Essential Physics II

英語で物理学の

エッセンス II

Lecture 4:   19-10-15

News

Last week's homework:

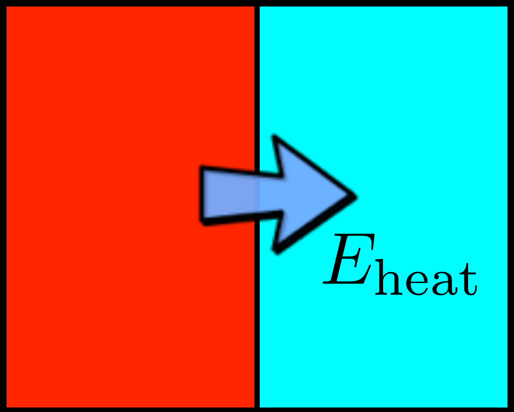
2015 / 10 / 26 (１０月２６日)

This week's homework:

Also due: 2015 / 10 / 26 (１０月２６日)

on http://masteringphysics.com

Last lectures...



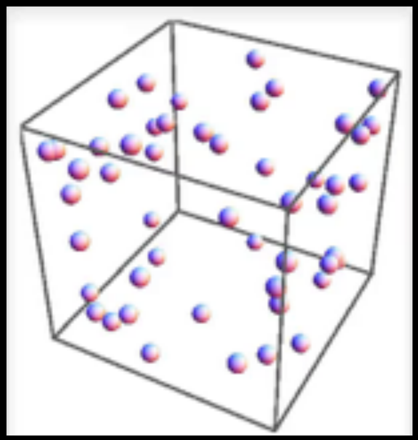
Temperature difference, ΔT → internal energy increases, ΔU

↑

moving energy: heat

E_{heat}

Ideal gases:



Ideal gas law: $pV = nRT$

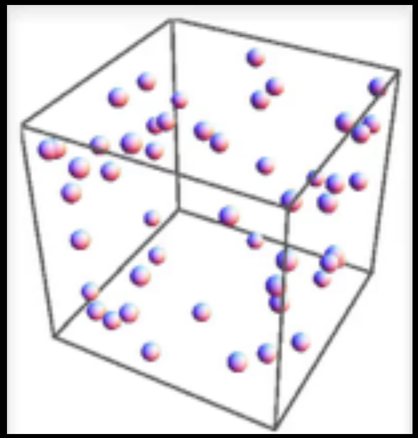
mols

$R = 8.314\,\mathrm{J/K\cdot mol}$

T measures K for random motion of molecules: $\frac{3}{2}kT = \frac{1}{2}m\bar{v}^2$

Last lectures...

Quiz



An ideal gas has volume, V and pressure, p.

The thermal speed of the gas molecules is v.

If both volume and pressure are doubled to 2V and 2p, what is the thermal speed of the gas molecules?

(a) v

(b) 2v

(c) 4v

(d) v/2

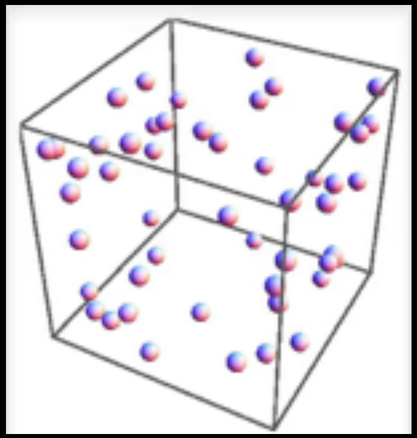
(e) v/4

Last lectures...

Quiz

An ideal gas has volume, V and pressure, p.

The thermal speed of the gas molecules is v.



If both volume and pressure are doubled to 2V and 2p, what is the thermal speed of the gas molecules?

(a) v

(b) 2v

(c) 4v

(d) v/2

(e) v/4

Ideal gas law: $pV = nRT \longrightarrow T_1 = \dfrac{p_1V_1}{nR}$

$$T_2 = \frac{p_2V_2}{nR} = \frac{4p_1V_1}{nR} = 4T_1$$

Since: $\dfrac{3}{2}kT = \dfrac{1}{2}m\bar{v}^2 \longrightarrow \bar{v} = \sqrt{\dfrac{3kT}{m}}$

$$\bar{v_2} = \sqrt{\frac{3kT_2}{m}} = \sqrt{\frac{3k(4T_1)}{m}} = 2\bar{v_1}$$

Heat and Work

Joule's apparatus Quiz

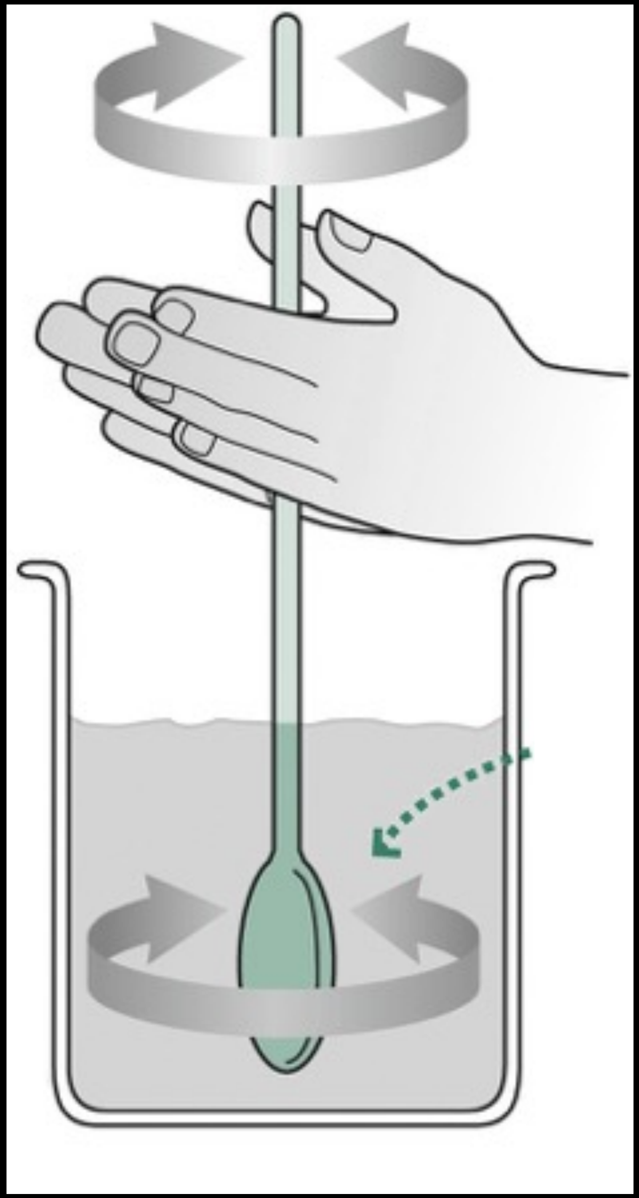
You stir water (vigorously) with a spoon.

The water's temperature, T ...

(A) increases

(B) decreases

(C) unchanged



No temperature difference between spoon and water: $\Delta T = 0$

No heat is transferred. Spoon does work on the water.

Mechanical work → increases water's internal energy

Joule’s apparatus

Joule's apparatus

Quiz

What is the temperature increase, ΔT ?

(Assume all energy is converted to heat)

$$c_{\text{oil}} = 1800\,\mathrm{J/kg\cdot K}$$

(A) $10^\circ\mathrm{C}$

(B) $1^\circ\mathrm{C}$

(C) $0.1^\circ\mathrm{C}$

(D) $0.01^\circ\mathrm{C}$

(E) $0^\circ\mathrm{C}$

Joule's apparatus

Quiz

What is the temperature increase, ΔT ?

(Assume all energy is converted to heat)

$$c_{\mathrm{oil}} = 1800\,\mathrm{J/kg \cdot K}$$

Potential energy: $\Delta U = mg\Delta h$

$$= (2\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(1\,\mathrm{m})$$

$$= 19.62\,\mathrm{J}$$

Heat energy: $Q = m_{\mathrm{oil}} c \Delta T$

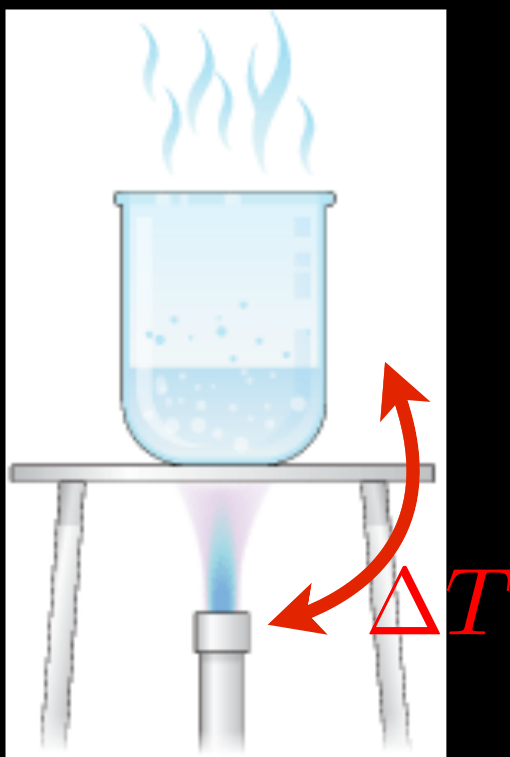
$$\Delta T = \frac{19.62\,\mathrm{J}}{(1\,\mathrm{kg})(1800\,\mathrm{J/kg \cdot K})} \simeq 0.01\,\mathrm{K}$$

$$= 0.01^{\circ}\mathrm{C}$$

(max)

Joule's apparatus

Temperature can be increased by...

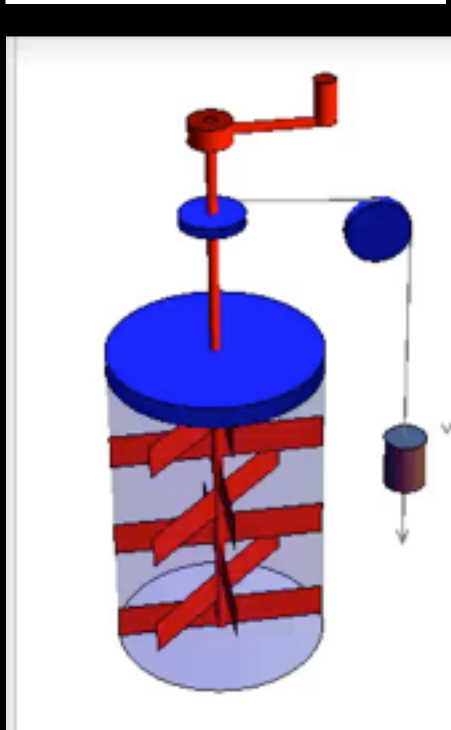


Heating:

Temperature difference between flame and water

Heat energy → internal energy

$Q = mc\Delta T$ higher T



Doing work:

Water is stirred

Mechanical energy → internal energy

W higher T

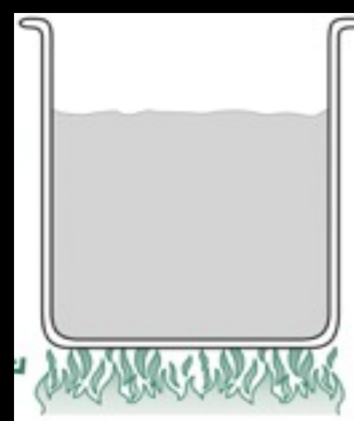
Same final state: T is higher

First Law of Thermodynamics

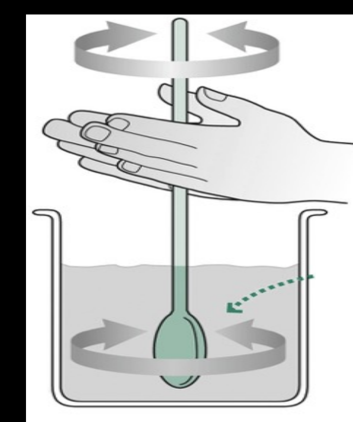
Change in internal energy

$$\Delta U = Q + W$$

heat transferred



work done on the system



1st law of thermodynamics:

The **change in internal energy** of a system depends only on the **net heat** transferred to the system and the **net work done** on the system, independent of the particular processes involved.

How the energy moves is **not important**

First Law of Thermodynamics

$$\Delta U = Q + W$$

Rate of energy flow

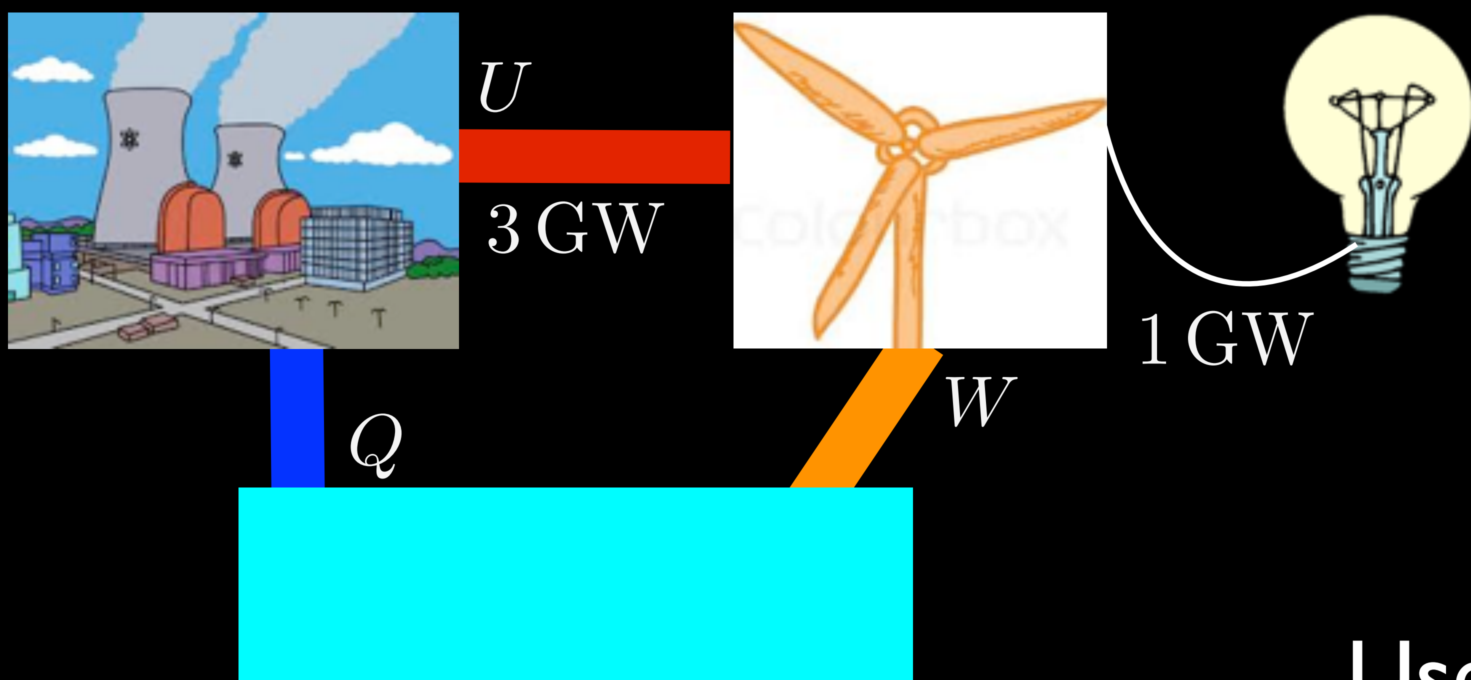
$$\frac{dU}{dt} = \frac{dQ}{dt} + \frac{dW}{dt}$$

Rate of change of internal energy

Rate of heat transfer

Rate at which work is done on the system

First Law of Thermodynamics Ex.



