# Essential Physics II

## 英語で物理学の
## エッセンス II

Lecture 13:   12-01-16

# Exam (2 weeks)

## Equations from the course:

Essential Physics II

List of equations covered in this course.

Equations in boxes ☐ will be provided in the exam. Please check you know where/when/how to use these equations.

Students are expected to **know** equations not in boxes.

Part I: Electromagnetism

$$\bar{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}_{12}$$ Coulomb's law

$\boxed{k = 9.0 \times 10^9 \mathrm{Nm}^2/\mathrm{C}^2}$

$$\bar{E} = \frac{kq}{r^2}\hat{r}$$

$$\bar{E} = \sum_i d\bar{E} = \int \frac{kdq}{r^2}\hat{r}$$

$\boxed{p = qd}$ Dipole moment

$\boxed{\bar{\tau} = \bar{p} \times \bar{E}}$ Torque on dipole

$\boxed{U = -pE\cos\theta = -\bar{p}\cdot\bar{E}}$ Potential energy

If equation is in a box...

It will be given to you

If equation is NOT in a box....

You must know it!

# Exam (2 weeks)

WARNING!

You must know HOW to use the equation!

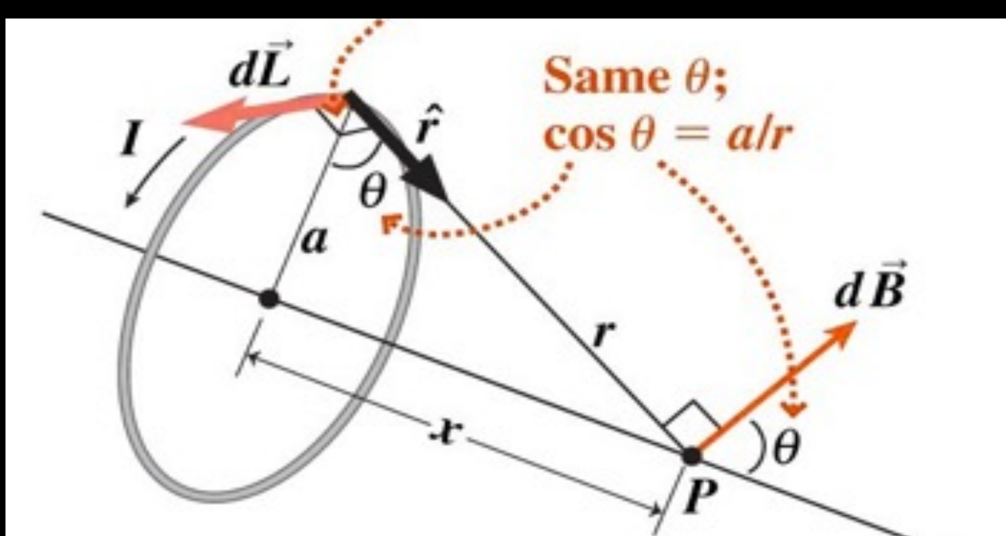
e.g. Biot-Savart Law given

$$d\bar{B} = \frac{\mu_0}{4\pi}\frac{I d\bar{l} \times \bar{r}}{r^2}$$

Biot-Savart Law (know solutions for current loop, straight wire)

$$\mu_0 = 4\pi \times 10^{-7} \mathrm{N/A^2}$$

BUT if question asks about current loops....



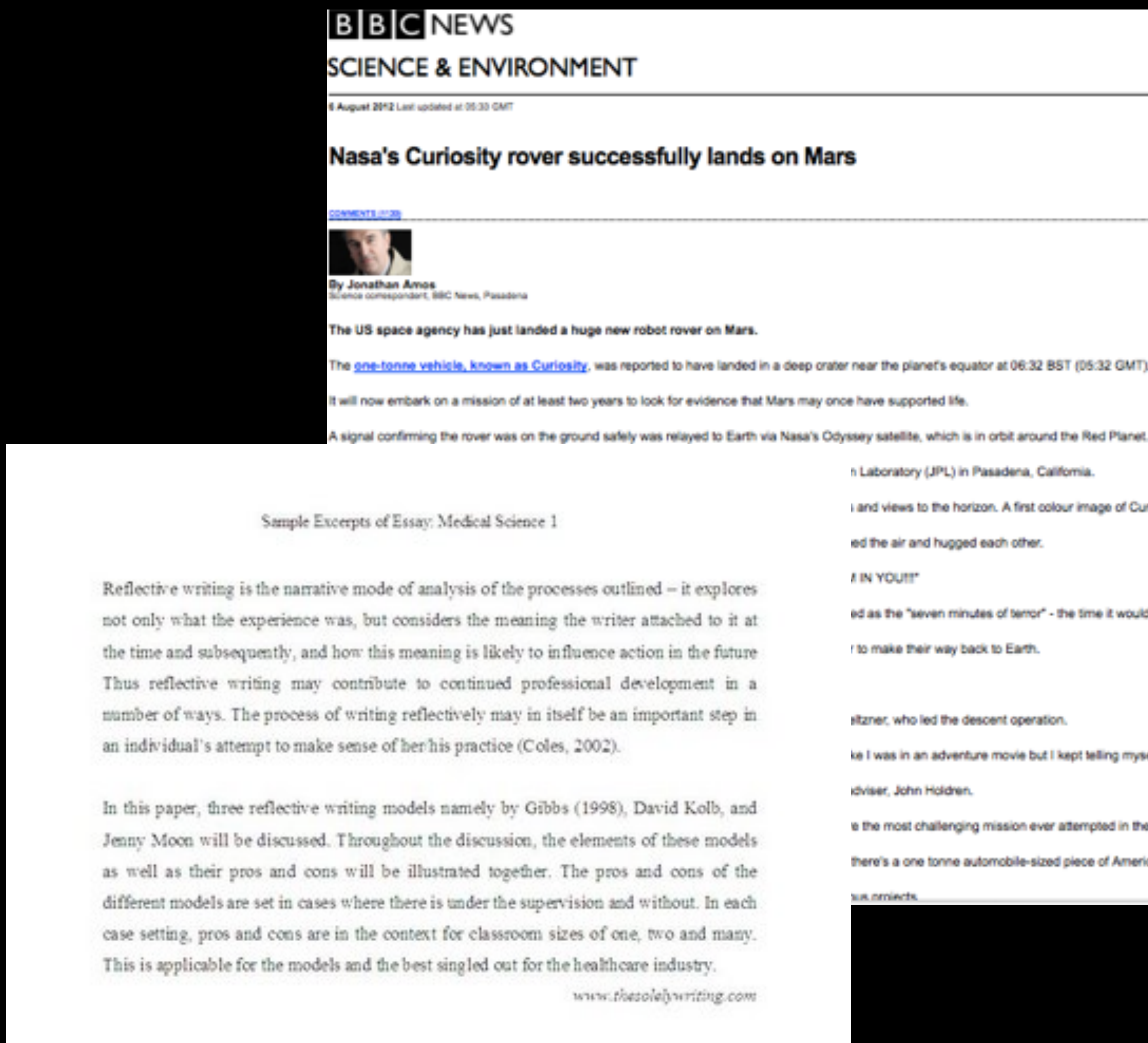

$$d\bar{B} = \frac{\mu_0}{4\pi}\frac{I d\bar{l} \times \bar{r}}{r^2}$$

know this step!!

$$B = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

# Reminder!! 1 weeks!

<u>250 word essay</u>



BBC NEWS

SCIENCE & ENVIRONMENT

Nasa's Curiosity rover successfully lands on Mars

By Jonathan Amos

The US space agency has just landed a huge new robot rover on Mars.

The one-tonne vehicle, known as Curiosity, was reported to have landed in a deep crater near the planet's equator at 06:32 BST (05:32 GMT).

It will now embark on a mission of at least two years to look for evidence that Mars may once have supported life.

A signal confirming the rover was on the ground safely was relayed to Earth via Nasa's Odyssey satellite, which is in orbit around the Red Planet.

Sample Excerpts of Essay: Medical Science 1

Reflective writing is the narrative mode of analysis of the processes outlined – it explores not only what the experience was, but considers the meaning the writer attached to it at the time and subsequently, and how this meaning is likely to influence action in the future. Thus reflective writing may contribute to continued professional development in a number of ways. The process of writing reflectively may in itself be an important step in an individual's attempt to make sense of her/his practice (Coles, 2002).

In this paper, three reflective writing models namely by Gibbs (1998), David Kolb, and Jenny Moon will be discussed. Throughout the discussion, the elements of these models as well as their pros and cons will be illustrated together. The pros and cons of the different models are set in cases where there is under the supervision and without. In each case setting, pros and cons are in the context for classroom sizes of one, two and many. This is applicable for the models and the best singled out for the healthcare industry.

www.thesolelywriting.com

Read a physics article (in English) on a topic that interests you

This can be one we have covered in class, or a new one.

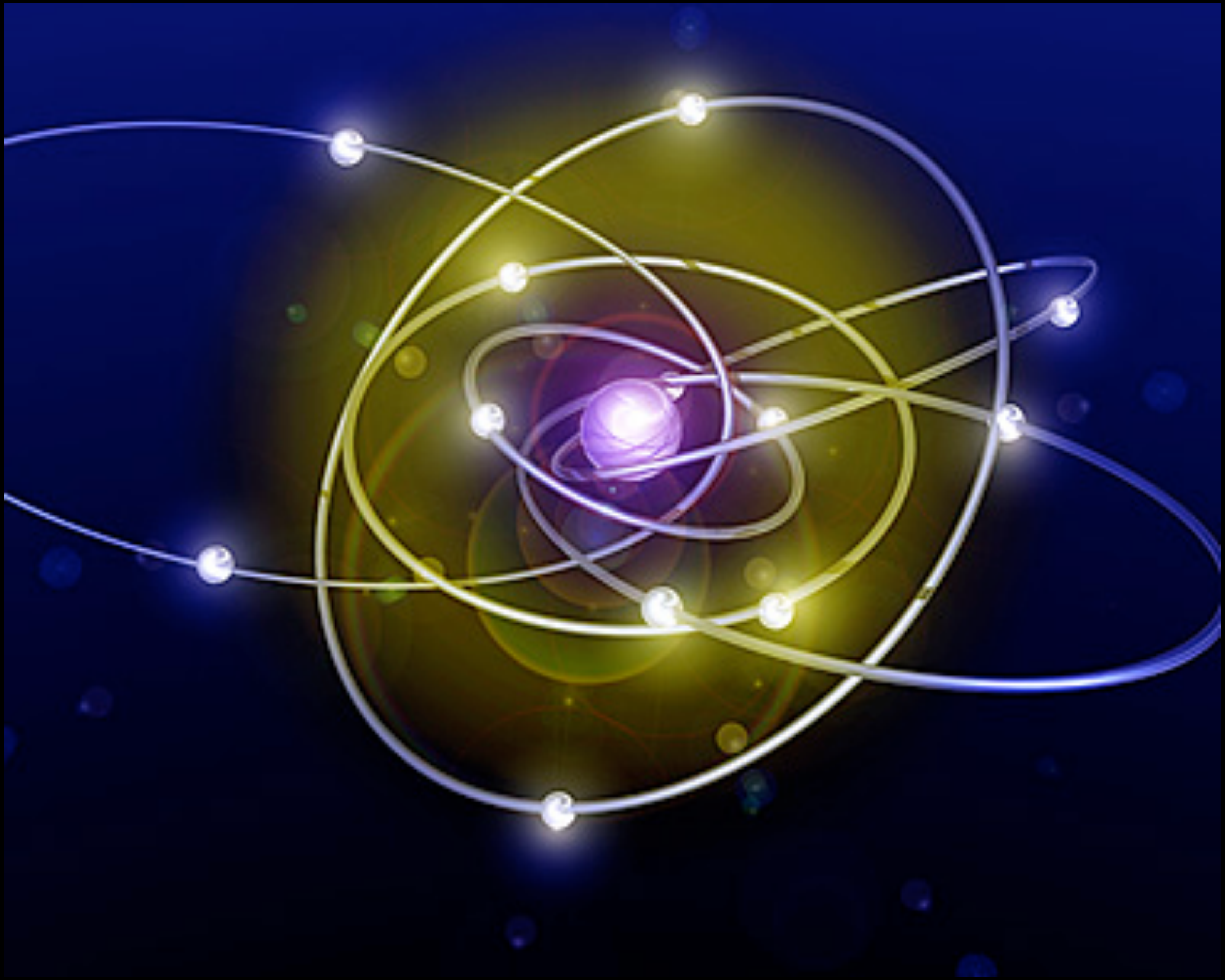
Describe its main points in 250 words.

Hand in BOTH essay and article

Due 2016/1/18

NO EXTENSIONS!

# Modern Physics



# Quantum Mechanics

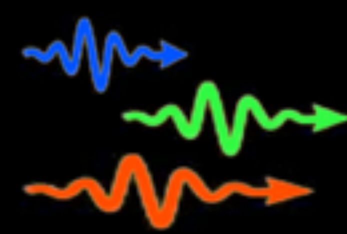
# Last Lecture

1 Blackbody radiation

2 Photoelectric effect

3 Stability and atom size

EM radiation is quantized into photon particles



$$E = hf$$

waves (e.g. light)

can show

particle properties

particles (e.g. electron)

can show

wave properties

$$\lambda = \frac{h}{p}$$

very small for big particles (e.g. baseballs) : particle properties dominant

# Last Lecture

1. Blackbody radiation
2. Photoelectric effect
3. Stability and atom size

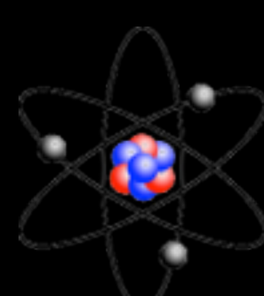
wave - particle properties explain these...

$$E = hf$$

$$\lambda = \frac{h}{p}$$

But is there a theory that predicts properties for all atomic scales?

➡ Quantum mechanics



# Last Lecture

1 Blackbody radiation

wave - particle properties

Last week:

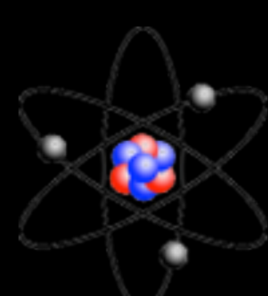
To explain observations: energy quantization
wave - particle duality

This week:

What theory gives this?

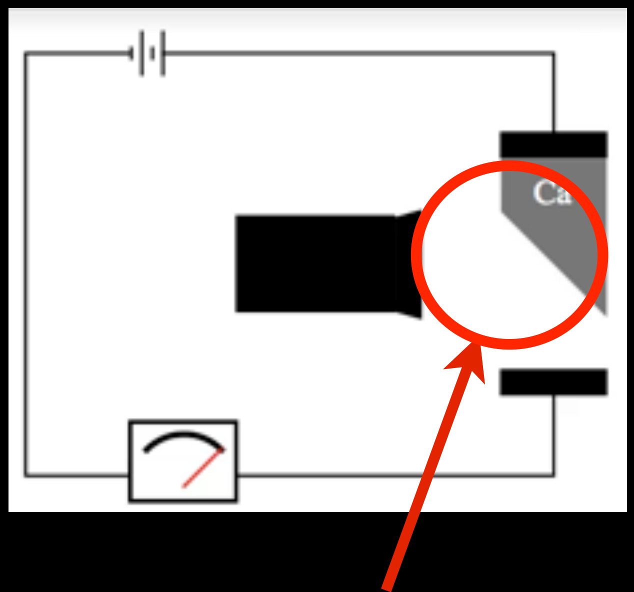
But is there a theory that predicts properties for all atomic scales?

→ Quantum mechanics

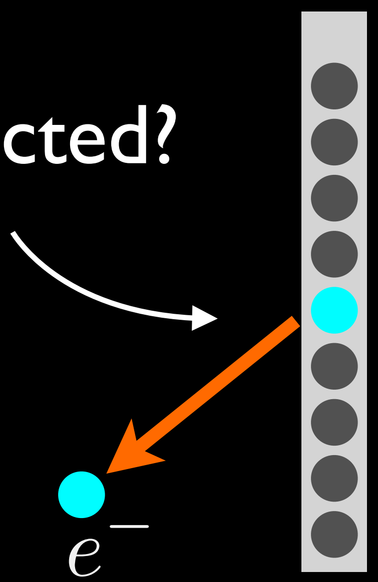


# Connecting waves and particles

## Photoelectric experiment




Can we predict when this electron will be ejected?




We know:

total rate of ejection $\propto$ light intensity

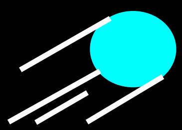
But.....

$$\Delta x \Delta p \geq \hbar$$

uncertainty principal

cannot follow photon accurately

No! Cannot know when/where electron will be ejected



$t = ?$

$(x, y, z) = ?$

Can only say:

Probability of an electron being ejected $\propto$ light intensity

chance of being found

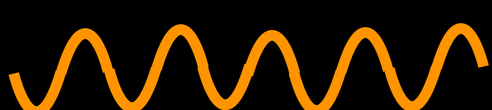
# Connecting waves and particles

Probability of an electron being ejected $\propto$ light intensity

High wave intensity,
high chance of photon hitting electron,
high chance of an electron being ejected

Because exact motion of photon is unknown

Equally (and more generally) we can ask:

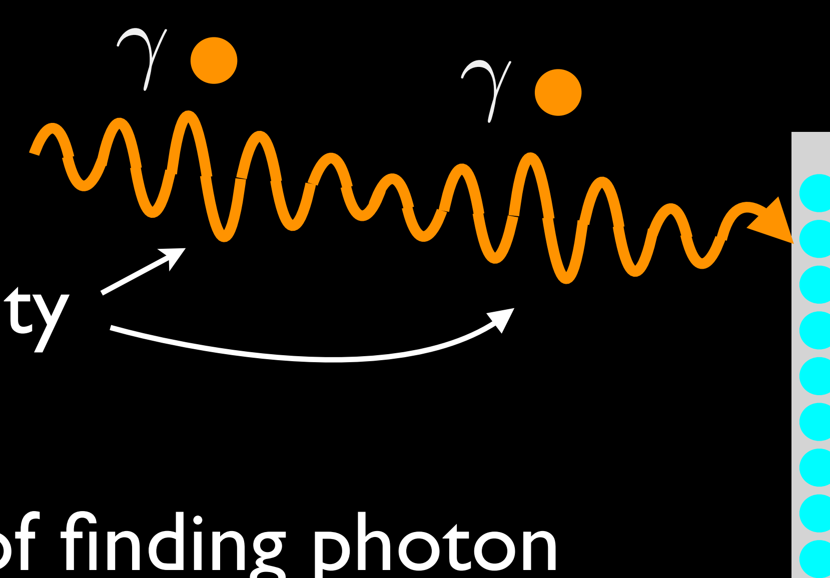


In a continuous wave...

$\gamma$ ●

where is photon?

