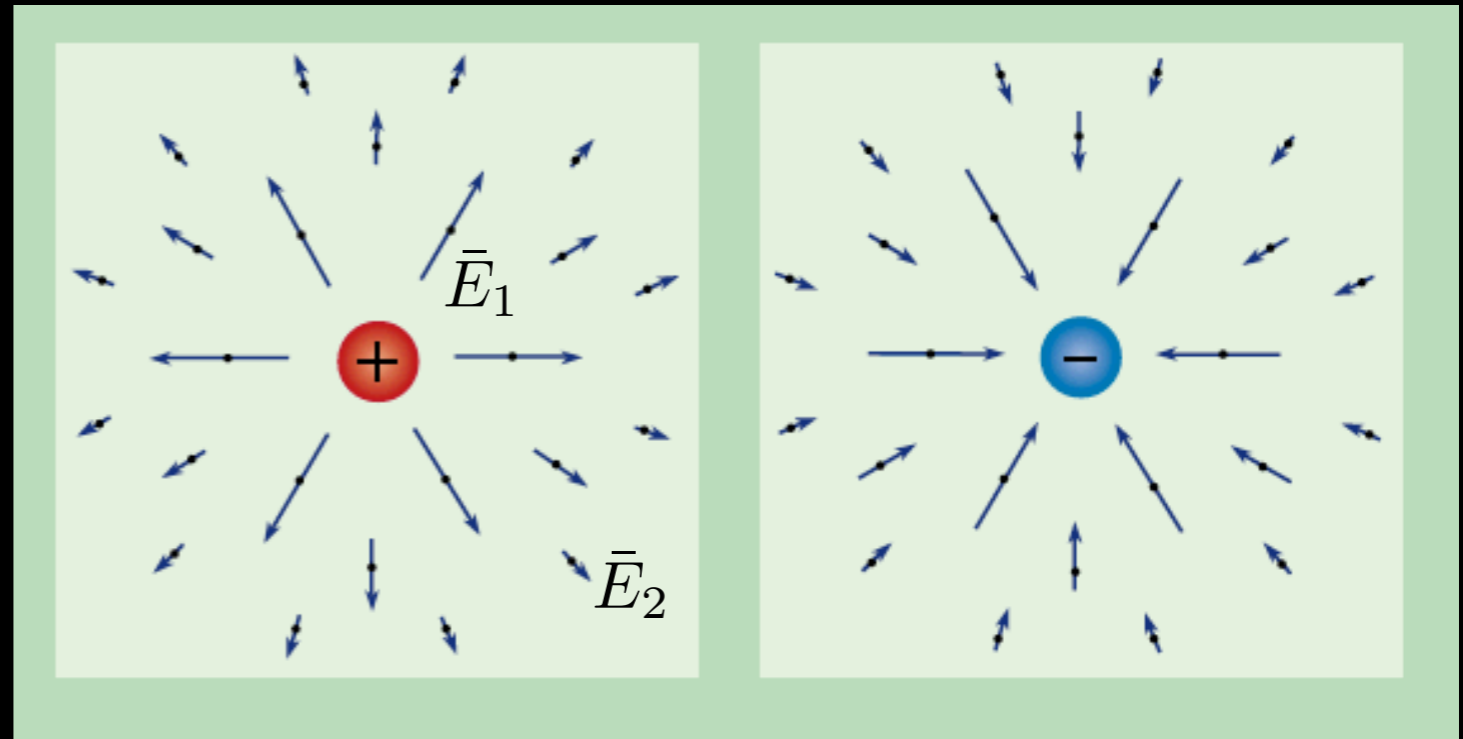
Close to the charge, the electric field is \vec{E}_1 . Here, point charge q , would feel $\vec{F} = q\vec{E}_1$

Further from the charge, the electric field is \vec{E}_2 . Here, point charge q would feel a weaker force, $\vec{F} = q\vec{E}_2$ in a different direction.

The electric field

The field for a point charge is radial.

It points outwards for a positive charge and inwards for a negative charge.

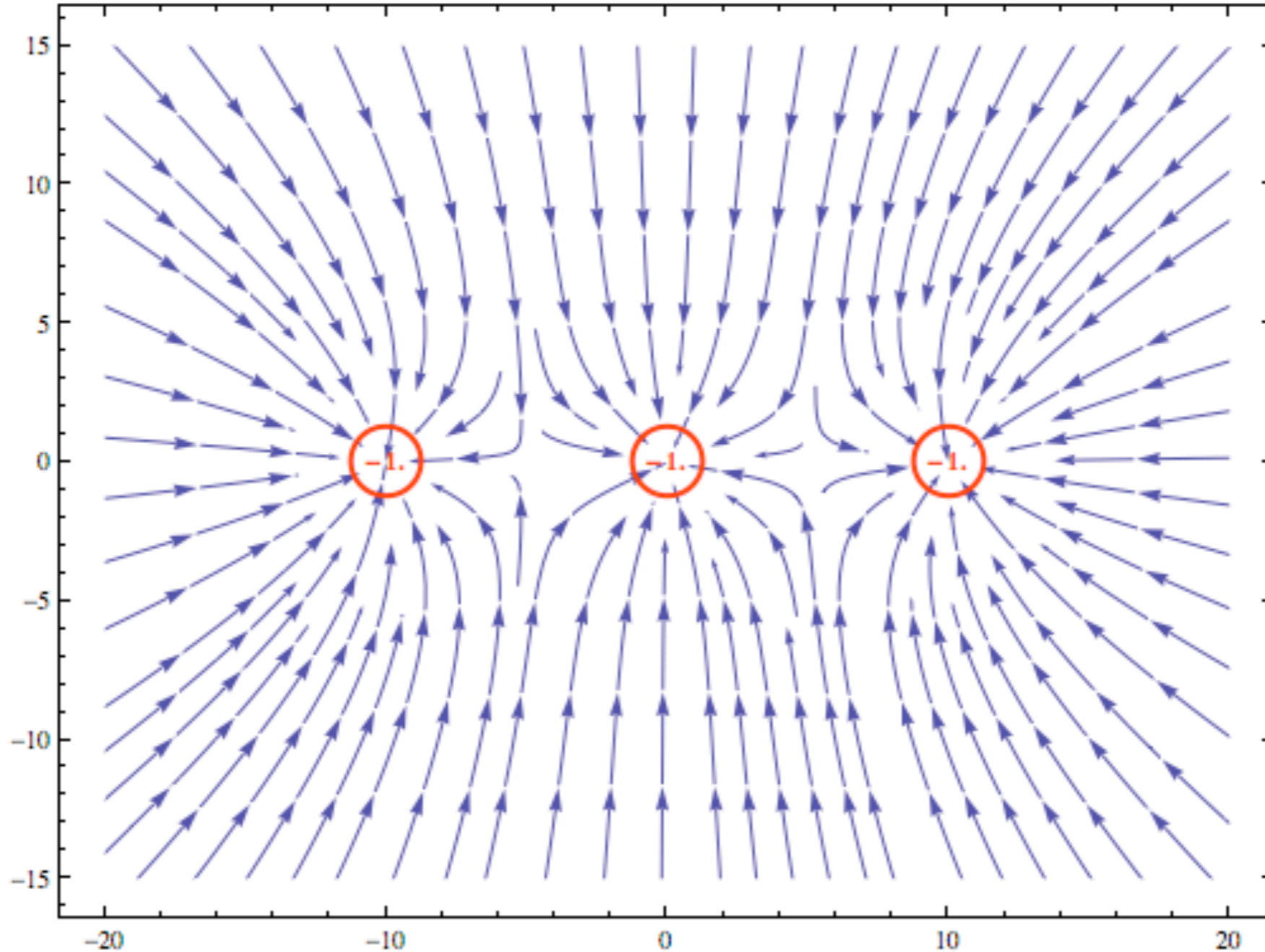


Since $\vec{E} = \frac{\vec{F}}{q}$ and Coulomb's law: $\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}_{12}$

We get: $\vec{E} = \frac{kq}{r^2}\hat{r}$ field of a point charge
(also called Coulomb's law)

The electric field

Electric field for 3 equal point charges



The electric field

Like the electric force, electric fields can be added:

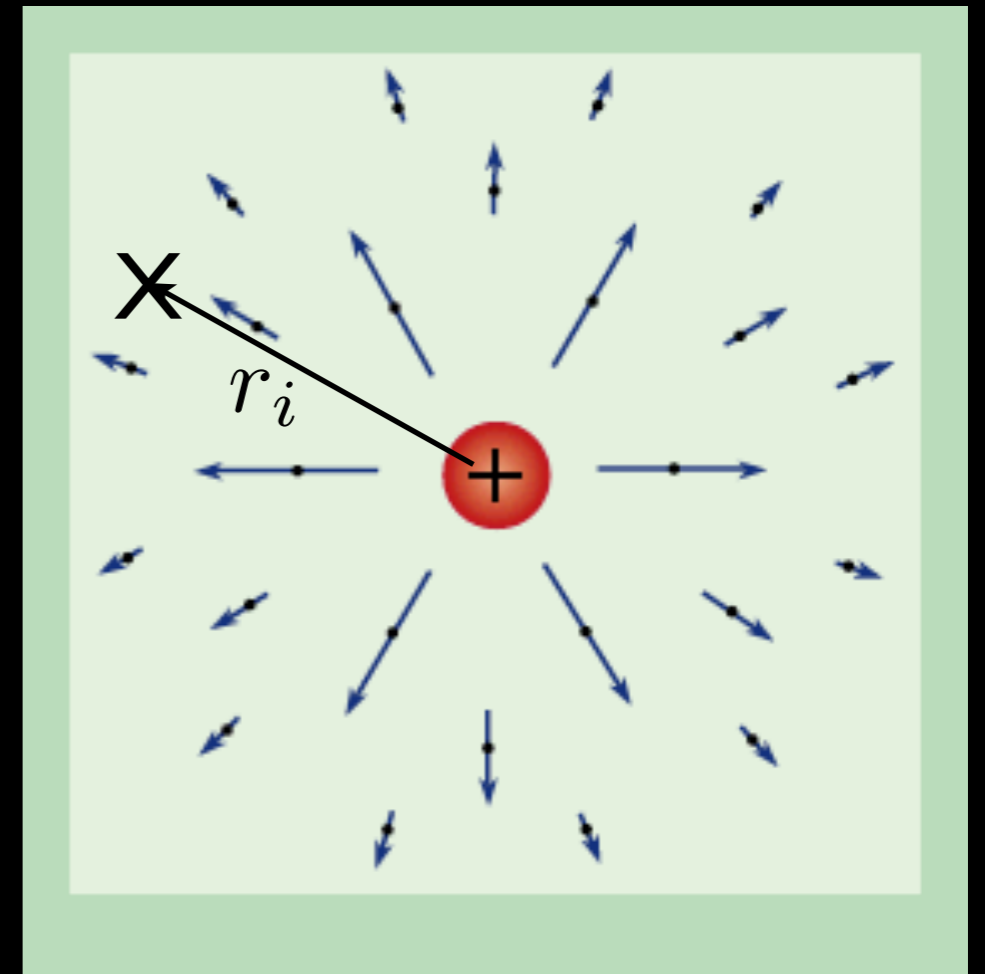
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots = \sum_i \vec{E}_i$$

$$= \sum_i \frac{kq_i}{r_i} \hat{r}_i$$

The place where the field is measured is the **field point**.

r_i is the distance from the source to the field point.

\hat{r}_i is the unit vector pointing from the source towards the field point.



The electric field

Quiz

Two charges $+q$ and $-q$ are on the y -axis, symmetric about the origin.

What is the electric field direction at field point A?

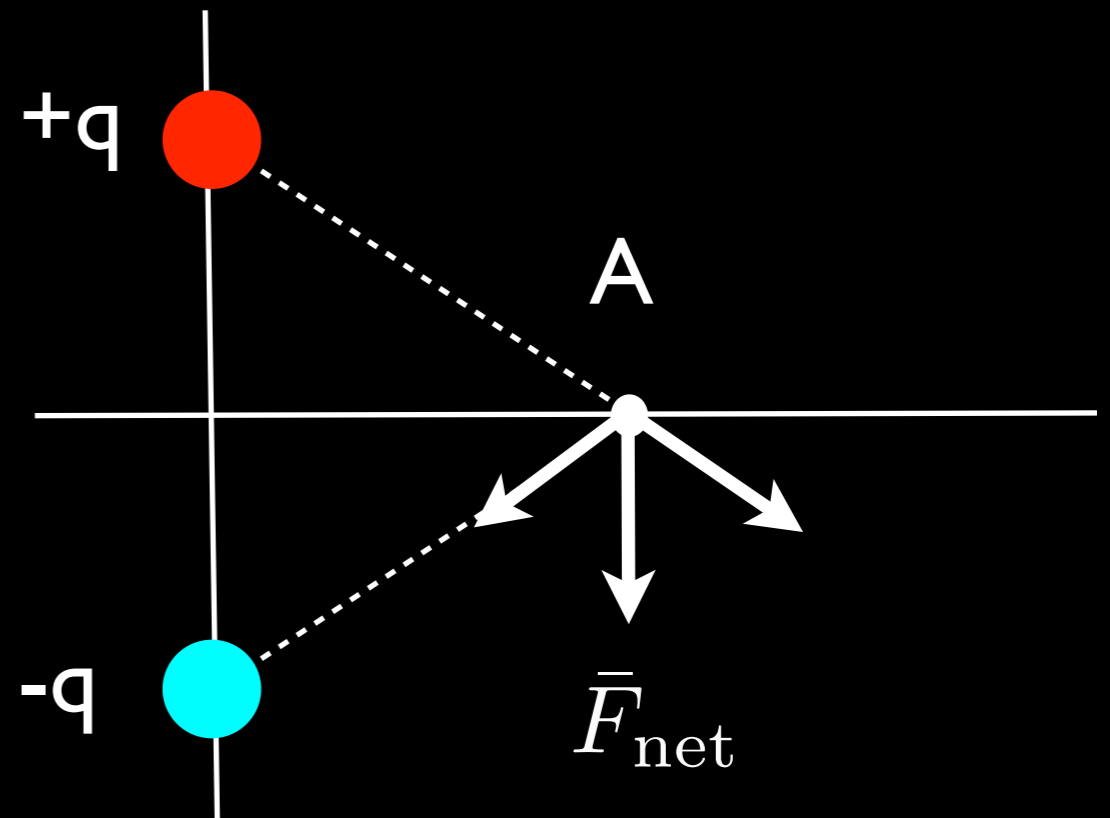
(A) Up

(B) Down

(C) Left

(D) Right

(E) Other



The electric field

Quiz

An electron is at the origin and an ion with charge $+5e$ is at $x = 10 \text{ nm}$. Find the point where the electric field is 0.

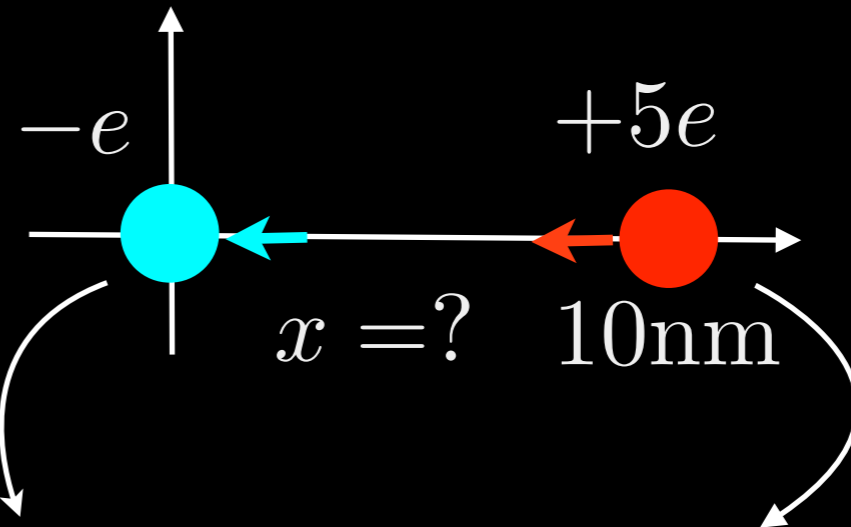
(A) 2nm

$$\vec{E} = \frac{kq}{r^2} \hat{r} = \frac{kq}{r^2} \frac{\vec{r}}{|\vec{r}|} = kq \frac{(x - x_q) \hat{i}}{|x - x_q|^3}$$

(B) 3.1nm

(C) 5nm

(D) -8.1nm



$$0 = \frac{k(-e)x}{x^3} \hat{i} + \frac{k(5e)(x - 10 \text{ nm})}{|x - 10 \text{ nm}|^3} \hat{i}$$

$$= \left(-\frac{1}{x^2} + \frac{5}{|x - 10 \text{ nm}|^2} \right) \hat{i}$$

$$= (4x^2 + 20x - 100) \hat{i}$$

The electric field

Quiz

An electron is at the origin and an ion with charge $+5e$ is at $x = 10 \text{ nm}$. Find the point where the electric field is 0.