A power plant supplies energy at a rate of 3.0 GW.

↓ steam

drives turbine for electricity

↓

Used steam is cooled in the water

If the power plant produces electrical energy at a rate of 1.0 GW, what is the **rate of heat transfer** to the water?

$$\frac{dU}{dt} = \frac{dQ}{dt} + \frac{dW}{dt} \rightarrow \frac{dQ}{dt} = \frac{dU}{dt} - \frac{dW}{dt} = -3.0\,\mathrm{GW} - (-1.0\,\mathrm{GW})$$

extracting energy

doing work

$$= -2.0\,\mathrm{GW}$$

First Law of Thermodynamics Quiz

A heat source supplies heat to a gas at a rate of 187.0 W

The gas does work at a rate of 130.9 W

What rate does the internal energy (dU/dt) of the gas change?

(a) 56.1 W

(b) 318 W

(c) -56.1 W

(d) 187 W

$$\frac{dU}{dt} = \frac{dQ}{dt} + \frac{dW}{dt}$$

$$= 187\,\mathrm{W} - 130.9\mathrm{W}$$

$$= 56.1W$$

First Law of Thermodynamics Quiz

A gas expands at constant T to twice (x2) its original volume.

During the expansion, the gas absorbs 200 kJ of heat.

What is the change in internal energy of the gas during the expansion?

(a) 0 kJ

(b) 100 kJ

(c) 200 kJ

(d) 400 kJ

What measures internal energy (U) ?

Temperature (T).

Does T change? No.

Therefore does U change? No.

First Law of Thermodynamics Quiz

A gas expands at constant T to twice (x2) its original volume.

During the expansion, the gas absorbs 200 kJ of heat.

How much work does the gas do during the expansion?

$$\Delta U = Q + W$$

(a) 0 kJ

$$0 = 200\,\mathrm{kJ} + W$$

(b) 100 kJ

(c) 200 kJ

work done on the gas: $W = -200\,\mathrm{kJ}$

(d) 400 kJ

work done by the gas: $200\,\mathrm{kJ}$

Reversible & Irreversible

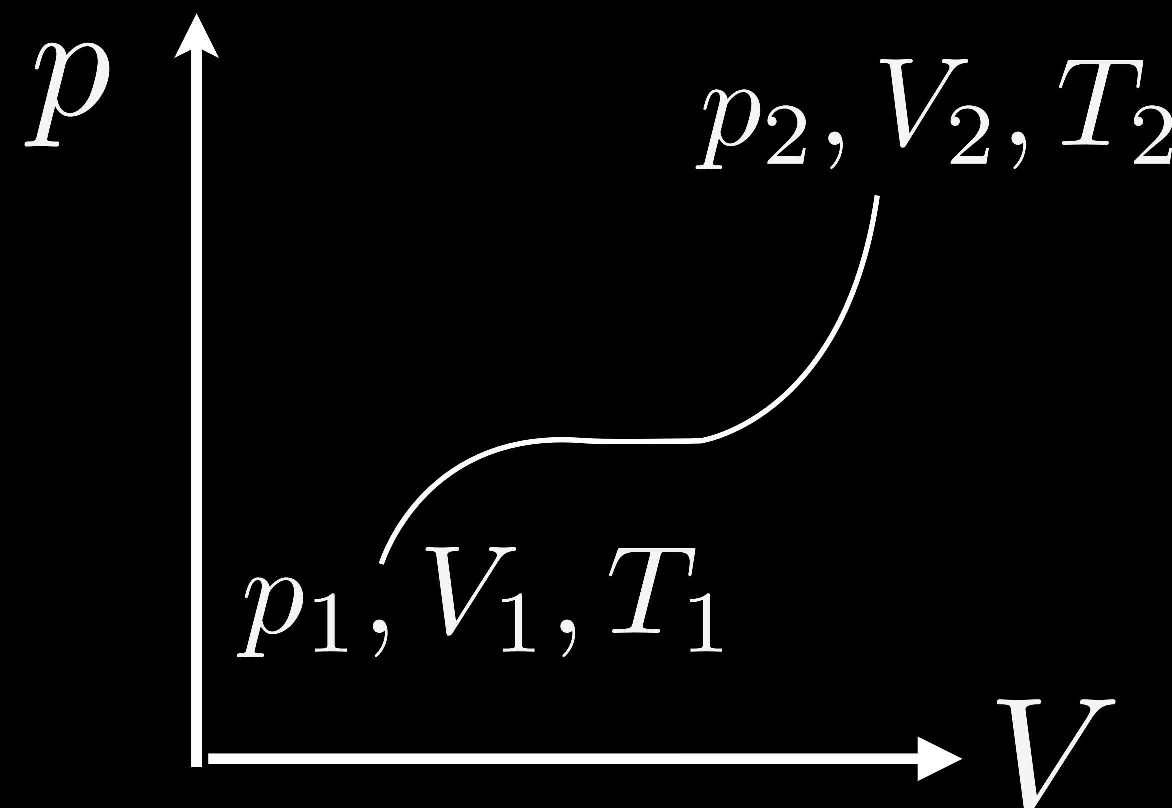
$\Delta U = Q + W$ First law applies to *any* system

but...

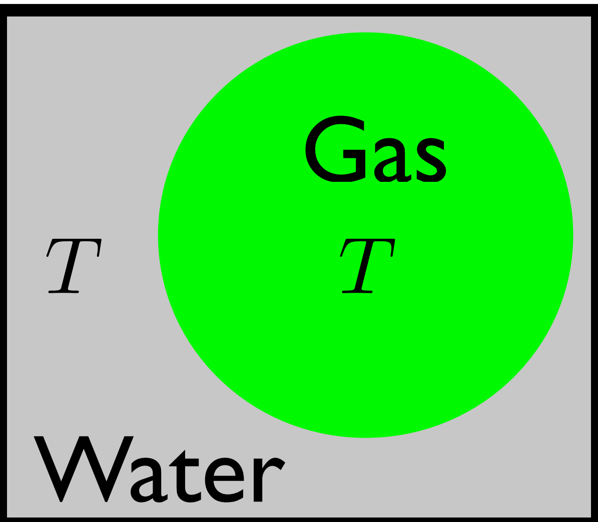
Ideal gas: $pV = nRT$

Only need 2 variables
[(p, V) or (V, T) or (T, p)]

can use pV diagram to show state:



Reversible Process



System: Gas ball immersed (covered by) water in equilibrium

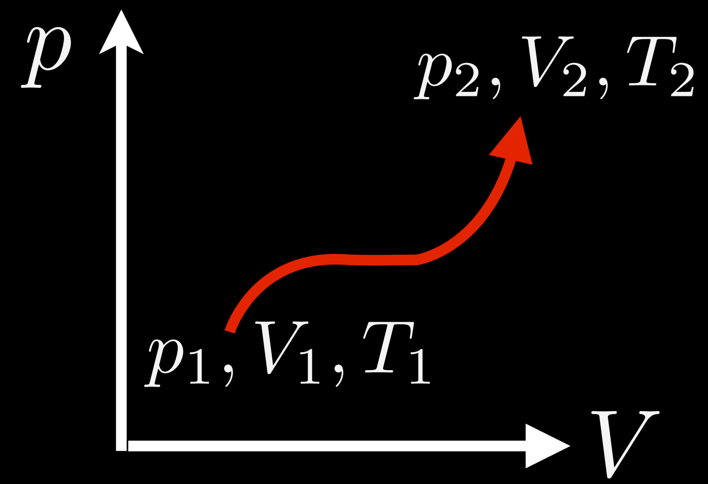
If water T increases *slowly*:

Gas and water change T together

Always in equilibrium

Change follows curve

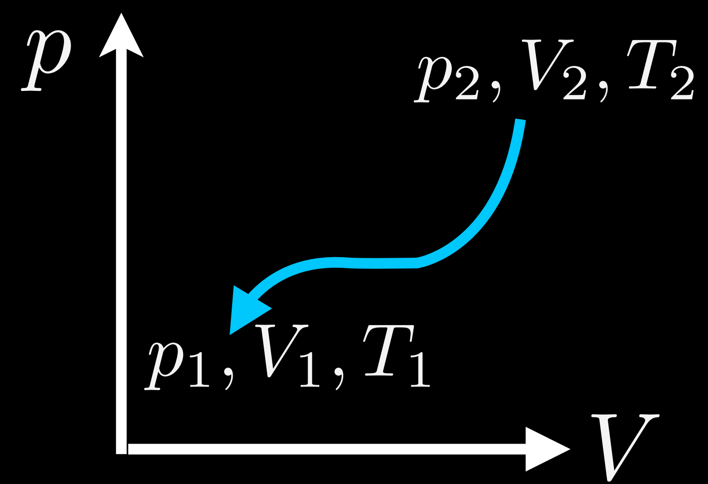
quasi-static process



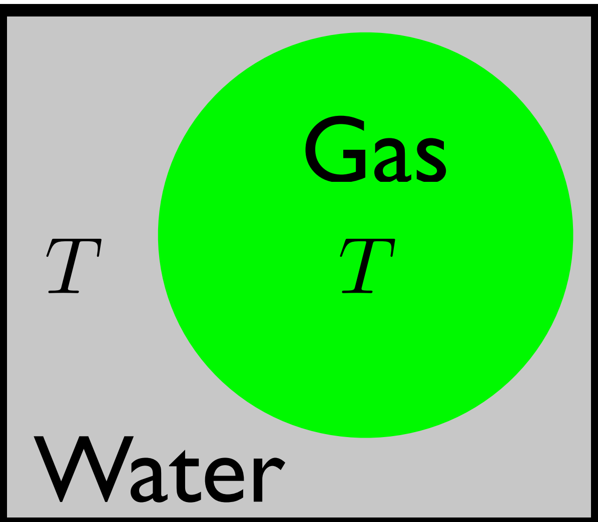
If water T decreases *slowly*:

Change follows same curve

reversible process



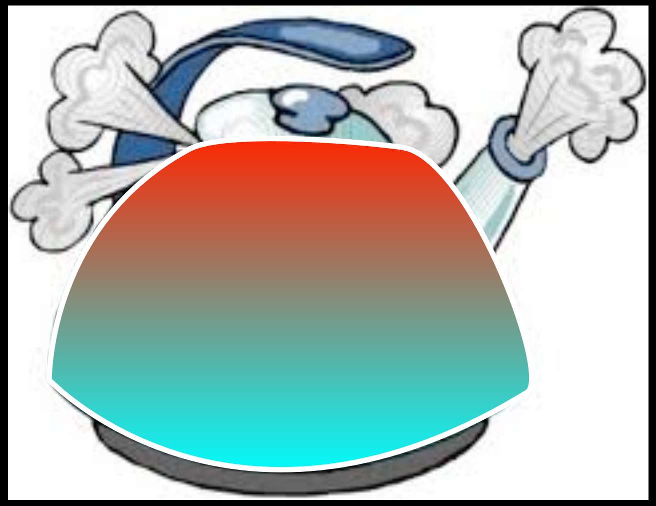
Irreversible Process



System: Gas ball immersed (covered by) water in equilibrium

If water T increases *quickly*:

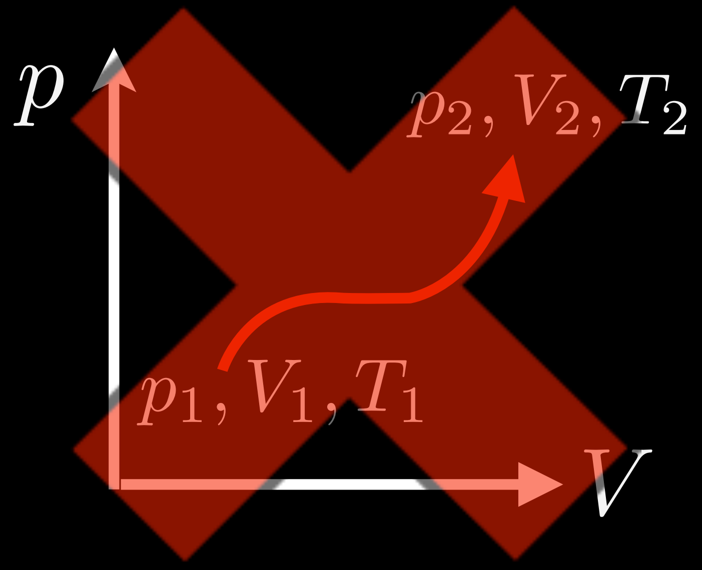
e.g. pour boiling water over cold gas ball



Gas and water NOT in equilibrium

Different p and T in different regions

→ not well-defined values



→ irreversible process

Work

Ideal gas: $pV = nRT$

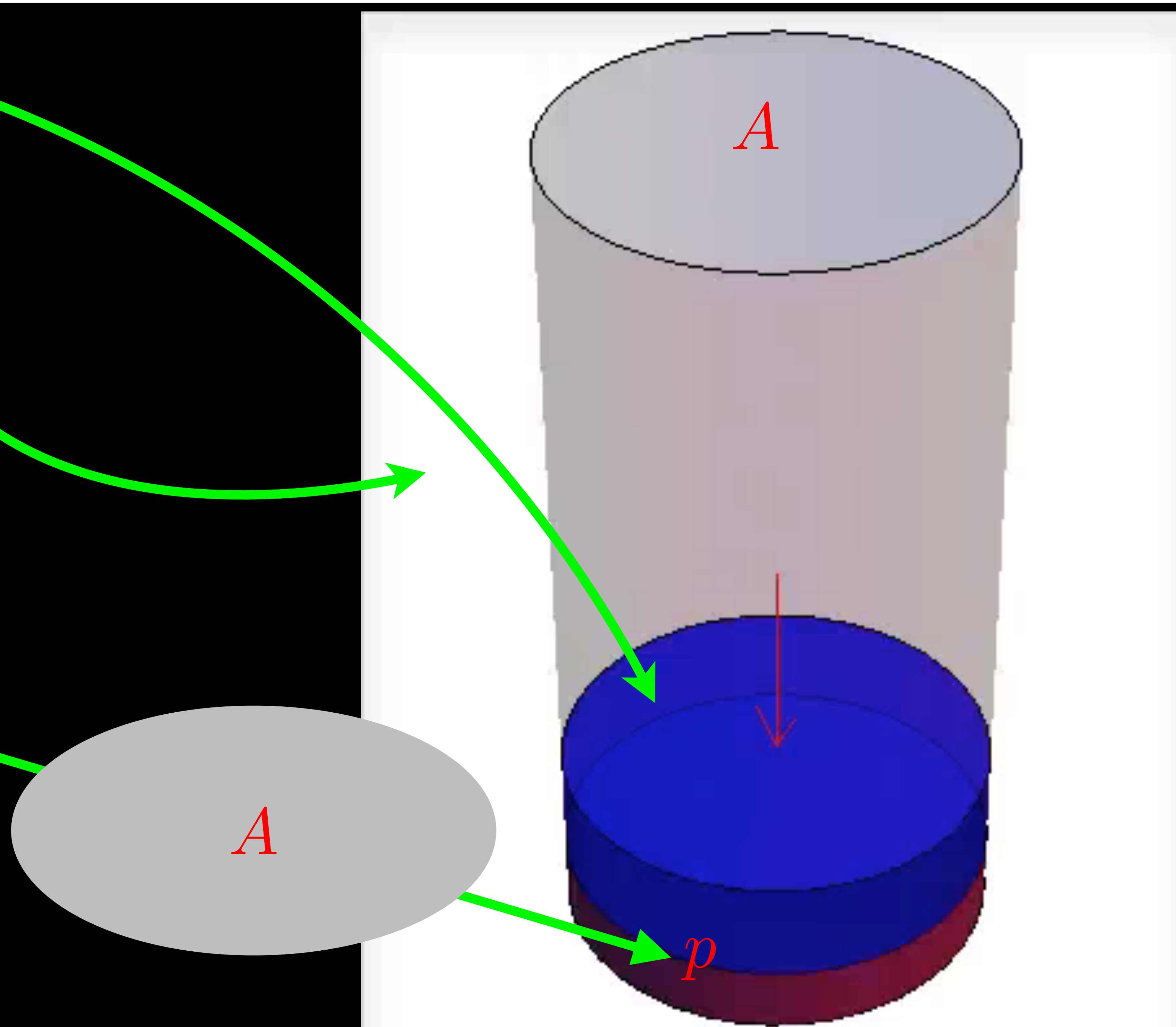
Insulated cylinder
(no heat loss, $E_{\mathrm{out}} = 0$)