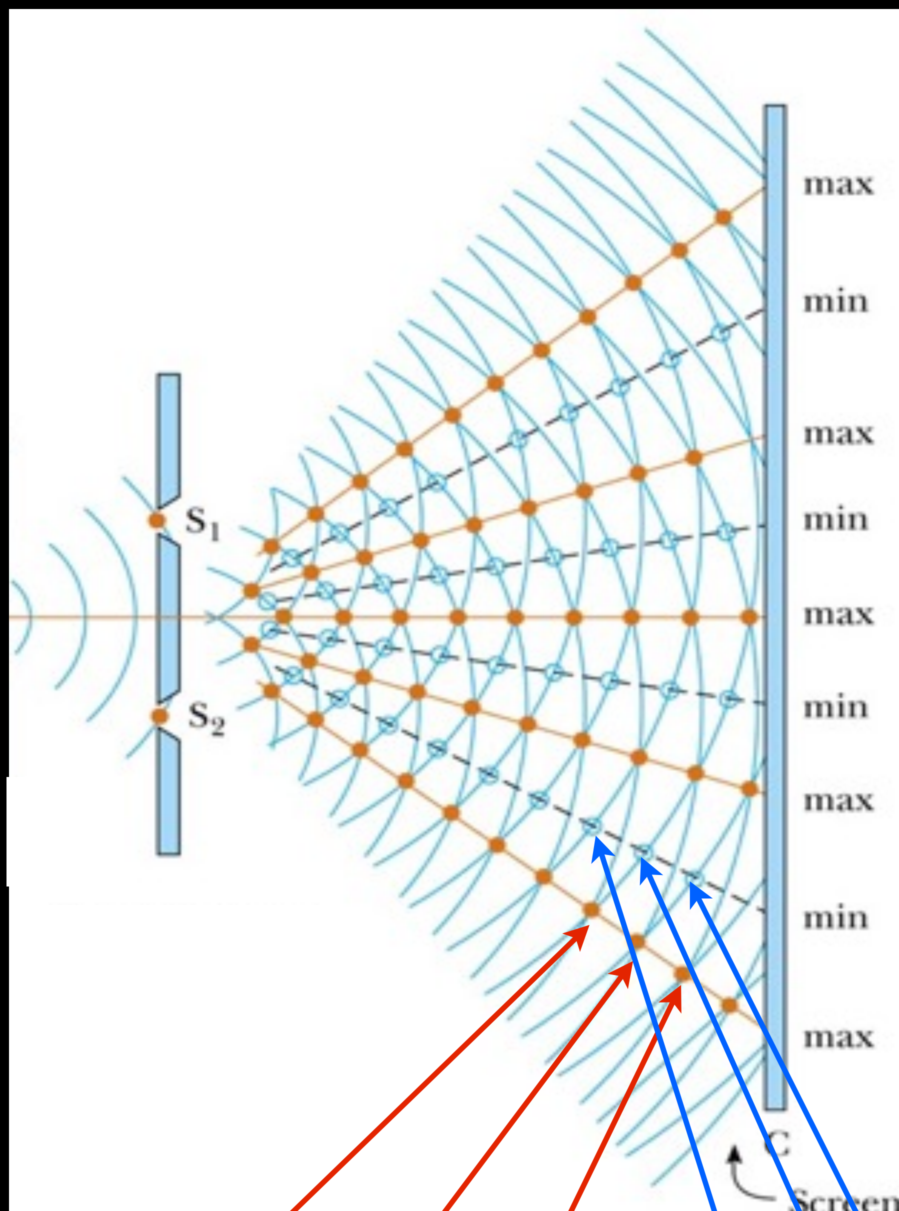
high wave intensity

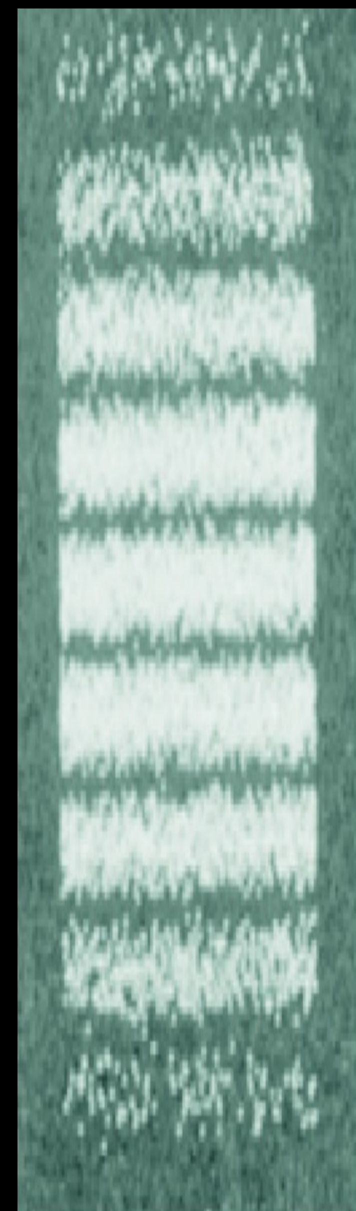



high probability of finding photon

Probability of finding a photon in a wave $\propto$ wave intensity

# Connecting waves and particles



High chance of finding photon

Low chance of finding photon

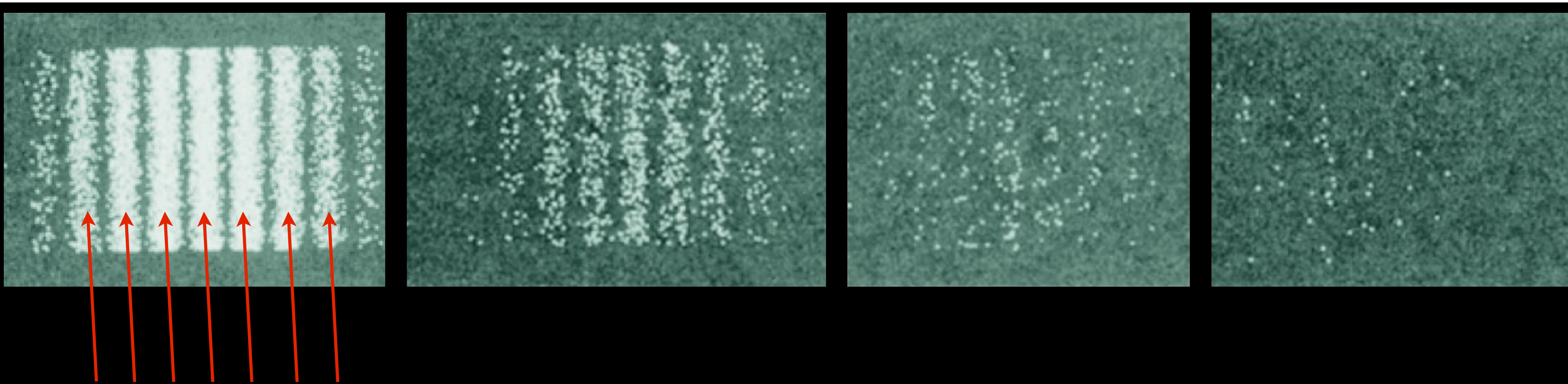
Classical description:

Interference between light waves

Quantum description:

Wave field gives the probability that a photon will be detected.

# Connecting waves and particles



High chance of finding photon

↓

Many photon hit screen

↓

Bright fringes

Low light intensity

↓

Less photons

↓

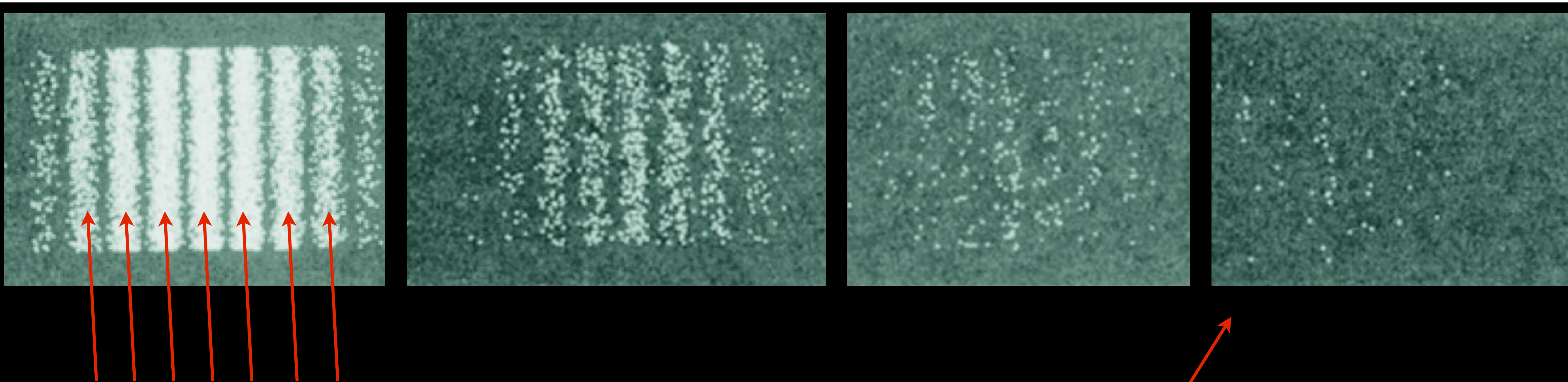
Weaker version of interference pattern?

classical prediction (light is a wave)

Random pattern?

quantum prediction

# Connecting waves and particles



High chance of finding photon

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Random pattern?

classical prediction (light is a wave)

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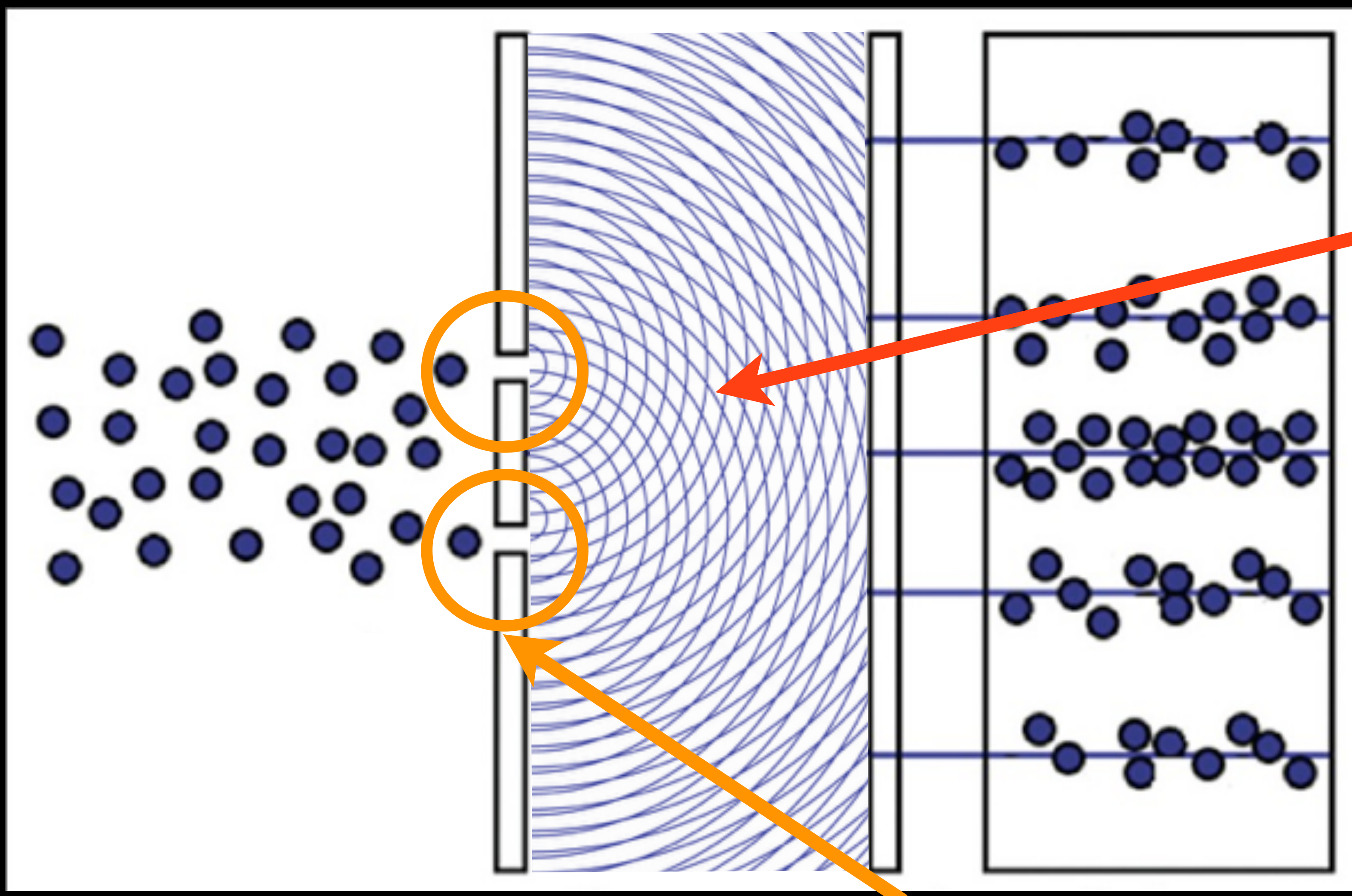
# Connecting waves and particles

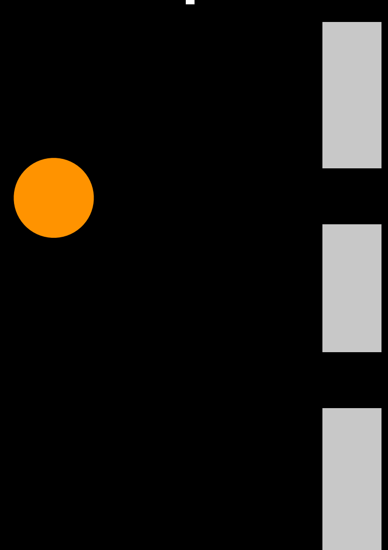


wave properties

(interference pattern)

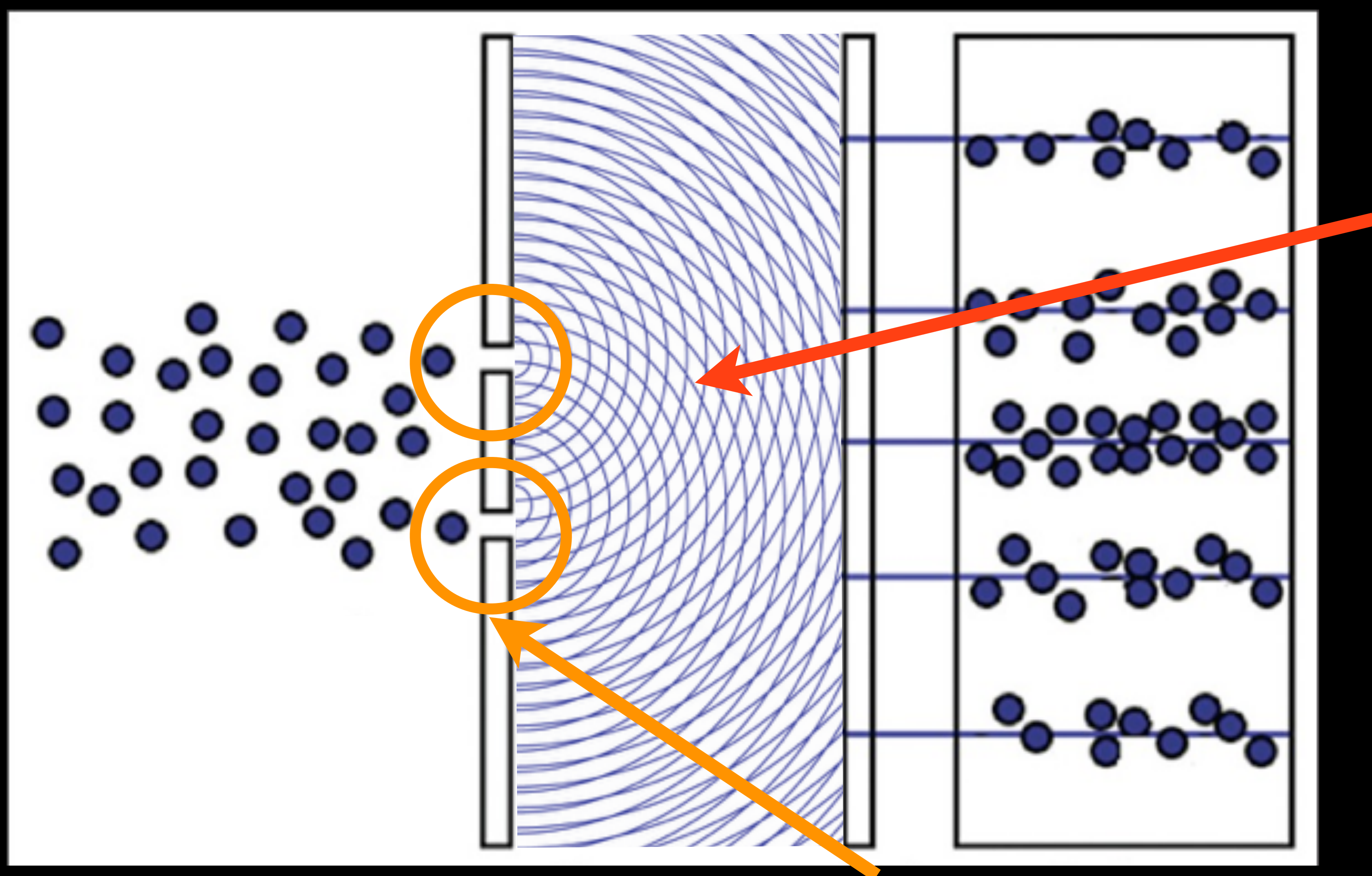
probability of finding photon (particle)

But if the photon is a particle….



