Essential Physics II

英語で物理学のエッセンス II

Lecture 12: 21-12-15

News

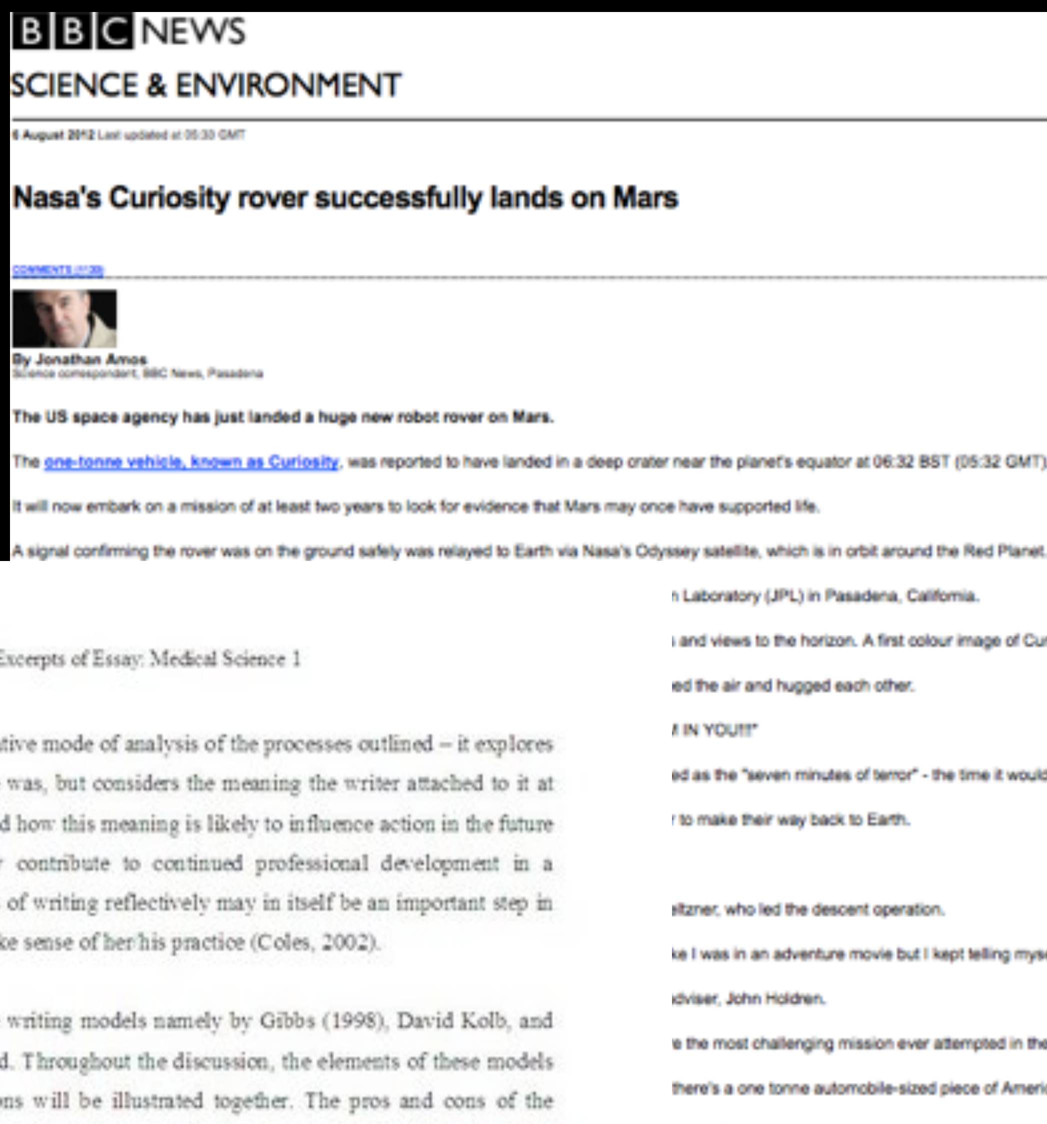
Next lecture:

Tuesday 12th January (１月１２日)

火曜日

Essay

<u>250 word essay</u>


BBC NEWS

SCIENCE & ENVIRONMENT

Nasa's Curiosity rover successfully lands on Mars

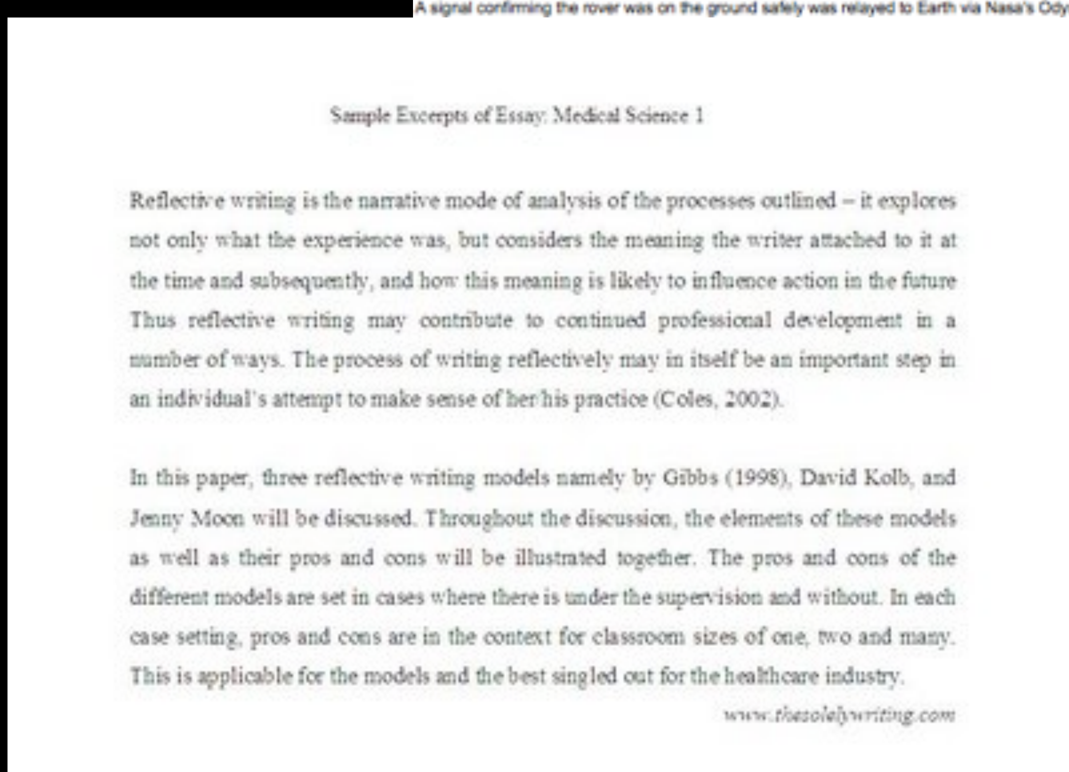
By Jonathan Amos

The US space agency has just landed a huge new robot rover on Mars.

The one-tonne vehicle, known as Curiosity, was reported to have landed in a deep crater near the planet's equator at 06:32 BST (05:32 GMT).

It will now embark on a mission of at least two years to look for evidence that Mars may once have supported life.

A signal confirming the rover was on the ground safely was relayed to Earth via Nasa's Odyssey satellite, which is in orbit around the Red Planet.


Sample Excerpts of Essay: Medical Science 1

Reflective writing is the narrative mode of analysis of the processes outlined – it explores not only what the experience was, but considers the meaning the writer attached to it at the time and subsequently, and how this meaning is likely to influence action in the future. Thus reflective writing may contribute to continued professional development in a number of ways. The process of writing reflectively may in itself be an important step in an individual's attempt to make sense of her/his practice (Coles, 2002).

In this paper, three reflective writing models namely by Gibbs (1998), David Kolb, and Jenny Moon will be discussed. Throughout the discussion, the elements of these models as well as their pros and cons will be illustrated together. The pros and cons of the different models are set in cases where there is under the supervision and without. In each case setting, pros and cons are in the context for classroom sizes of one, two and many. This is applicable for the models and the best singled out for the healthcare industry.

www.thesolelywriting.com

Read a physics article (in English) on a topic that interests you

This can be one we have covered in class, or a new one.

Describe its main points in 250 words. Use your own words (don't copy!)

Hand in BOTH essay and article

Due 2016/1/18

Modern Physics

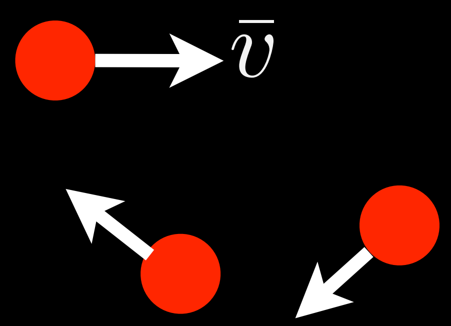
The problem with Physics....

Physics until 1900:

The Universe consists of:

Matter: Newton's Laws of motion $\bar{F} = m\frac{d\bar{v}}{dt} = m\bar{a}$

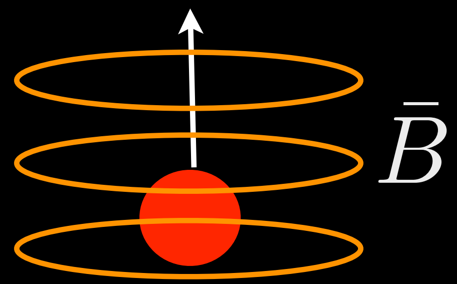
Particles

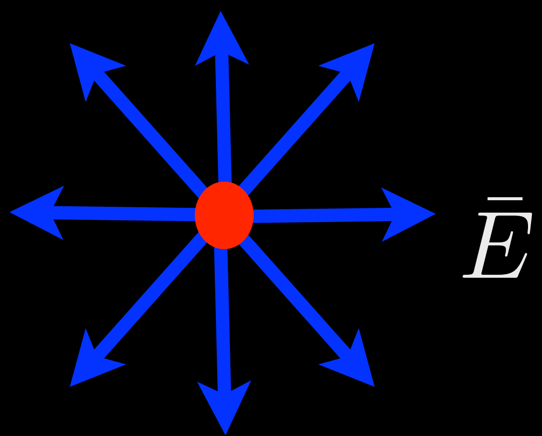


Momenta and position of particles at single time, t, gives motion

Electromagnetic radiation: Maxwell's equations

Waves





Light is a wave of electric and magnetic fields.