Volume can change

Pressure, p

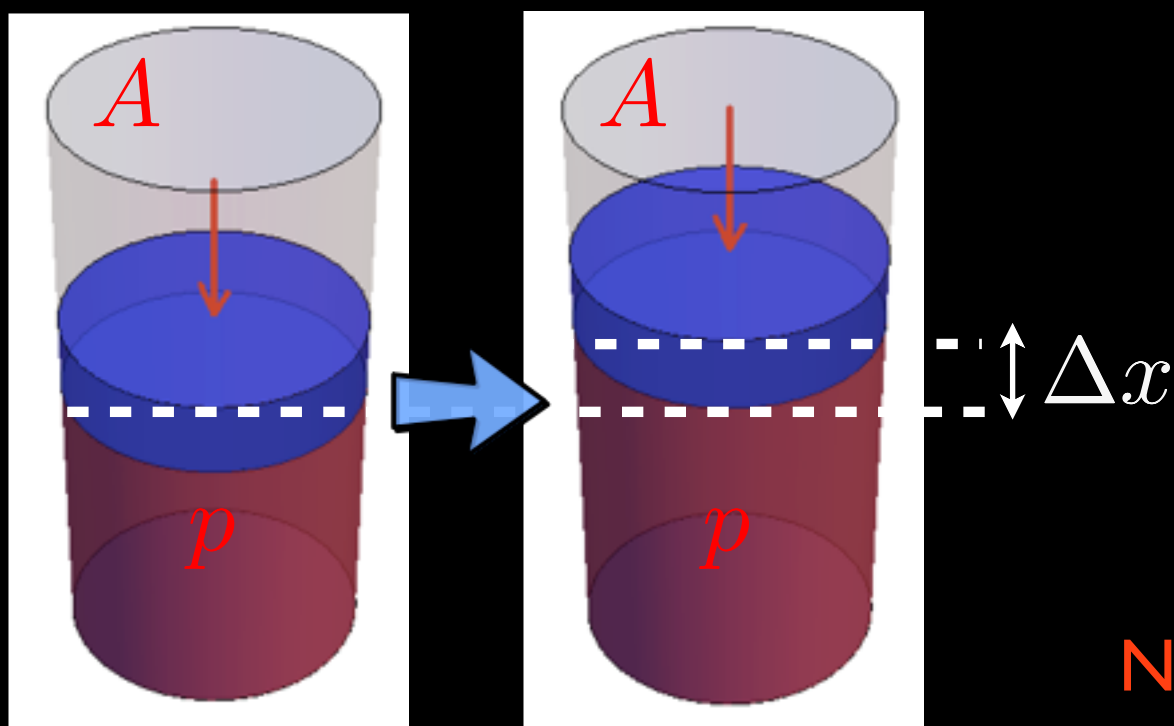
Cross-section area, A

Reversible process



How much work is done on the gas?

Work



Force from gas: $F_{\text{gas}} = pA$

Gas does work: $\Delta W_{\text{gas}} = F_{\text{gas}} \Delta x$

$$= pA\Delta x$$

$$= p\Delta V$$

Newton's 3rd Law

Work done *on* the gas: $\Delta W = -\Delta W_{\text{gas}} = -p\Delta V$

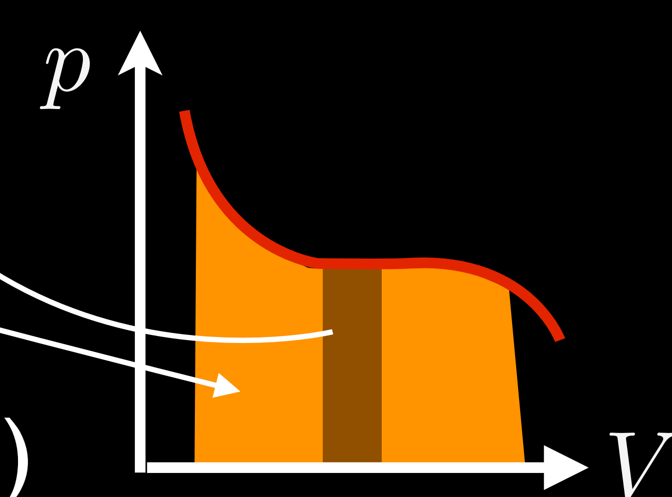
But... p maybe changes as V changes: $dW = -pdV$

small volume change:
p ~ constant

Total work: $W = \int dW = -\int_{V_1}^{V_2} pdV$

= - (area under pv curve)

(+ if gas is compressed $V_2 < V_1$, - if expands $V_1 < V_2$)



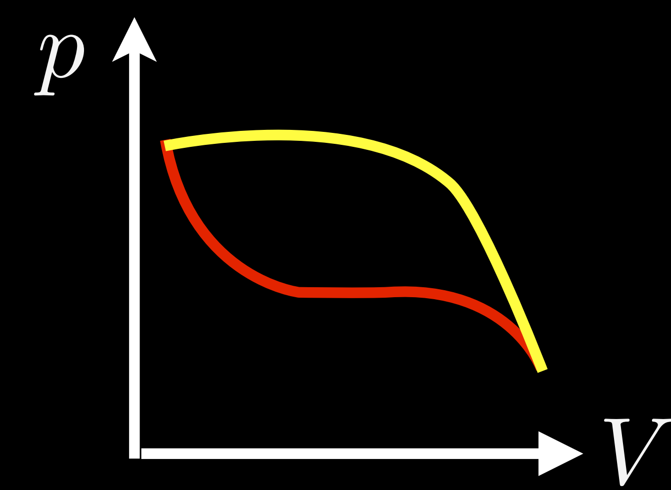
Work

Quiz

2 gas cylinders start and end in the same state.

$$(p_i, V_i) \to (p_f, V_f)$$

They move between the 2 states by different processes.



What is the same for both cylinders?

(a) work done on or by the gas

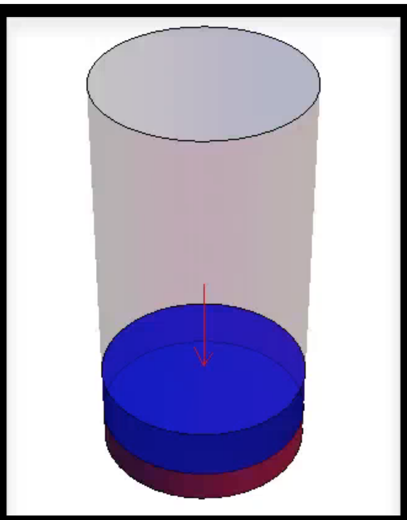
(b) heat added or removed

(c) change in internal energy

quantities that do not depend on the HOW you reach a state (path taken) = state variables

depends only on current p, V

Work



$$W = -\int_{V_1}^{V_2} p dV$$

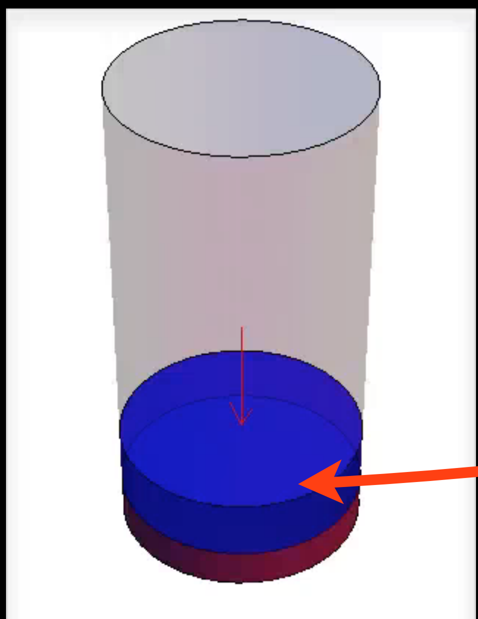
work done on gas during volume change

What happens when 1 of

p	(pressure)
V	(volume)
T	(temperature)
Q	(heat)

is constant (no change) ?

Isothermal (T = constant)

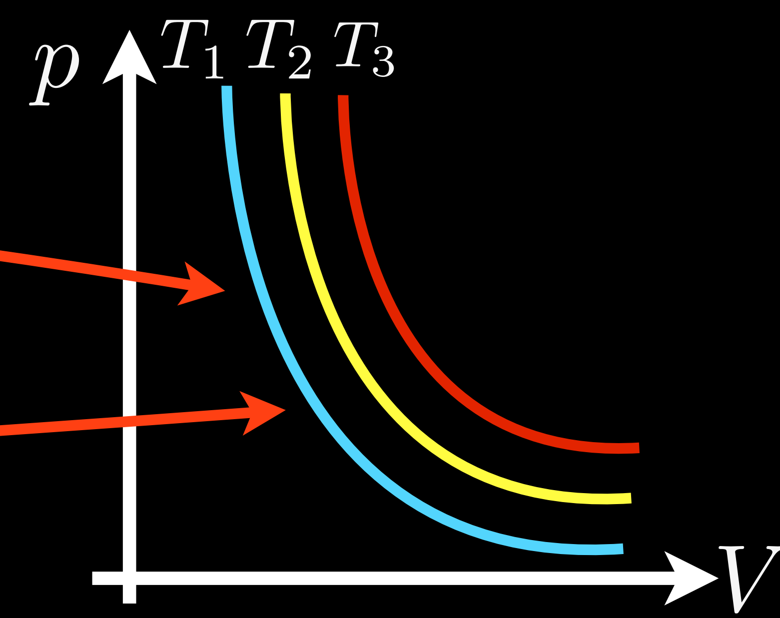


T = constant

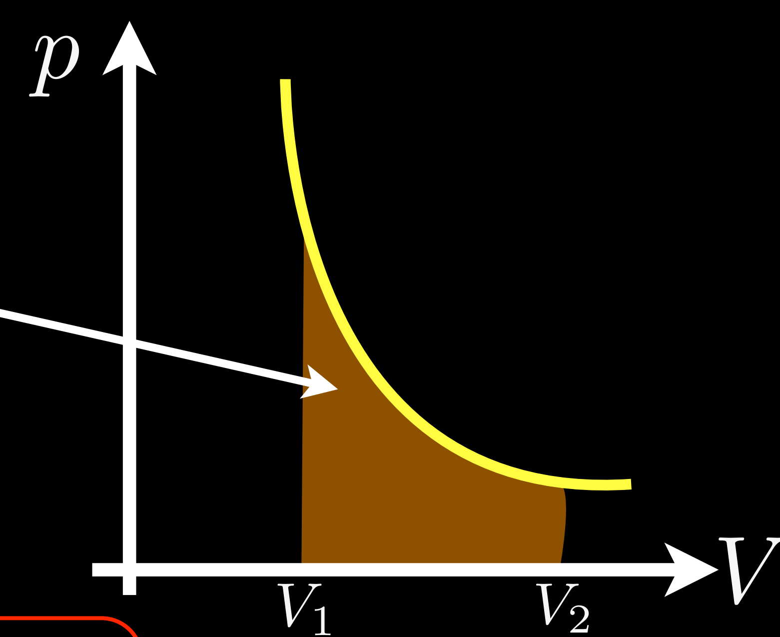
$$pV = nRT \Rightarrow p \propto \frac{1}{V}$$

System changes along **isotherm**:

curve of constant T: $T_3 > T_2 > T_1$



Work done on gas = -(area)

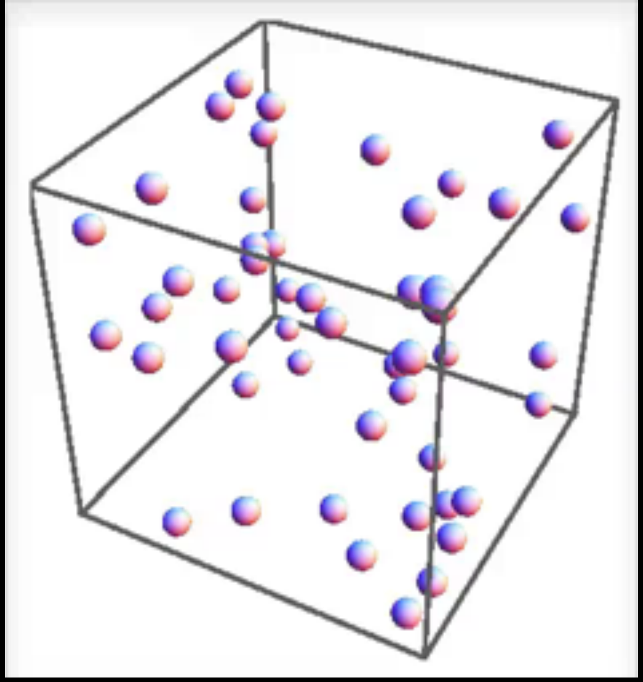


$$W = -\int_{V_1}^{V_2} p dV = -\int_{V_1}^{V_2} \frac{nRT}{V} dV$$

$$\boxed{\int \frac{1}{x} dx = \ln x} \qquad \boxed{\ln x_2 - \ln x_1 = \ln\left(\frac{x_2}{x_1}\right)}$$

$$= -nRT \int_{V_1}^{V_2} \frac{dV}{V} = -nRT \ln V \Big|_{V_1}^{V_2} = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

Isothermal (T = constant)



Internal energy, U from molecules' kinetic energy (motion)

from Gas T

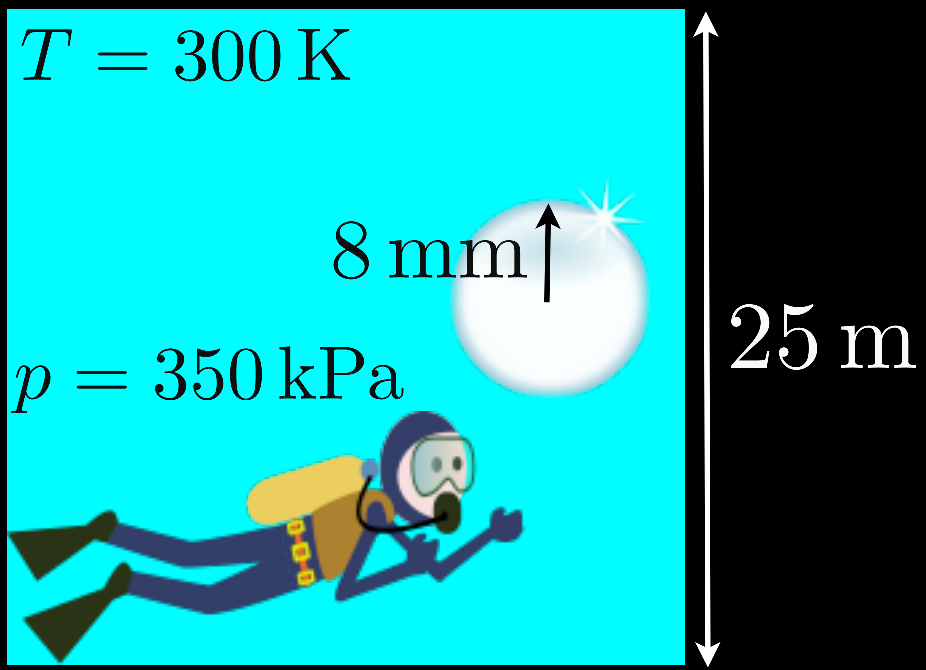
Therefore, constant T = constant U

1st law of thermodynamics: $\Delta U = 0 = Q + W$ $\Rightarrow$ $Q = -W$

since: $W = -nRT \ln\left(\frac{V_2}{V_1}\right)$

$\Rightarrow$ $$\boxed{Q = -W = nRT \ln\left(\frac{V_2}{V_1}\right)}$$ isothermal process

Isothermal Ex.