Then it must go through the top slit…

# Connecting waves and particles

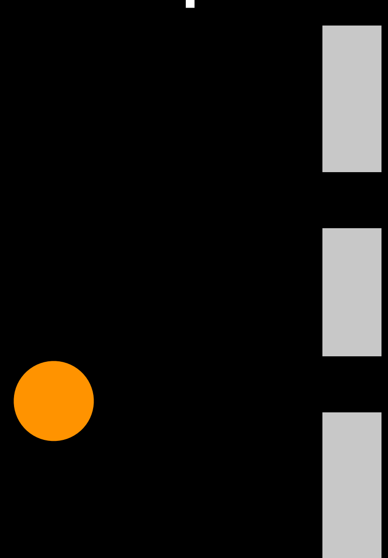


wave properties

(interference pattern)

probability of finding photon (particle)

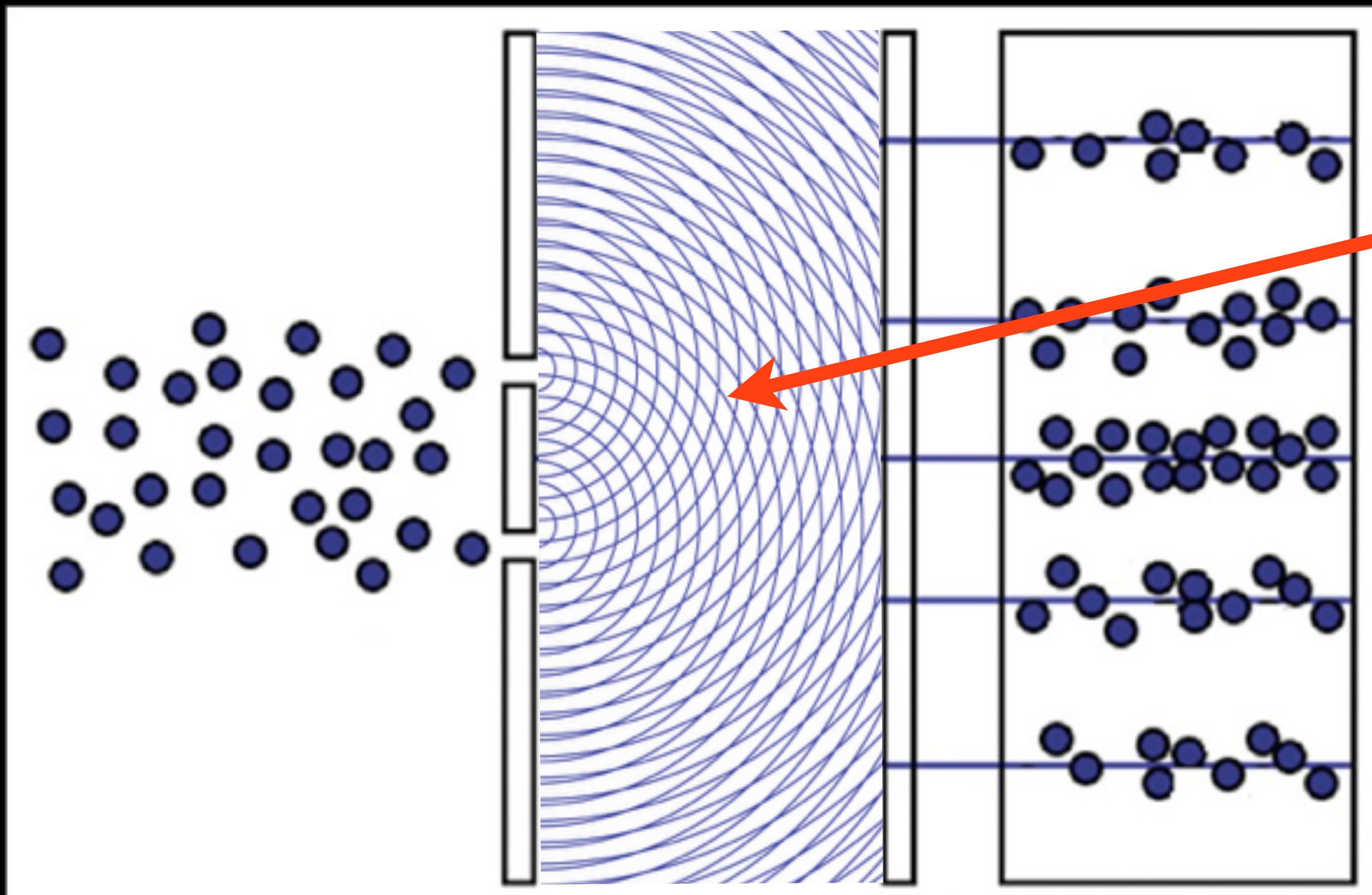
But if the photon is a particle….



Then it must go through the top slit…

… or the bottom slit

# Connecting waves and particles

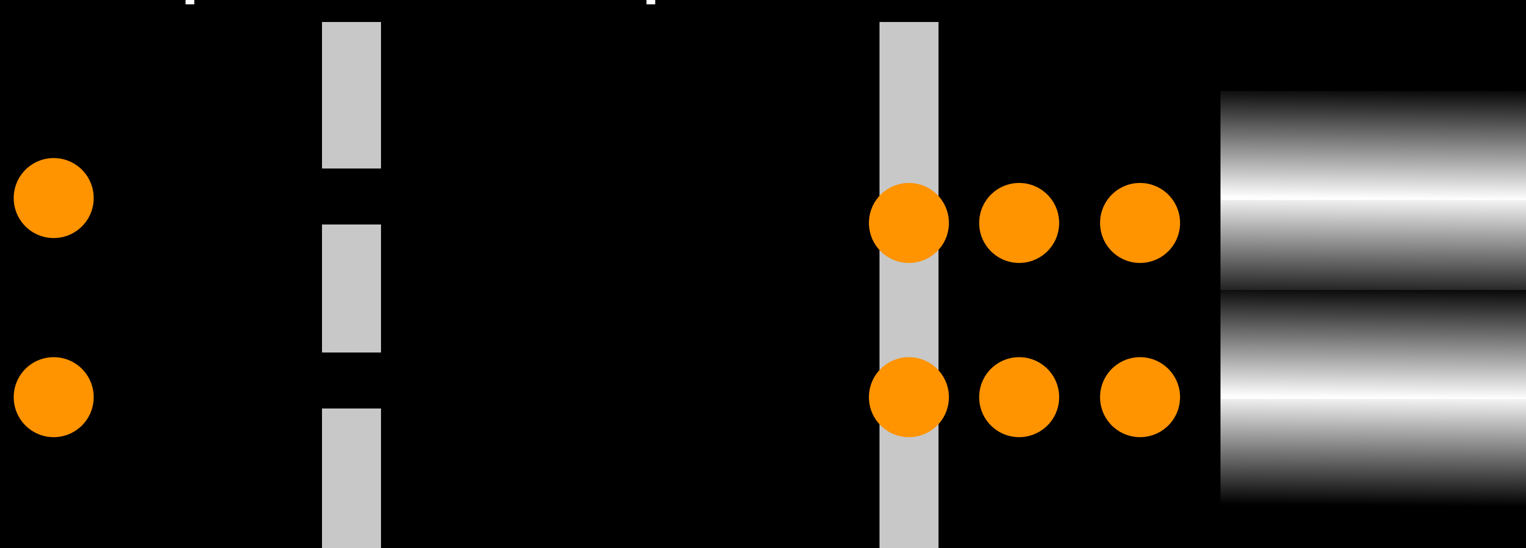


wave properties

(interference pattern)

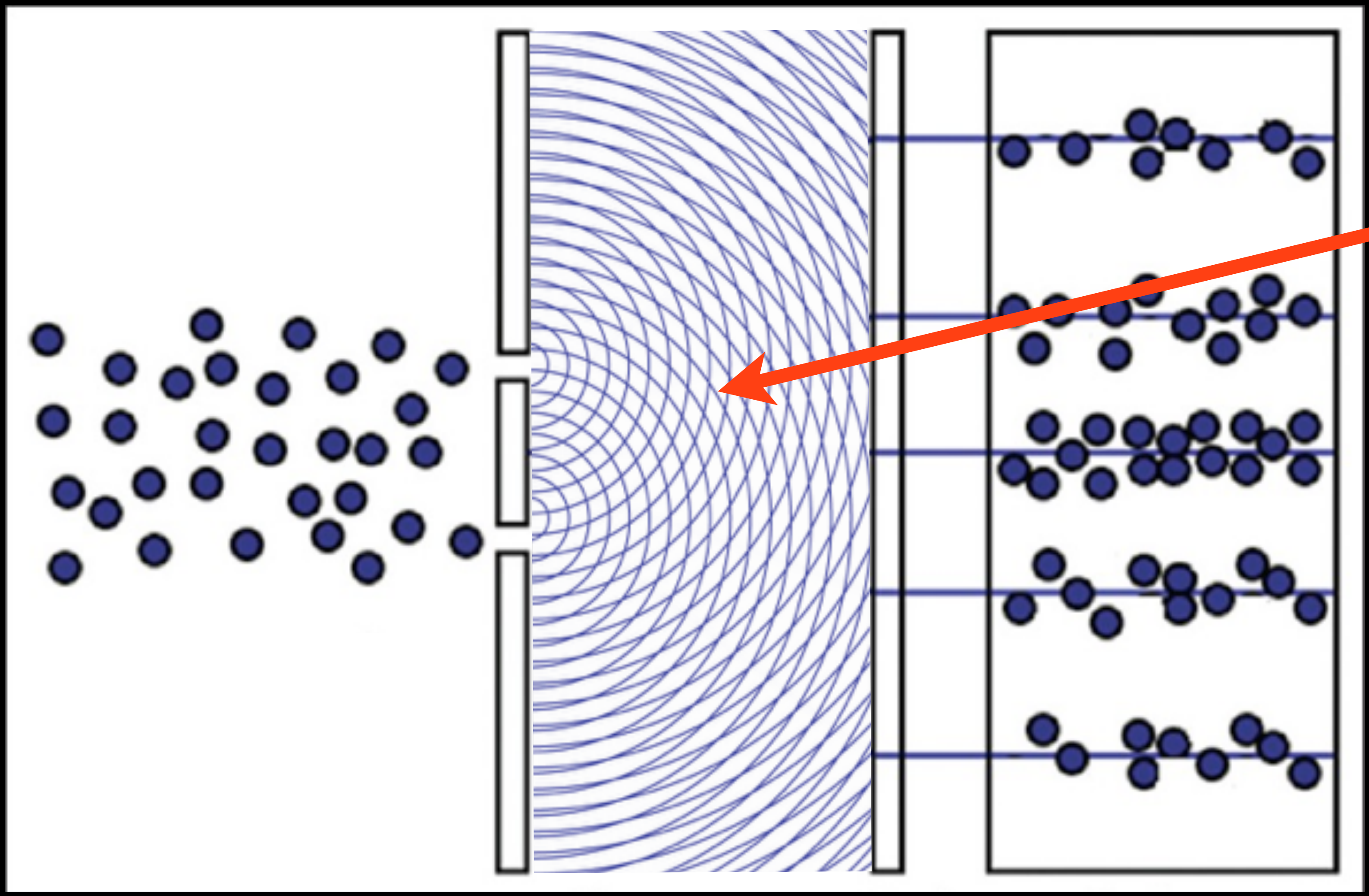
probability of finding photon (particle)

But if the photon is a particle….



and make only 2 bright fringes

# Connecting waves and particles



wave properties

(interference pattern)

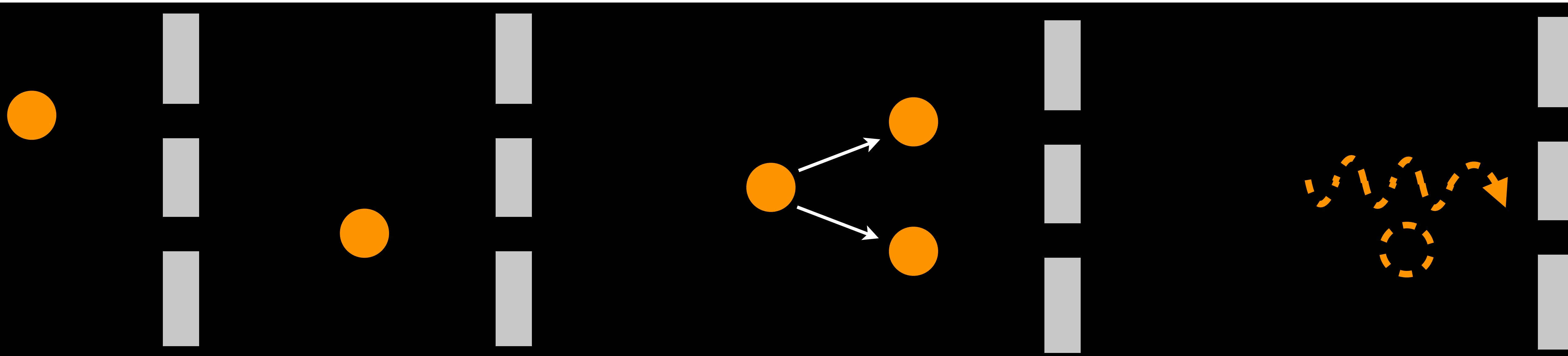
probability of finding photon (particle)

So which slit did the photon pass through?

The top slit?

or the bottom slit?

# Connecting waves and particles



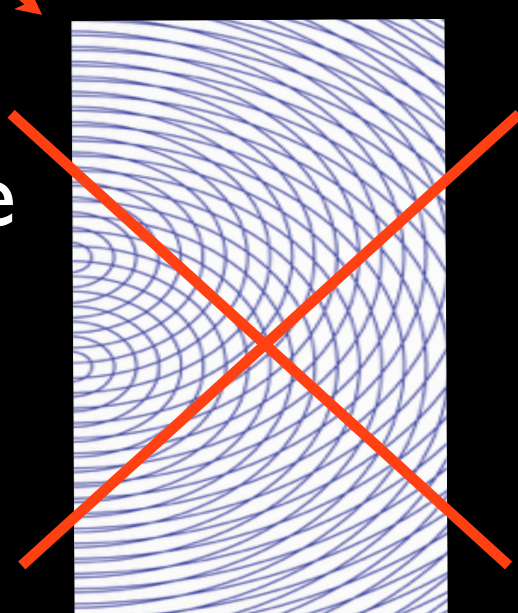
Top?

Bottom?

Both?

You can't ask that question.

Can't produce interference pattern



The photon is in two places at once??

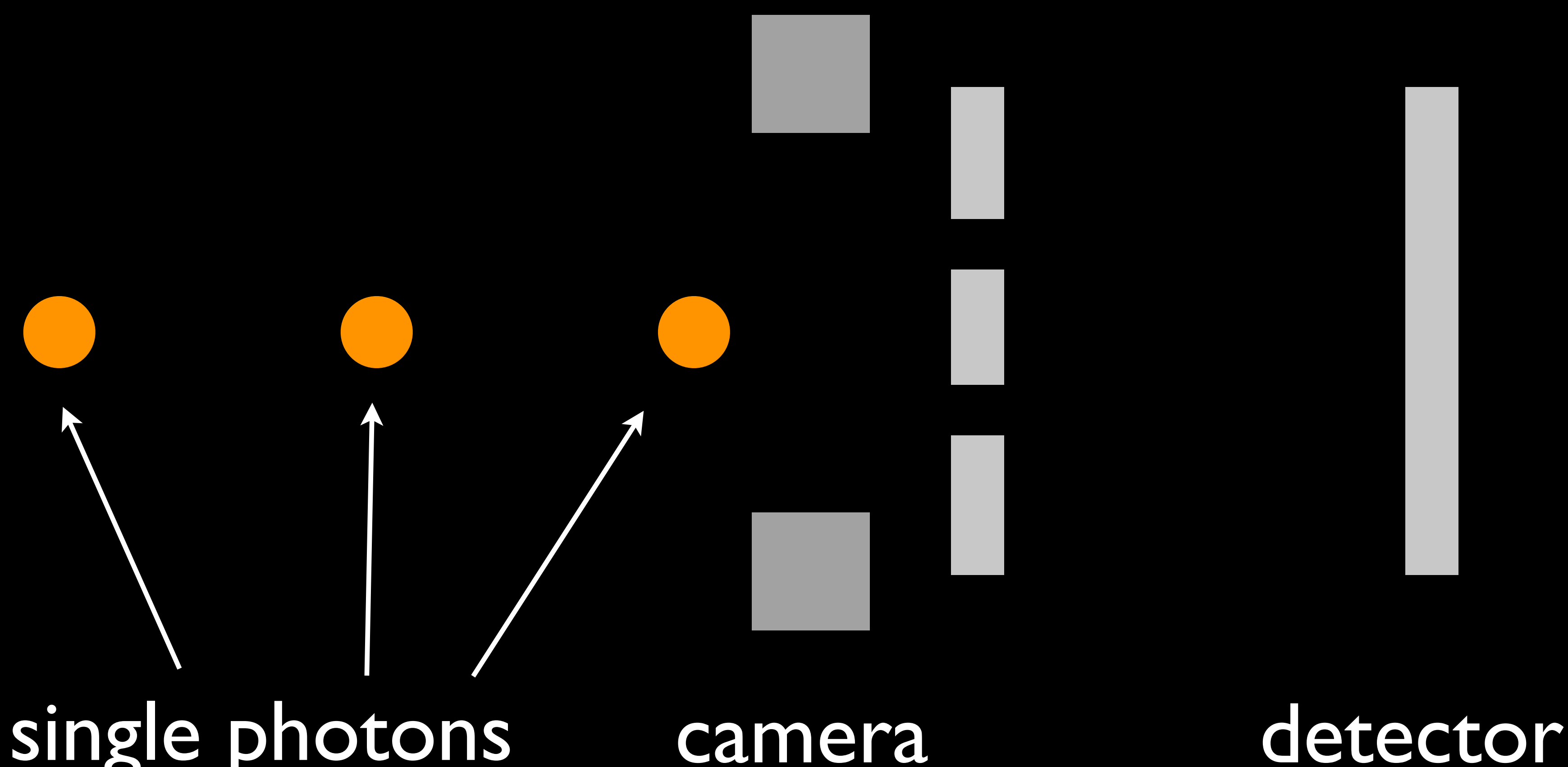
What?

# Connecting waves and particles


What if we try and see which slit the photon goes through?

## Let's see!

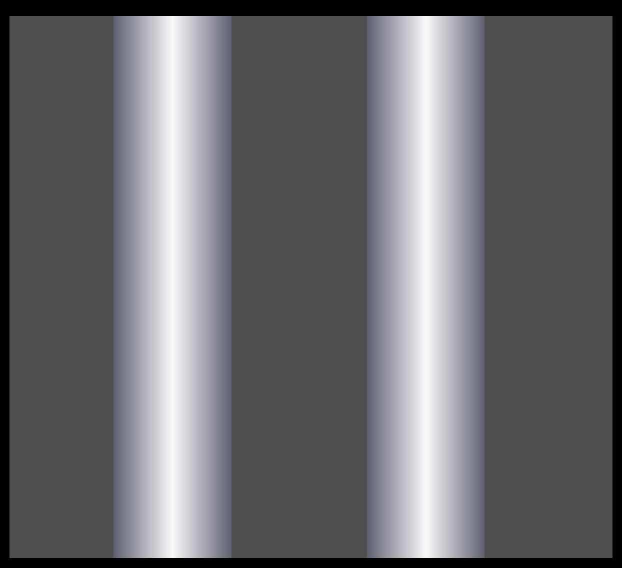
The experiment:



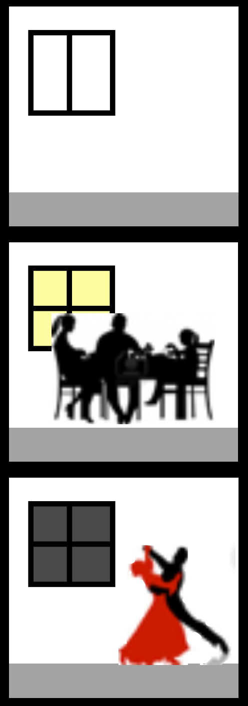
# Connecting waves and particles

# Connecting waves and particles

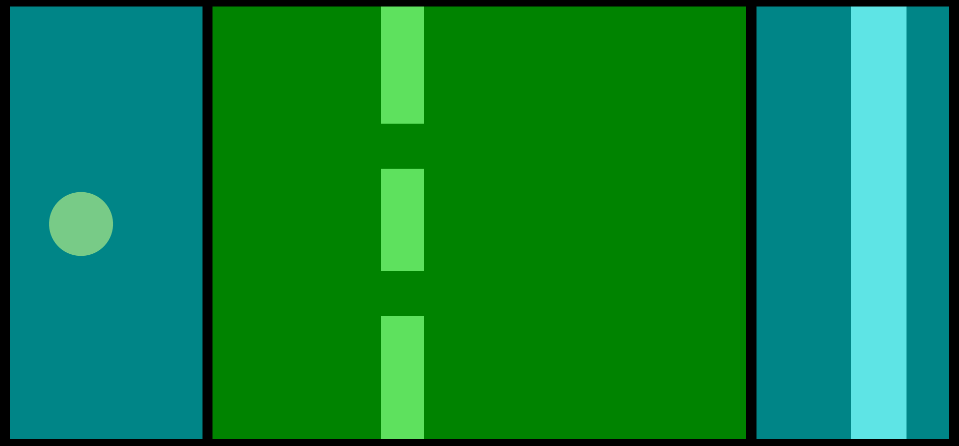

What if we try and see which slit the photon goes through?