Physics until 1900:

But...

There were a few *minor* problems that were not explained

1. Blackbody radiation

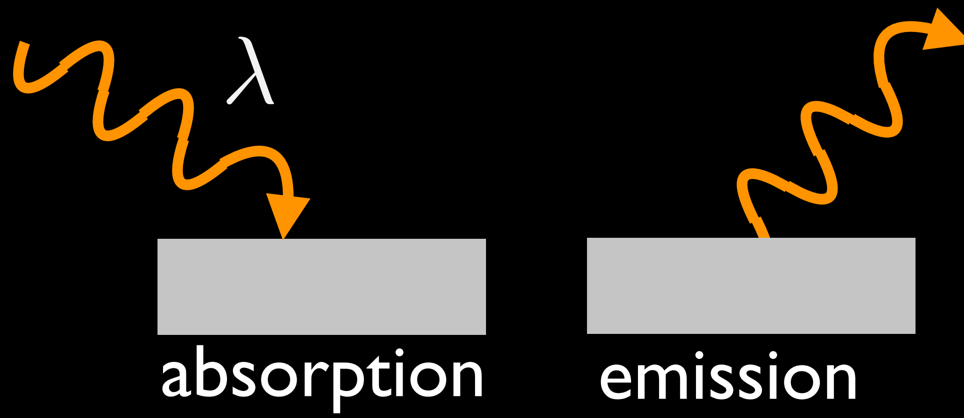
2. Photoelectric effect

3. Stability and atom size

1 Blackbody radiation

What is blackbody radiation?

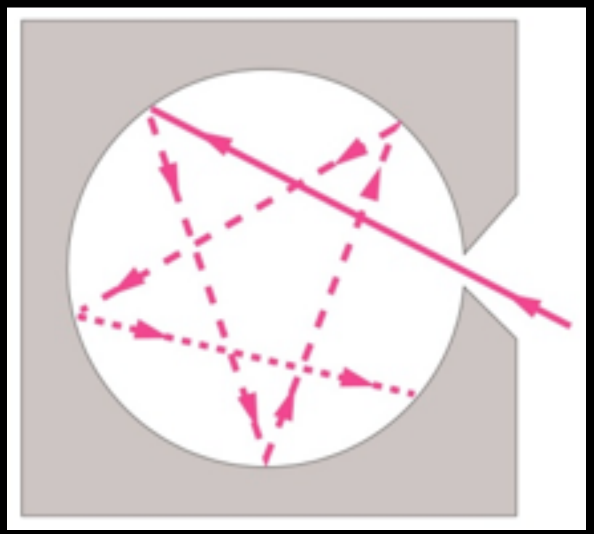
Every body (object) emits and absorbs electromagnetic radiation at all λ .



(If you heat an object, it glows)

A blackbody absorbs ALL radiation → looks black

Cavity with a small hole is a good blackbody →



Radiation gets reflected many times.
Finally absorbed.

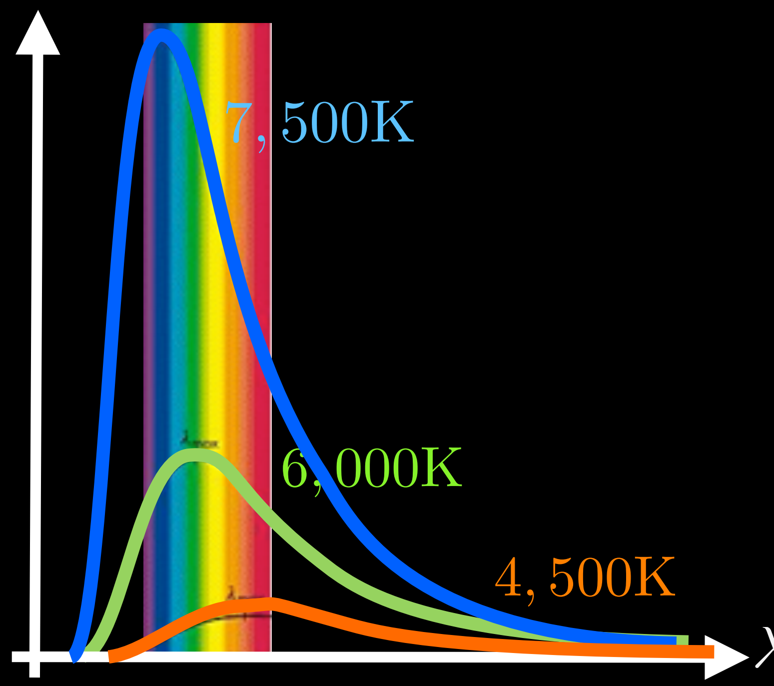
Blackbody radiation

When a blackbody is heated, it emits blackbody radiation.

Sun, electric stove ~ blackbodies

From experiment:

Spectrum depends only on temperature, T



Peak at wavelength: $\lambda_{\text{peak}} \propto \frac{1}{T}$

$$\lambda_{\text{peak}} T = 2.898 \text{mm} \cdot \text{K}$$

(Wien's law)

Total power from all λ: $P_{\text{blackbody}} = \sigma A T^4$

(Stefan-Boltzmann law)

$$\sigma = 5.67 \times 10^{-8} \text{Js}^{-1}\text{m}^{-2}\text{J}^{-5}$$

Blackbody radiation

Quiz

2 identical blackbodies are heated until A's temperature is twice B's

How do their radiated power compare?

(A) $P_A = 2P_B$

(B) $P_A = 8P_B$

(C) $P_A = 4P_B$

(D) $P_A = 16P_B$

$$P_{\mathrm{blackbody}} = \sigma A T^4$$

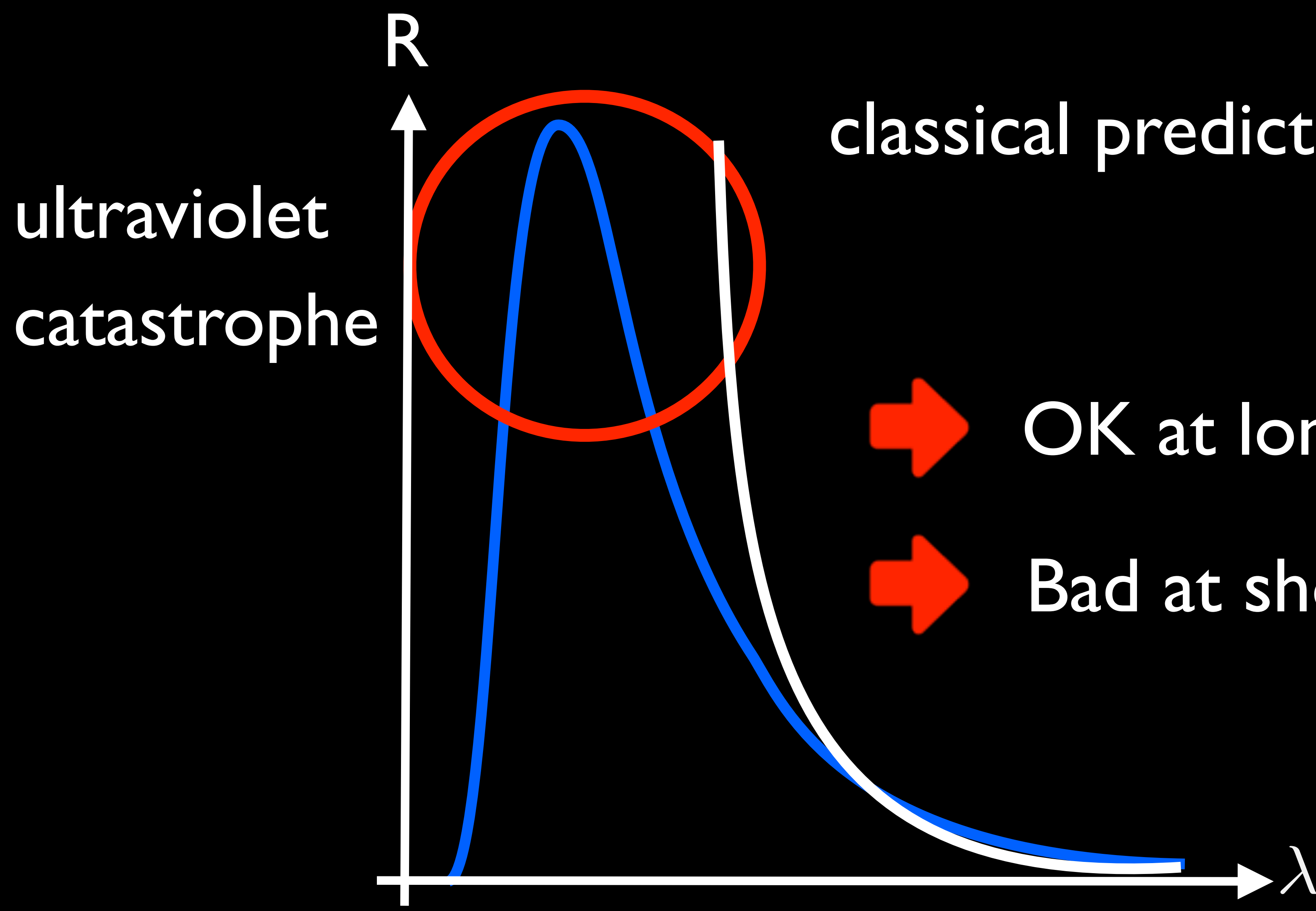
(Stefan-Boltzmann law)

Blackbody radiation

Physicists tried to explain the shape of blackbody radiation ...

using electromagnetism and statistics...

and failed.



classical prediction: $$R(\lambda, T) = \frac{2\pi c k T}{\lambda^4}$$

(Rayleigh-Jean Law)

- OK at long λ
- Bad at short λ

Blackbody radiation

In 1900, Physicist Max Planck produced:

$$R(\lambda, T) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)}$$

$$k = 1.38 \times 10^{-23} \mathrm{J/K}$$

$$c = 299,792,458 \mathrm{m/s}$$

new constant

Planck's constant, $h = 6.63 \times 10^{-34} \mathrm{J \cdot s}$

It fit the data, but Planck called it an 'empirical formula'.

based on observation, not underlying theory

Why? Because the physical meaning was unbelievable...