How much work does the bubble do as it rises to the surface?

Assume T = constant = 300 K

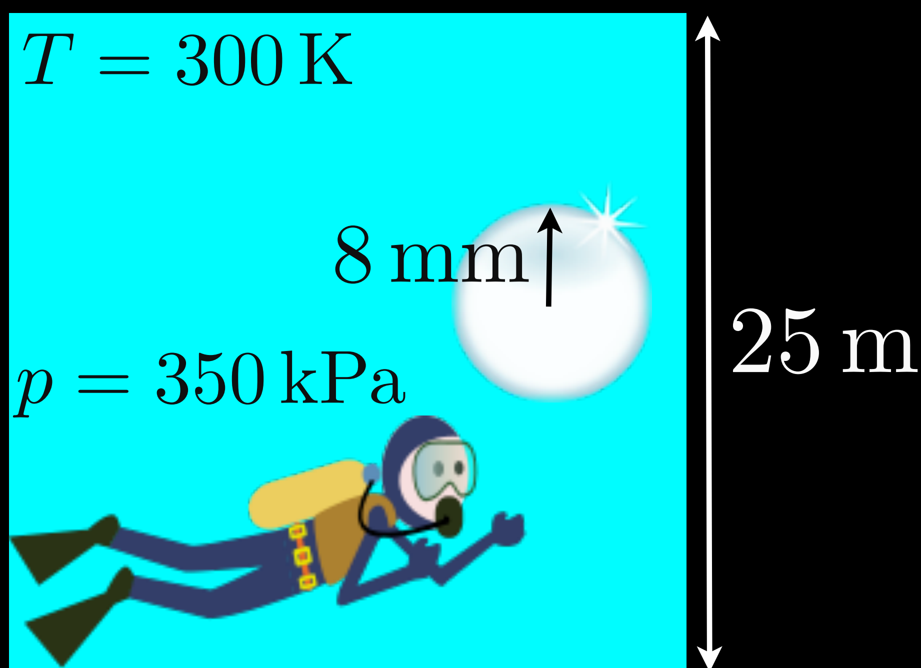
$$p_{\mathrm{atm}} = 101\,\mathrm{kPa}$$

Isothermal process: $$-W = nRT\ln\left(\frac{V_2}{V_1}\right)$$

?? ??

Isothermal

Ex.



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$$p_{\mathrm{atm}} = 101\,\mathrm{kPa}$$

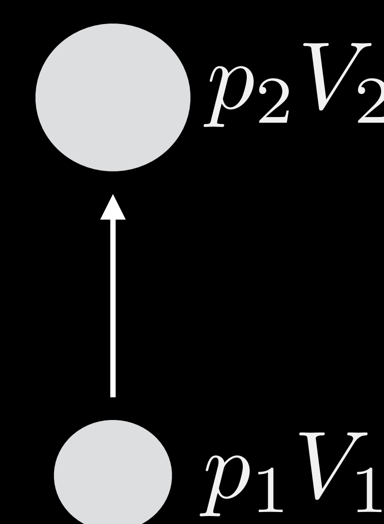
Isothermal process: $-W = nRT \ln\left(\frac{V_2}{V_1}\right)$

ideal gas: $pV = nRT = \frac{4}{3}\pi r^3 p$

$\frac{4}{3}\pi r^3$ constant

$$p_1 V_1 = \text{constant} = p_2 V_2 \quad \Rightarrow \quad \frac{V_2}{V_1} = \frac{p_1}{p_2} = 3.5$$

$$-W = \frac{4}{3}\pi r^3 p \ln 3.5 = -0.94\,\mathrm{J}$$



Isothermal Quiz

An ideal gas expands isothermally at 300 K.

Its volume increased from $0.020\,\mathrm{m}^3$ to $0.040\,\mathrm{m}^3$.

The final pressure is 120 kPa. The heat transfer to the gas....?

$$R = 8.314\,\mathrm{J/mol\cdot K}$$

(A) $3.3\,\mathrm{kJ}$

(B) $1.7\,\mathrm{kJ}$

(C) $-3.3\,\mathrm{kJ}$

(D) $-1.7\,\mathrm{kJ}$

(E) $0.0\,\mathrm{kJ}$

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(E) $0.0\,\mathrm{kJ}$

$$pV = nRT \leftarrow \text{constant}$$

$$(120 \times 10^3\,\mathrm{Pa})(0.040\,\mathrm{m}^3) = nRT$$

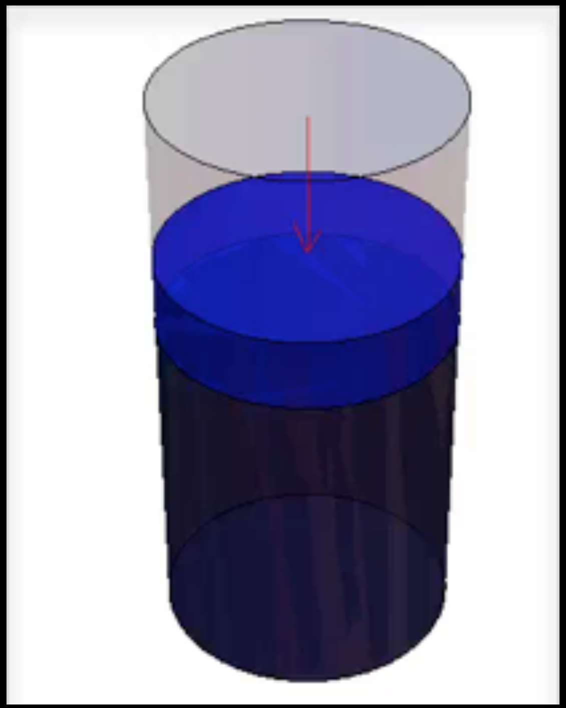
$$-W = nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$= (120 \times 10^3\,\mathrm{Pa})(0.040\,\mathrm{m}^3) \ln\left(\frac{0.040}{0.02}\right)$$

$$= 3.3\,\mathrm{kJ}$$

$$Q = -W = 3.3\,\mathrm{kJ}$$

Constant volume, V



constant volume: isometric
isochoric
isovolume

volume does not change $\Rightarrow$ $W = 0$

1st law of thermodynamics: $\Delta U = Q$

Introduce molar specific heat at constant volume, C_V :

$$Q = nC_V \Delta T$$ (constant-volume)

moles

C per unit mass (last lecture)
C_V per mol

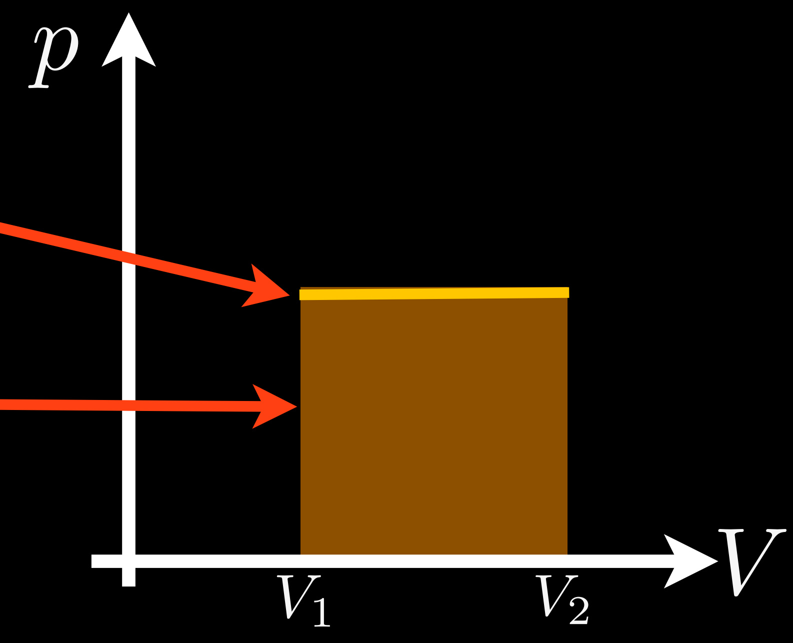
$\Rightarrow$ $\boxed{\Delta U = nC_V \Delta T \quad \text{(any process)}}$

U only depends on T:
$$\frac{\Delta U}{\Delta T} = \text{constant}$$

Isobaric (p = constant)

System changes along isobar:

curve of constant P



Work done on gas = -(area)

$$\boxed{W = -p(V_2 - V_1) = -p\Delta V}$$

1st law of thermodynamics: $Q = \Delta U - W = \Delta U + p\Delta V$

Since: $\Delta U = nC_V \Delta T$ ➡ $Q = nC_V \Delta T + p\Delta V$

Introduce molar specific heat at constant pressure, C_P:

$$\boxed{Q = nC_P \Delta T}$$

➡ $nC_p \Delta T = nC_V \Delta T + p\Delta V$ isobaric process

Isobaric (p = constant)

How are C_V and C_p related?

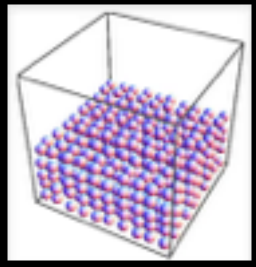
Ideal gas law: $pV = nRT$ $\Rightarrow$ $p\Delta V = nR\Delta T$

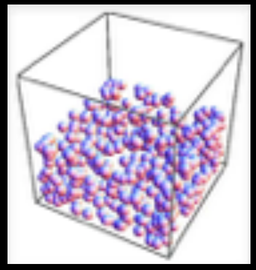
Therefore:

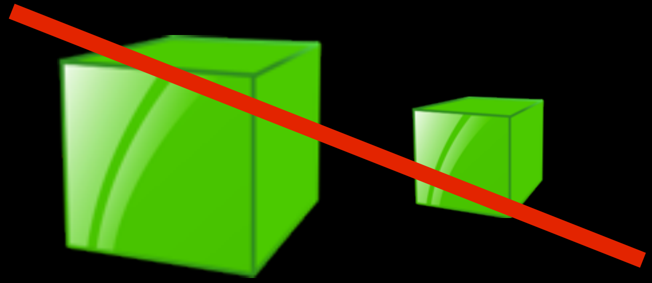
$$nC_p\Delta T = nC_V\Delta T + p\Delta V \quad \Rightarrow \quad nC_p\Delta T = nC_V\Delta T + nR\Delta T$$

So: $C_p = C_V + R$ molar specific heat

For solids and liquids, expansion is small

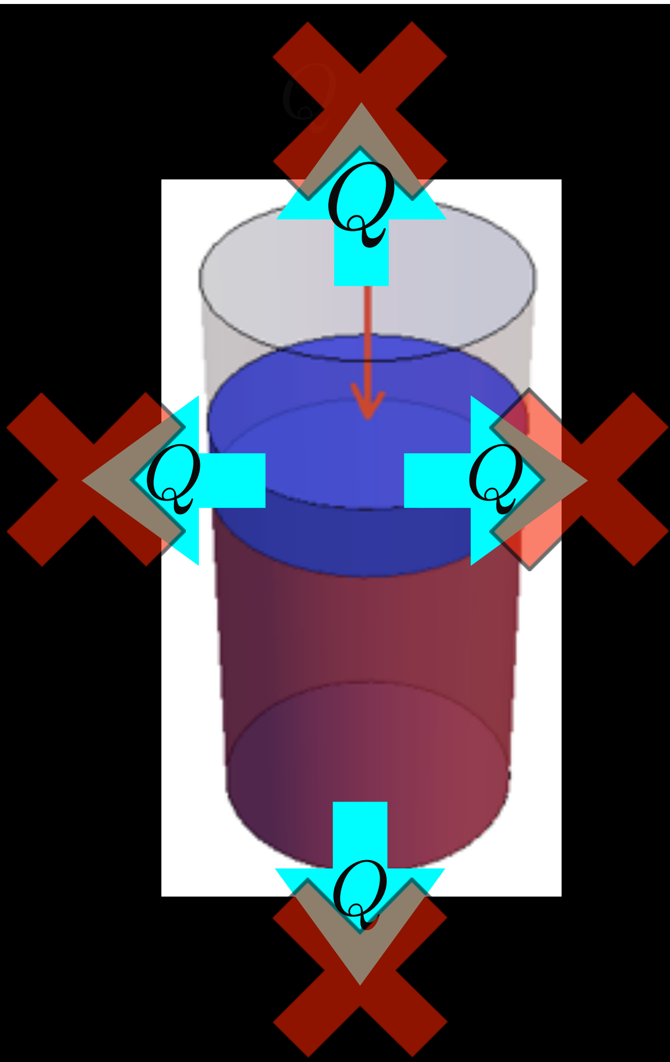






Therefore, work is small: $C_p \sim C_V$

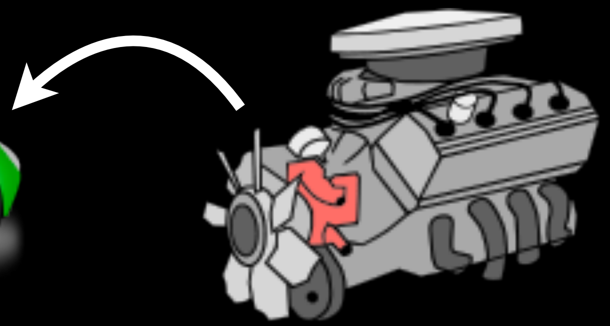
Adiabatic (Q = 0)



No heat flow : $Q = 0$

Quickly occurring processes ~ adiabatic
(finished before heat transfer occurs)

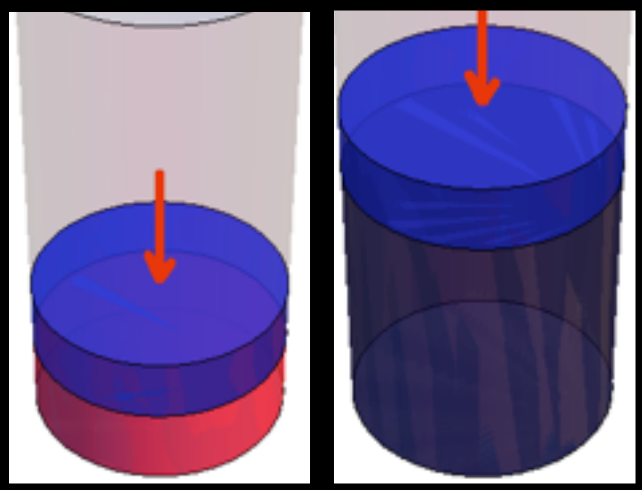
e.g.
combustion engine



1st law of thermodynamics: $\boxed{\Delta U = W \quad \text{adiabatic process}}$

Adiabatic (Q = 0)