No interference pattern!



If we try to detect the particle, it acts like a particle.



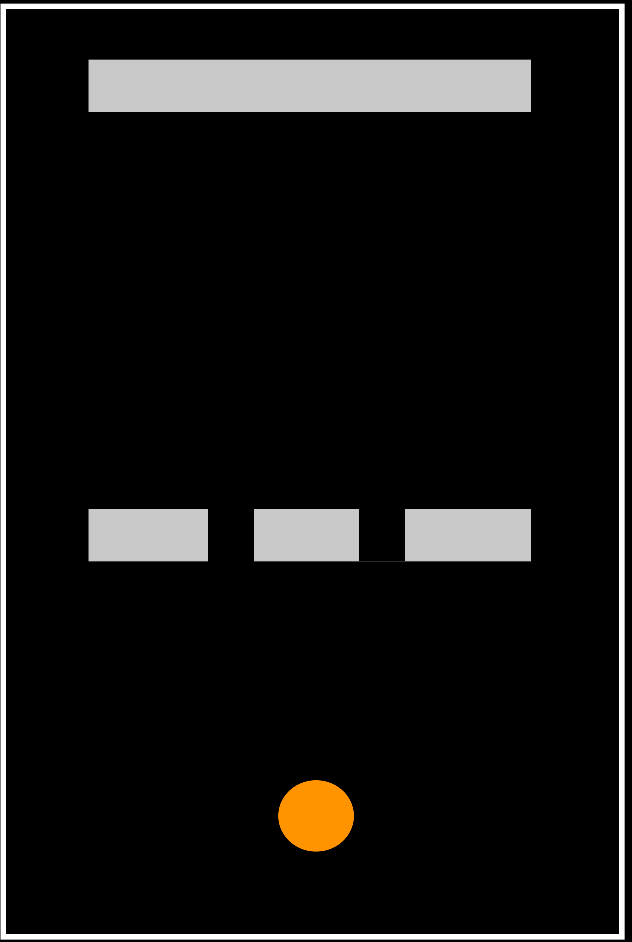
Only when we don't observe the photon can it be both particle and wave

# Connecting waves and particles Quiz

If you shoot a photon through 2 slits to hit a screen it...:

(A) Cannot hit the middle: path is blocked

(B) Hit a random point:
Equal probability of hitting anywhere



(C) Must hit maximum of interference pattern

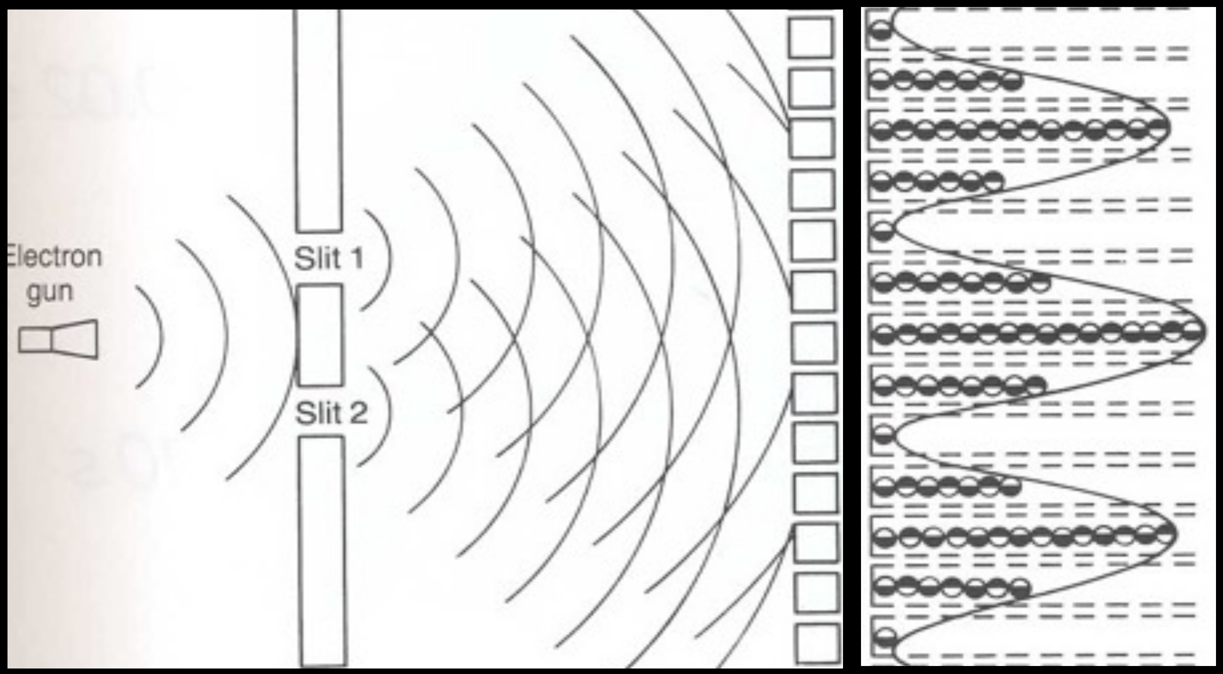
(D) Can hit anywhere, but most likely to hit where interference pattern is brightest

# Connecting waves and particles

Last lecture: matter and light show both wave and particle properties.

$$\lambda = \frac{h}{p}$$ (de Broglie wavelength)

Double slit experiment also works with electrons.




If Maxwell's equations describe the wave properties of light

(and probability of finding a photon)

What describes matter waves?

# The Schrodinger Equation

1926, Erwin Schrodinger described matter waves with wave function:

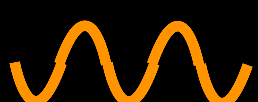
$$\psi(\bar{x}, t)$$

space — time

Let's simplify

$$\psi(x)$$

1D space



Sinusoidal wave: $\psi(x) = A \sin kx$ with $k = \frac{2\pi}{\lambda}$

Differentiate twice: $$\frac{d^2\psi(x)}{dx^2} = -Ak^2 \sin kx = -k^2\psi(x)$$

# The Schrodinger Equation

From de Broglie $\lambda = \frac{h}{p}$ ➡ $k = \frac{2\pi p}{h} = \frac{p}{\hbar}$

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ $(p = mv)$

Total energy: $E = K + U$ ➡ $E - U = \frac{p^2}{2m}$ ➡ $p^2 = 2m(E - U)$

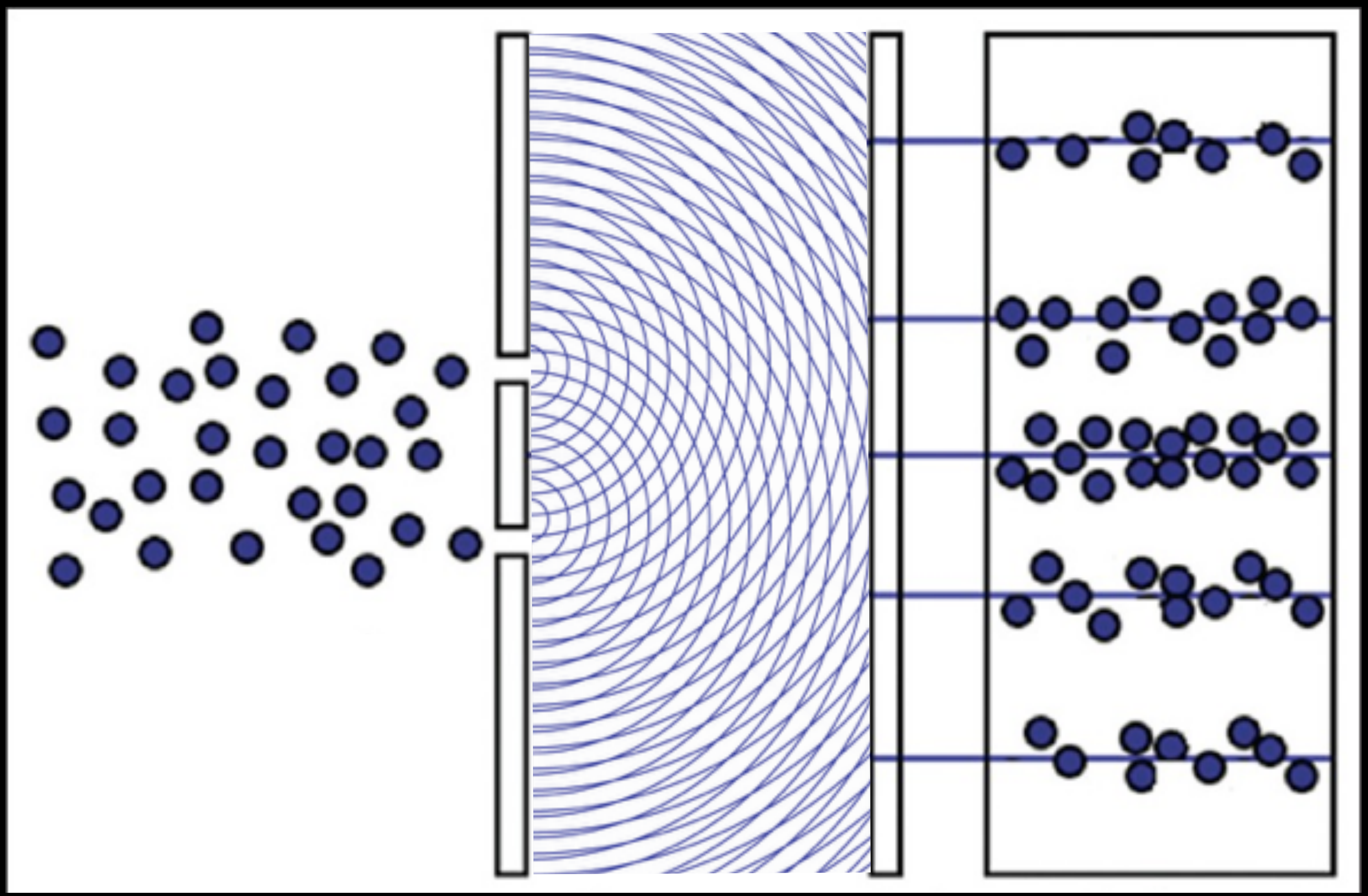
Therefore: $k^2 = \frac{p^2}{\hbar^2} = \frac{2m(E-U)}{\hbar^2}$

➡
$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

(Time-independent Schrodinger Equation)

# What does $\psi$ mean?

For light...



Maxwell's wave equations

↓

Give light intensity

↓

Probability of finding photon

From Maxwell: $E^2 \propto I \propto P$

field — intensity — probability

Similarly for matter: $P \propto \psi^2$

# What does $\psi$ mean?

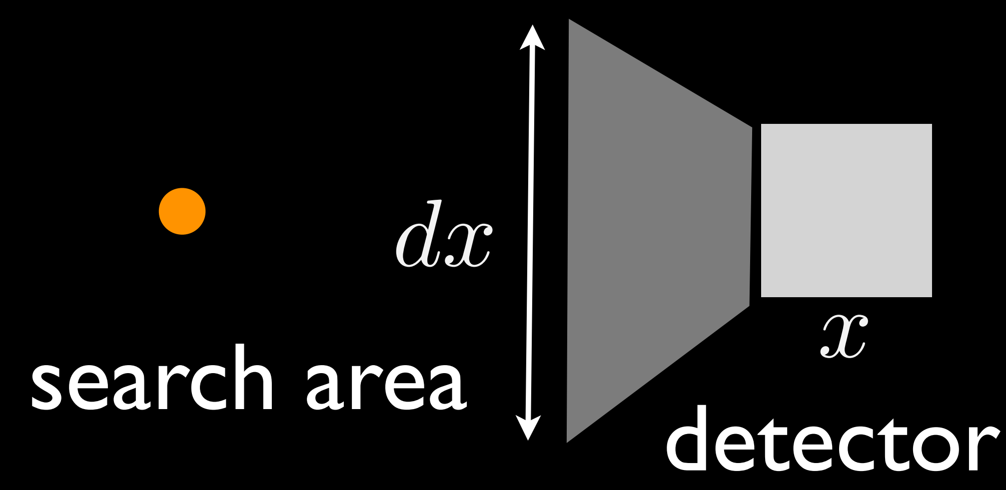
Probability of finding particle in small length dx:

$$P(x) = \psi^2(x)dx$$

Therefore:

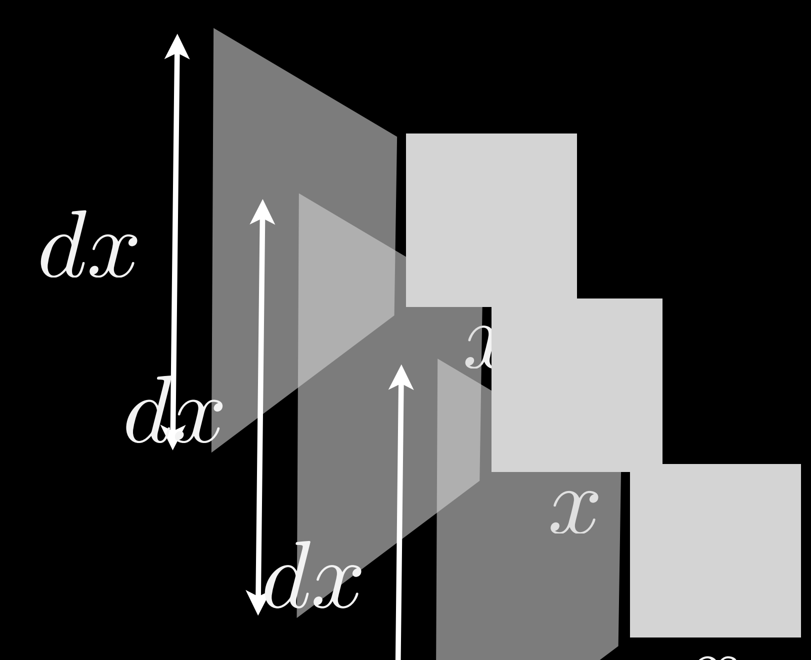
In 1 experiment:

$P(x)$ = probability of finding particle by a detector at $x$ searching over length $dx$

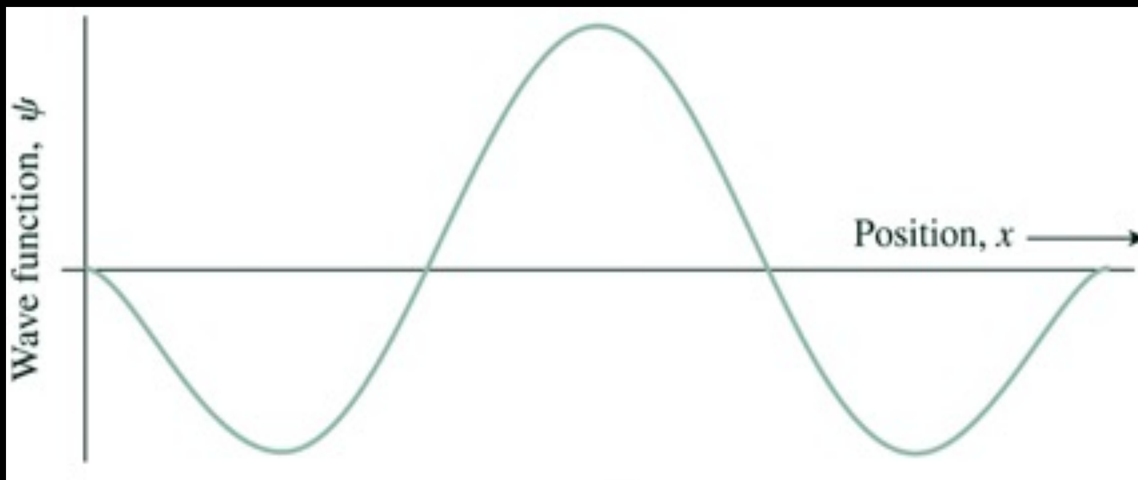



In many experiments:

$P(x)$ = fraction of experiments that will find particles in the detector at $x$

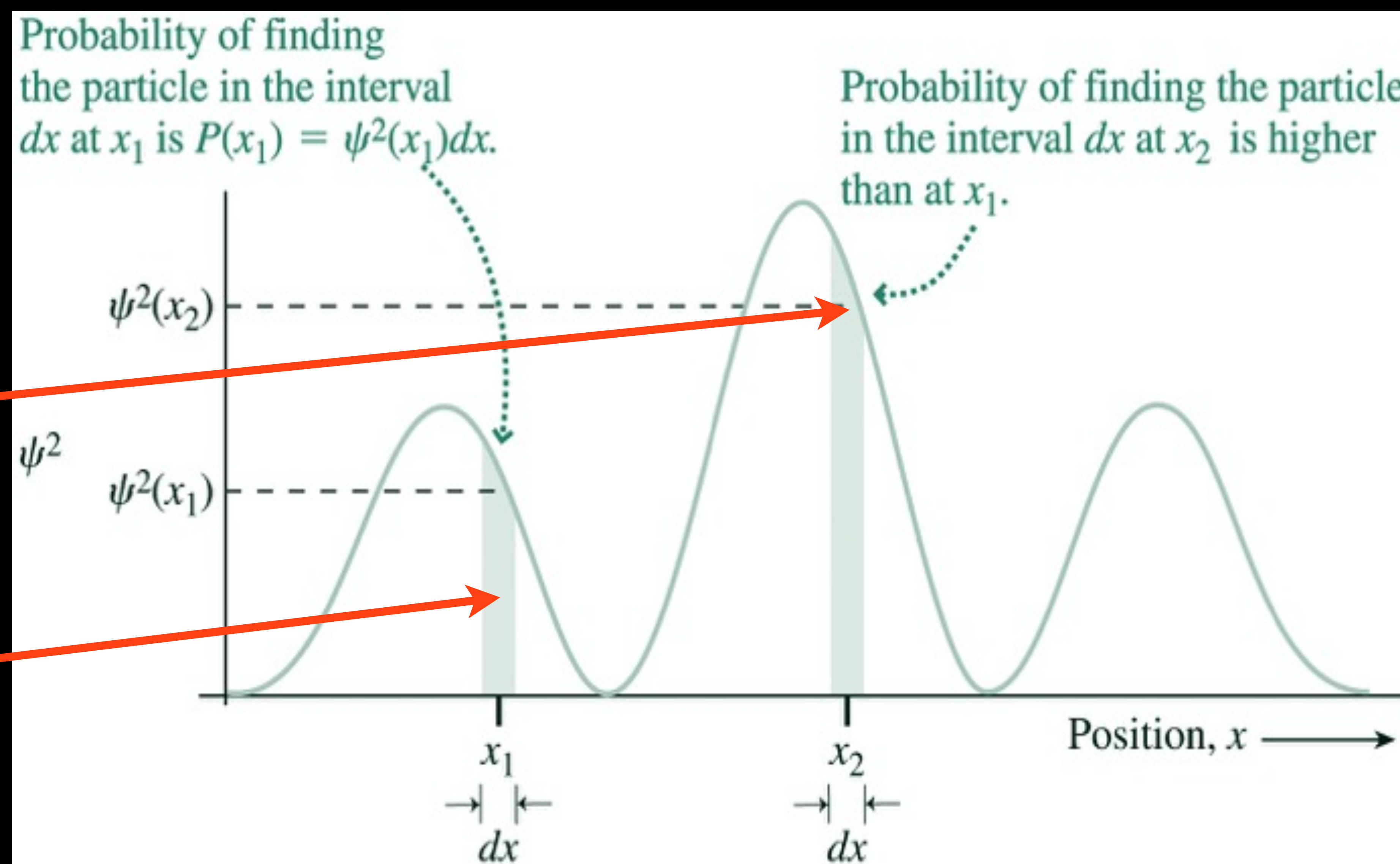


# What does $\psi$ mean?




← $\psi(x)$

$\psi^2(x)$ →




more likely to find a particle here

than here

# What does $\psi$ mean? Quiz

This is the wave function for a neutron.