Blackbody radiation

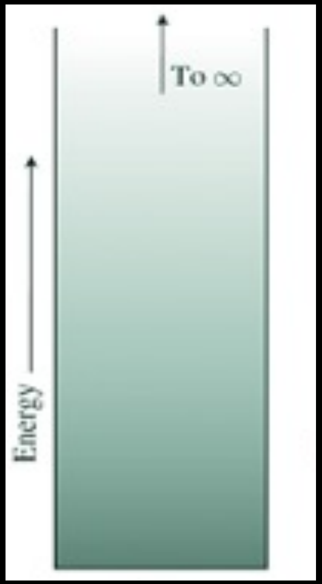
To derive this formula

$$R(\lambda, T) = \frac{2\pi h c^2}{\lambda^5 (e^{hc/\lambda k T} - 1)}$$

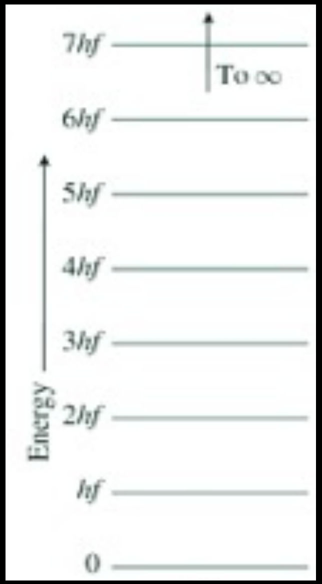
Planck HAD TO assume: $E = nhf \qquad n = 0, 1, 2, 3, \ldots$

Energy absorbed/emitted from a molecule can only take certain values

Energy is quantized!



Classical physics



Planck's theory

Blackbody radiation

$$R(\lambda, T) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)}$$

When $\frac{hc}{\lambda}(= hf) >> kT$

$$e^{hf/kT} - 1 = \left(1 + \frac{hf}{kT} + \ldots\right) - 1$$

series expansion for $e^{hf/kT}$

$$\simeq \frac{hf}{kT} \quad = \frac{hc}{\lambda kT}$$

$$R(\lambda, T) = \frac{2\pi ckT}{\lambda^4}$$

classical result!

Blackbody radiation

Example

A lightbulb's filament temperature is 3000 K.

(a) Find wavelength of peak radiance

(b) compare radiance (R) at 550 nm with peak radiance.

Wien's law: $\lambda_{\text{peak}} T = 2.898\text{mm} \cdot \text{K}$

$$\lambda_{\text{peak}} = \frac{2.898\text{mm} \cdot \text{K}}{3000\text{K}} = 966\text{nm}$$

Blackbody radiation

Example

A lightbulb's filament temperature is 3000 K.

(a) Find wavelength of peak radiance

(b) compare radiance (R) at 550 nm with peak radiance.

Planck: $R(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)}$

$$\frac{R(\lambda_2, T)}{R(\lambda_1, T)} = \frac{\lambda_1^5(e^{hc/\lambda_1 kT} - 1)}{\lambda_2^5(e^{hc/\lambda_2 kT} - 1)}$$

$$= \frac{(966\text{nm})^5(e^{hc/966kT} - 1)}{(550\text{nm})^5(e^{hc/550kT} - 1)} = 0.38$$

Blackbody radiation

Example

For a 2.0 kK blackbody, by what percentage is the Rayleigh-Jeans (classical) law in error at a wavelength of 1.0 mm?

$$R(\lambda, T) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$$

$$R(\lambda, T) = \frac{2\pi ckT}{\lambda^4}$$

$$\frac{R_{\mathrm{RJ}}}{R_{\mathrm{p}}} = \frac{(2\pi ckT/\lambda^4)\lambda^5(e^{hc/\lambda kT} - 1)}{2\pi hc^2}$$

$$= \frac{e^{hc/\lambda kT} - 1}{hc/\lambda kT}$$

$$T = 2000K$$

$$\lambda = 1\mathrm{mm}$$

$$= \frac{e^{7.245 \times 10^{-3}} - 1}{7.245 \times 10^{-3}} = 1.00363$$

➡ 0.36%

Blackbody radiation

Quiz

For a 2.0 kK blackbody, by what percentage is the Rayleigh-Jeans (classical) law in error at a wavelength of $1\mu m$?

(A) 30%

(B) 0.36%

(C) 0.5%

(D) $1.9 \times 10^4\%$

$$\frac{R_{\mathrm{RJ}}}{R_{\mathrm{p}}} = \frac{(2\pi ckT/\lambda^4)\lambda^5(e^{hc/\lambda kT} - 1)}{2\pi hc^2}$$

$$= \frac{e^{hc/\lambda kT} - 1}{hc/\lambda kT}$$

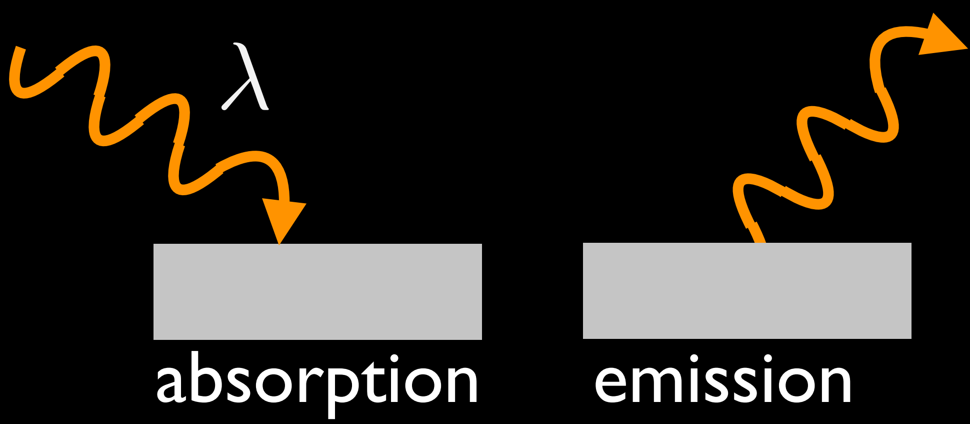
$$= \frac{e^{7.245} - 1}{7.245} = 193$$

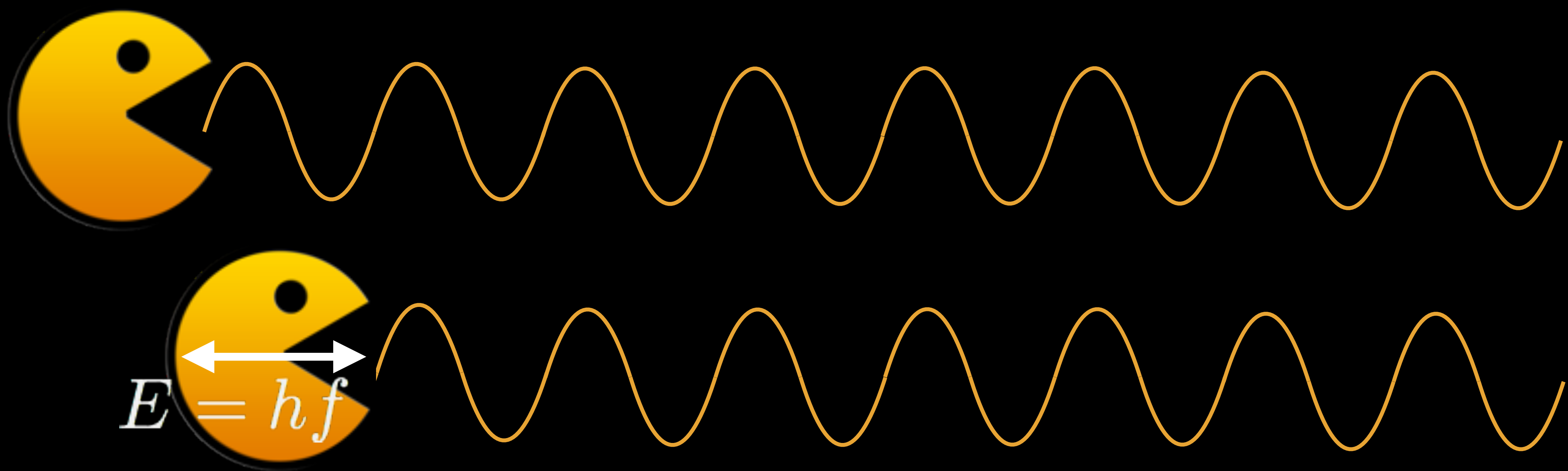
$$\Rightarrow 1.9 \times 10^4\%$$

Blackbody radiation

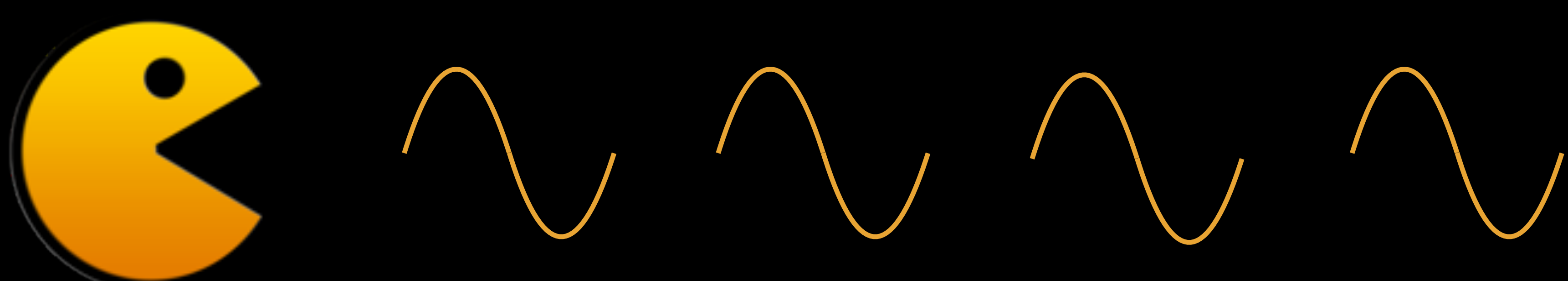
So...