(A) 2 nm

(B) 3.1 nm

(C) 5 nm

(D) -8.1 nm

Quadratic Eq. $0 = 4x^2 + 20x - 100$

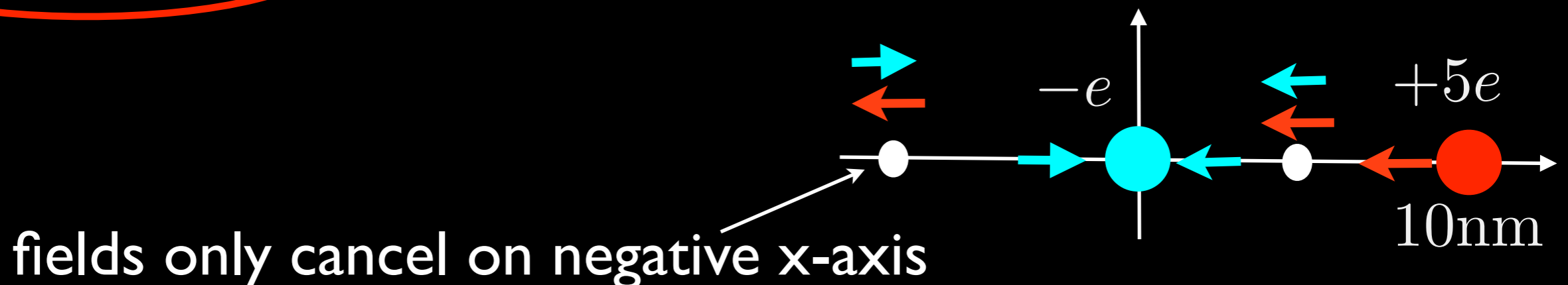
$$\left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

a b c

$$x = \frac{-20 \pm \sqrt{20^2 - 4(4)(-100)}}{2(4)}$$

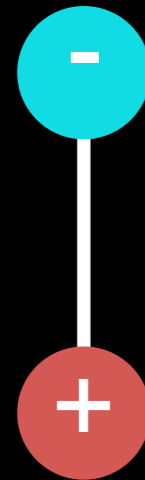
$$= 3.1 \text{ or } -8.1$$

which is right?



The electric dipole

Charge distributions that are 2 point charges of equal magnitude but opposite sign are **electric dipoles**.

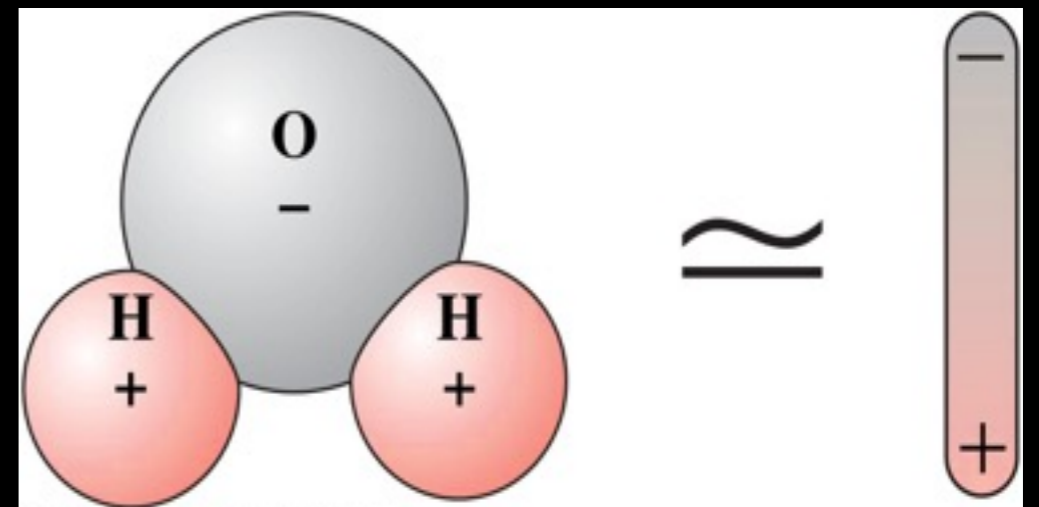


The electric dipole

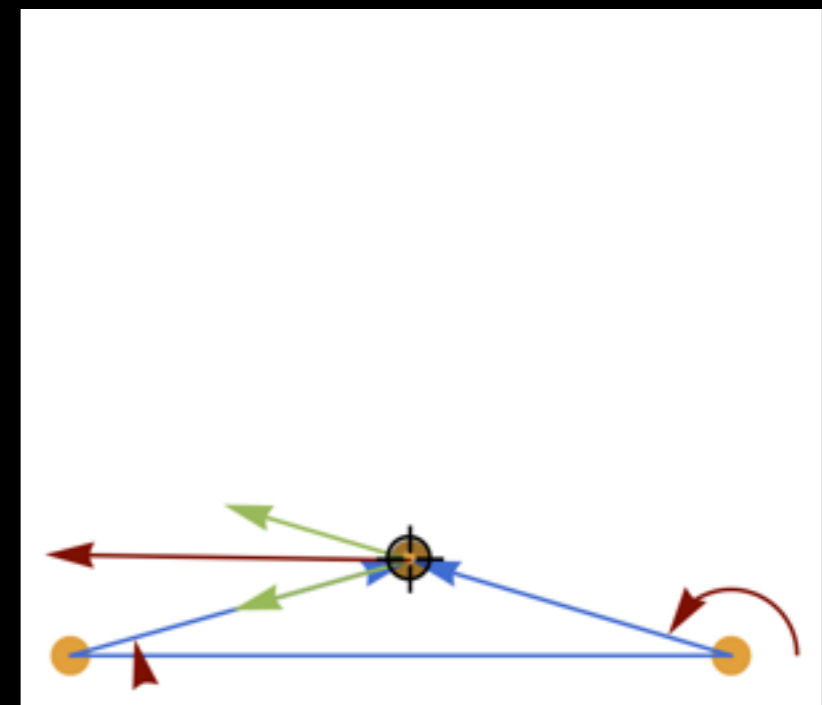
Charge distributions that are 2 point charges of equal magnitude but opposite sign are **electric dipoles**.

Many molecules are dipoles.

The molecule is neutral but the separation of its charges means it has an electric field.



The field drops as you move further from the dipole.