At which point is the neutron most likely to be found?

(A) $x = 0$

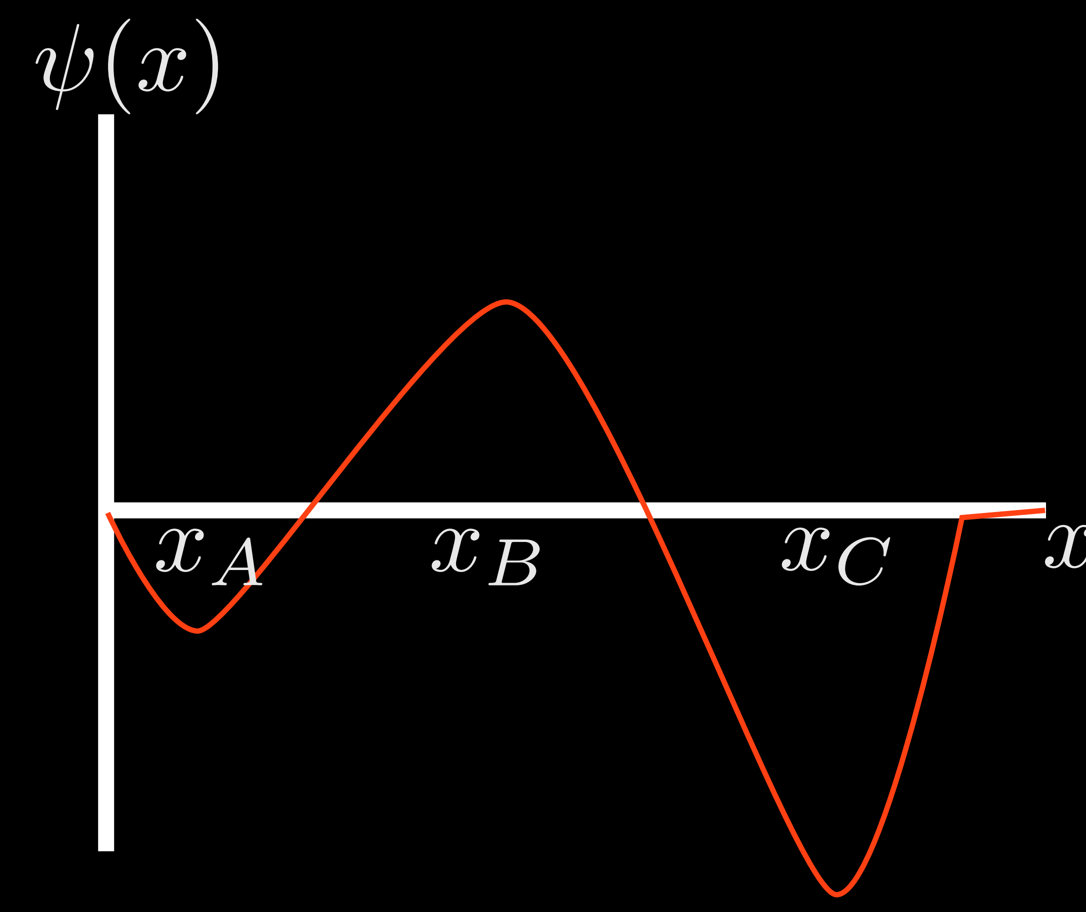
(B) $x = x_A$

(C) $x = x_B$

(D) $x = x_C$

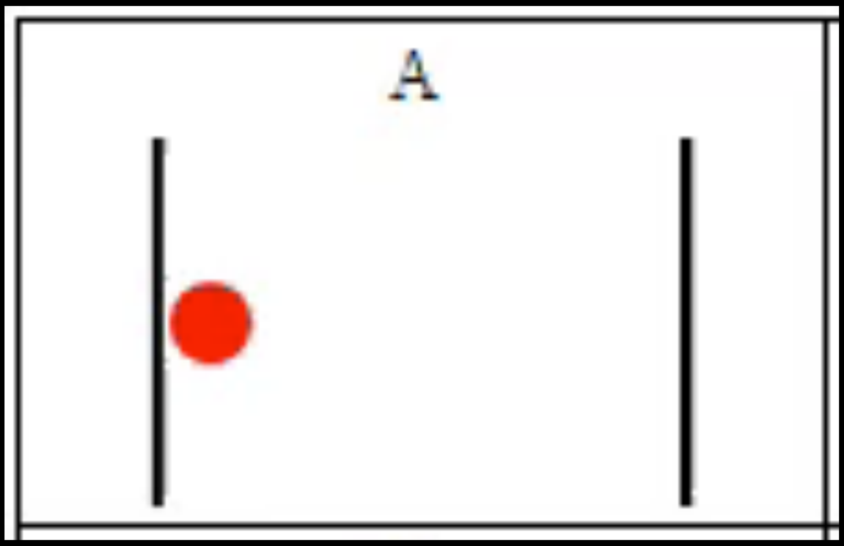
$$P(x) = \psi^2(x)dx$$

(E) There is no most likely place



# Solving the Schrodinger Equation

System 1: The infinite square well




Particle trapped in a 1D box with infinitely high walls.

e.g. electron in a superconductor:

particle can move inside superconductor, but can't leave.

Question: Where is the particle most likely to be found?

Classical answer:

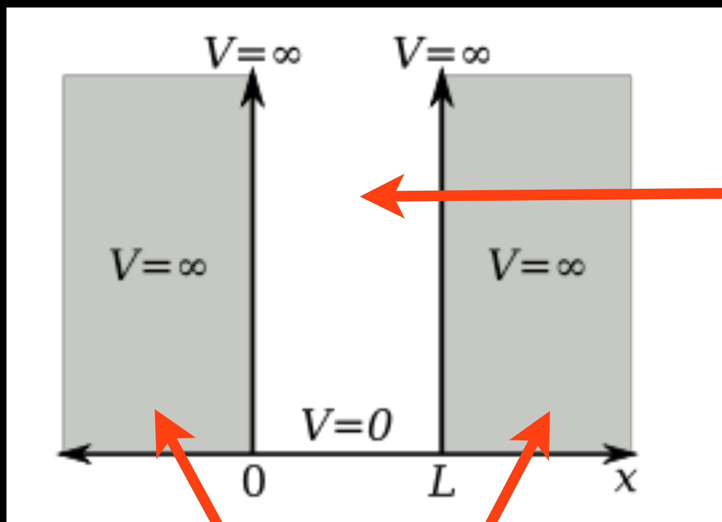
Particle moves back and forth with constant speed: $v = \text{constant}$

Every location equally likely.

Without friction, its energy is constant. $E = \text{constant}$

Energy can take any value. $E : [0, \infty]$

# Solving the Schrodinger Equation




Inside well:

Particle feels no forces.

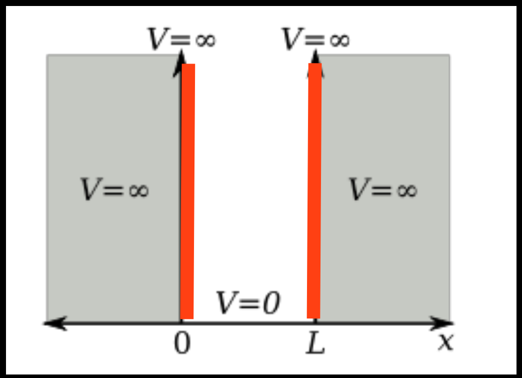
Potential Energy is constant, choose $U = 0$

for $0 < x < L$

Outside well: $U = \infty$ for $x > L$ and $x < 0$

Particle *cannot* move here: $\psi = 0$

Inside well: $0 < x < L$




At well walls: $U$ changes from $0 \rightarrow \infty$

so $\psi \rightarrow 0$

Boundary conditions: $\psi = 0$ at $x = 0$ and $x = L$

# Solving the Schrodinger Equation

Inside well: $0 < x < L$

Schrodinger Eq: $$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

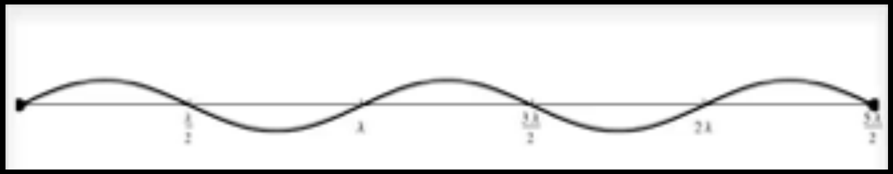
$U(x) \rightarrow 0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

From the boundary conditions: $\psi(0) = 0$

$\psi(L) = 0$

standing wave!



Therefore: $k = \frac{n\pi}{L}$ so $\psi(x) = A\sin kx \Rightarrow \psi(x) = A\sin\left(\frac{n\pi x}{L}\right)$

Likely solution to the Schrodinger equation

# Solving the Schrodinger Equation

$$\psi(x) = A\sin\left(\frac{n\pi x}{L}\right)$$

does this solution satisfy the Schrodinger Equation?

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

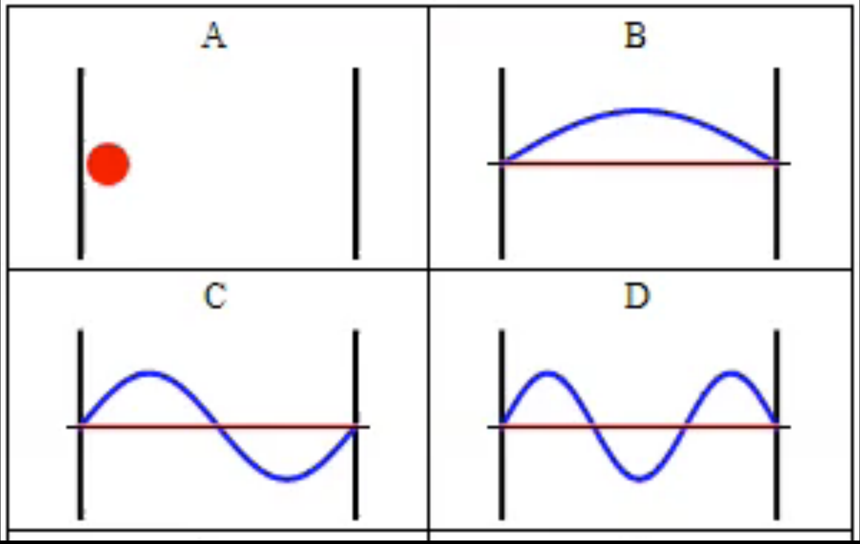
$$\left(-\frac{\hbar}{2m}\right)\left[-A\frac{n^2\pi^2}{L^2}\sin\left(\frac{n\pi x}{L}\right)\right] = EA\sin\left(\frac{n\pi x}{L}\right)$$

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2}$$

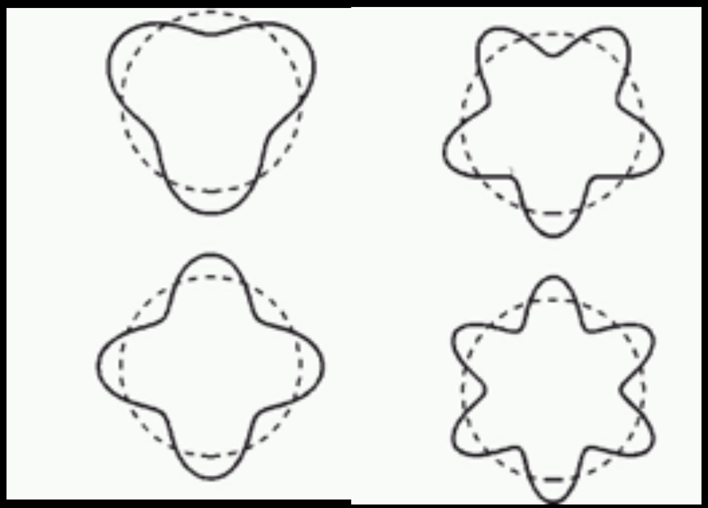
energy levels for an infinite square well potential

Schrodinger Equation ➡ energy quantisation

# Solving the Schrodinger Equation

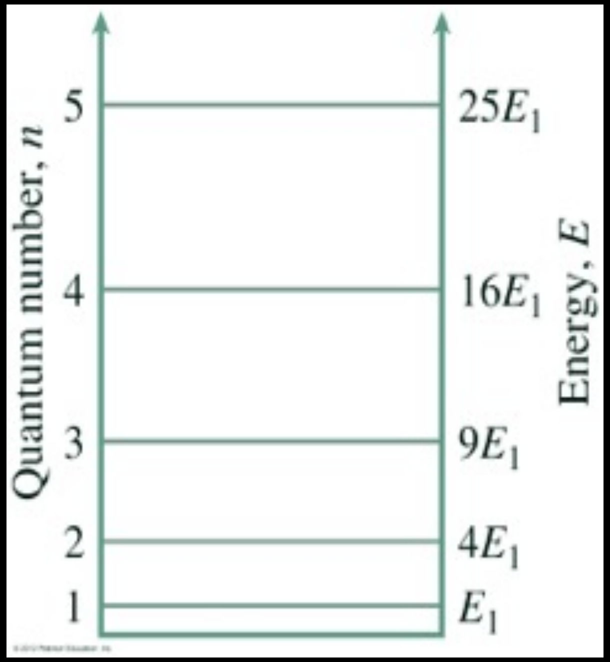



Same result at de Broglie →



Matter in a confined system must be standing waves.

(e.g. superconductors, simple atoms...)




$$E = \frac{n^2 h^2}{8mL^2}$$

$n$ is the quantum number.

$n = 0$ gives $\psi = 0$, no chance of finding particle

Therefore, $n > 0$ $\quad n = 1, 2, 3...$

Lowest possible energy = ground state energy $\quad E_1 = \frac{h^2}{8mL^2}$

Unlike classical mechanics, $E = 0$ is not possible.

# Solving the Schrodinger Equation Quiz

Electron A is in a square well 1 nm wide $10^{-9}\text{m}$

Electron B is in a square well 1 pm wide $10^{-12}\text{m}$

How do their ground-state energies compare?

(A) $E_B = E_A$

(B) $E_B = 10^2 E_A$

(C) $E_B = 10^3 E_A$

(D) $E_B = 10^6 E_A$

$$E = \frac{n^2 h^2}{8mL^2} \propto \frac{1}{L^2}$$

# Solving the Schrodinger Equation Quiz

Find the width of an infinite square well in which a proton's minimum energy is 100 eV.

$$hc = 1240\text{eV}\cdot\text{nm}$$
$$mc^2 = 938\text{MeV}$$

(A) $2.1\text{nm}$

(B) $1.43\text{nm}$

(C) $1.43\text{pm}$

(D) $2.1\text{pm}$

$$E_1 = \frac{h^2}{8mL^2}$$

$$L = hc\sqrt{\frac{1}{8mc^2E}}$$

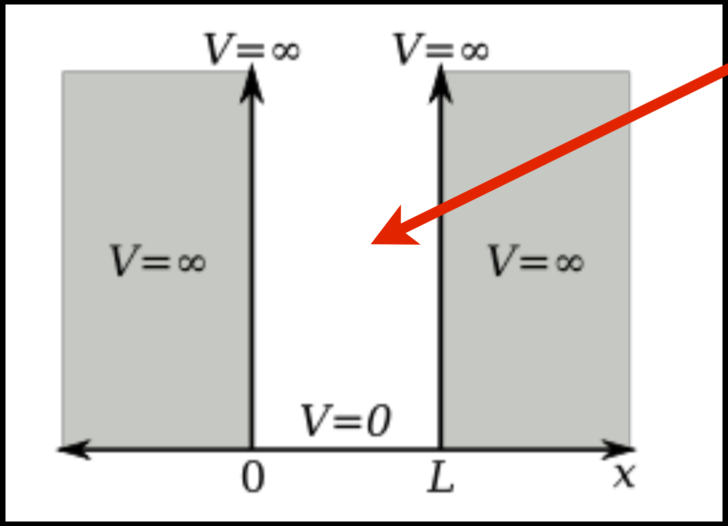
$$= (1240\text{eVnm})\sqrt{\frac{1}{8(939\text{MeV})(100\text{eV})}}$$

$$= 1.43\text{pm}$$

# Solving the Schrodinger Equation

$$\psi(x) = A \sin\left(\frac{n\pi x}{L}\right)$$

what is $A$?