Bodies (objects) absorb / emit EM radiation in quantized amounts of size hf.



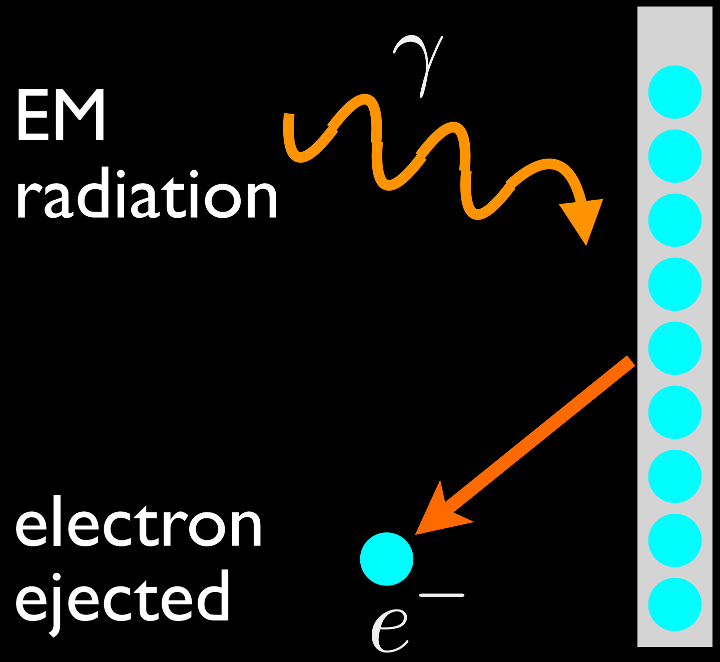


Is the actual radiation quantized too?



Photoelectric Effect

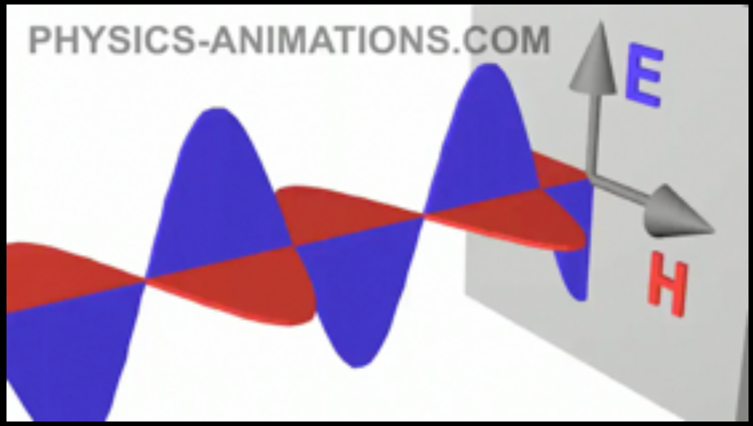
What is the photoelectric effect?



Metals emit electrons when struck by light.

Classical explanation:

Electron feels a force in light's oscillating electric field



Electron absorbs the wave's energy and *eventually*, has enough energy to escape.

But is this what is happening?

Photoelectric Effect

Experiment:

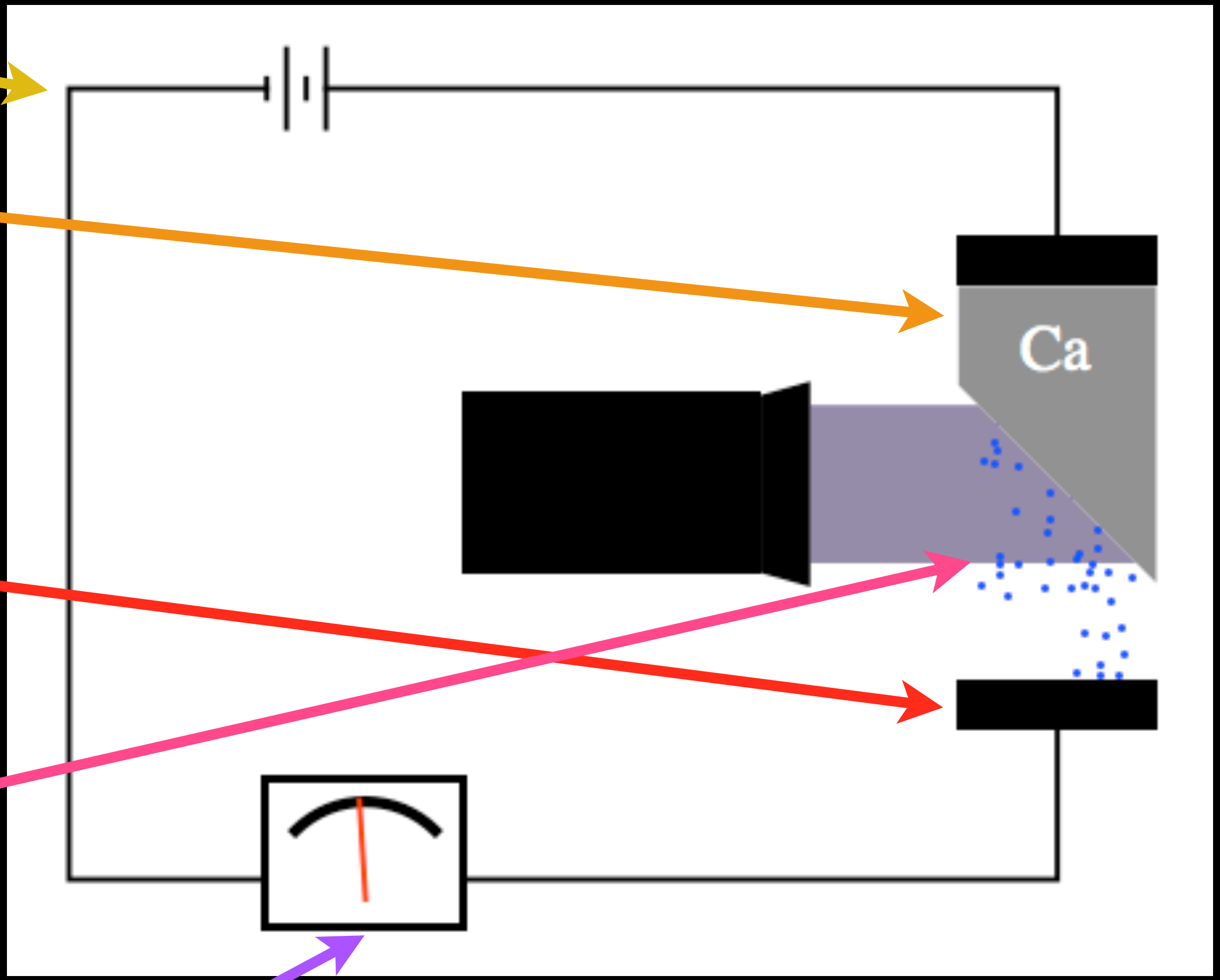
Electric circuit

Illuminate metal with light (eject electrons)

Give +ve charge to attract electrons

Current flows with ejected electrons

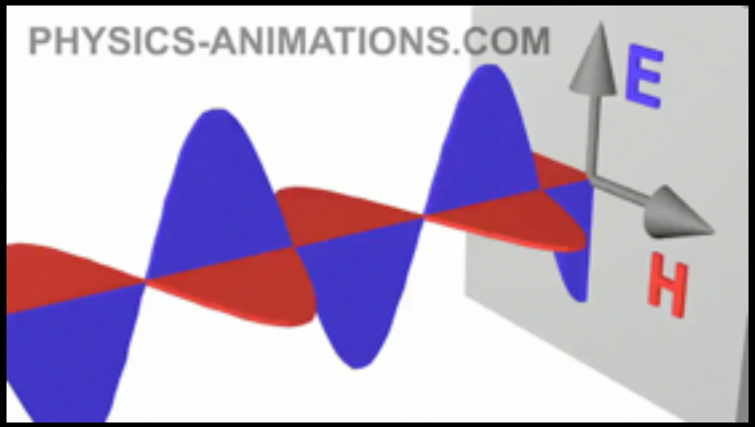
Voltage needed to stop current: stopping potential



Measures maximum energy of electrons: $K_{\max} = eV_s$

Photoelectric Effect

If the classical explanation is right...