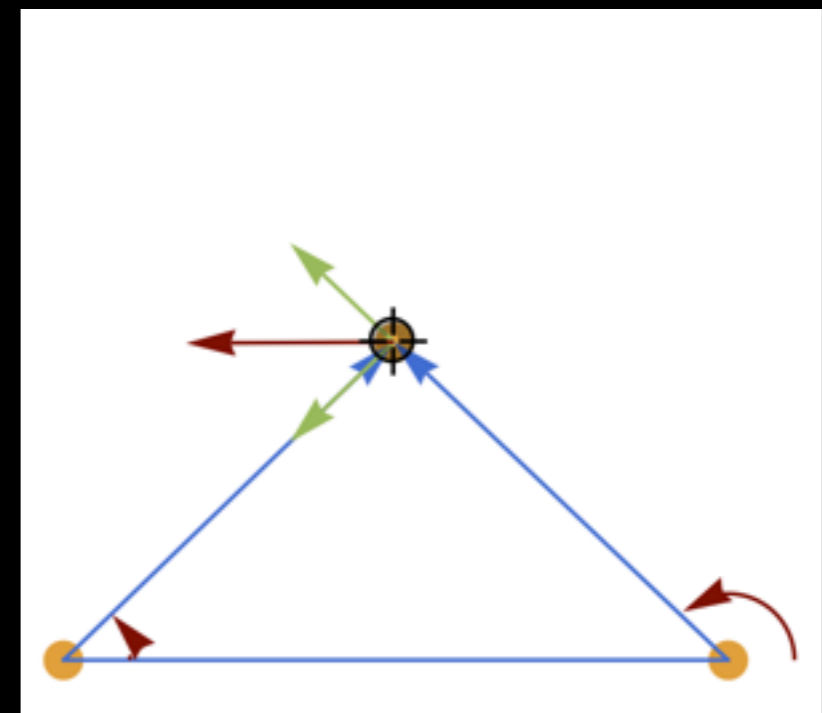
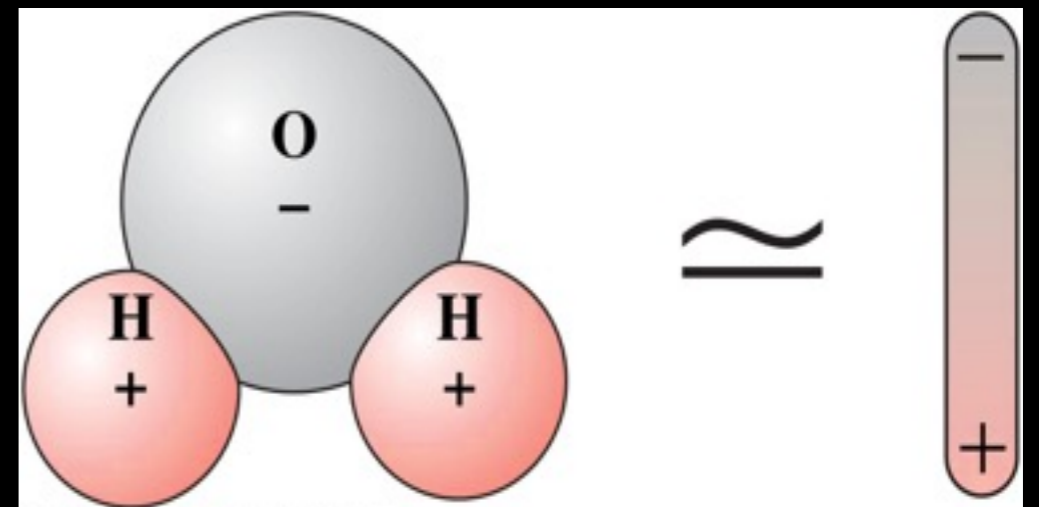
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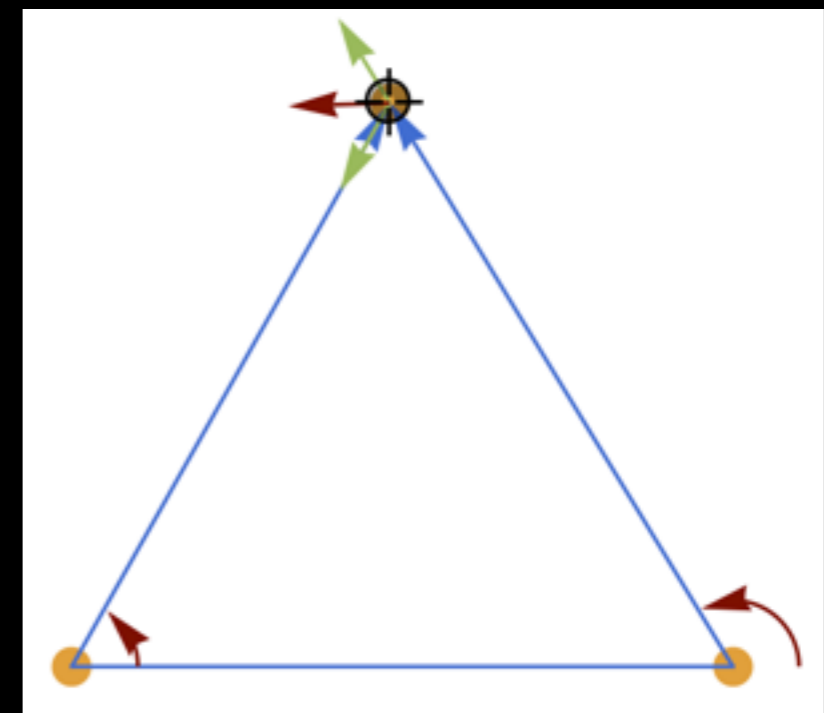
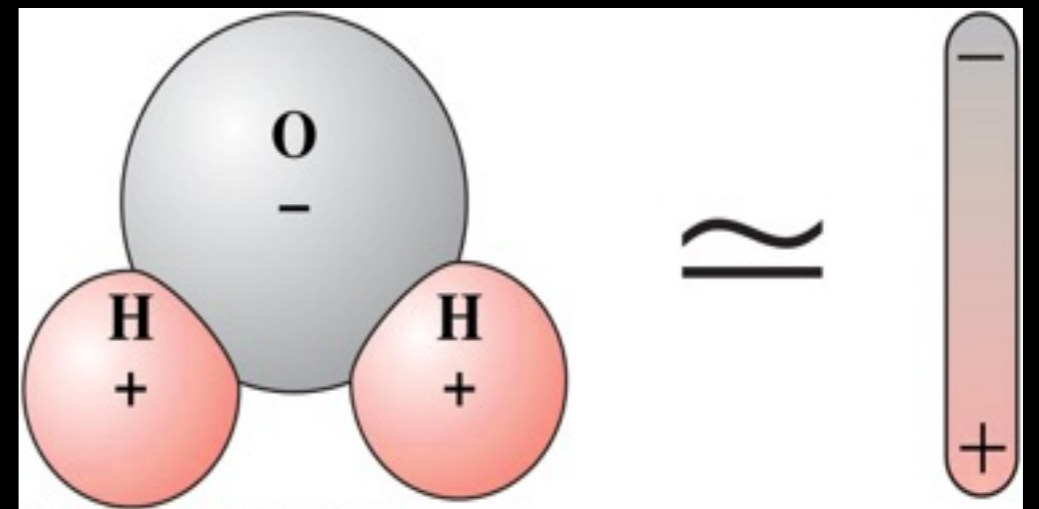
The electric dipole

Charge distributions that are 2 point charges of equal magnitude but opposite sign are **electric dipoles**.

Many molecules are dipoles.

The molecule is neutral but the separation of its charges means it has an electric field.

The field drops as you move further from the dipole.



The electric dipole

y-axis field component cancels,
net electric field is along x-axis.

$$\bar{\mathbf{E}} = \bar{\mathbf{E}}_{-,x} + \bar{\mathbf{E}}_{+,x}$$

$$= \frac{k(-q)}{r^2} \hat{\mathbf{r}}_{-,x} + \frac{k(q)}{r^2} \hat{\mathbf{r}}_{+,x}$$

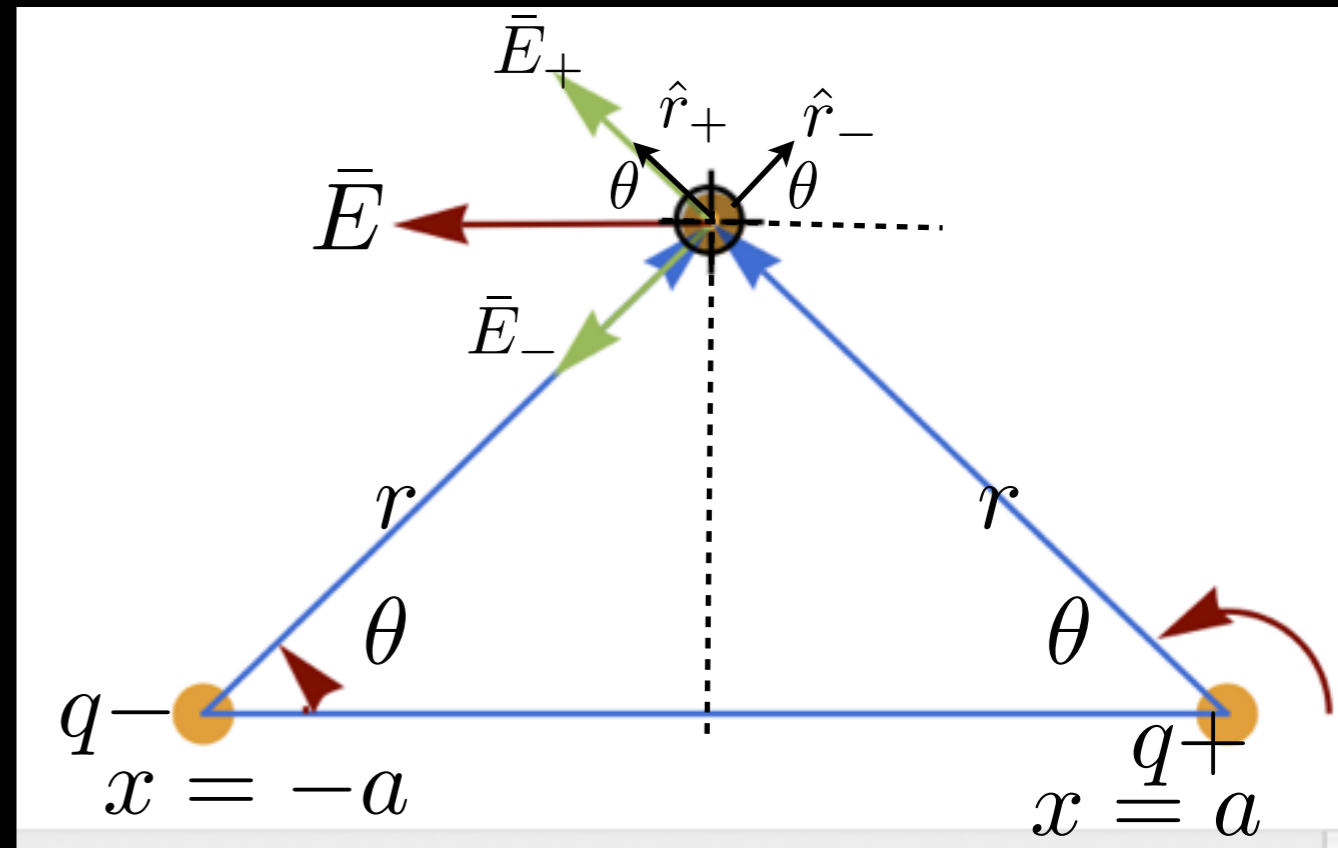
$$\hat{\mathbf{r}}_{-,x} = \hat{\mathbf{r}}_- \cdot \hat{\mathbf{i}} = \cos \theta = \frac{a}{r}$$