Particle *must* be in well somewhere

$$\int_0^L \psi^2(x)dx = \int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

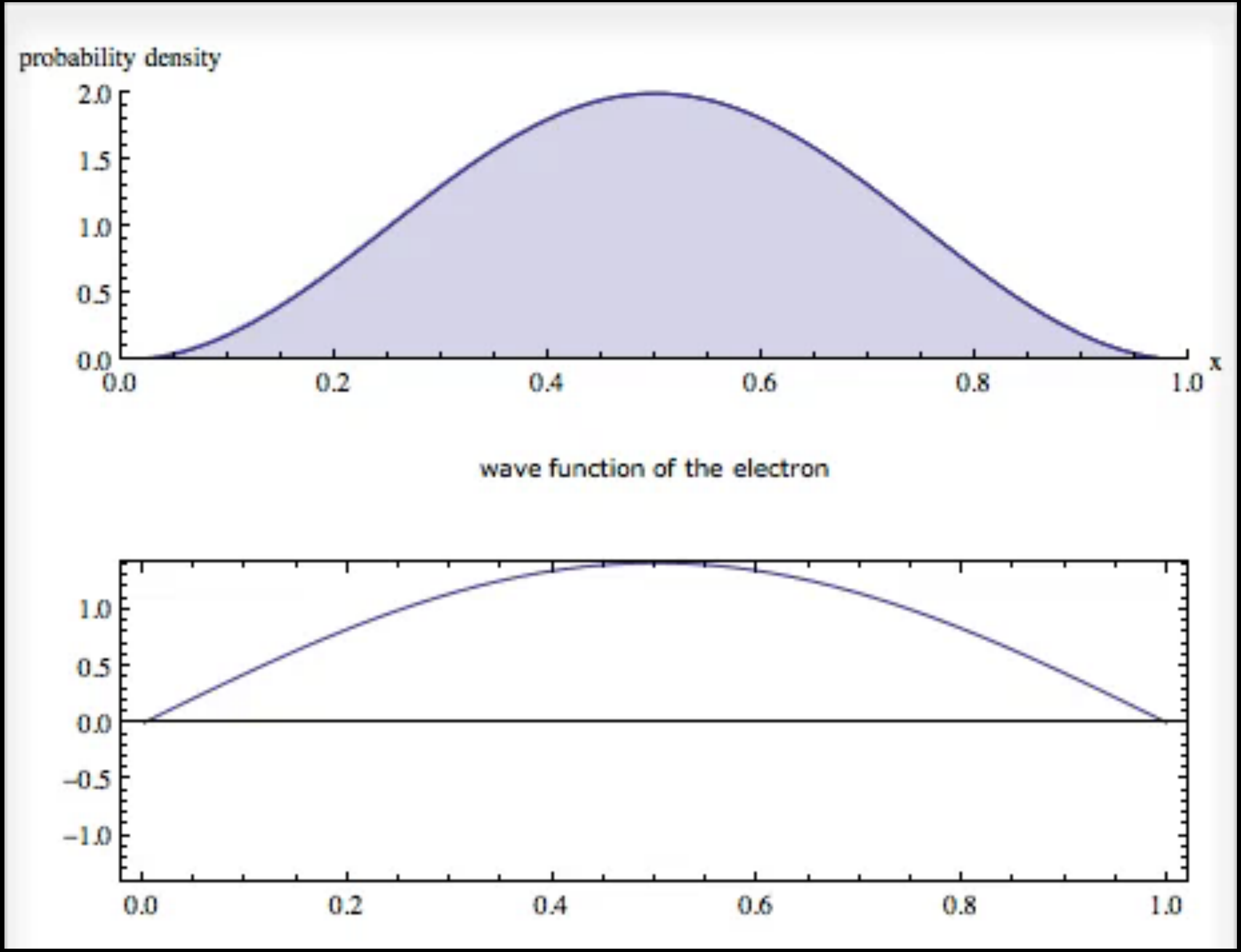
$$A^2 \frac{L}{2} = 1 \Rightarrow A = \sqrt{\frac{2}{L}}$$

$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

nth quantum state

# Solving the Schrodinger Equation

Question: Where is the particle most likely to be found?

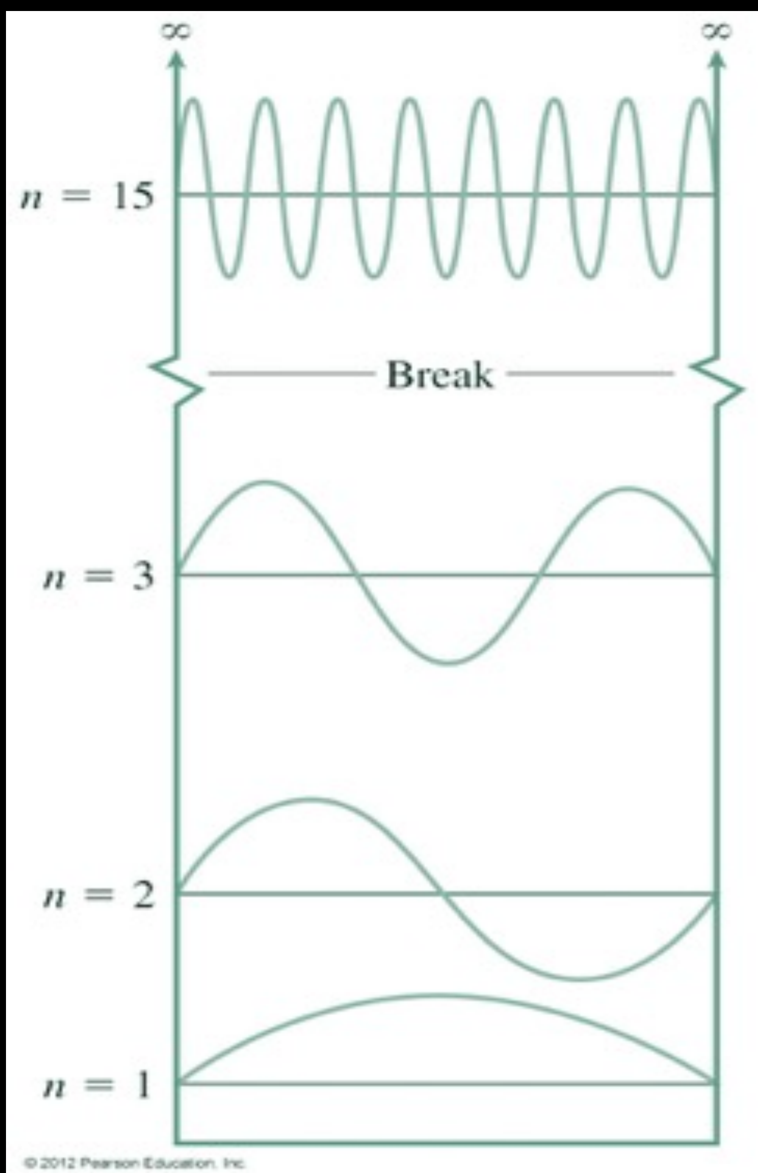



$\psi^2(x)$

$\psi(x)$

# Solving the Schrodinger Equation

Question: Where is the particle most likely to be found?




$$\psi(x) \longrightarrow \psi^2(x)$$

Classical prediction:

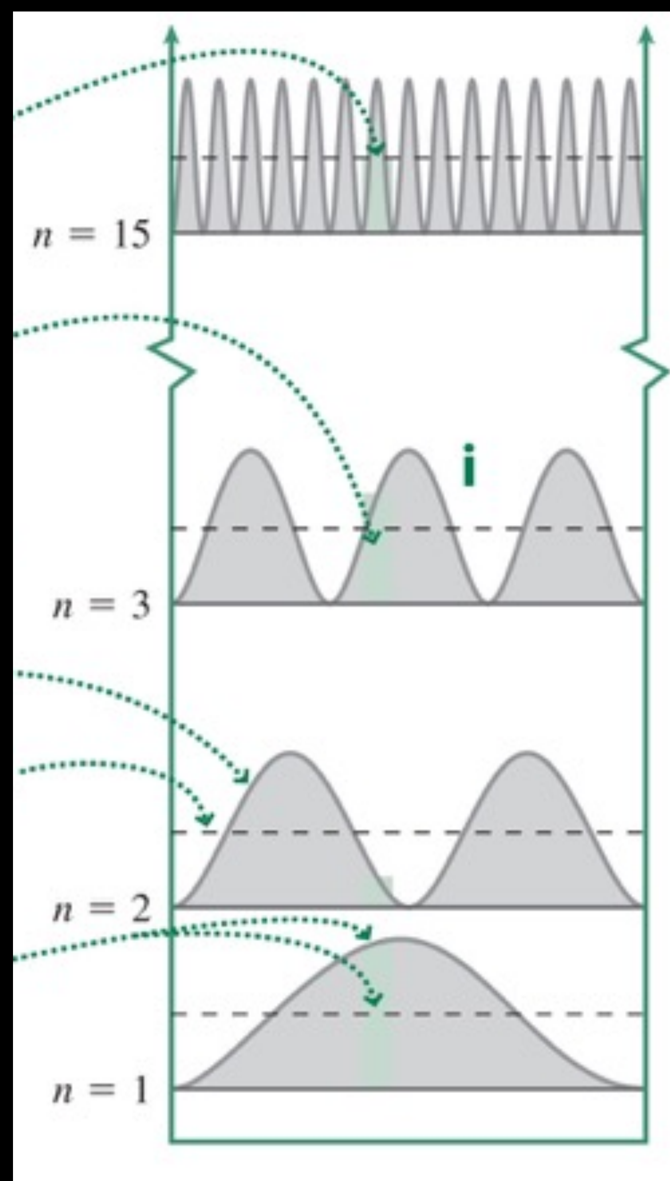
Every location equally likely.

High n:

Classical prediction OK

Low n:

Classical prediction very bad.



# Solving the Schrodinger Equation Ex.

A particle is in the ground state of an infinite square well.

Find the probability that it is found in the left-hand quarter of the well

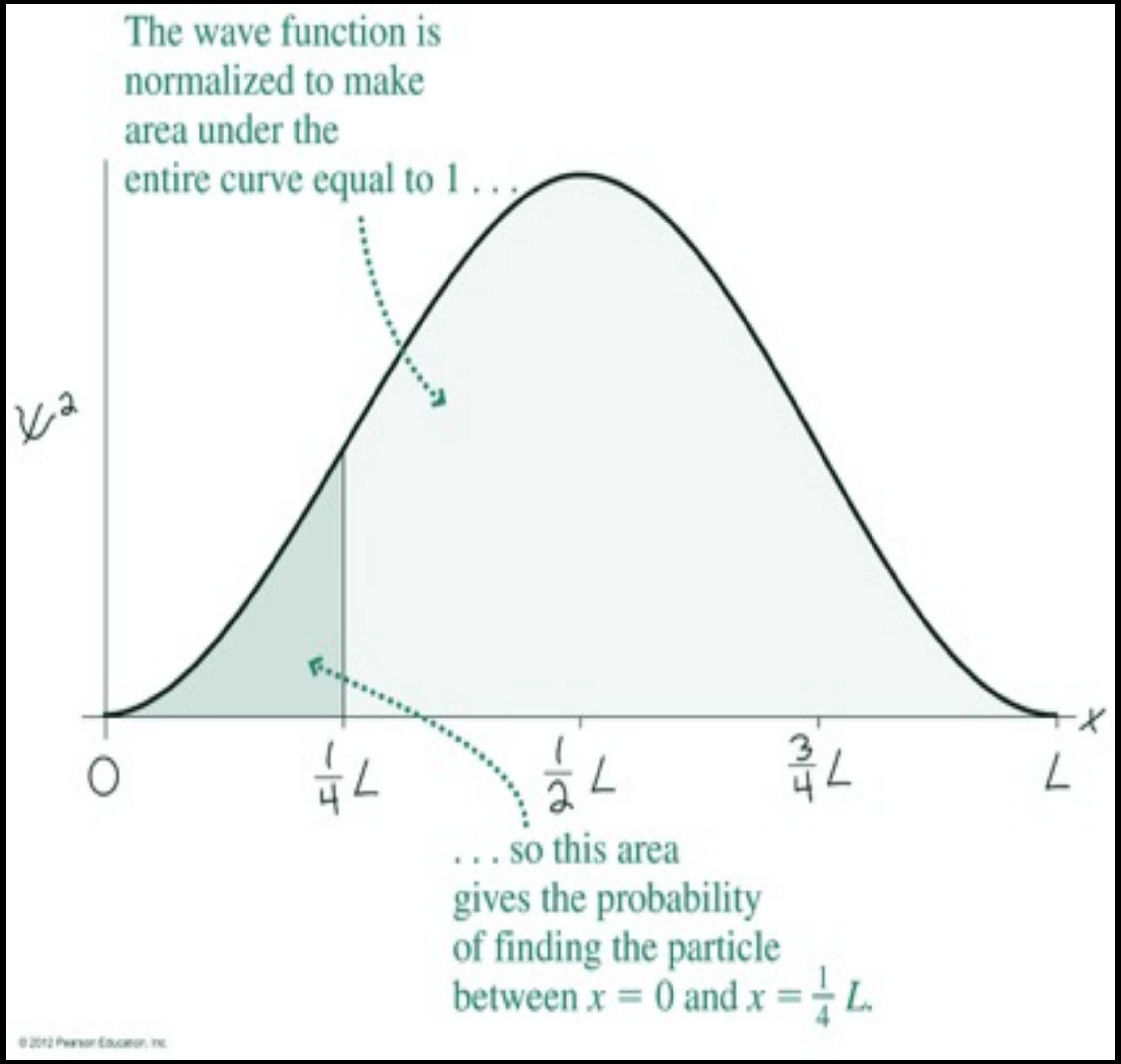
Ground state: $$\psi_1 = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

$$P = \frac{2}{L}\int_0^{L/4} \sin^2\left(\frac{\pi x}{L}\right) dx$$

standard integral

$$\int \sin^2 ax = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$P = \frac{2}{L}\left(\frac{L}{8} - \frac{L}{4\pi}\right) = 0.091$$




much lower than classical probability of 0.25.

# Solving the Schrodinger Equation Quiz

A particle is in the ground state of an infinite square well.

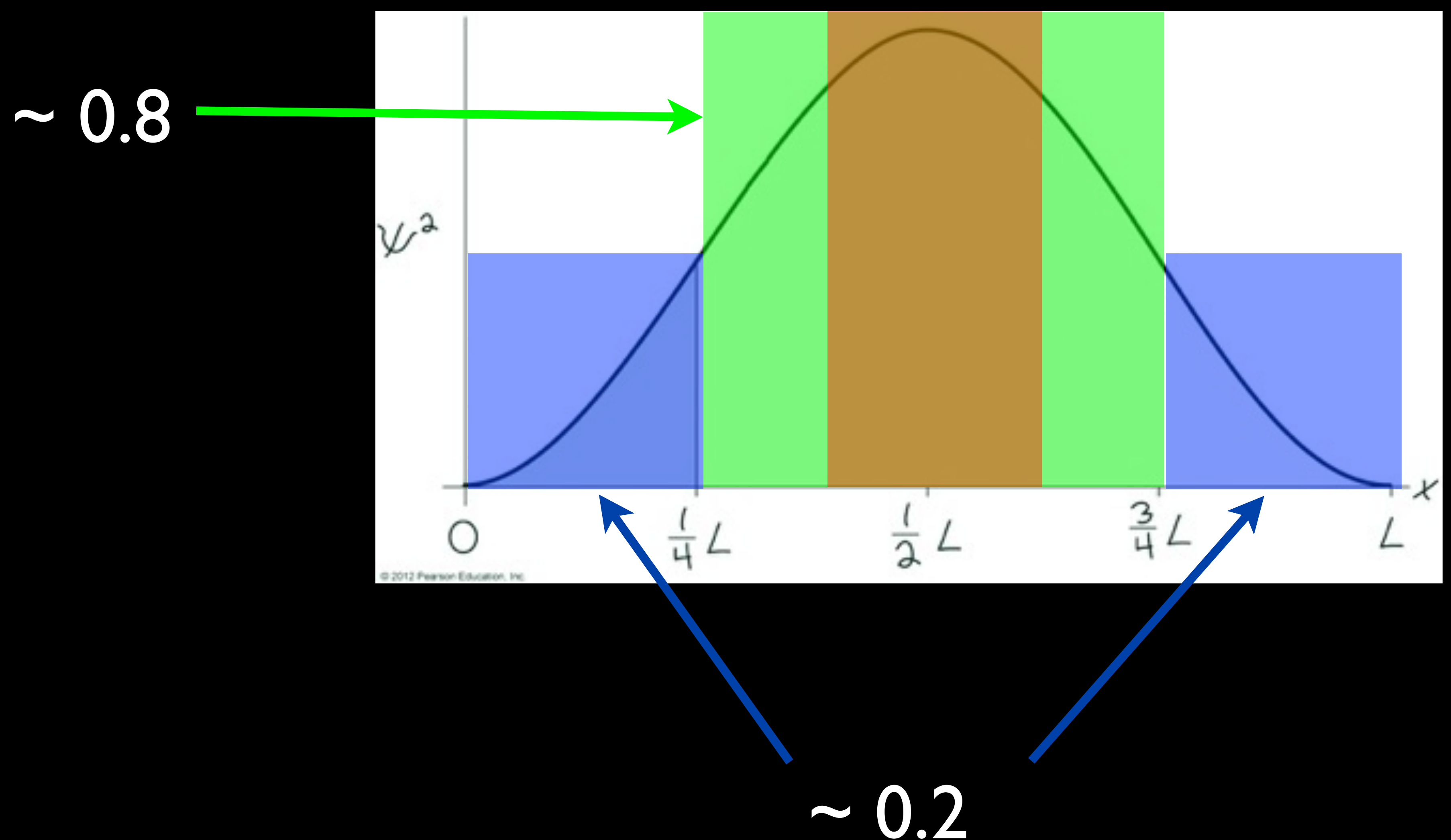
Which of the following is a reasonable estimate of the probability that the particle would be found in the central quarter of the well?

(A) 0.091

(B) 0.250

(C) 0.475

(D) 0.9



# Solving the Schrodinger Equation Quiz

A 3g snail crawls at 0.5 mm/s between 2 rocks 15 cm apart.

If this system is an infinite square well, find the quantum number, n.

$h = 6.63 \times 10^{-34} \mathrm{J} \cdot \mathrm{s}$

(A) $1.8 \times 10^{28}$

(B) $7 \times 10^{26}$

(C) $3.5 \times 10^{31}$

(D) $4 \times 10^{12}$

# Solving the Schrodinger Equation Quiz

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(D) $4 \times 10^{12}$

$$E = \frac{1}{2}mv^2 = \frac{1}{2}(3 \times 10^{-3}\mathrm{kg})(5 \times 10^{-4}\mathrm{m/s})^2$$

$$= 3.75 \times 10^{-10}\mathrm{J}$$

$$E = \frac{n^2 h^2}{8mL^2} \Rightarrow n = \frac{\sqrt{8mL^2E}}{h}$$

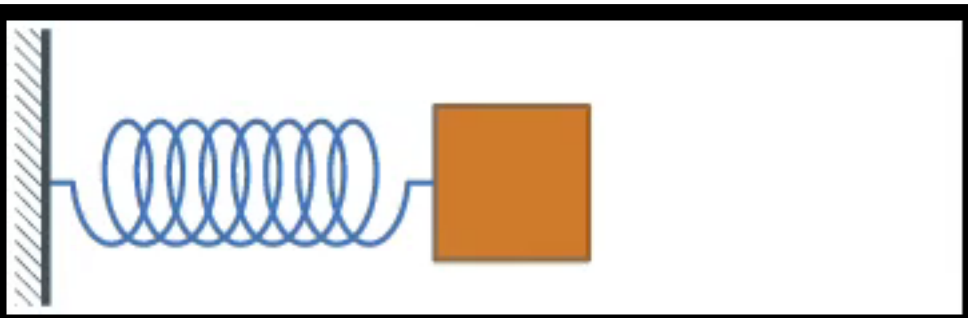
$$n = \frac{(0.15\mathrm{m})\sqrt{8(3 \times 10^{-3}\mathrm{kg})(3.75 \times 10^{-10}\mathrm{J})}}{6.63 \times 10^{-34}\mathrm{J \cdot s}}$$

$$= 7 \times 10^{26}$$

Classical approximation fine!