Increasing light intensity should increase electric field

➡ e^- Electron escape quicker and move faster

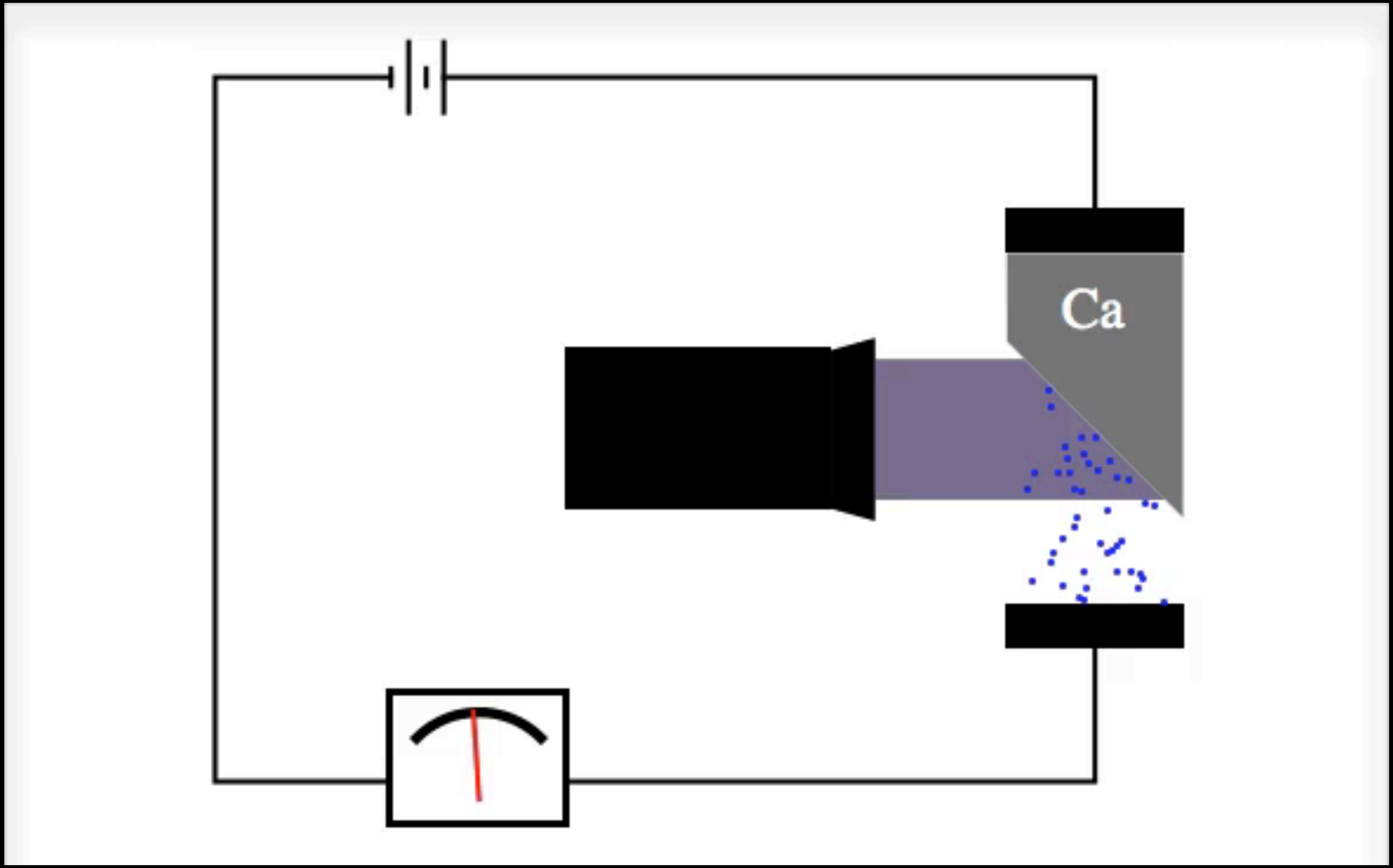
⬇ current, I, starts faster

⬇ $K_{\max} = eV_s$ higher

Independent of wave frequency

➡ same result with UV .... red ... blue ... etc

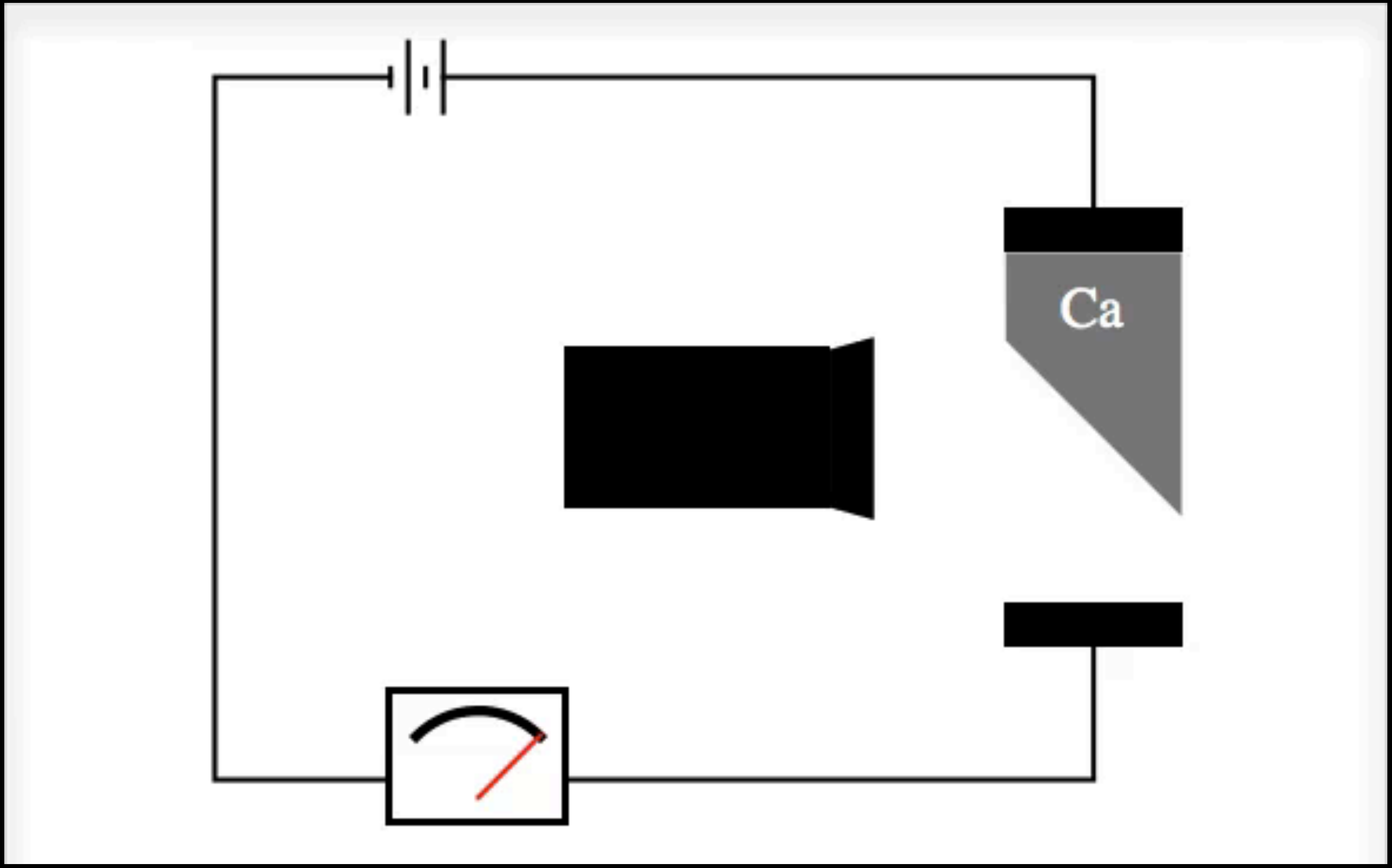
Photoelectric Effect



Let's change the light wave frequency....

Classical theory: frequency DOES NOT change the result

Photoelectric Effect

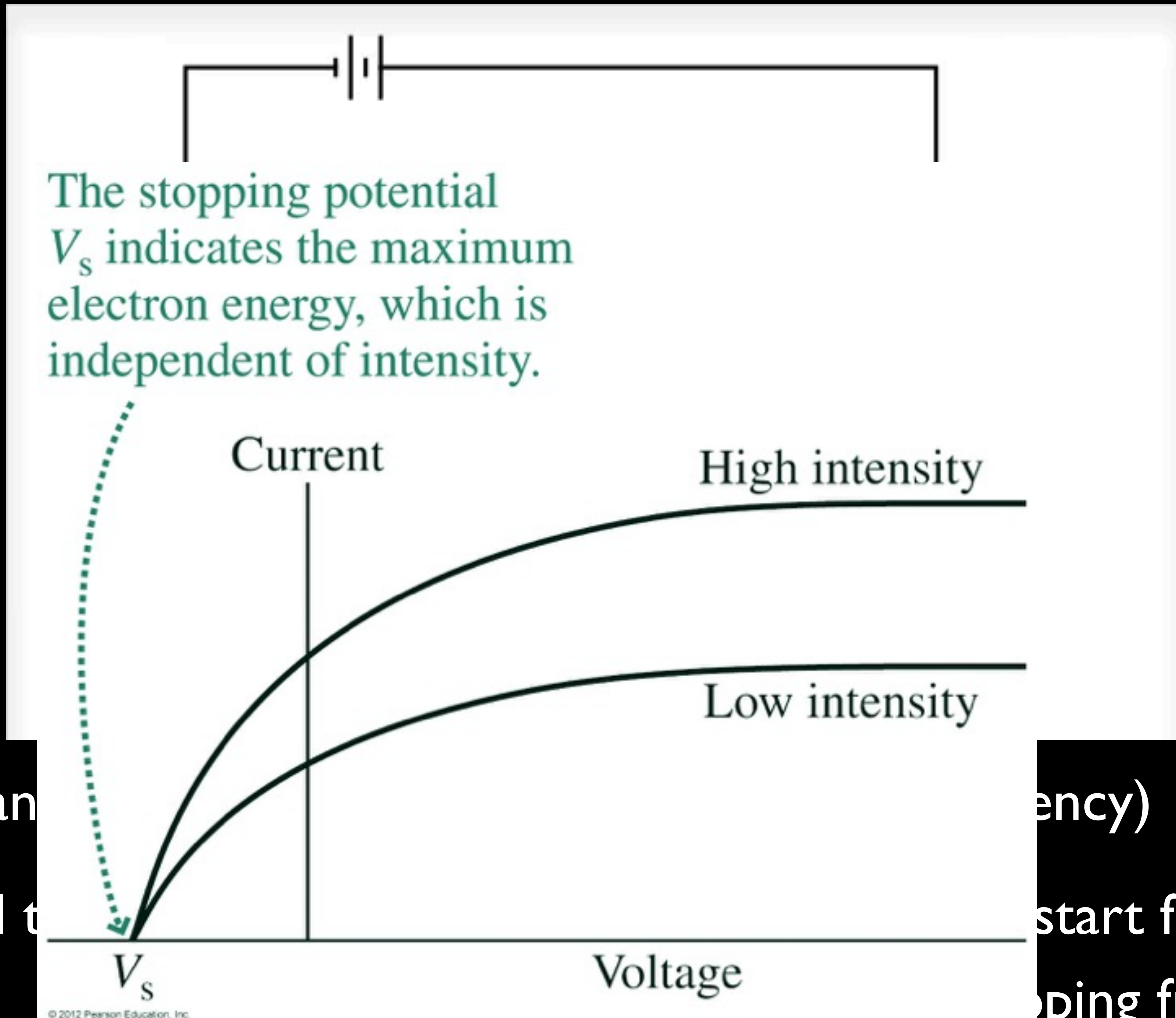


Let's change the light wave intensity.... (Fixed frequency)