Work $\propto$ internal energy → W done by gas lowers gas U

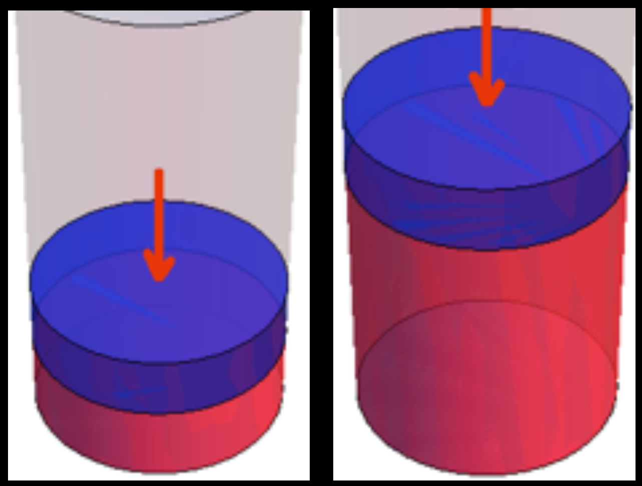


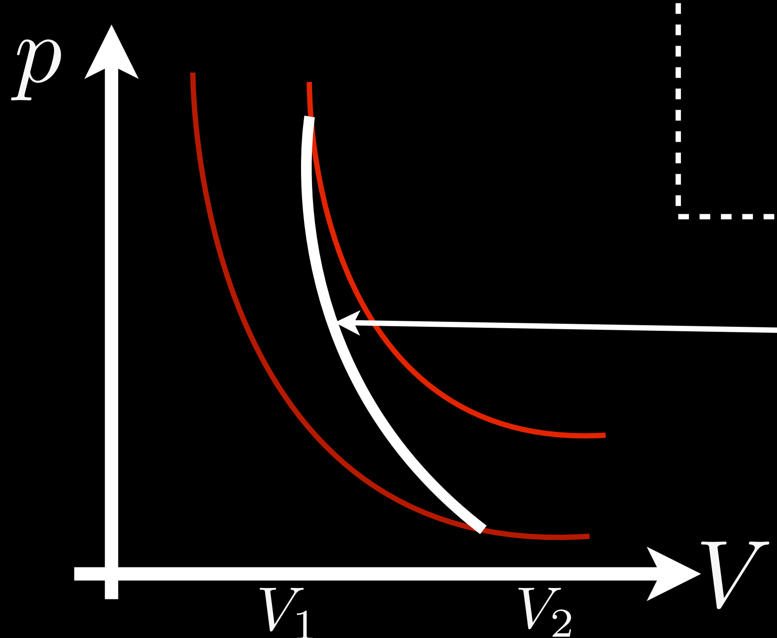
If V increases, gas does W → U decreases

Since Q = 0 → T decreases

Since $pV = nRT$ → p decreases

Isothermal changes only V and p





Adiabat : steeper than isotherms

$$\boxed{pV^{\gamma} = \text{constant} \qquad \gamma = C_p/C_V}$$

Adiabatic (Q = 0) Quiz

$pV^\gamma = \text{constant}$

Rewrite this equation in terms of T :

(a) $V^\gamma = T$

(b) $TV^\gamma = \text{constant}$

(c) $TV^{\gamma-1} = \text{constant}$

(d) $T^{\gamma-1}V = \text{constant}$

Adiabatic (Q = 0)

Quiz

$$pV^\gamma = \text{constant}$$

Rewrite this equation in terms of T :

Ideal gas: $pV = nRT$ → $p = \dfrac{nRT}{V}$

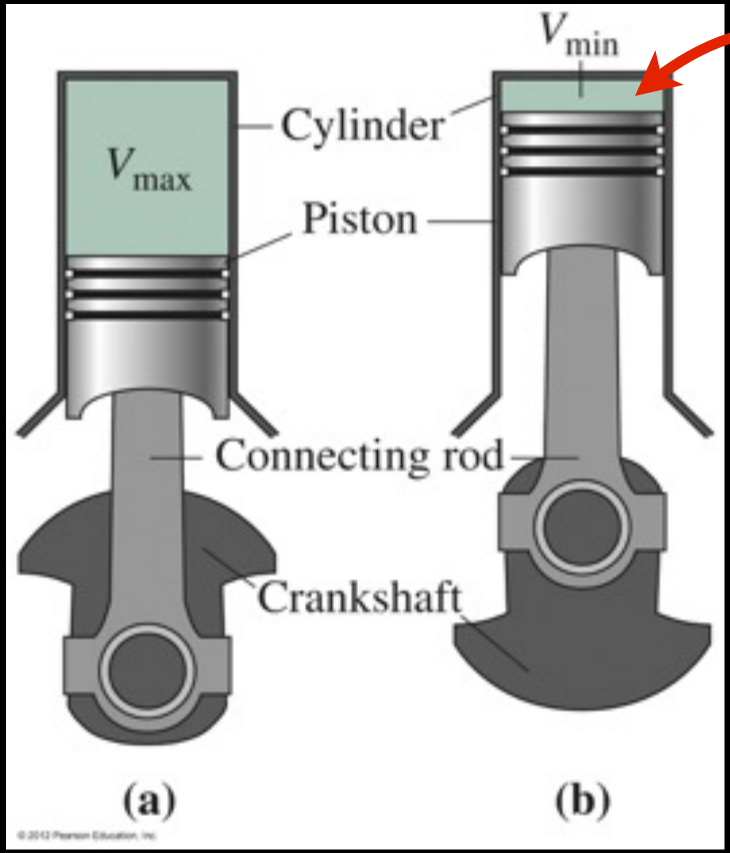
constant

$$\frac{nRT}{V}V^\gamma = \text{constant}$$

$$\boxed{TV^{\gamma-1} = \text{constant}}$$

Adiabatic (Q = 0)   Ex.

Diesel Engine



Compressed gas raises $T \to 500^\circ\mathrm{C}$
and ignites (starts burning) fuel

What is compression ratio, $\dfrac{V_b}{V_a}$?

Initial $T = 20^\circ\mathrm{C}$ and $\gamma = 1.4$

$$TV^{\gamma-1} = \text{constant} \quad \Rightarrow \quad T_\mathrm{b} V_\mathrm{b}^{\gamma-1} = T_\mathrm{a} V_\mathrm{a}^{\gamma-1}$$

before   after

$$\frac{V_b^{\gamma-1}}{V_a^{\gamma-1}} = \frac{T_a}{T_b} \quad \Rightarrow \quad \frac{V_b}{V_a} = \left(\frac{T_a}{T_b}\right)^{1/\gamma-1} = \left(\frac{500 + 273\,\mathrm{K}}{20 + 273\,\mathrm{K}}\right)^{1/0.4} = 11$$

Adiabatic (Q = 0)

Since $W = -\int_{V_1}^{V_2} p dV$ and $pV^\gamma = \text{constant} = C$

What is the work done by an adiabatic gas?

$$p = \frac{C}{V^\gamma} \Rightarrow W = -\int_{V_1}^{V_2} \frac{C}{V^\gamma} dV = -\int_{V_1}^{V_2} CV^{-\gamma} dV$$

$$pV^\gamma = C \qquad = \left[-\frac{CV^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2}$$

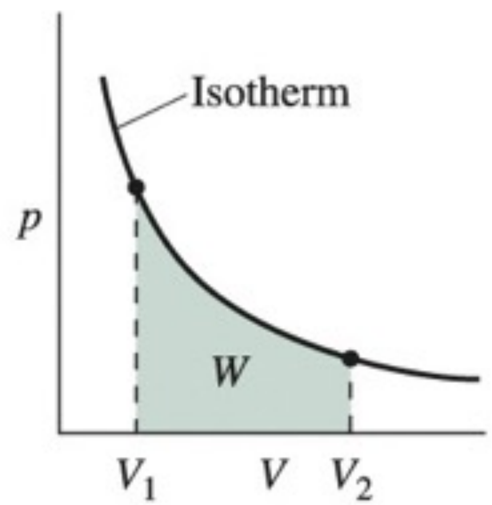
$$\boxed{W = \frac{p_2 V_2 - p_1 V_1}{\gamma - 1}}$$

Ideal gas processes

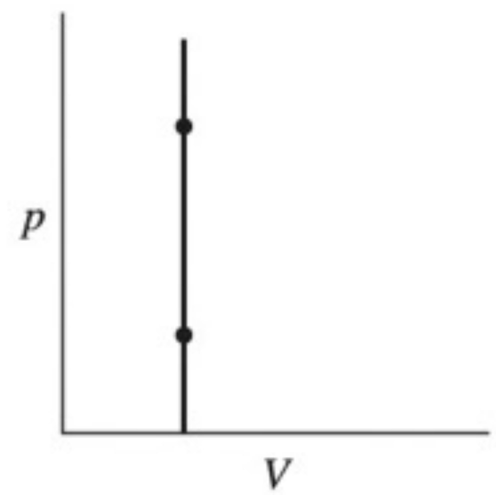
Table 18.1 Ideal-Gas Processes

pV diagram

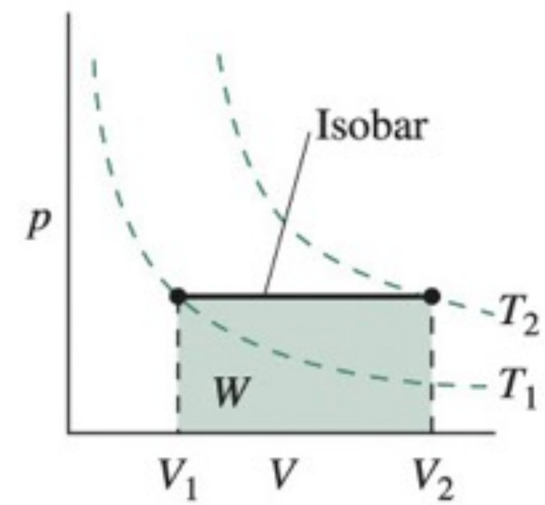
ISOTHERMAL



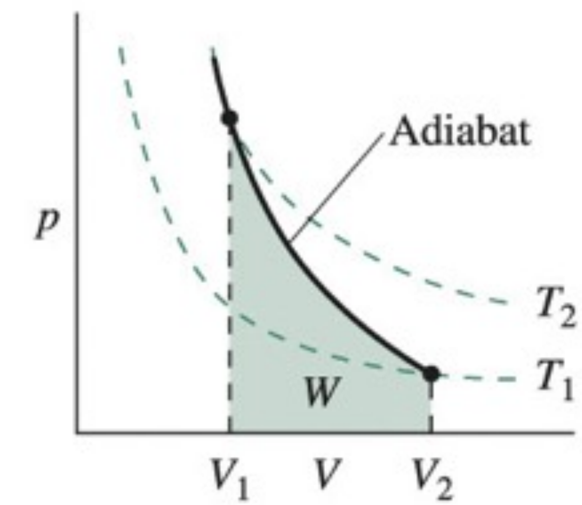
CONSTANT-VOLUME



ISOBARIC



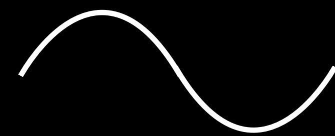
ADIABATIC



	ISOTHERMAL	CONSTANT-VOLUME	ISOBARIC	ADIABATIC
Defining characteristic	$T = \text{constant}$	$V = \text{constant}$	$p = \text{constant}$	$Q = 0$
First law	$Q = -W$	$Q = \Delta U$	$Q = \Delta U - W$	$\Delta U = W$
Work done on gas	$W = -nRT \ln\left(\frac{V_2}{V_1}\right)$	$W = 0$	$W = -p\,(V_2 - V_1)$	$W = \frac{p_2V_2 - p_1V_1}{\gamma - 1}$
Other relationships	$pV = \text{constant}$	$Q = nC_V\Delta T$	$Q = nC_p\Delta T$ $C_p = C_V + R$	$pV^\gamma = \text{constant}$ $TV^{\gamma-1} = \text{constant}$

Cyclic Processes

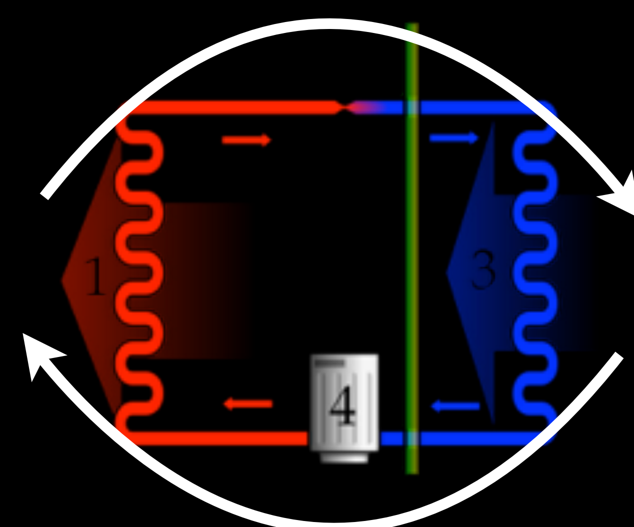
Systems that return periodically to the same state



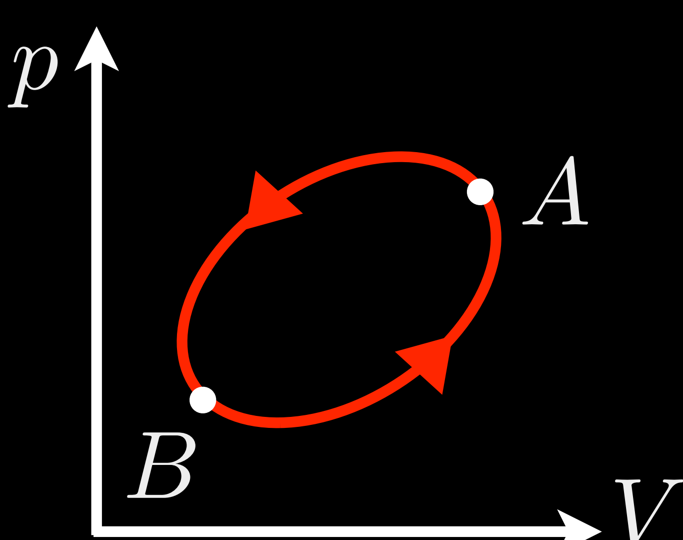
(p, V, T)

e.g. refrigerator

liquid looses heat outside



liquid removes heat from inside

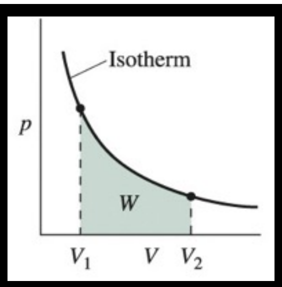


System moves between ‘A’ and ‘B’

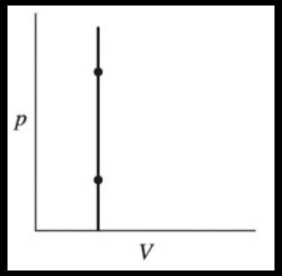
Cyclic Processes

Cyclic processes can involve:

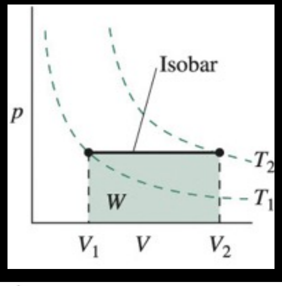
Isothermal



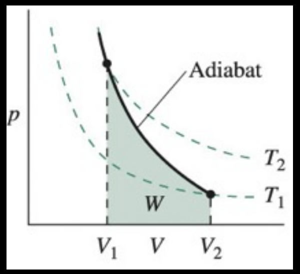
constant-volume



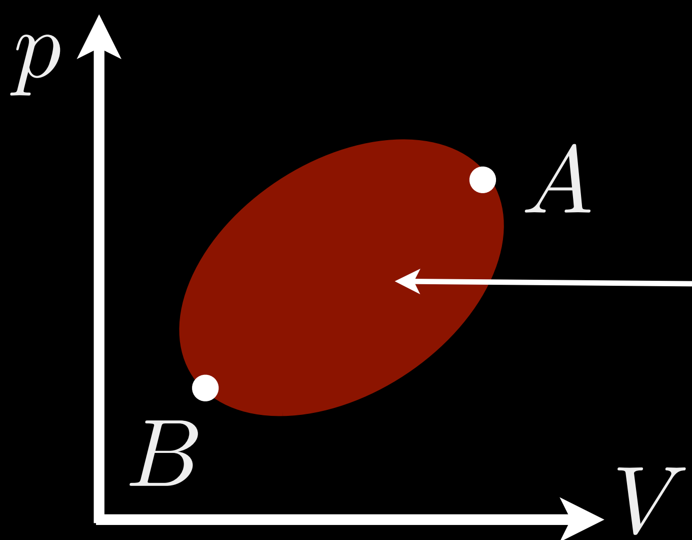
Isobaric



Adiabatic



processes



Net (total) work done on the gas

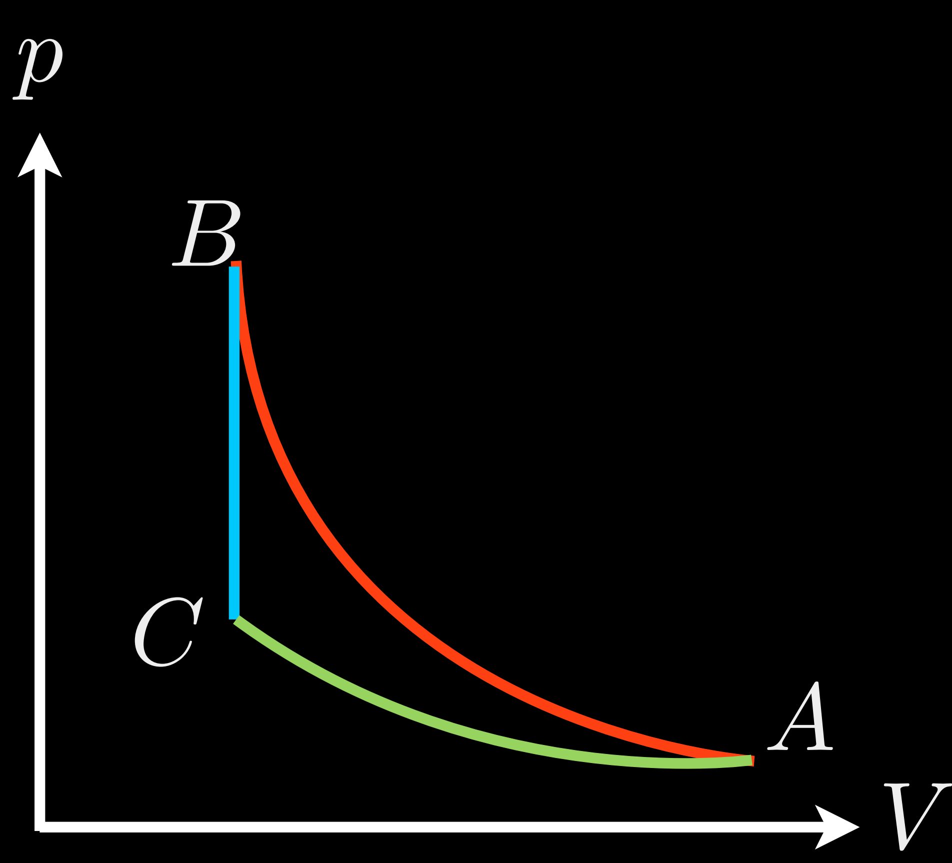
Cyclic Processes

Ex.

An ideal gas has $\gamma = 1.4$, V = 4.0 L, T = 300 K and p = 100 kPa.

1. Compressed adiabatically to 0.25 V $\quad A \rightarrow B$
2. Cooled at constant-volume back to 300K $\quad B \rightarrow C$
3. Expands isothermally to V $\quad C \rightarrow A$

How much work is done on the gas?



Adiabat $A \rightarrow B$

$$W = \frac{p_B V_B - p_A V_A}{\gamma - 1}$$

???

$$pV^\gamma = C \longrightarrow \quad p_B V_B^\gamma = p_A V_A^\gamma$$

$$p_B = p_A (V_A / V_B)^\gamma \quad = 696.4\,\mathrm{kPa}$$

$$W = \frac{(696.4 \times 10^3)(1 \times 10^{-3}) - (100 \times 10^3)(4 \times 10^{-3})}{1.4 - 1}$$

$$= 741\,\mathrm{J}$$

Cyclic Processes

Ex.

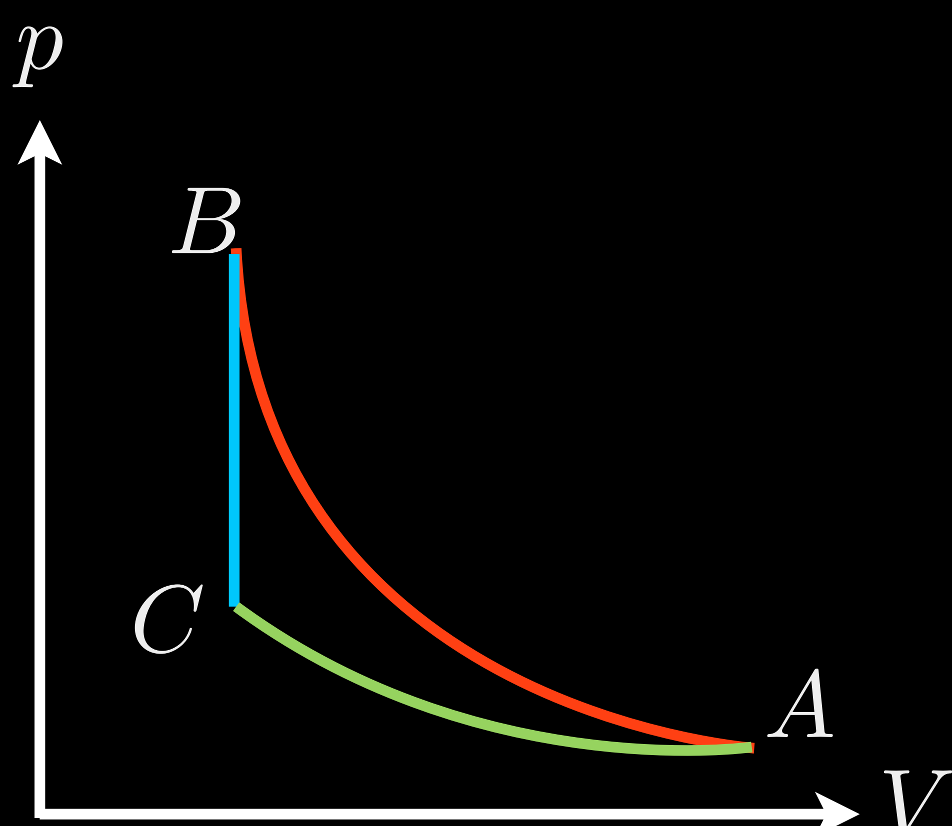
An ideal gas has $\gamma = 1.4$, V = 4.0 L, T = 300 K and p = 100 kPa.

1. Compressed adiabatically to 0.25 V $A \rightarrow B$

2. Cooled at constant-volume back to 300K $B \rightarrow C$

3. Expands isothermally to V $C \rightarrow A$

How much work is done on the gas?



Constant-volume $B \rightarrow C$

volume does not change ➡ $W = 0$

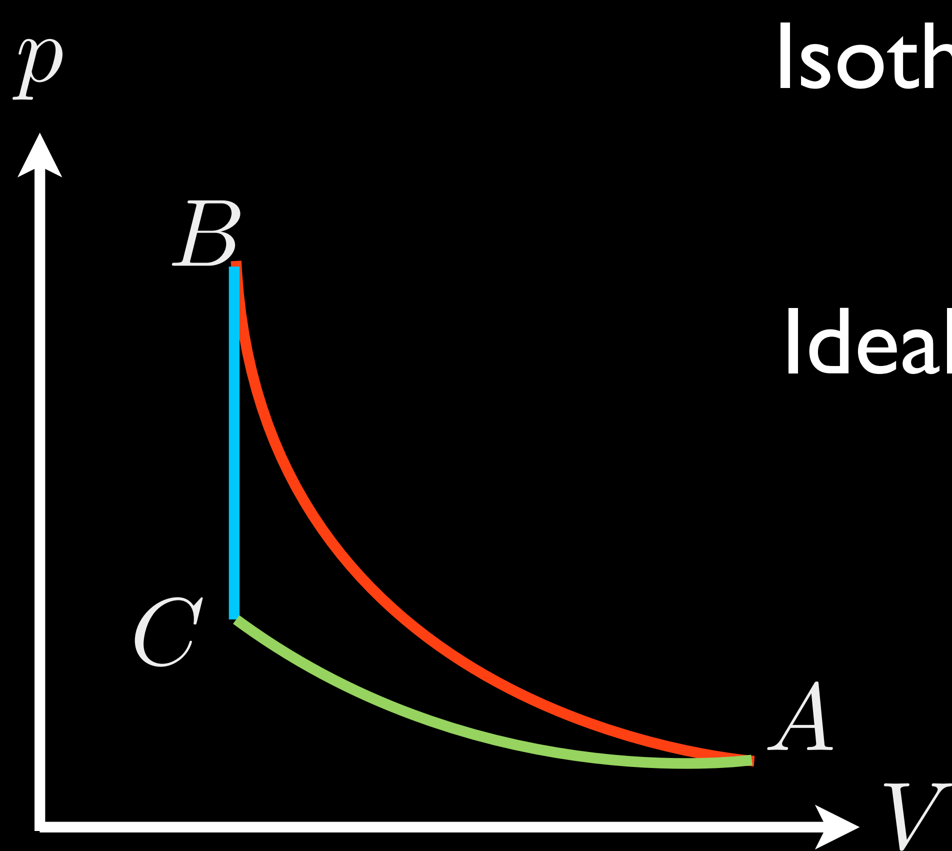
Cyclic Processes

Ex.

An ideal gas has $\gamma = 1.4$, V = 4.0 L, T = 300 K and p = 100 kPa.

1. Compressed adiabatically to 0.25 V $\quad A \rightarrow B$
2. Cooled at constant-volume back to 300K $\quad B \rightarrow C$
3. Expands isothermally to V $\quad C \rightarrow A$

How much work is done on the gas?



Isotherm: $C \rightarrow A$ $\qquad W = -nRT \ln\left(\frac{V_A}{V_C}\right)$

constant

Ideal gas: $pV = nRT$

$$= p_A V_A = (100 \times 10^3)(4 \times 10^{-3})$$

$$= 400\,\mathrm{J}$$

$$W = -(400\,\mathrm{J})(\ln 4) = -555\,\mathrm{J}$$

Cyclic Processes

Ex.

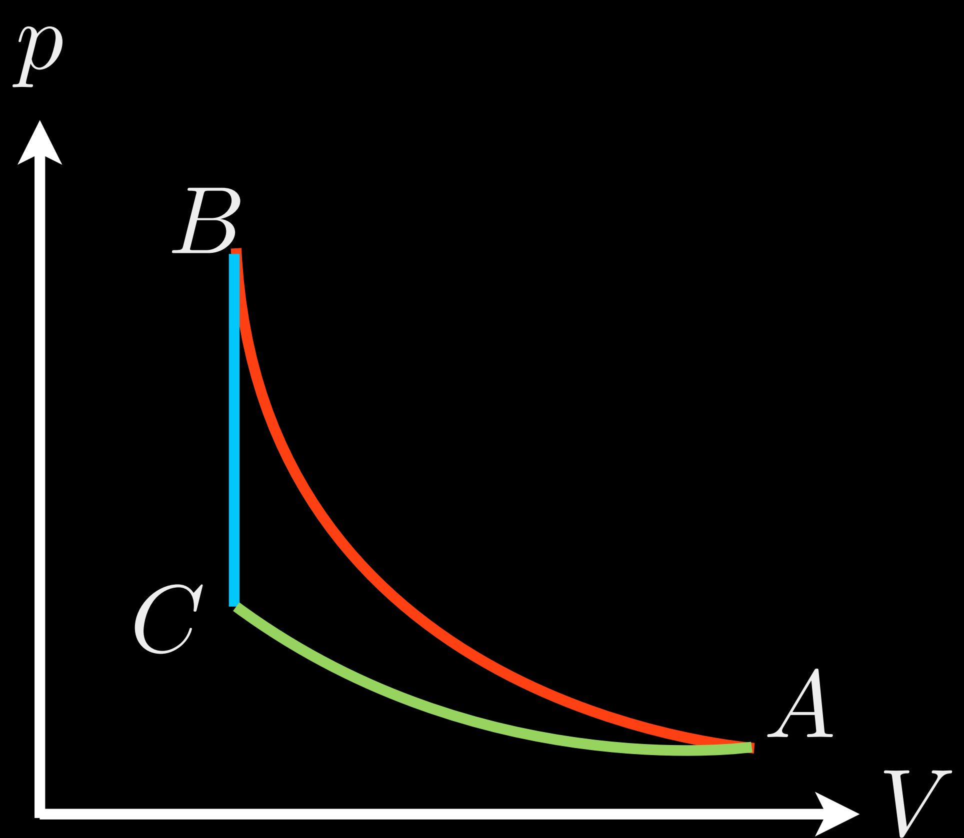
An ideal gas has $\gamma = 1.4$, V = 4.0 L, T = 300 K and p = 100 kPa.

1. Compressed adiabatically to 0.25 V $\quad A \rightarrow B$

2. Cooled at constant-volume back to 300K $\quad B \rightarrow C$

3. Expands isothermally to V $\quad C \rightarrow A$

How much work is done on the gas?