# Solving the Schrodinger Equation

System 2: The Harmonic Oscillator



e.g. molecules

Force $\propto$ displacement $(\Delta x)$

mass-spring system: $U = \frac{1}{2}kx^2$ (k: spring constant )

$\omega = \sqrt{k/m}$ (angular frequency)

$$U = \frac{1}{2}m\omega^2 x^2$$

Schrodinger: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$

(solving difficult but result is...)

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

ground state: $n = 0$

# Solving the Schrodinger Equation Quiz

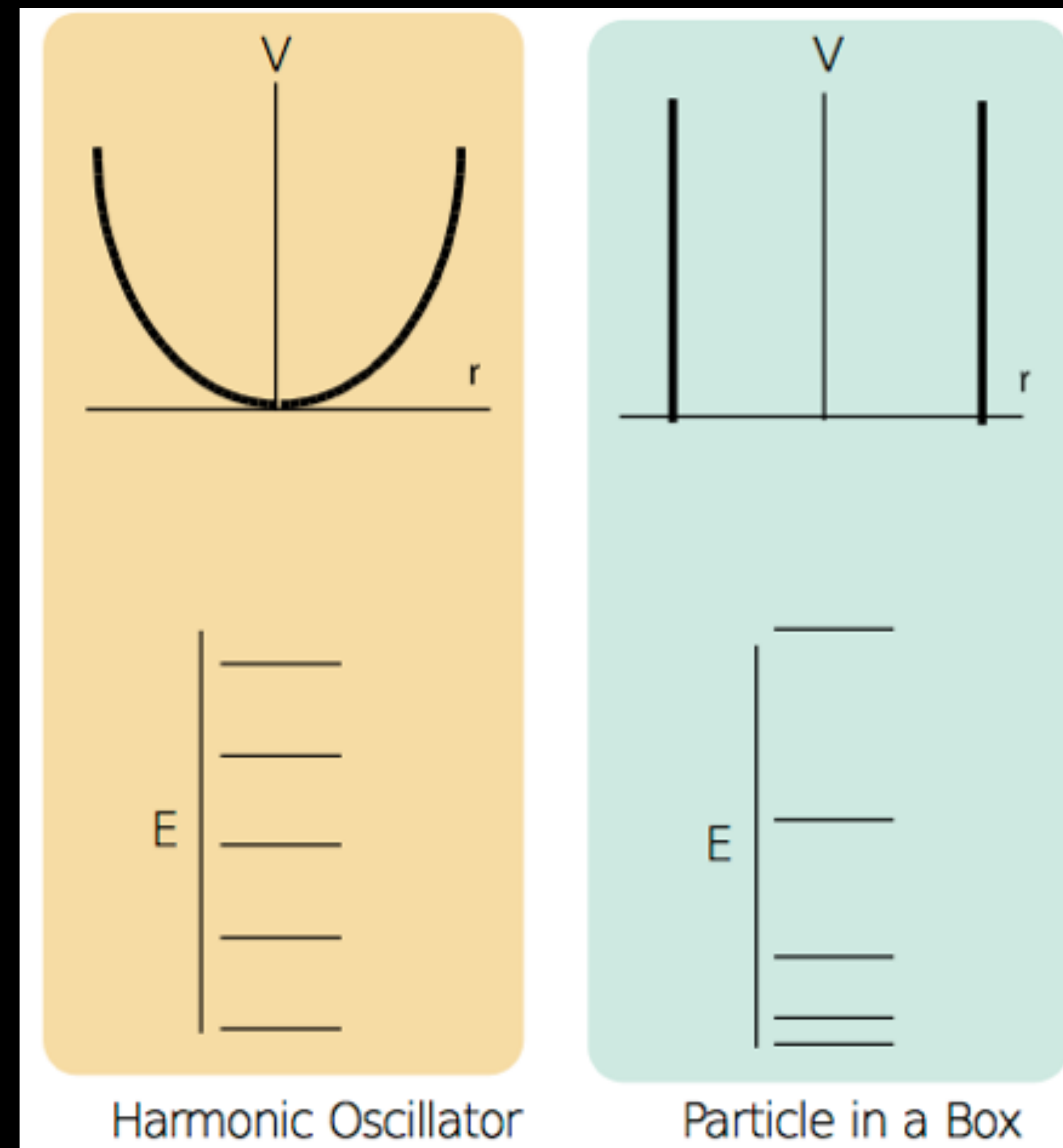
Are the energy levels of a harmonic oscillator compared to the square well....

(A) Evenly spaced

(B) More widely spaced at the bottom, closer at the top

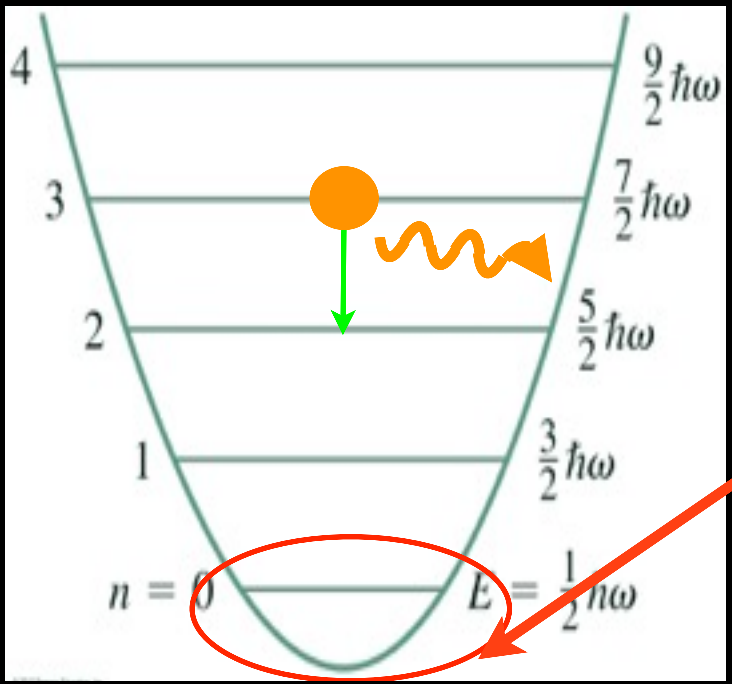
(C) More widely spaced at the top, closer at the bottom

(D) identical




$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \qquad E = \frac{n^2 h^2}{8mL^2}$$

# Solving the Schrodinger Equation




$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

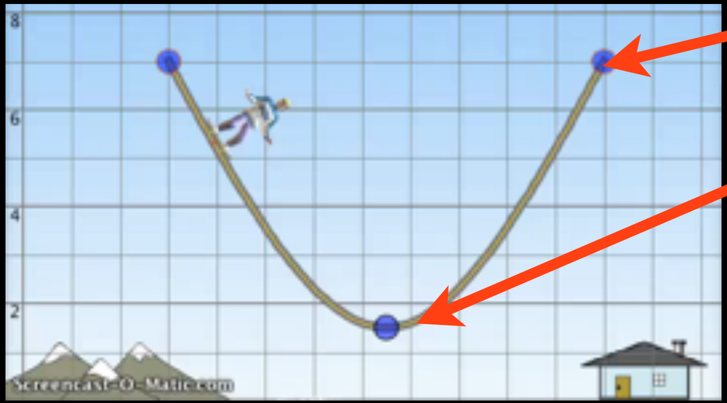
**Planck prediction:**
(from Blackbody radiation)

$$E = nhf$$

Planck didn't predict ground state > 0

Even spacing ➡ emitted/absorbed photons always same energy

Classical:



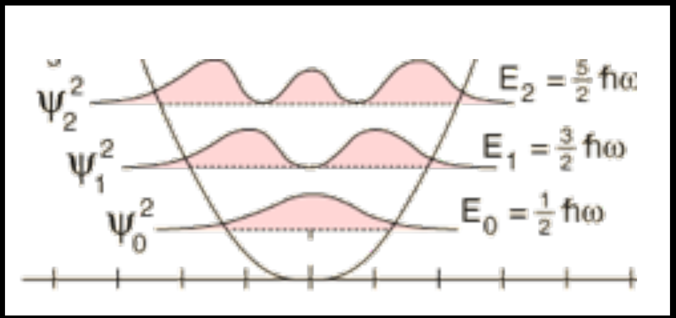

velocity minimum at turning points

velocity maximum at bottom (equilibrium)

⬇

Particle *least* likely to be found at equilibrium

Quantum:

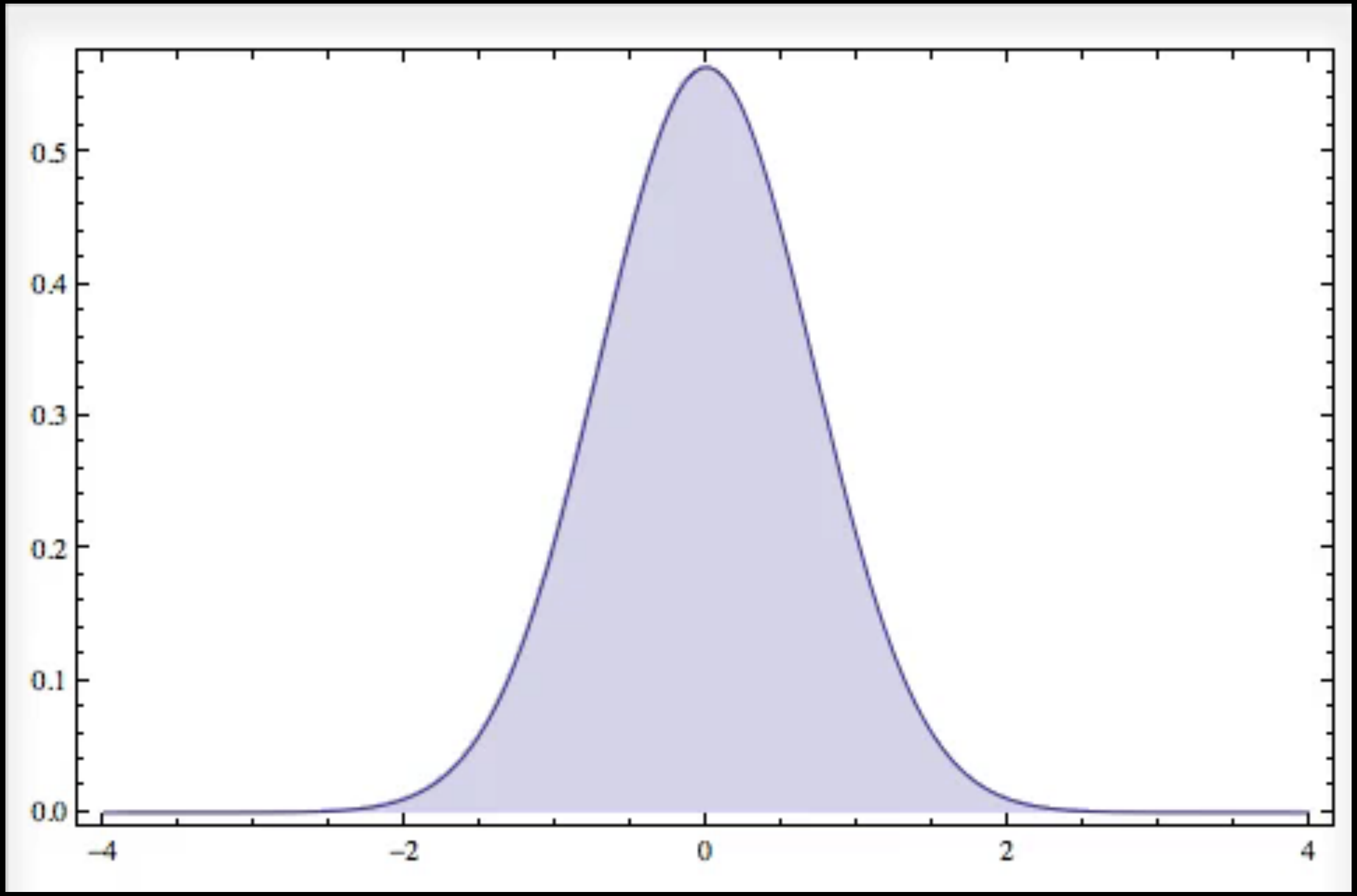



Particle *most* likely to be found at equilibrium in ground state.

➡ Low n states very different from classical

# Solving the Schrodinger Equation

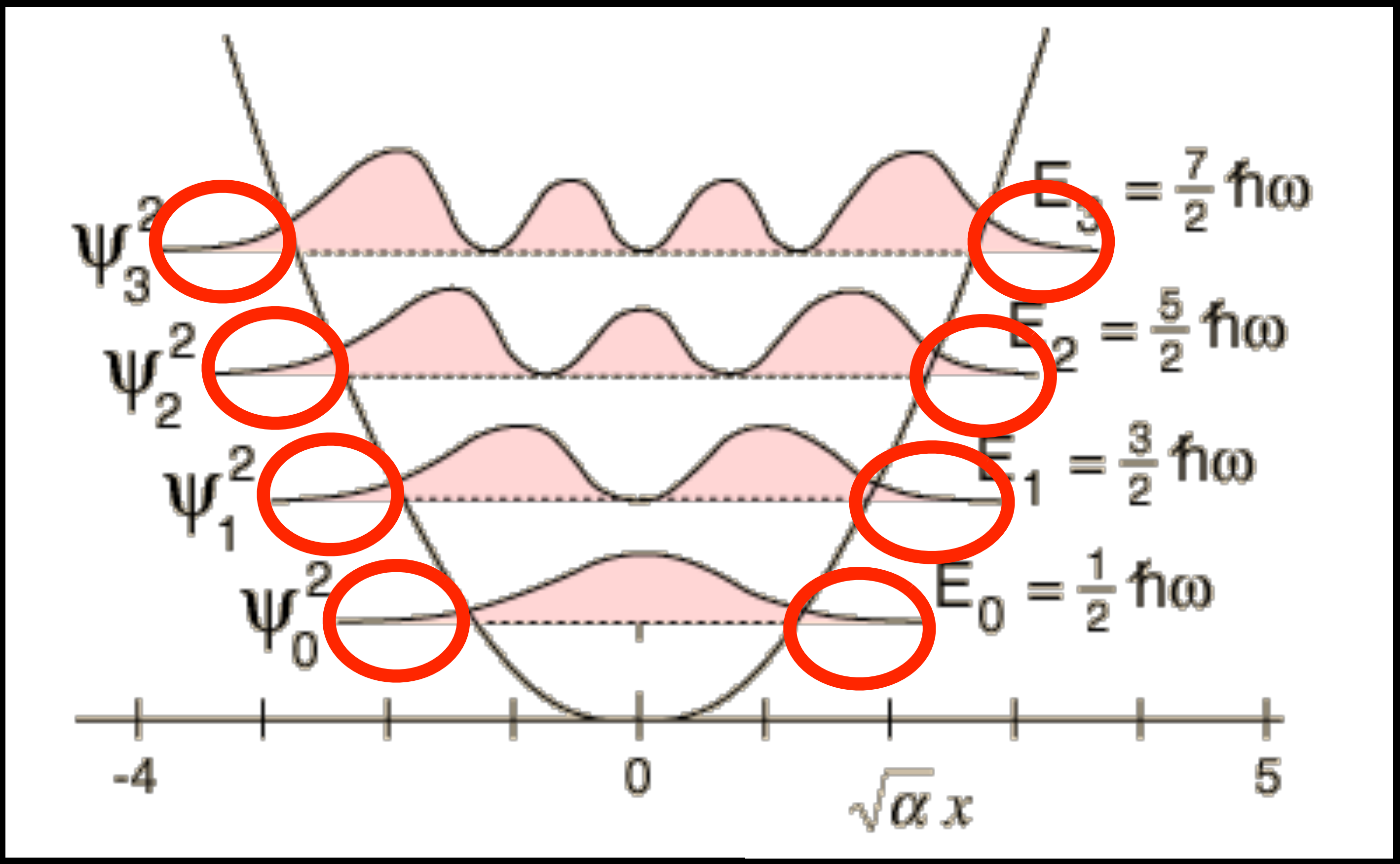
Question: Where is the particle most likely to be found?




Low n : very different from classical prediction

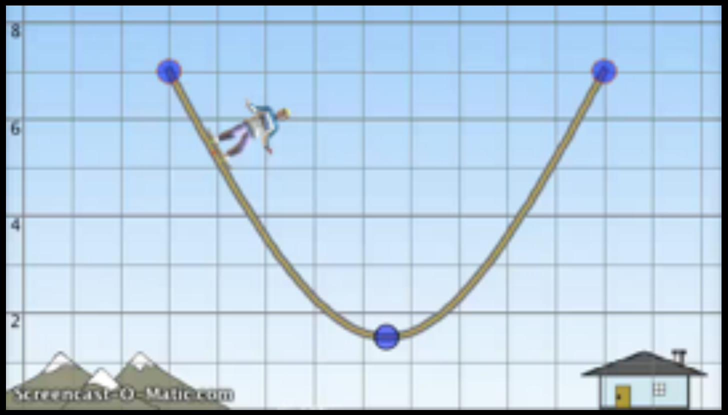
High n : close to classical prediction

# Quantum Tunnelling




Ehhhh?