$$\hat{\mathbf{r}}_{+,x} = \hat{\mathbf{r}}_+ \cdot -\hat{\mathbf{i}} = -\cos \theta = -\frac{a}{r}$$

unit vectors, magnitude 1

$$\bar{\mathbf{E}} = \frac{k(-q)}{r^2} \left(\frac{a}{r} \right) \hat{\mathbf{i}} + \frac{kq}{r^2} \left(-\frac{a}{r} \right) \hat{\mathbf{i}} = -\frac{2kqa}{(a^2 + y^2)^{3/2}} \hat{\mathbf{i}}$$

$$\bar{\mathbf{E}} = -\frac{2kqa}{y^3} \hat{\mathbf{i}} \quad y \gg a$$



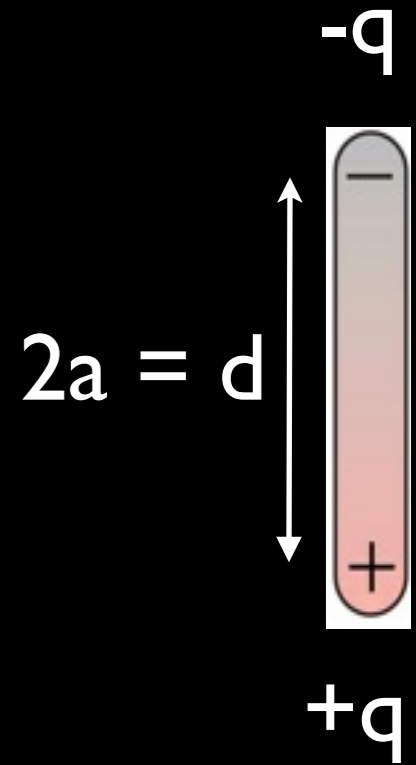
Dipole field decreases faster
than point charge field.

The electric dipole

$$\vec{E} = -\frac{kq2a}{y^3}\hat{i}$$

Dipole properties
(charge and distance
between charges)

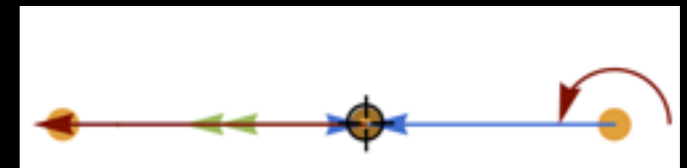
$$p = qd \quad \text{dipole moment}$$



$$\vec{E} = -\frac{kp}{y^3}\hat{i} \quad \text{dipole field for } y \gg a, \text{ perpendicular to dipole.}$$

But... calculating the dipole field along the dipole gives:

$$\vec{E} = -\frac{2kp}{y^3}\hat{i} \quad \text{orientation matters!}$$



Therefore, make the dipole
moment a vector:



The electric dipole

The water molecule's dipole moment is $6.2 \times 10^{-30} \text{ C} \cdot \text{m}$

What is the separation distance (d) if the molecule consists of 2 charges, $+e$ and $-e$?

(A) 0.039nm

(B) 0.02nm

(C) 0.1nm

(D) 0.15nm

$$p = qd$$

$$d = \frac{p}{q}$$

$$= \frac{6.2 \times 10^{-30} \text{ C} \cdot \text{m}}{1.6 \times 10^{-19} \text{ C}}$$

$$= 0.039 \text{ nm}$$

Continuous charge distribution

A typical piece of matter might contain 10^{23} particles.

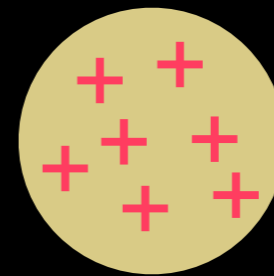


We could add these to find the electric field...

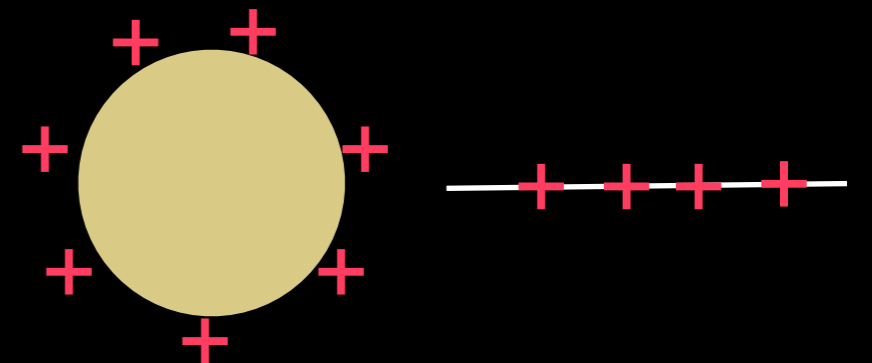
... but it would be slow!

Assume charge is spread evenly through matter.

If charge is spread through a volume, we use **volume charge density** [C/m^3]



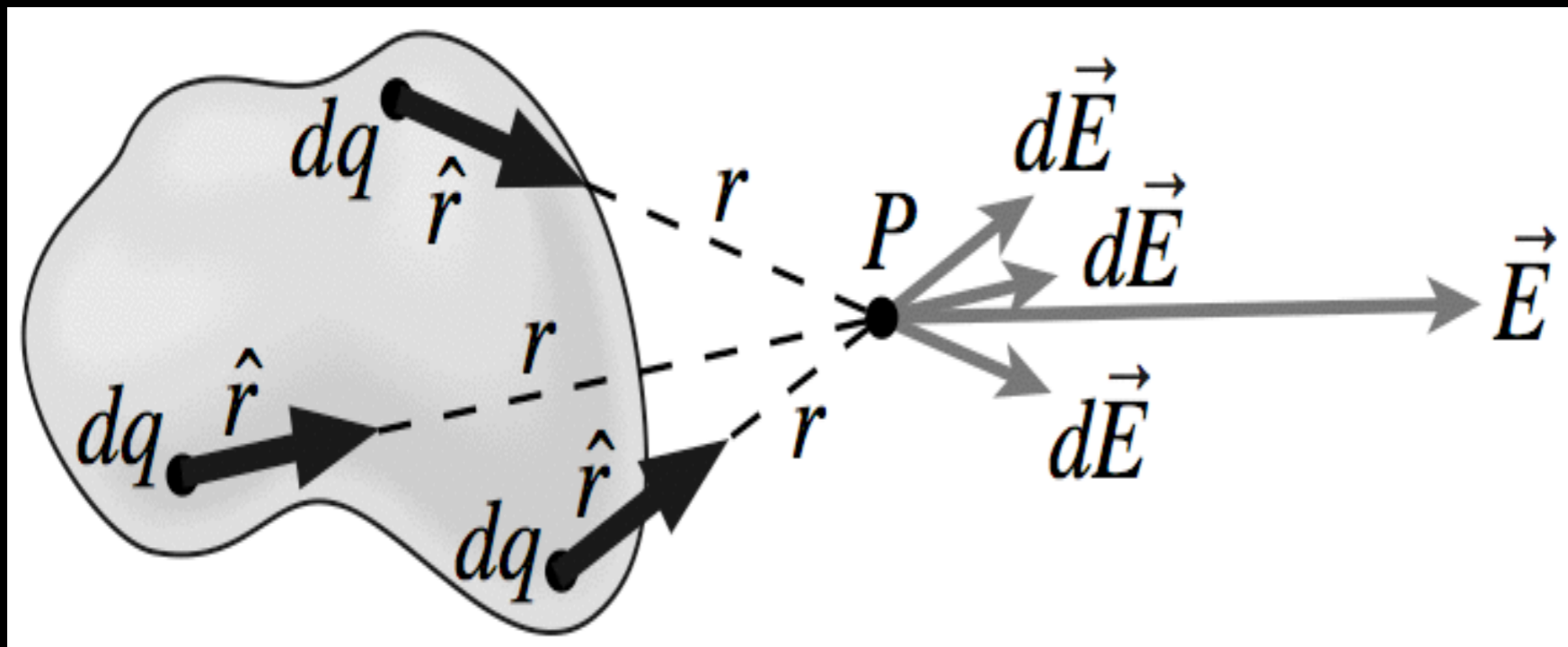
If charge is spread over a surface or a line, we use **surface charge density** [C/m^2]
or **line charge density** [C/m]



Continuous charge distribution

Divide the charged region into small elements, dq .