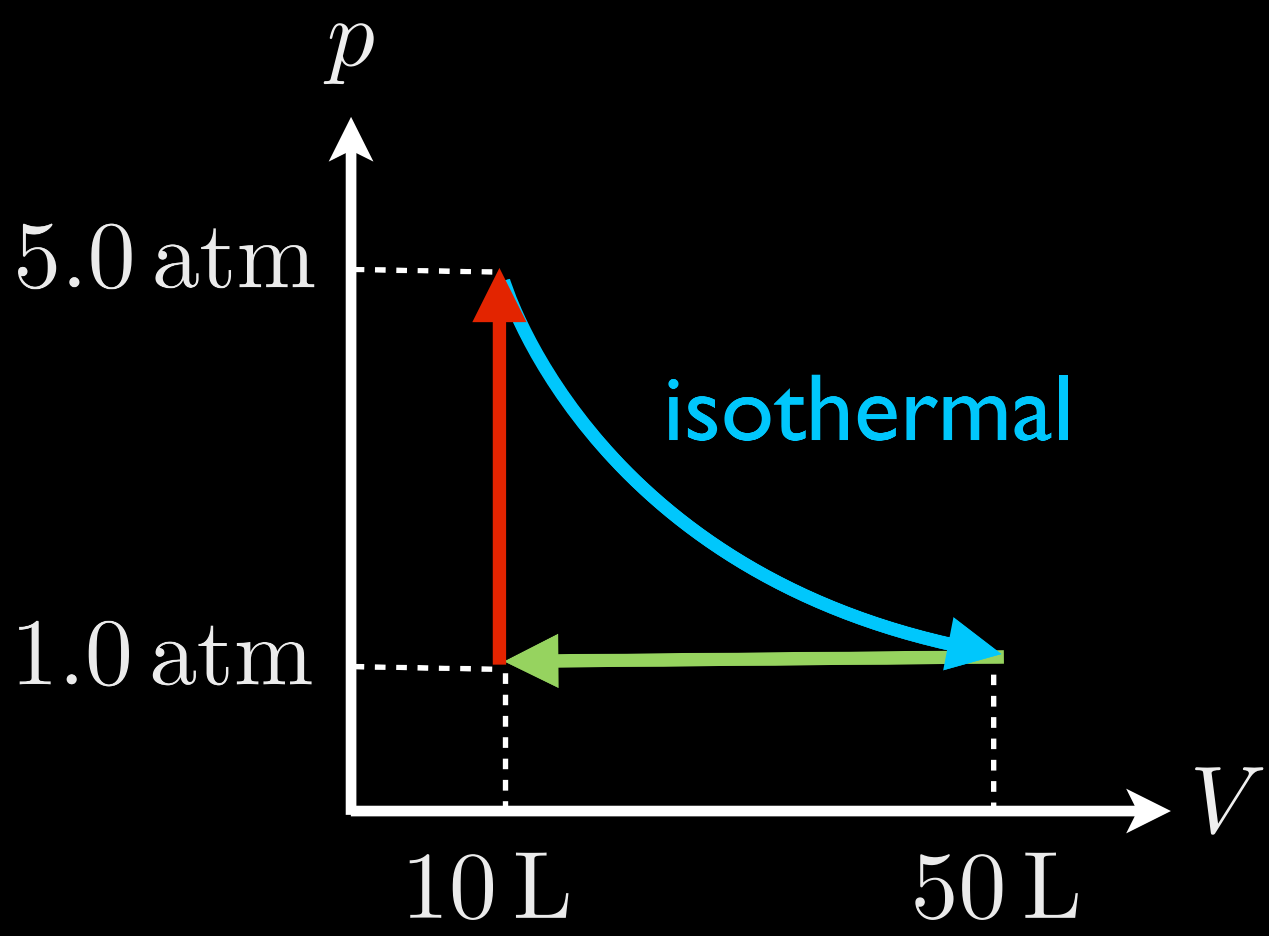
$$W_{\mathrm{ABCA}} = W_{\mathrm{AB}} + W_{\mathrm{BC}} + W_{\mathrm{CA}}$$

$$= 741\,\mathrm{J} + 0\,\mathrm{J} - 555\,\mathrm{J}$$

$$= 186\,\mathrm{J}$$

Cyclic Processes

Quiz



What is the work done?

$$1\,\mathrm{L} = 0.001\,\mathrm{m}^3$$

$$1\,\mathrm{atm} = 101,325\,\mathrm{Pa}$$

(a) $-8120\,\mathrm{J}$

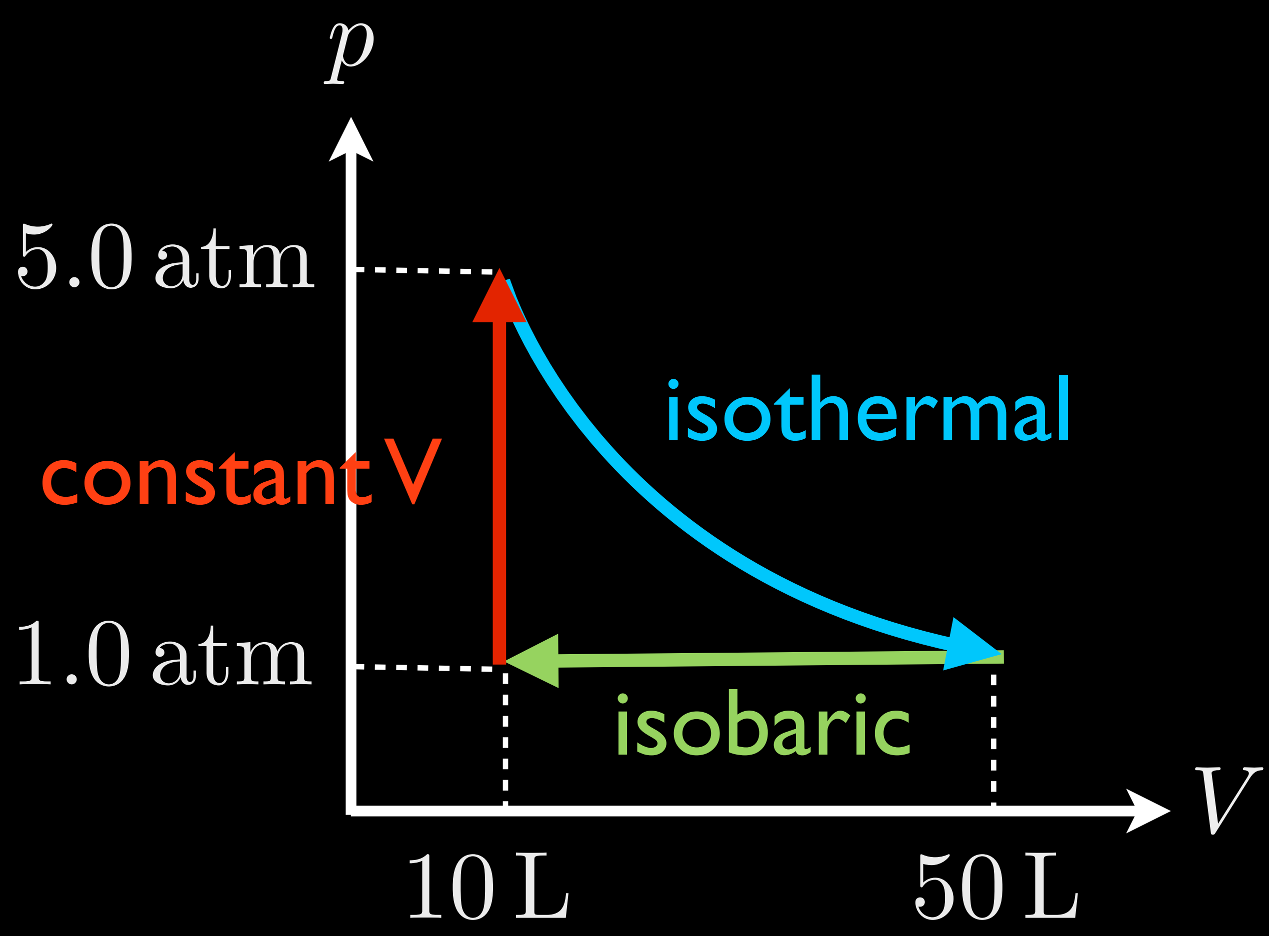
(b) $0\,\mathrm{J}$

(c) $4050\,\mathrm{J}$

(d) $-4101\,\mathrm{J}$

Cyclic Processes

Quiz



What is the work done on the ideal gas?

$$1\,\mathrm{L} = 0.001\,\mathrm{m}^3$$

$$1\,\mathrm{atm} = 101,325\,\mathrm{Pa}$$

isobaric: $W = -p(V_2 - V_1)$

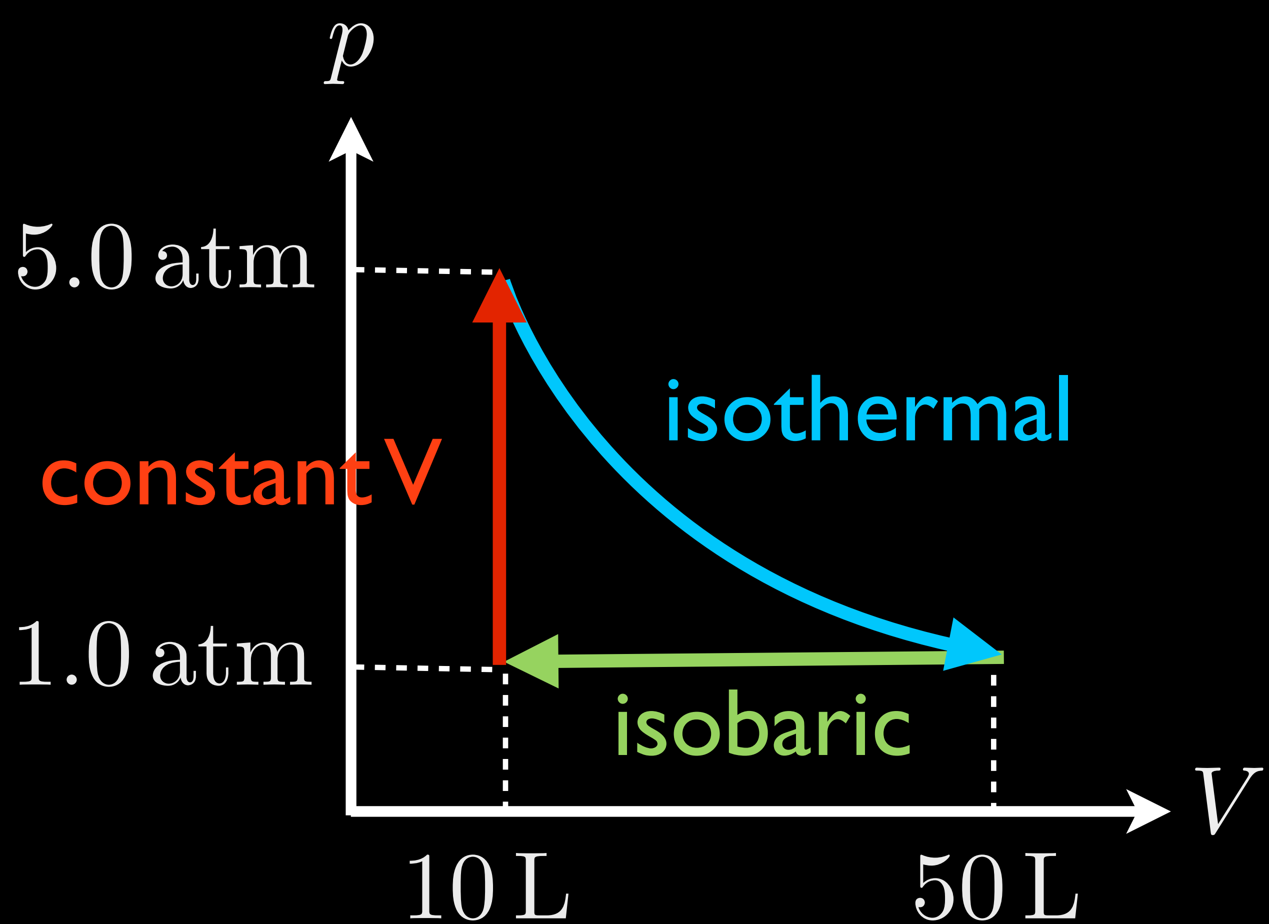
$$= -(101,325\,\mathrm{Pa})(-40 \times 10^{-3}\,\mathrm{m}^3)$$

$$= 4053\,\mathrm{J}$$

constant V: $W = 0$

Cyclic Processes

Quiz



What is the work done on the ideal gas?

$$1\,\mathrm{L} = 0.001\,\mathrm{m}^3$$

$$1\,\mathrm{atm} = 101,325\,\mathrm{Pa}$$

isothermal: $W = -nRT \ln\left(\frac{V_2}{V_1}\right)$

constant

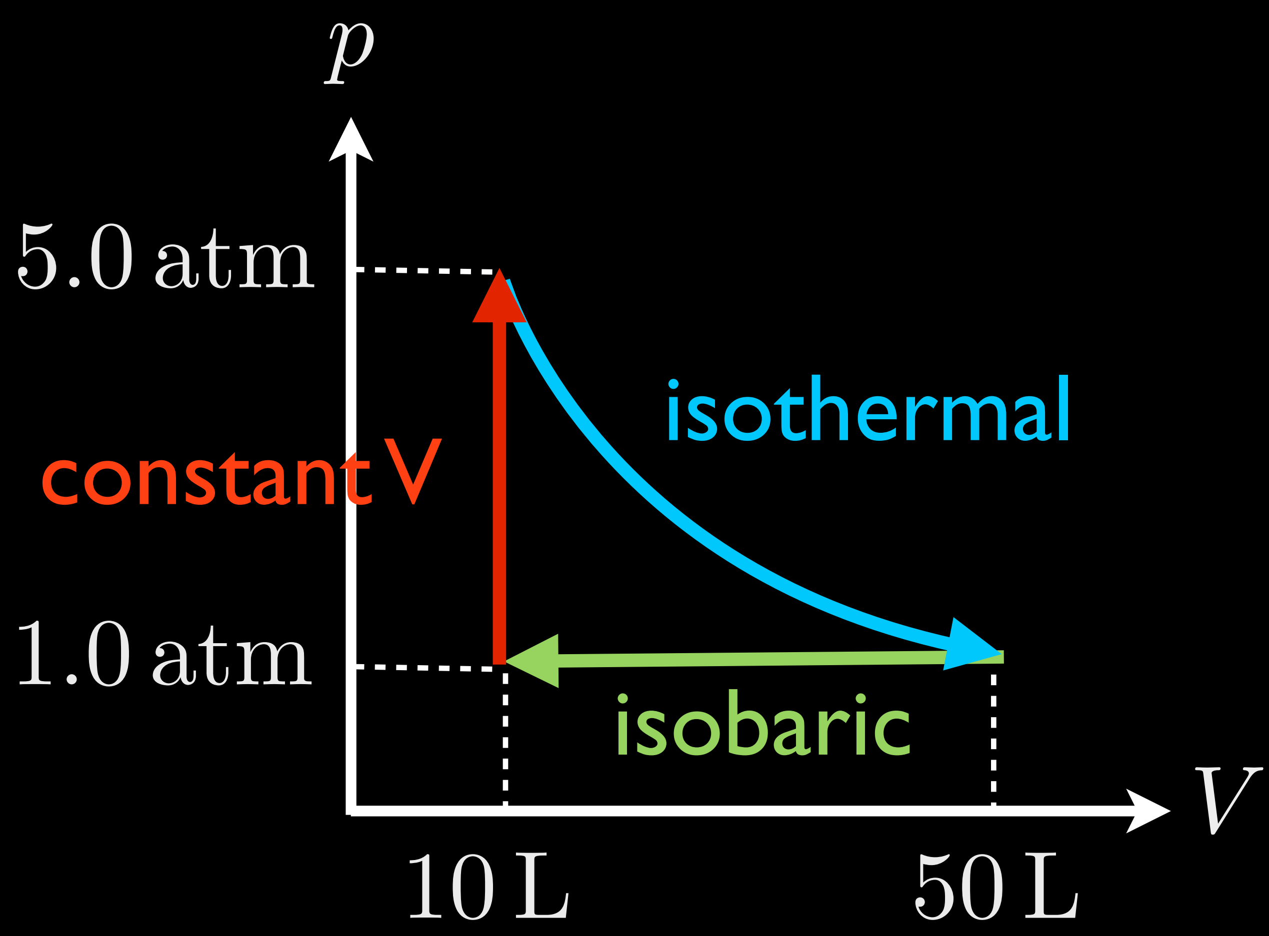
$$pV = (5.0 \times 101,325\,\mathrm{Pa})(10 \times 10^{-3}) = nRT$$

$$W = -(5.0 \times 101,325\,\mathrm{Pa})(10 \times 10^{-3}) \ln\left(\frac{50}{10}\right)$$

$$= -8153.8\,\mathrm{J}$$

Cyclic Processes

Quiz



What is the work done on the ideal gas?