Classical theory: higher intensity causes current to start faster

causes higher stopping frequency

Photoelectric Effect

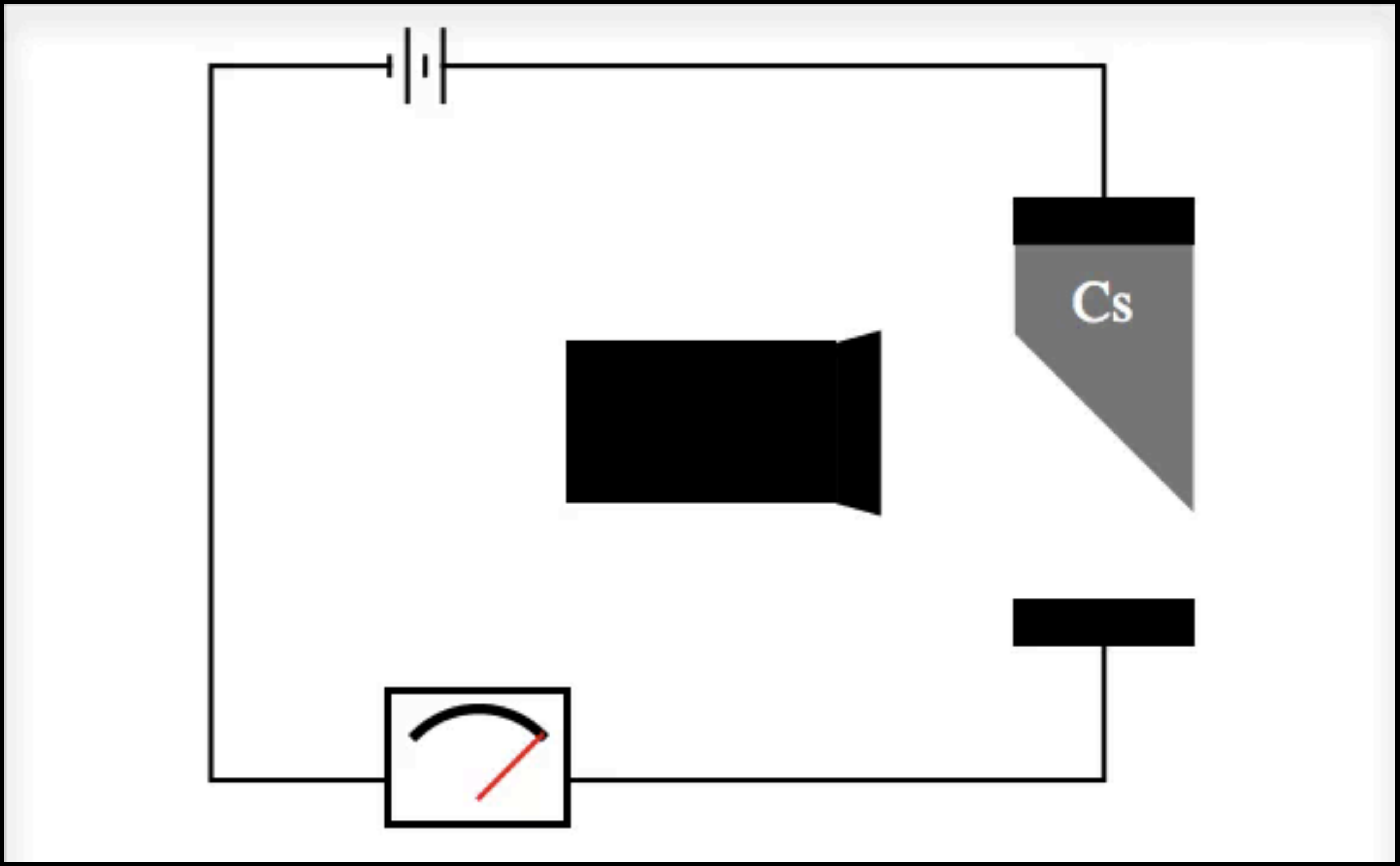


Let's chan … ency)

Classical t … start faster

… pping frequency

Photoelectric Effect



What about at a lower frequency....

How does intensity affect the current?

Photoelectric Effect

Oh dear.... 3 major disagreements with classical theory:

1 Current, I, starts *immediately*

➡ electrons escape immediately, even in dim light

2 The maximum electron energy, $K_{\max}$, *doesn't change* with intensity

measured by stopping potential V_s

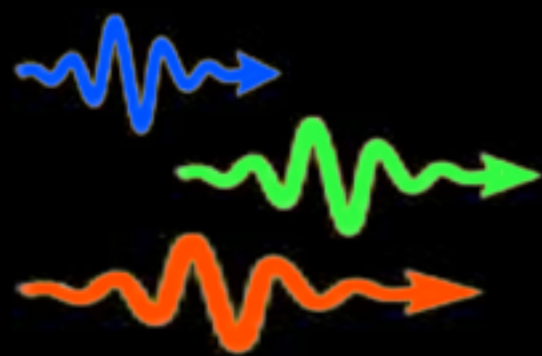
3 *Below* a cut-off frequency, *no electrons* escape at any intensity.

Above a cut-off frequency, electrons escape with a $K_{\max}$ that increases with frequency.

Photoelectric Effect

Einstein's explanation:

EM wave's energy made up of small packages of energy



"photons"

or "quanta"

Photon energy in light of frequency, f : $E = hf$

Higher intensity ➡ more photons, same energy

In 1921, Einstein won the Nobel Prize for this explanation

Photoelectric Effect

Oh dear.... 3 major disagreements with classical theory:

1 Current, I, starts

Photon gives all energy to electron

Energy doesn't 'add up' to eject electron

e ... ght

2 The maximum electron energy, K ... *doesn't change* with intensity

Higher intensity gives more photons

Photons all have the same energy

At low frequency, no photon has enough energy eject electron

Below a cut-off frequency, *no*

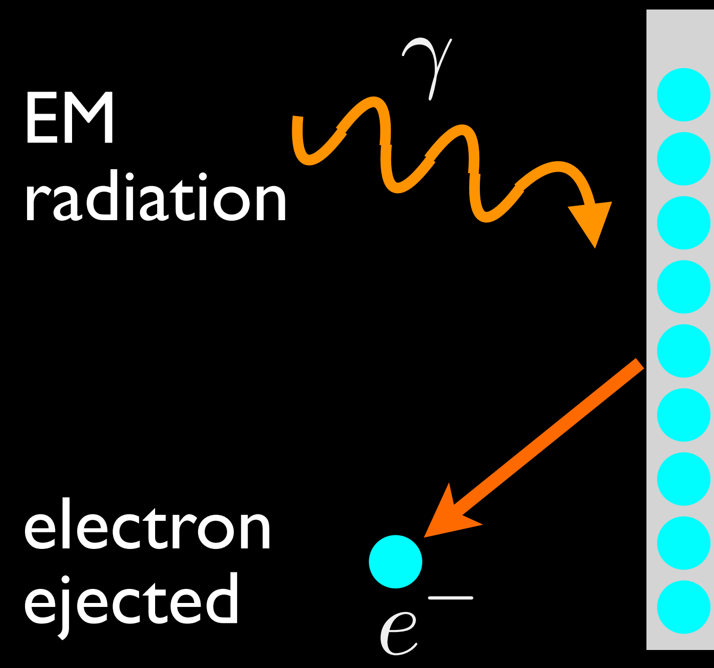
3

Higher frequency means higher energy photons, giving the electron more speed

e with a $K_{\max}$ that

Photoelectric Effect

If the frequency is too low, photons do not have enough energy to remove electrons