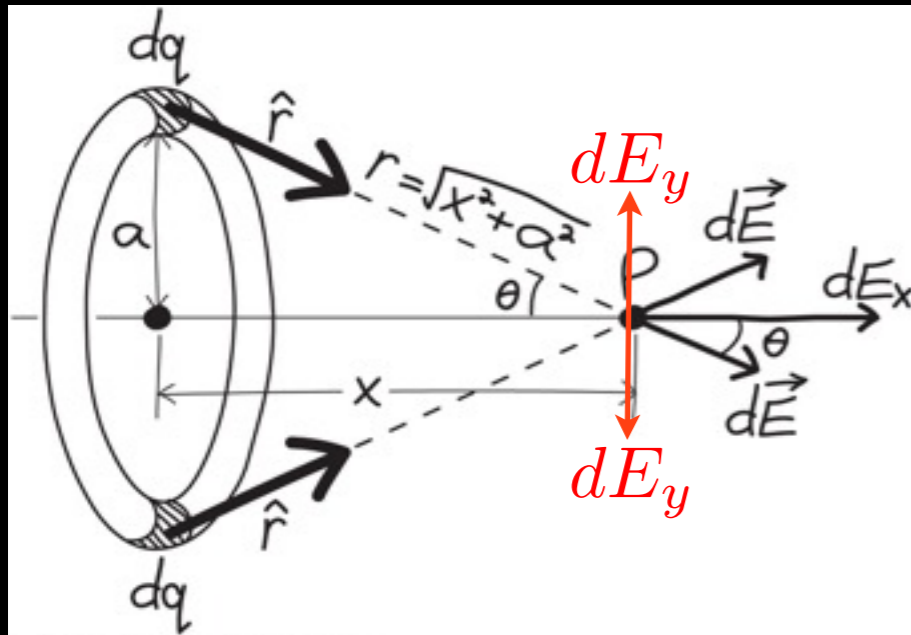
Then, treat like point charges, each with field $d\vec{E} = (kdq/r^2)\hat{r}$



$$\vec{E} = \sum_i d\vec{E} = \int d\vec{E} = \int \frac{kdq}{r^2} \hat{r}$$

Continuous charge distribution

The charged ring



A ring of radius a has a charge Q spread evenly over the ring.

What is the electric field at P?

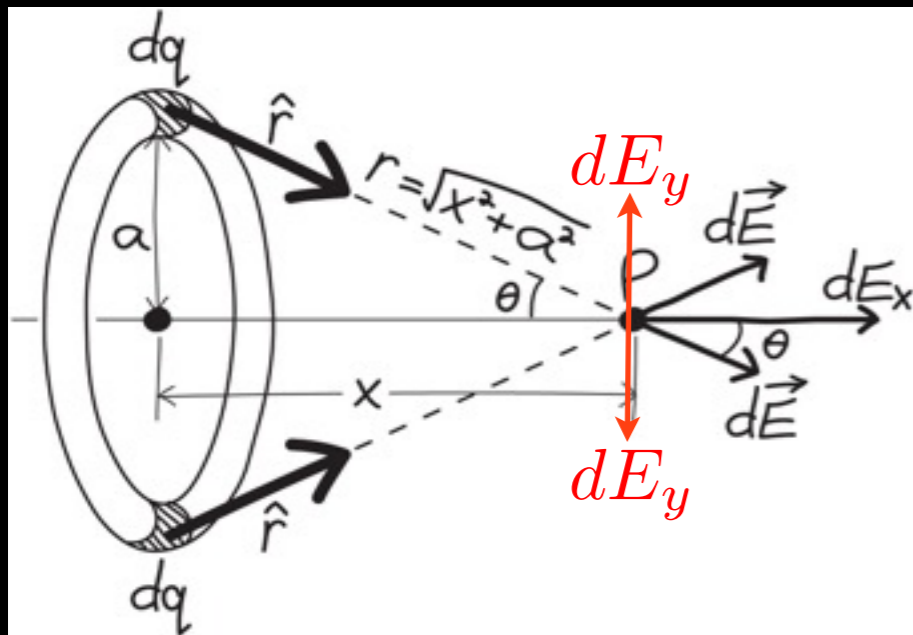
Which electric field component cancels?

(A) x- component = 0

(B) y- component = 0

Continuous charge distribution

The charged ring



A ring of radius a has a charge Q spread evenly over the ring.

What is the electric field at P?

y-component from opposite sides cancels. Net field in x-direction only.

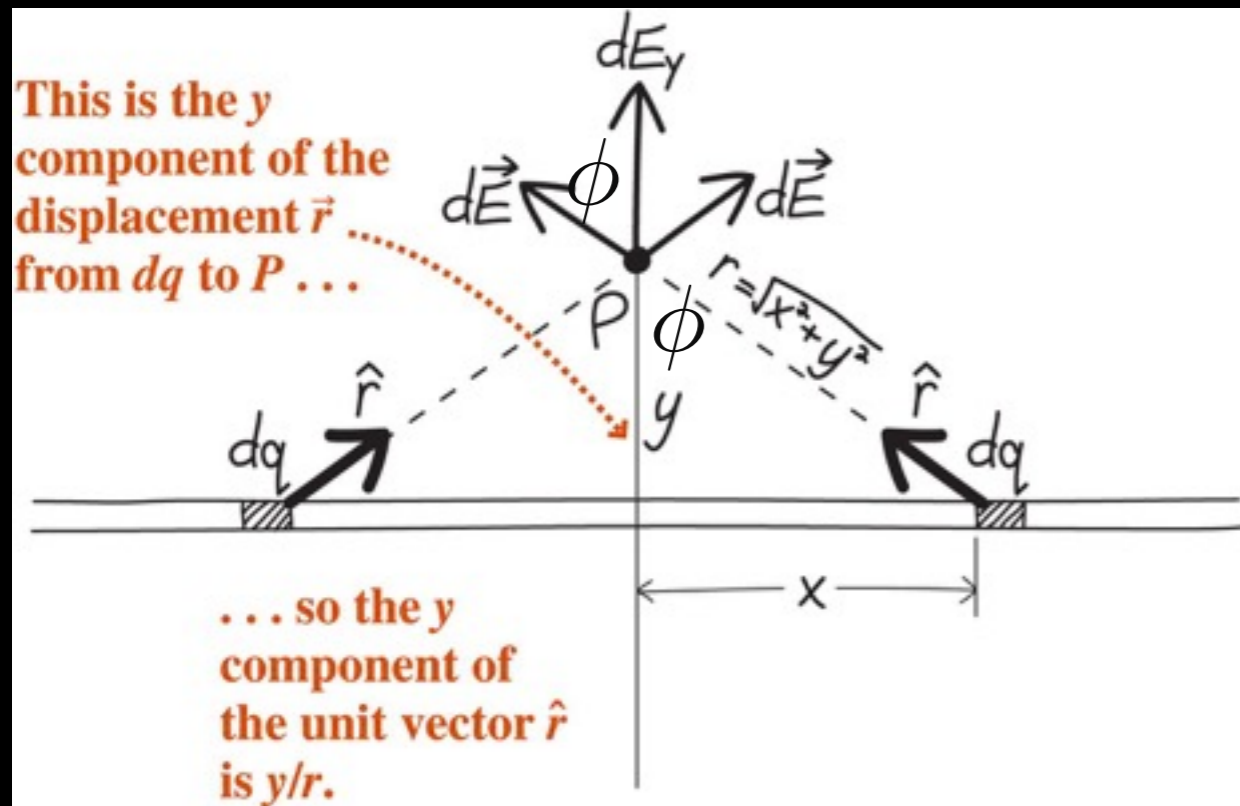
$$dE_x = dE \cdot \hat{i} = dE \cos \theta$$

$$\cos \theta = \frac{x}{\sqrt{x^2 + a^2}}$$

$$\begin{aligned} E &= \int_{\text{ring}} dE_x = \int_{\text{ring}} \frac{kx dq}{(x^2 + a^2)^{3/2}} = \frac{kx}{(x^2 + a^2)^{3/2}} \int_{\text{ring}} dq \\ &= \frac{kQx}{(x^2 + a^2)^{3/2}} \end{aligned}$$

Continuous charge distribution

Line charge



Long straight electric power line with uniform line density λ [C/m]

What is the electric field at P ?