Probability of particle being found outside well $> 0$

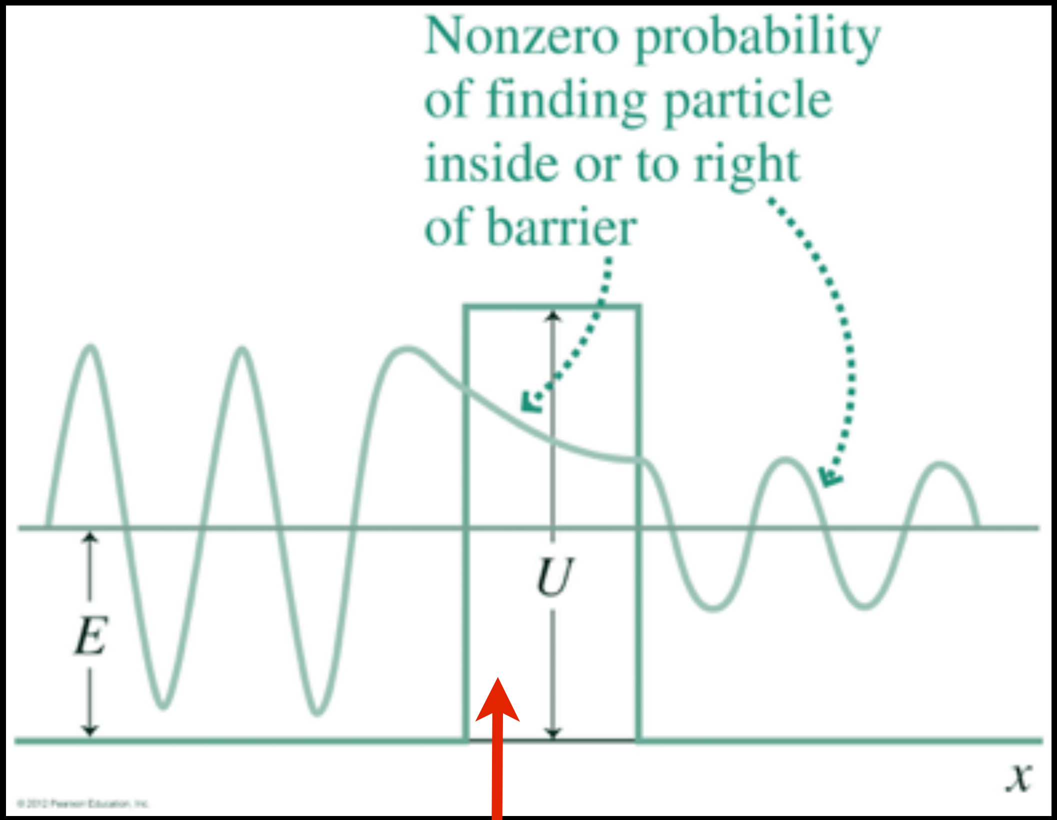


Classically impossible!

Particle does not have enough energy to escape potential.

➡ Quantum tunnelling

# Quantum Tunnelling




Also can occur through a potential barrier

e.g. gaps (space) between materials,


or an electric potential difference.

Inside barrier, $\psi(x)$ depends on: $e^{\pm\sqrt{2m(U-E)}x/\hbar}$

decrease fast unless....

particle energy, $E$, close to barrier potential, $U$

or particle mass, $m$, is small

You can't walk through a wall!

# Quantum Tunnelling

Quiz

A proton and an electron approach a barrier. Both have the same energy E, lower than the barrier potential U.

Which is the particles is more likely to get through?

(A) The proton

(B) The electron

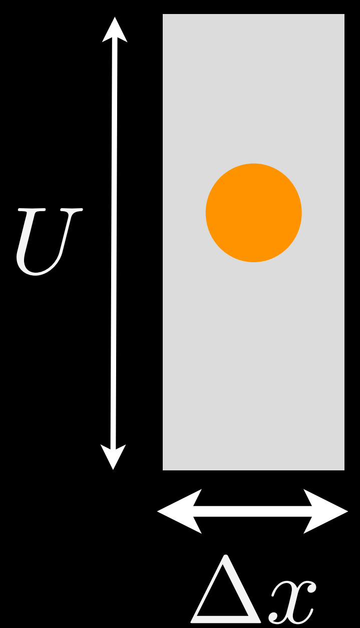
(C) Both are equally as likely to get through.

(D) Neither will get through the barrier.

# Quantum Tunnelling

Doesn't this break the Conservation of Energy?!

... prove it!




If we try and detect the particle inside the barrier....

Particle position is within $\Delta x$

Therefore, energy is uncertain: $\Delta x \Delta p \geq \hbar$

(Lecture 12: uncertainty principal)

It is found: $E + \Delta E > U$ No quantum tunnelling!

But, it we don't try and detect the *particle*...

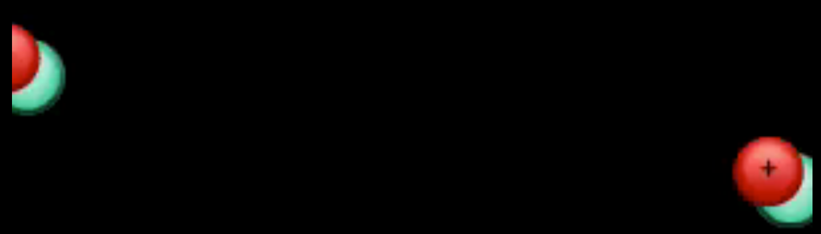
... it acts as a *wave*.

Quantum tunnelling possible!

# Quantum Tunnelling

Strange but true!

Quantum tunnelling makes the sun shine!

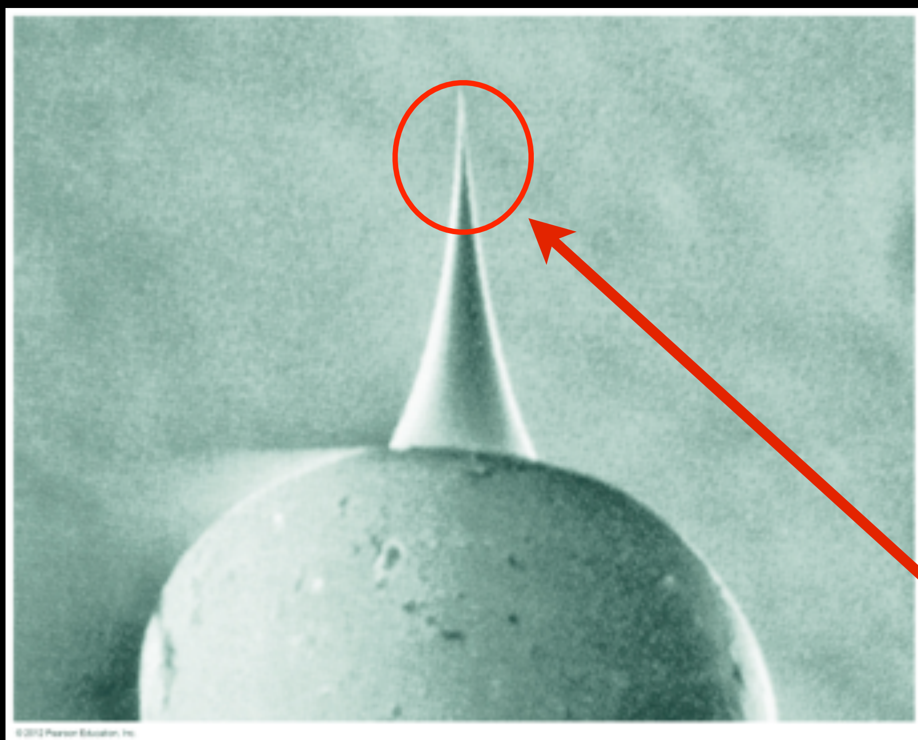


Classically, the sun’s nuclei don’t have enough energy to overcome the Coulomb force and fuse.

But, nuclei can quantum tunnel through the Coulomb barrier.

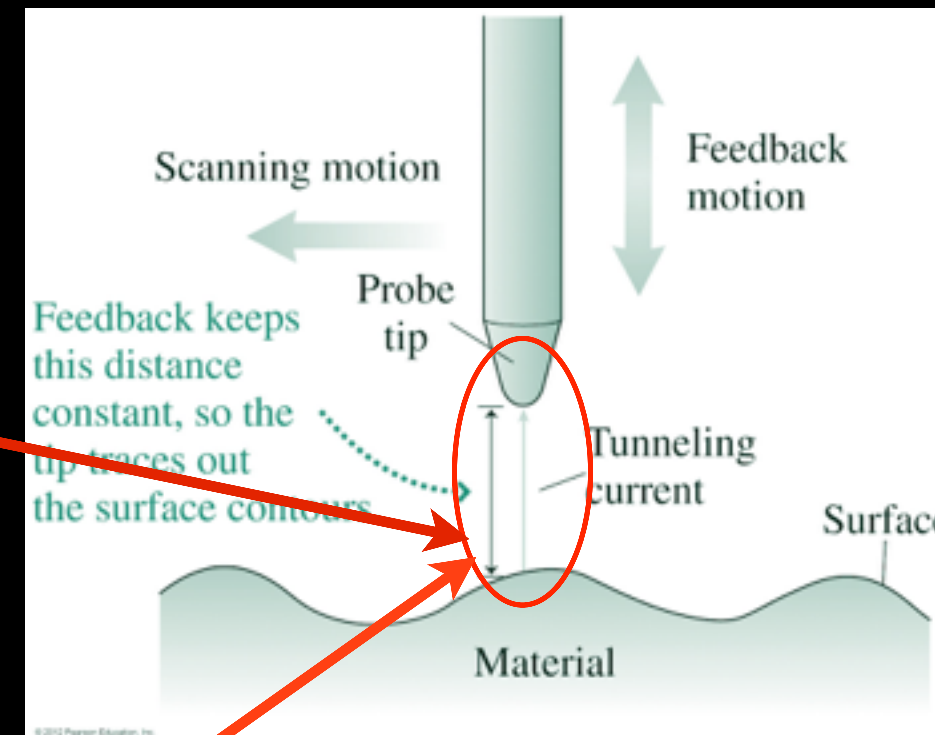
# Quantum Tunnelling

## Scanning Tunnelling Microscope

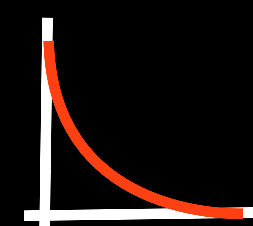


Quantum tunnelling of electrons between tip and surface

Tip ~ 1 atom wide




In gap, $\psi(x)$ decreases exponentially



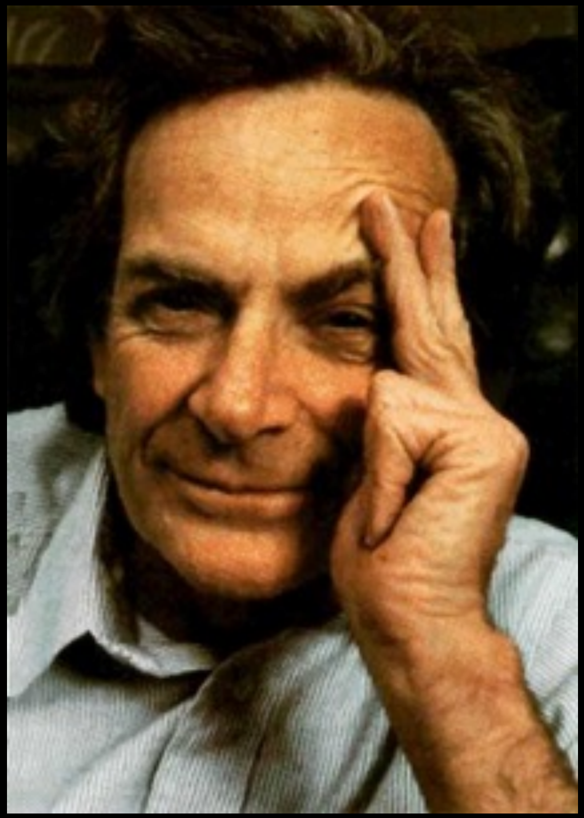
- Tunnelling current very very sensitive to gap size.
- Used to image a surface in incredible detail

# Quantum Mechanics

Confused about Quantum Mechanics?

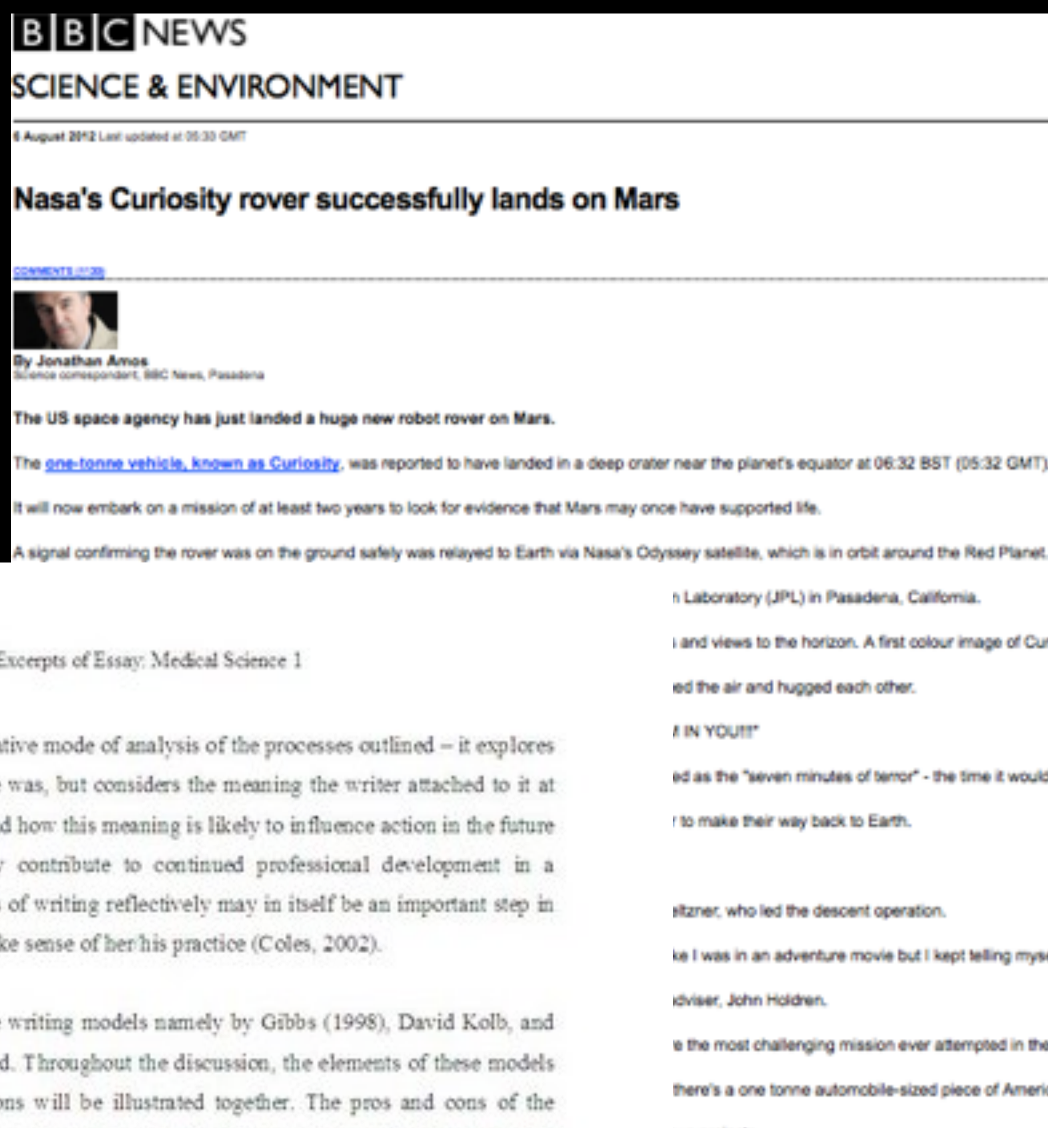

"I think I can safely say that nobody understands quantum mechanics"

-- Richard Feynman



# Reminder!! 1 weeks!

250 word essay



BBC NEWS

SCIENCE & ENVIRONMENT

**Nasa's Curiosity rover successfully lands on Mars**

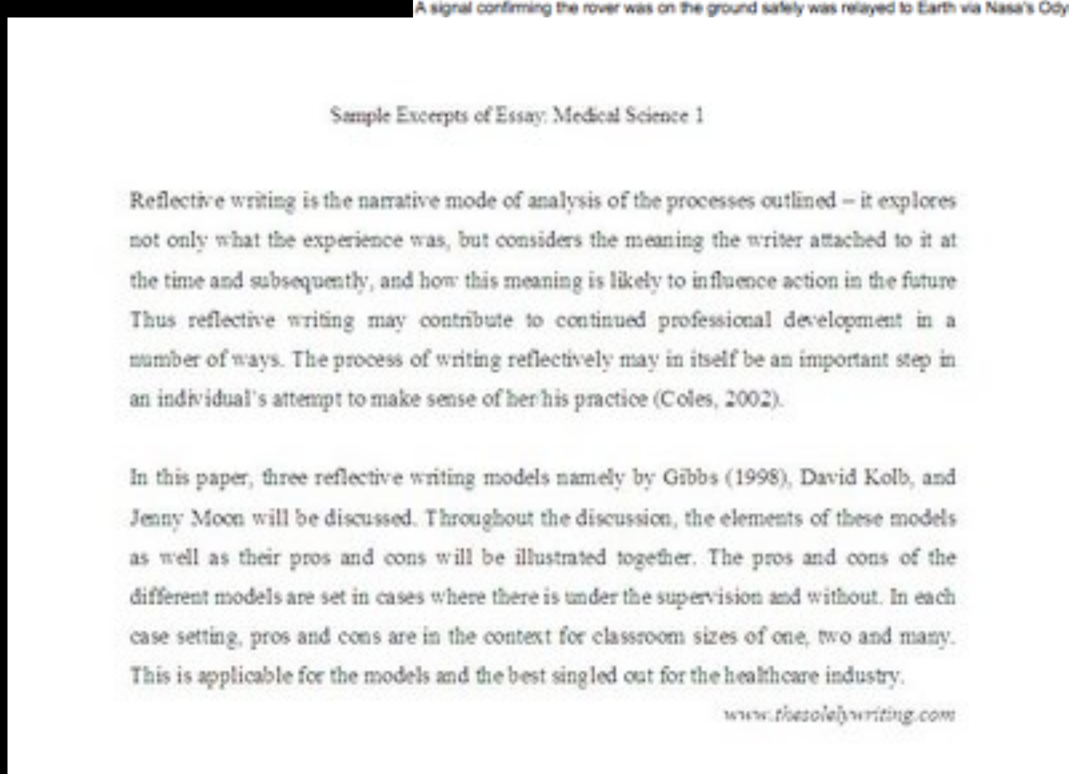
By Jonathan Amos

The US space agency has just landed a huge new robot rover on Mars.

The one-tonne vehicle, known as Curiosity, was reported to have landed in a deep crater near the planet's equator at 06:32 BST (05:32 GMT).

It will now embark on a mission of at least two years to look for evidence that Mars may once have supported life.

A signal confirming the rover was on the ground safely was relayed to Earth via Nasa's Odyssey satellite, which is in orbit around the Red Planet.



Sample Excerpts of Essay: Medical Science 1

Reflective writing is the narrative mode of analysis of the processes outlined – it explores not only what the experience was, but considers the meaning the writer attached to it at the time and subsequently, and how this meaning is likely to influence action in the future. Thus reflective writing may contribute to continued professional development in a number of ways. The process of writing reflectively may in itself be an important step in an individual's attempt to make sense of her/his practice (Coles, 2002).

In this paper, three reflective writing models namely by Gibbs (1998), David Kolb, and Jenny Moon will be discussed. Throughout the discussion, the elements of these models as well as their pros and cons will be illustrated together. The pros and cons of the different models are set in cases where there is under the supervision and without. In each case setting, pros and cons are in the context for classroom sizes of one, two and many. This is applicable for the models and the best singled out for the healthcare industry.

*www.thesolelywriting.com*

Read a physics article (in English) on a topic that interests you

This can be one we have covered in class, or a new one.

Describe its main points in 250 words.

Hand in BOTH essay and article

Due 2016/1/18

NO EXTENSIONS!