↓

Minimum energy needed to remove electrons

↑

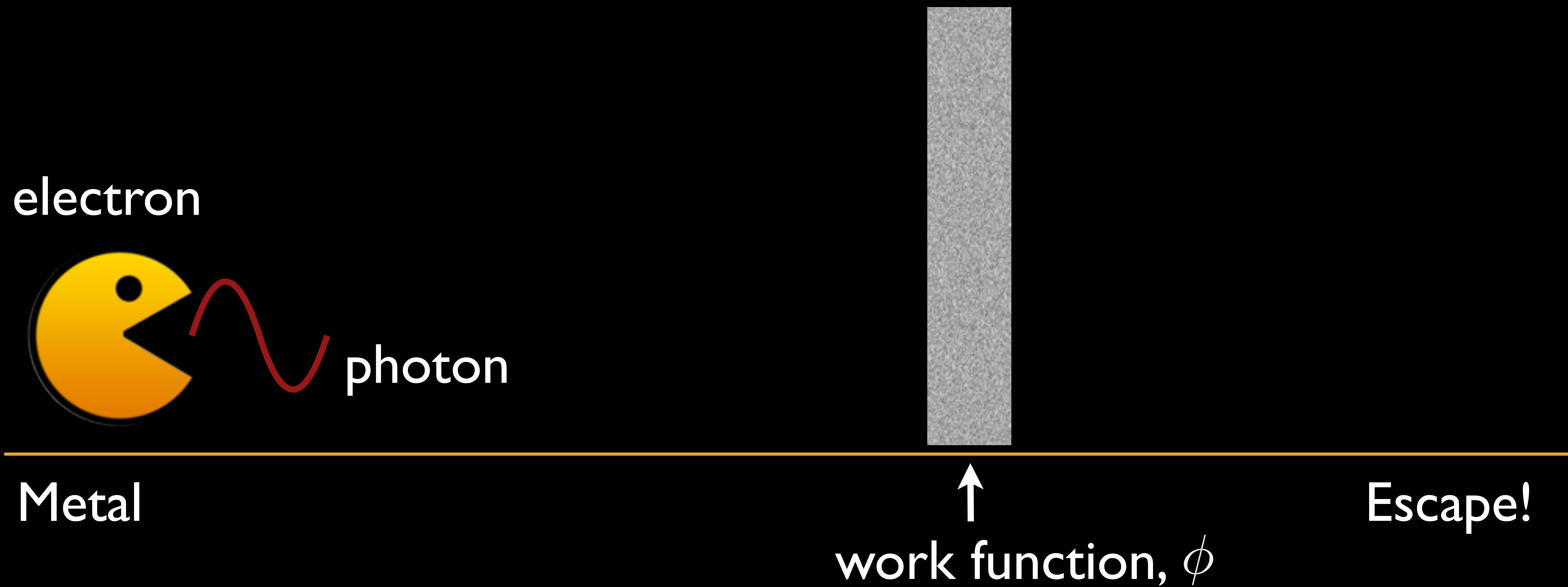
work function, ϕ

Cut-off frequency ➡ Photon energy = work function

$$E = \phi$$

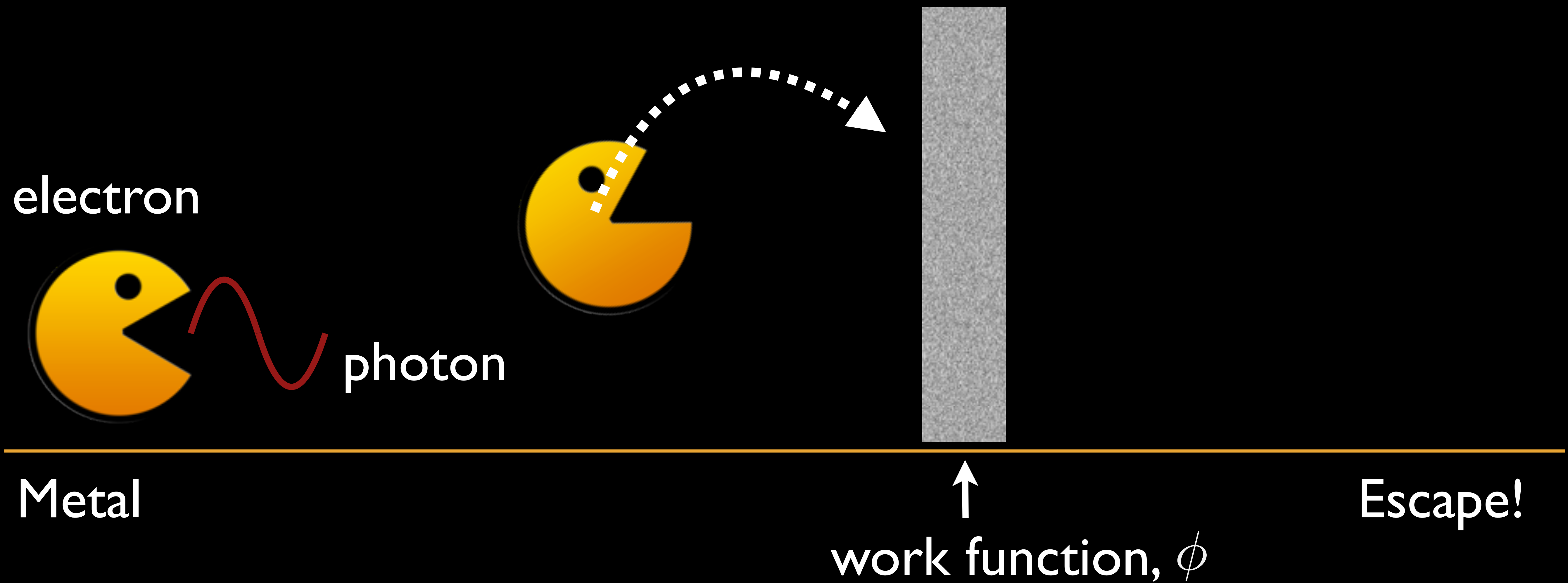
$f > f_{\text{cut-off}}$ ➡ Electrons gain energy $K_{\max} = hf - \phi$

Photoelectric Effect



Low frequency photon absorbed

Photoelectric Effect

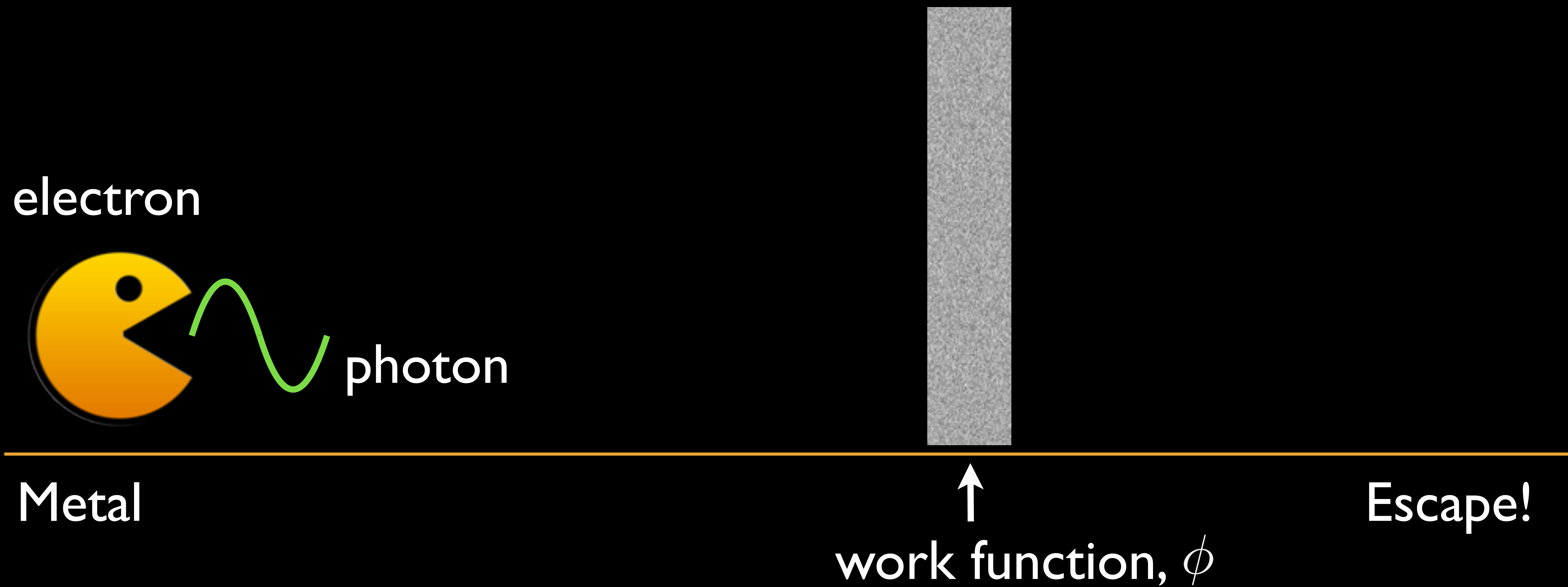


Low frequency photon absorbed

Energy < work function: $E < \phi$

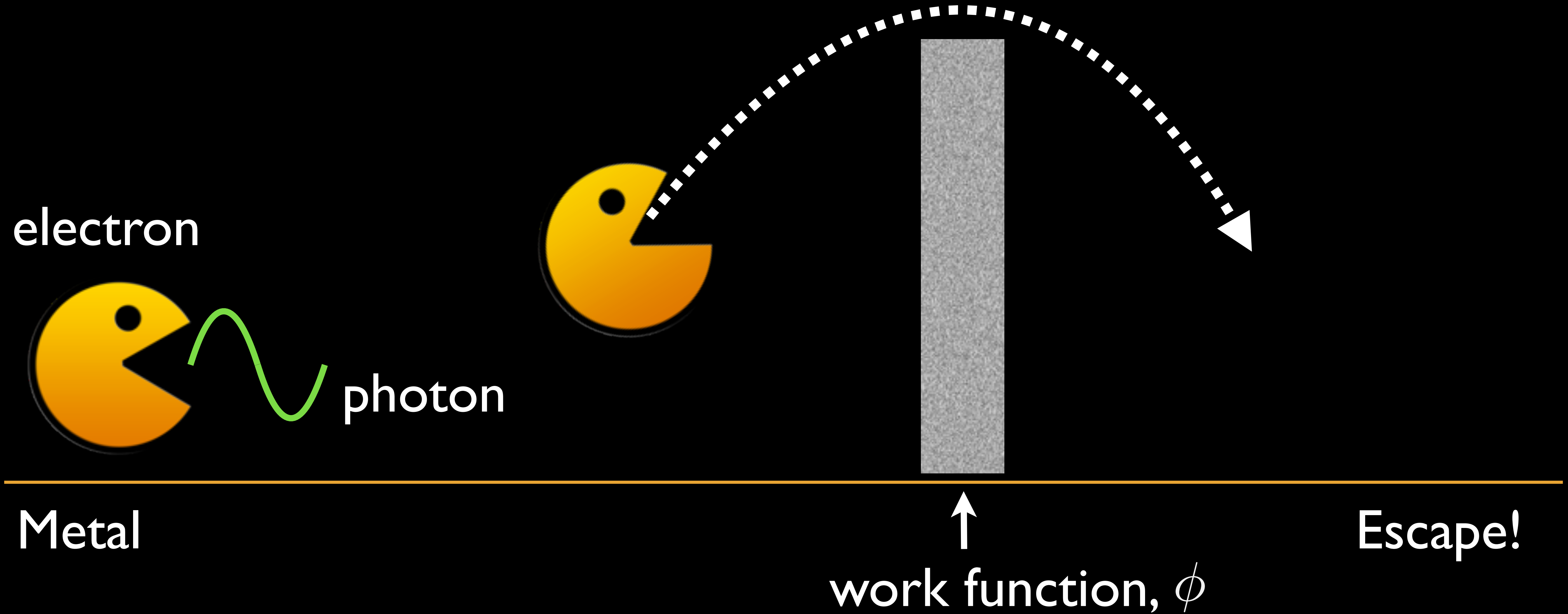
Cannot escape!