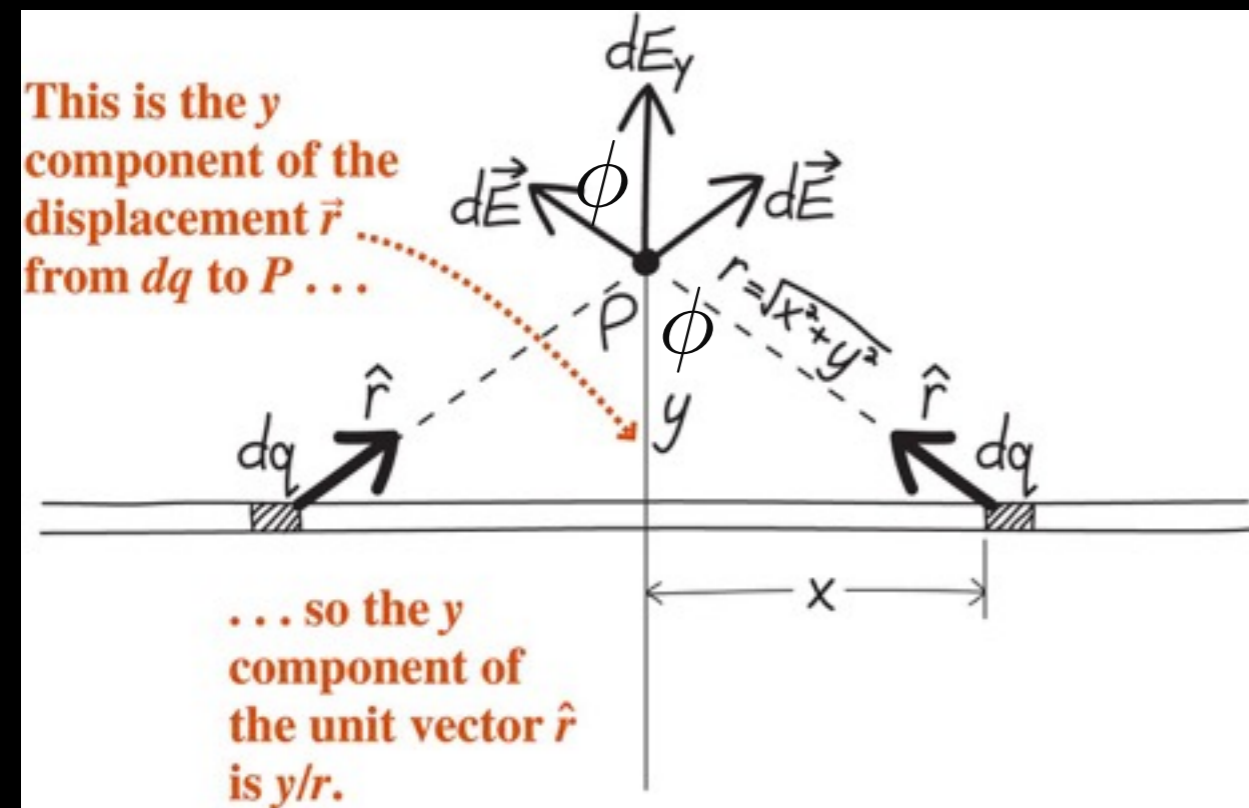
Which electric field component cancels?

(A) x - component = 0

(B) y - component = 0

Continuous charge distribution

Line charge



Long straight electric power line with uniform line density λ [C/m]

What is the electric field at P ?

x -component from opposite sides cancels. Net field in y -direction only.

$$dE_y = dE \cdot \hat{j} = dE \cos \phi \quad \cos \phi = \frac{y}{\sqrt{x^2 + y^2}} \quad dq = \lambda dx$$

$$d\bar{E}_y = \frac{k dq}{r^2} \frac{y}{\sqrt{x^2 + y^2}} = \frac{k \lambda y}{(x^2 + y^2)^{3/2}} dx$$

$$E = \int_{-\infty}^{+\infty} dE_y = \frac{2k\lambda}{y}$$

Matter in electric fields

A point charge q in an electric field obeys both

the electric force: $\vec{F} = q\vec{E}$

and Newton's law: $\vec{F} = m\vec{a}$

Combining gives the acceleration: $\vec{a} = \left(\frac{q}{m}\right)\vec{E}$

charge : mass ratio

Shows how easily particle accelerates in an electric field.

Electrons ~ 2000 x less massive than protons, but carrying same q are easy to accelerate.

Matter in electric fields

Quiz

A proton, an electron, a carbon-13 nucleus (6 protons, 7 neutrons) and a helium-4 nucleus (2 protons, 2 neutrons) are in a uniform electric field.

Which has the 2nd highest acceleration?

(proton mass \sim neutron mass)

(A) The proton

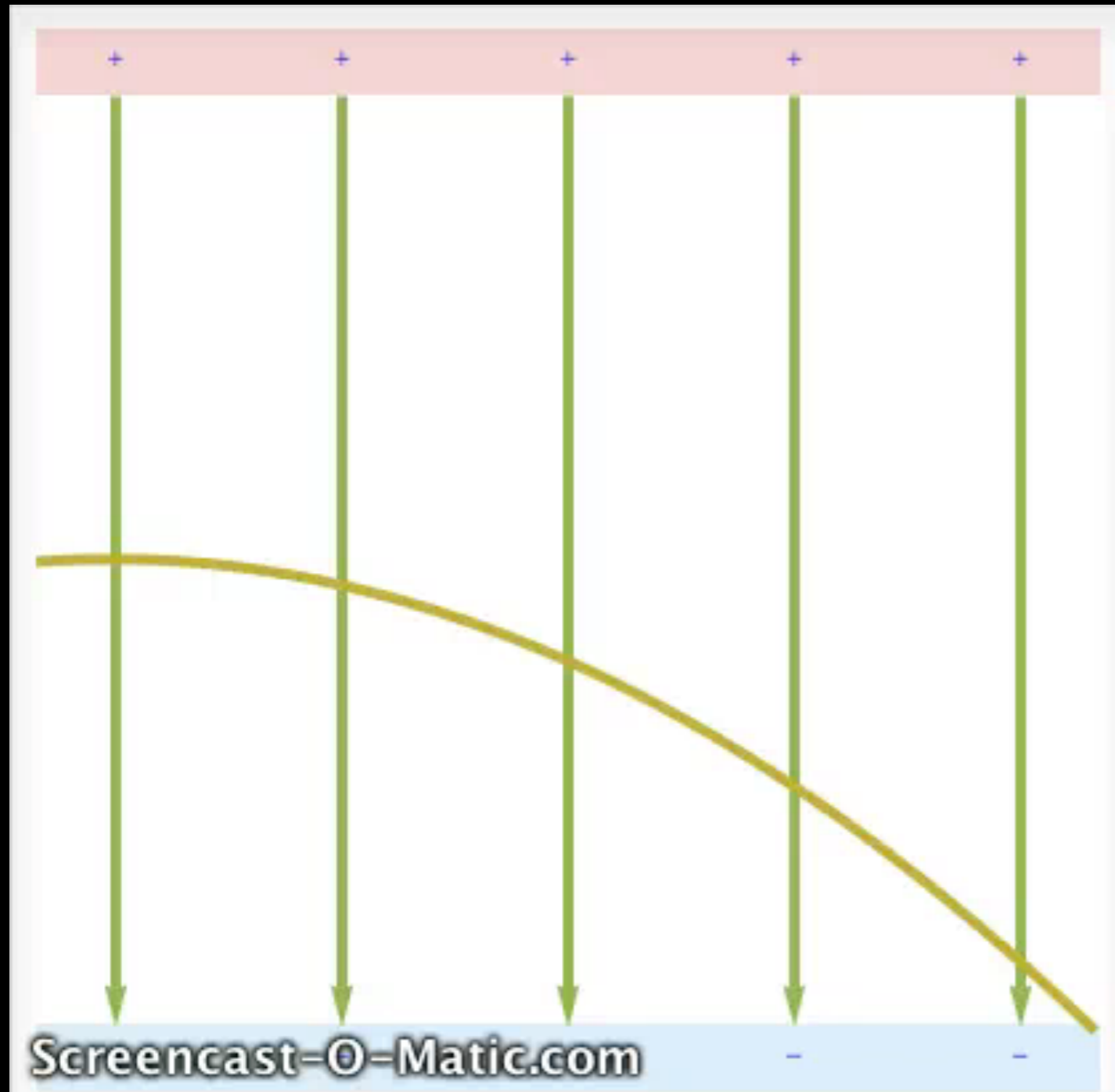
(B) The carbon-13 nucleus

(C) The electron

(D) The helium-4 nucleus

Matter in electric fields

In a uniform electric field, electron motion is a constant-acceleration problem.