$$1\,\mathrm{L} = 0.001\,\mathrm{m}^3$$

$$1\,\mathrm{atm} = 101,325\,\mathrm{Pa}$$

Total work: $= 4053 + 0 - 8153.8\,\mathrm{J}$

$$= -4100.8\,\mathrm{J}$$

Specific heats: C_V & C_p

Last lecture: $\frac{1}{2}m\bar{v^2} = \frac{3}{2}kT$ (for 1 molecule)

average K of molecule gas temperature

Total internal energy of n mols: $U = nN_A\left(\frac{1}{2}m\bar{v^2}\right)$

$$= \frac{3}{2}nN_AkT$$

R

$$= \frac{3}{2}nRT$$

From $\Delta U = nC_V\Delta T \longrightarrow C_V = \frac{1}{n}\frac{\Delta U}{\Delta T} = \frac{3}{2}R$

Specific heats: C_V & C_p

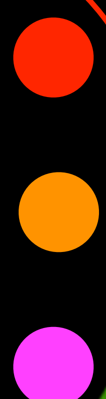
and: $$\gamma = \frac{C_p}{C_V} = \frac{C_V + R}{C_V}$$

Therefore: $$\gamma = \frac{\frac{5}{3}R}{\frac{3}{2}R} = \frac{5}{3} = 1.67$$

He

Ne

Ar

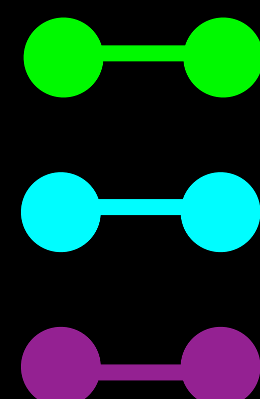


$$\gamma = \frac{5}{3}$$

H_2

N_2

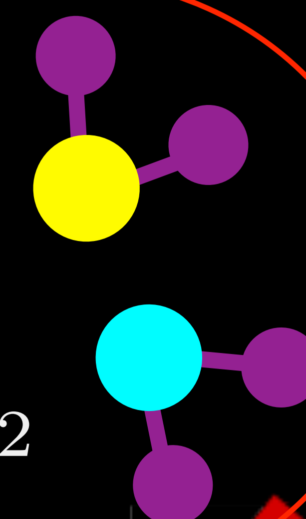
O_2



$$\gamma = \frac{7}{5}$$

SO_2

NO_2



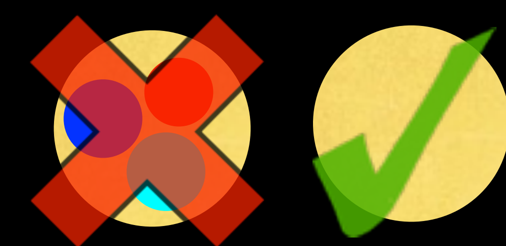
$$\gamma \simeq 1.3$$

Great! But....

Specific heats: C_V & C_p

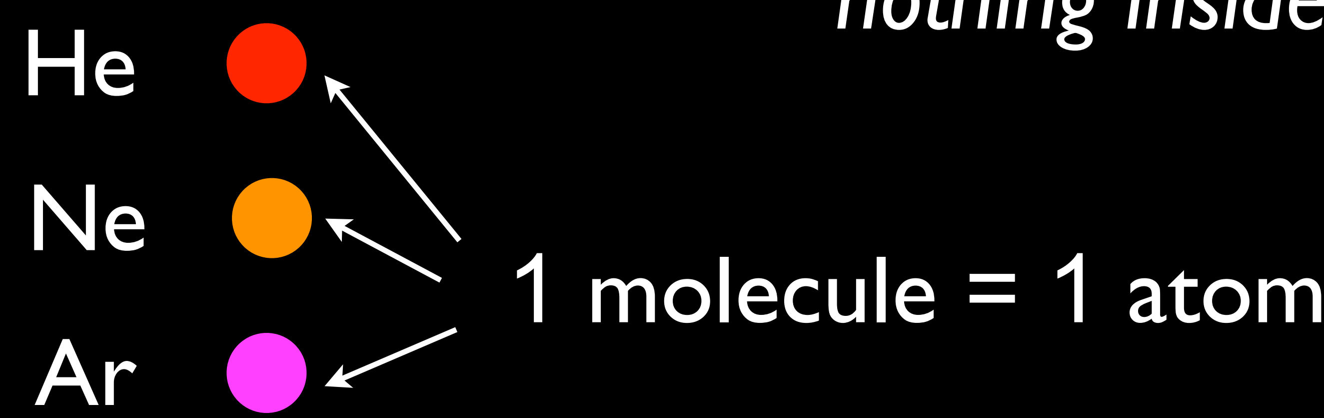
$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$ to find this...

Assumed: Gas molecules have no internal structure



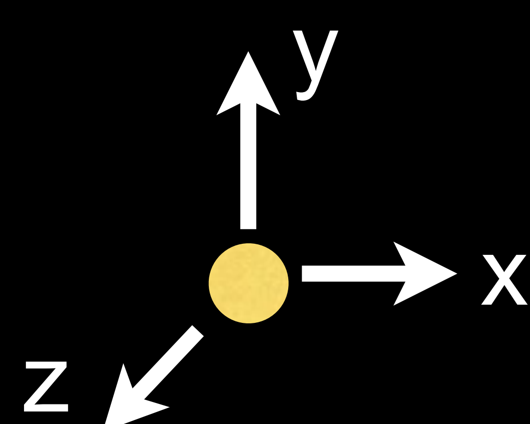
nothing inside

OK for monatomic molecules:



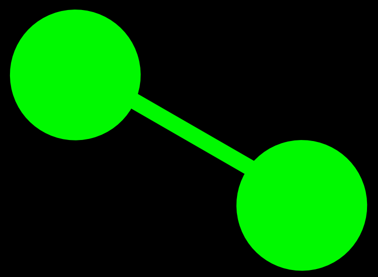
Can only move in 3 directions:

= 3 degrees of freedom

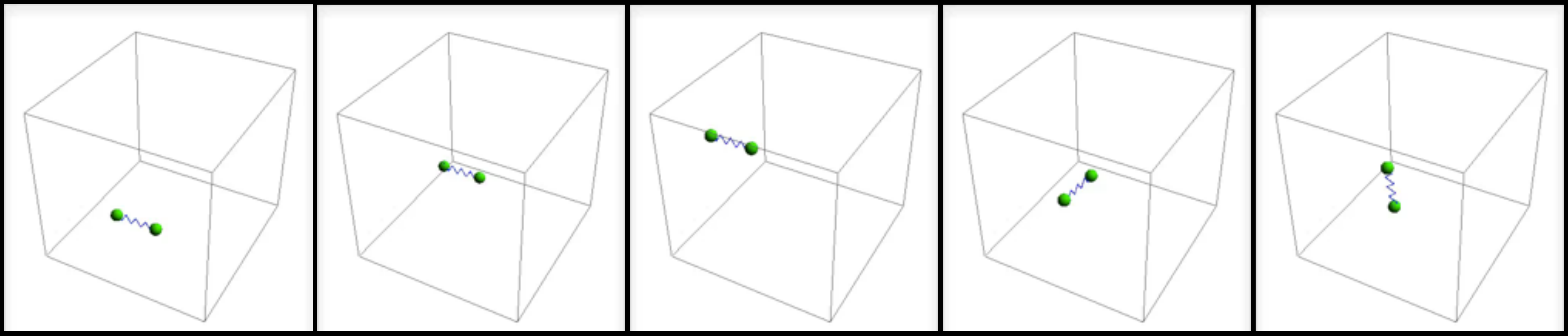


Specific heats: C_V & C_p

Diatomic molecules can move in 5 directions



1 molecule = 2 atoms



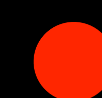
 5 degrees of freedom

Equipartition Theorem

In thermodynamic equilibrium,

$$\boxed{\text{Average energy / molecule} = \frac{1}{2}kT \quad \text{for each degree of freedom}}$$

Equipartition theorem

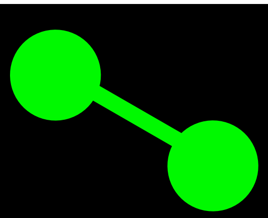
e.g. monatomic molecule, 3 degrees of freedom: 

$$U = 3\left(\frac{1}{2}kT\right) \quad = \frac{3}{2}kT$$

since all energy is kinetic: $U = K = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$

Equipartition Theorem

e.g. diatomic molecule, 5 degrees of freedom:



Av. energy / molecule: $U = 5\left(\frac{1}{2}kT\right) = \frac{5}{2}kT$

Total internal energy of n mols: $U = \frac{5}{2}nN_A kT$

$$= \frac{5}{2}nRT$$

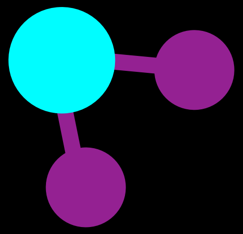
Therefore: $C_V = \frac{1}{n}\frac{\Delta U}{\Delta T} = \frac{5}{2}R$

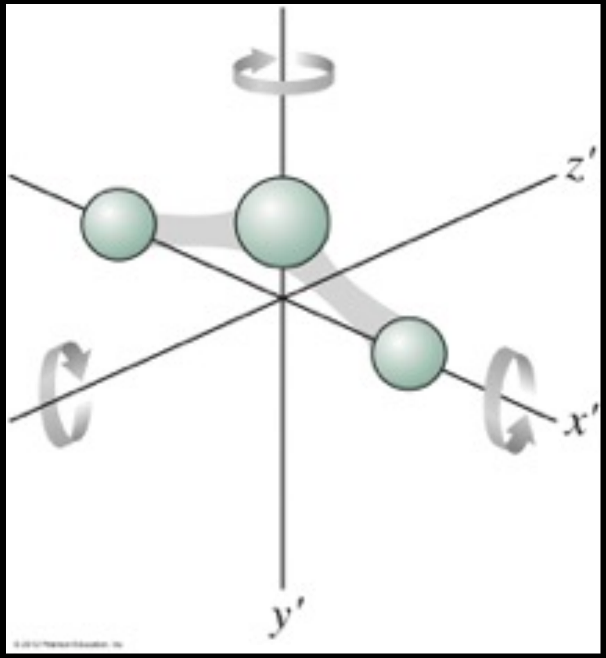
(as before)

and: $\gamma = \frac{C_p}{C_V} = \frac{C_V + R}{C_V} = \frac{7}{5} = 1.4$

Equipartition Theorem

e.g. polyatomic molecule, 6 degrees of freedom:





3 translational (x, y, z)

3 rotational

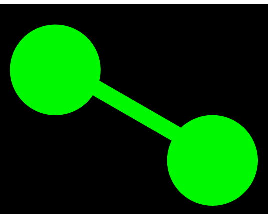
$$U = 3nRT$$

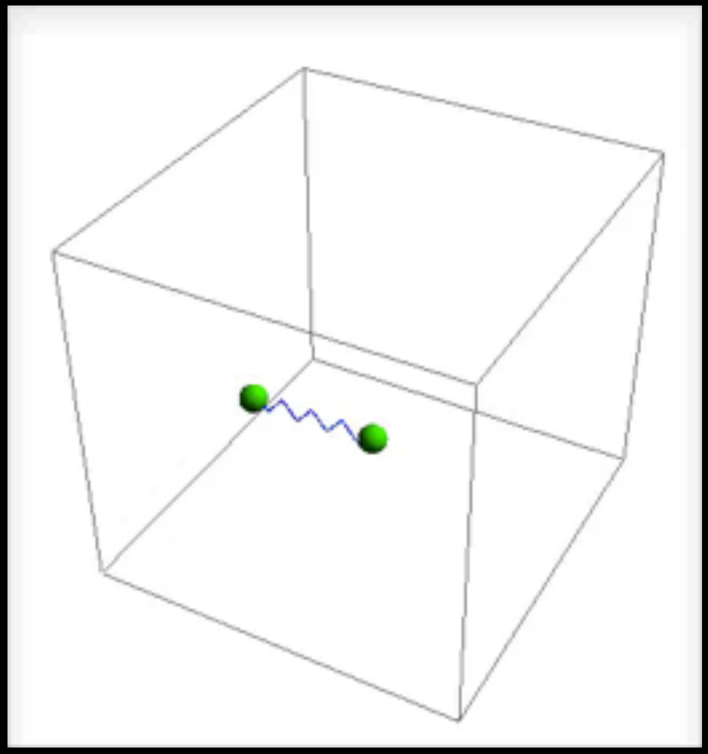
$$C_V = 3R$$

$$\gamma = \frac{4}{3} \simeq 1.33$$

Equipartition Theorem

At very high T, diatomic molecules can also vibrate:

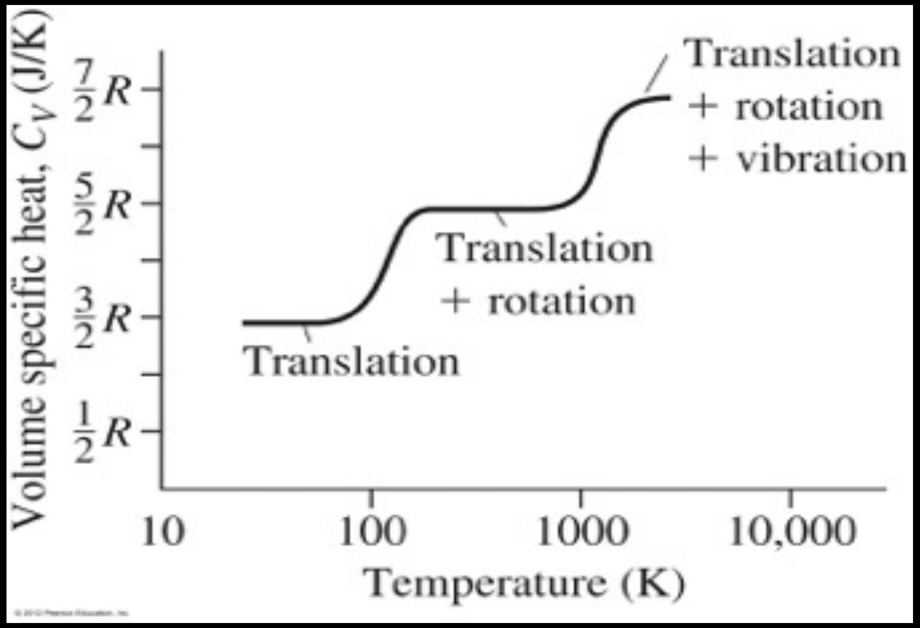




Adds kinetic energy (K)

... and potential energy

+ 2 degrees of freedom



Total: 7 degrees of freedom

(only at high T)

Equipartition Theorem

Ex.

A gas mixture has 2.0 mol of oxygen (O_2) and 1.0 mol of argon (Ar)

Find the volume specific heat, C_V

O_2 : 5 degrees of freedom $$U = \frac{5}{2}nRT = \frac{5}{2}2.0RT = 5RT$$

Ar : 3 degrees of freedom $$U = \frac{3}{2}nRT = \frac{3}{2}1.0RT = \frac{3}{2}RT$$

Total internal energy: $$U = 5RT + \frac{3}{2}RT = \frac{13}{2}RT$$

$$\frac{\Delta U}{\Delta T} = \frac{13}{2}R$$

$$C_V = \frac{1}{n}\frac{\Delta U}{\Delta T} = \frac{1}{2.0+1.0}\frac{13}{2}R = 2.2R$$