An ink jet printer uses oppositely charged plates to 'steer' the electrons to the right place.

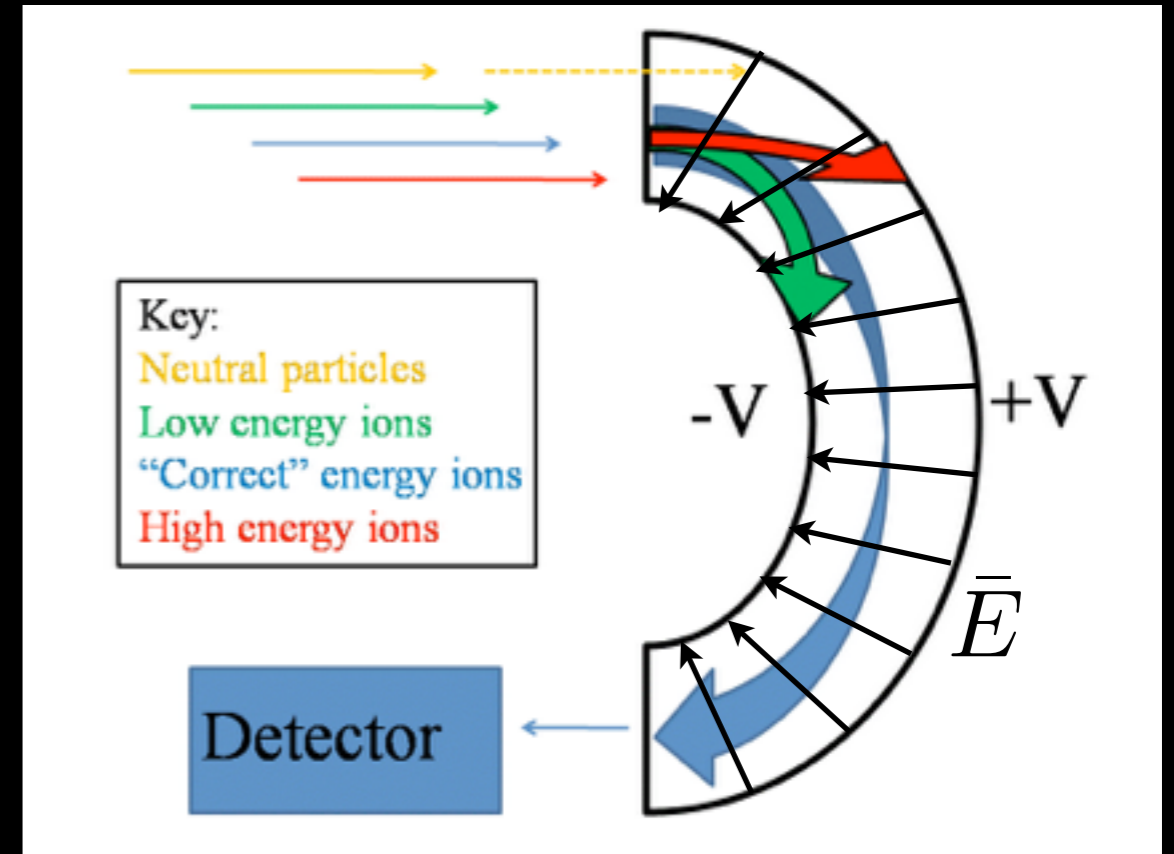
Matter in electric fields

In a radial field, a particle can perform uniform circular motion.

If the particle moves too fast, it hits the outer wall.

Too slow and it hits the inner wall.

Correct energy and the particle will exit horizontally.



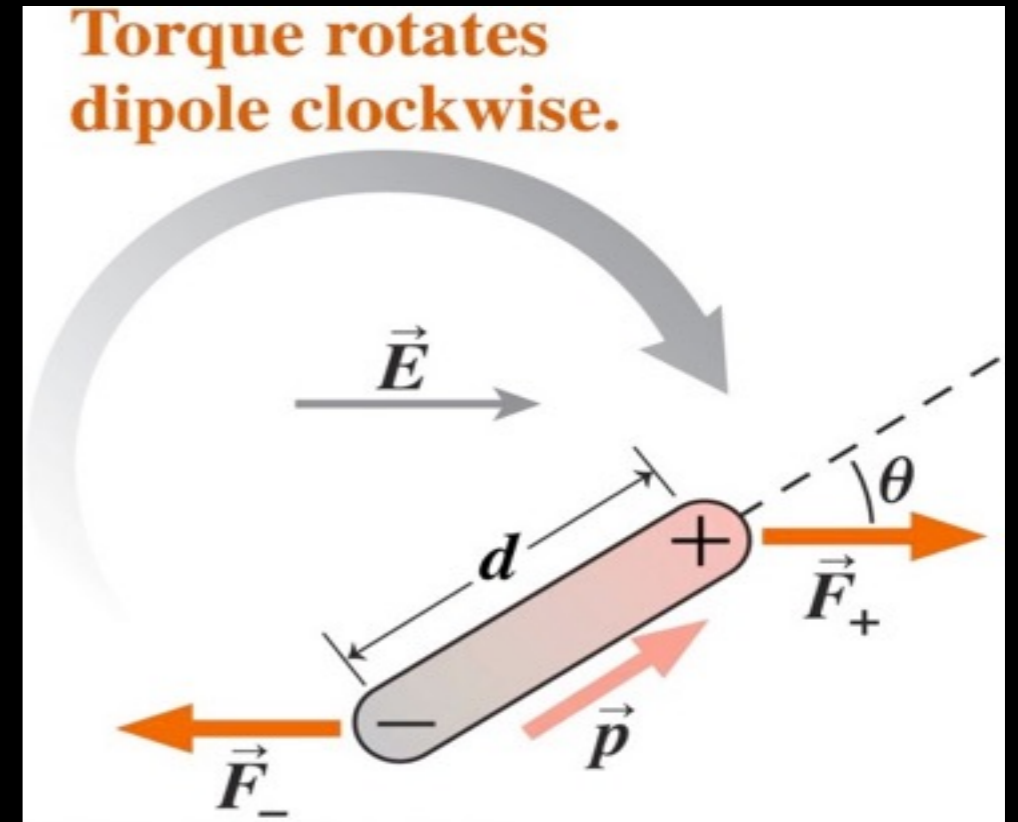
If $E = E_0 \frac{b}{r}$ where E_0 and b are constants:

$$a = \frac{v^2}{r} = \frac{eE}{m} = \frac{e}{m} E_0 \frac{b}{r} \longrightarrow v = \sqrt{eE_0 b / m}$$

Dipoles in electric fields

A dipole in an uniform electric field will experience a force at both ends.

Because the ends have opposite charge, the forces are equal and opposite.



This gives a torque (turning force) $\vec{\tau} = \vec{r} \times \vec{F}$

Magnitude: $\tau = rF \sin \theta$

The dipole will turn to align with the electric field.

Dipoles in electric fields

The force on the positive end of the dipole:

$$\tau_+ = rF \sin \theta = \left(\frac{1}{2}d\right)(qE) \sin \theta$$

The torque is the same on the negative end, giving a net torque:

$$\tau = qdE \sin \theta$$

But the dipole moment: $\bar{p} = qd$

$$\tau = pE \sin \theta$$

Which is the vector (cross) product:

$$\bar{\tau} = \bar{p} \times \bar{E}$$

Dipoles in electric fields

Work done in rotating dipole (from right angle to field)

$$W = \int_{\pi/2}^{\theta} \tau d\theta \quad (\text{Force x distance})$$

$$= \int_{\pi/2}^{\theta} pE \sin \theta d\theta$$

$$= -pE \cos \theta$$

The energy goes into potential energy, U:

$$U = -pE \cos \theta = -\vec{p} \cdot \vec{E}$$

Conductors & insulators

Materials in which charges **can** move freely are **conductors**.

Materials in which charges **cannot** move freely are **insulators**.

Insulators often contain molecular dipoles which feel torques in an electric field (= dielectrics).

Molecules that are not dipoles can become dipoles when stretched by the electric field.

Alignment of the dipole reduces the applied field inside the material