Photoelectric Effect



higher frequency photon absorbed

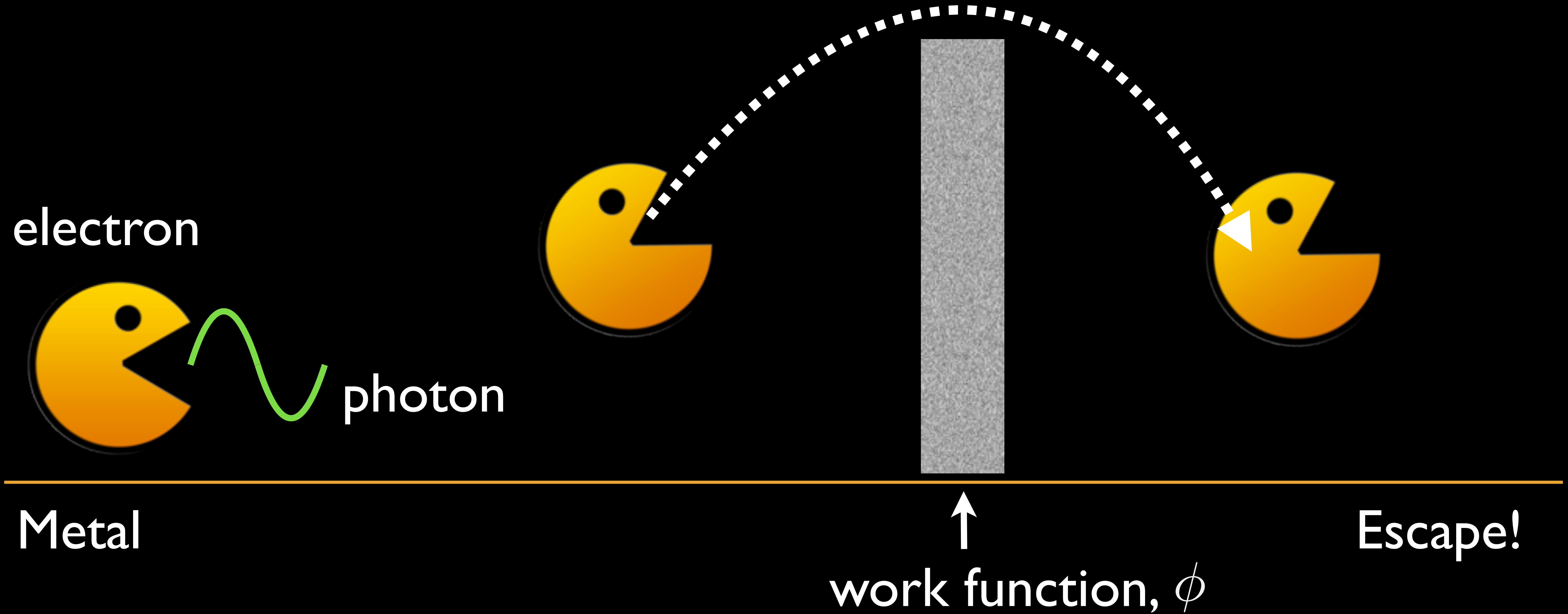
Photoelectric Effect



higher frequency photon absorbed

Energy = work function: $E = \phi$

Photoelectric Effect

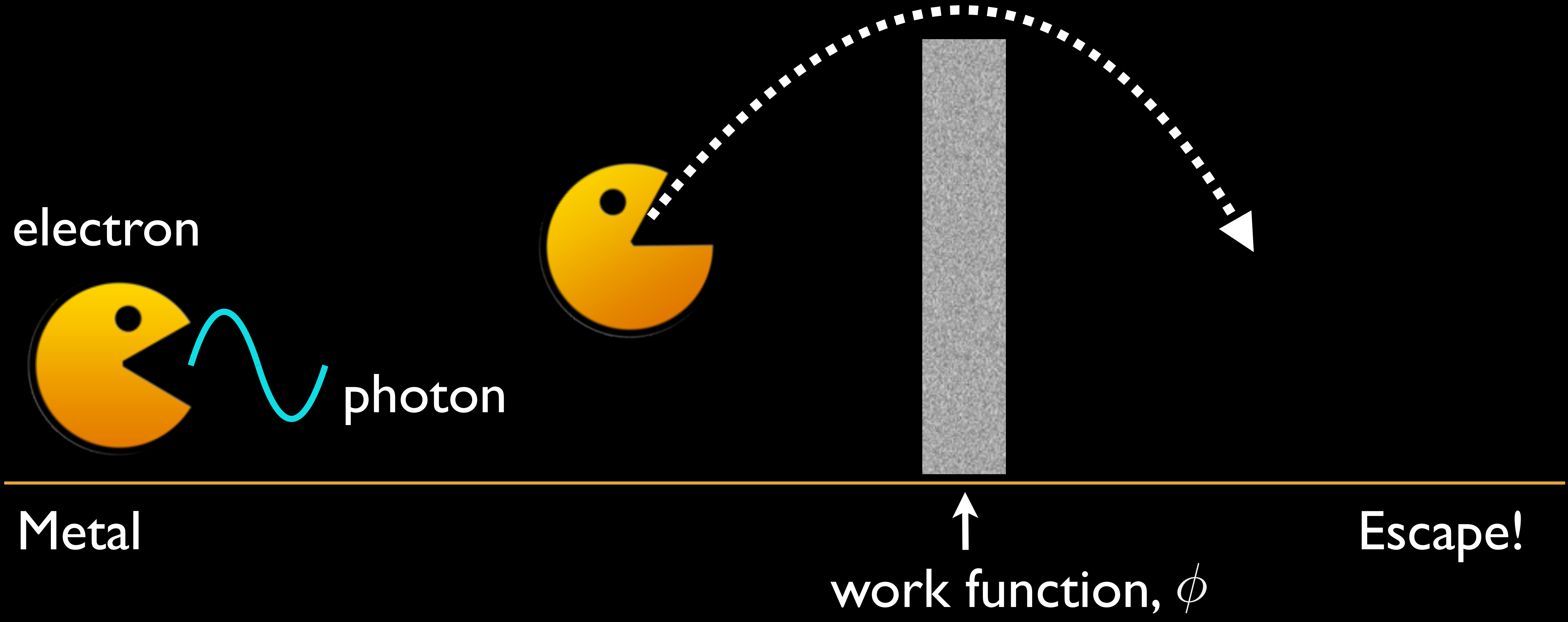


higher frequency photon absorbed

Energy = work function: $E = \phi$

Just escapes!

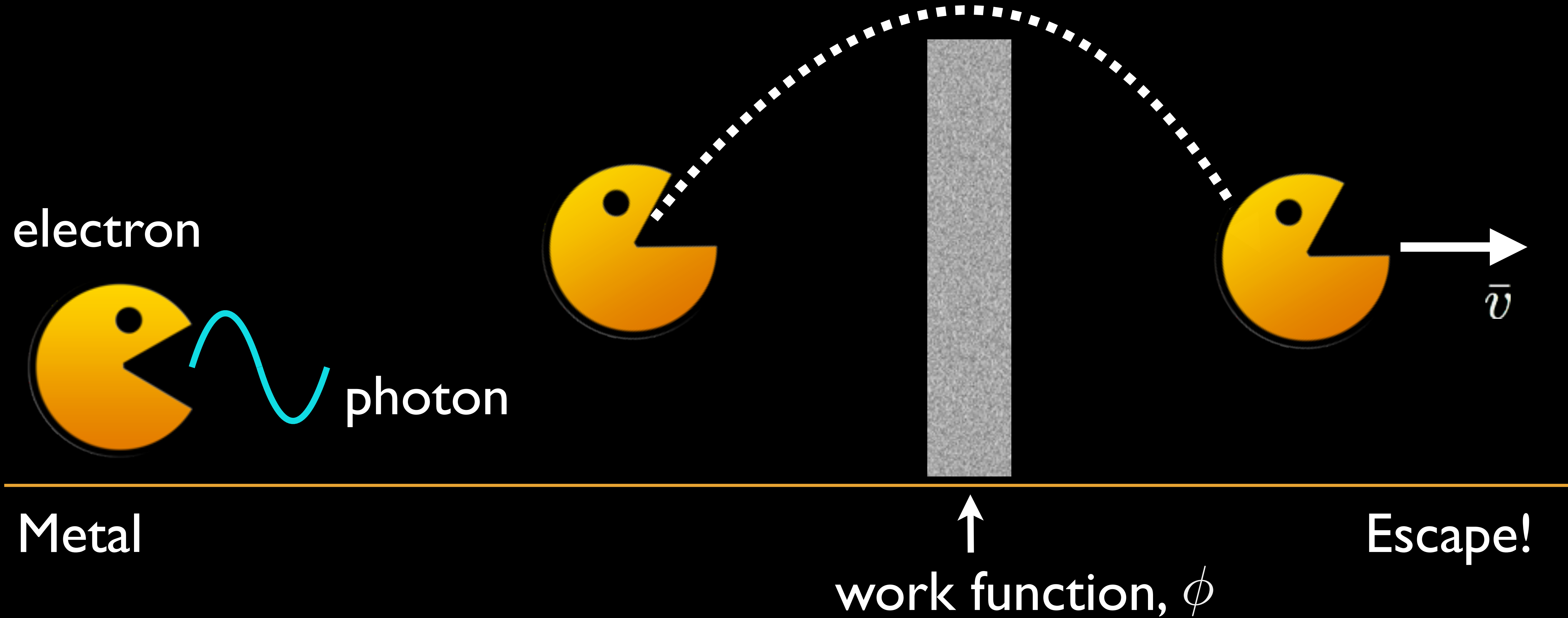
Photoelectric Effect



high frequency photon absorbed

Energy > work function: $E > \phi$

Photoelectric Effect

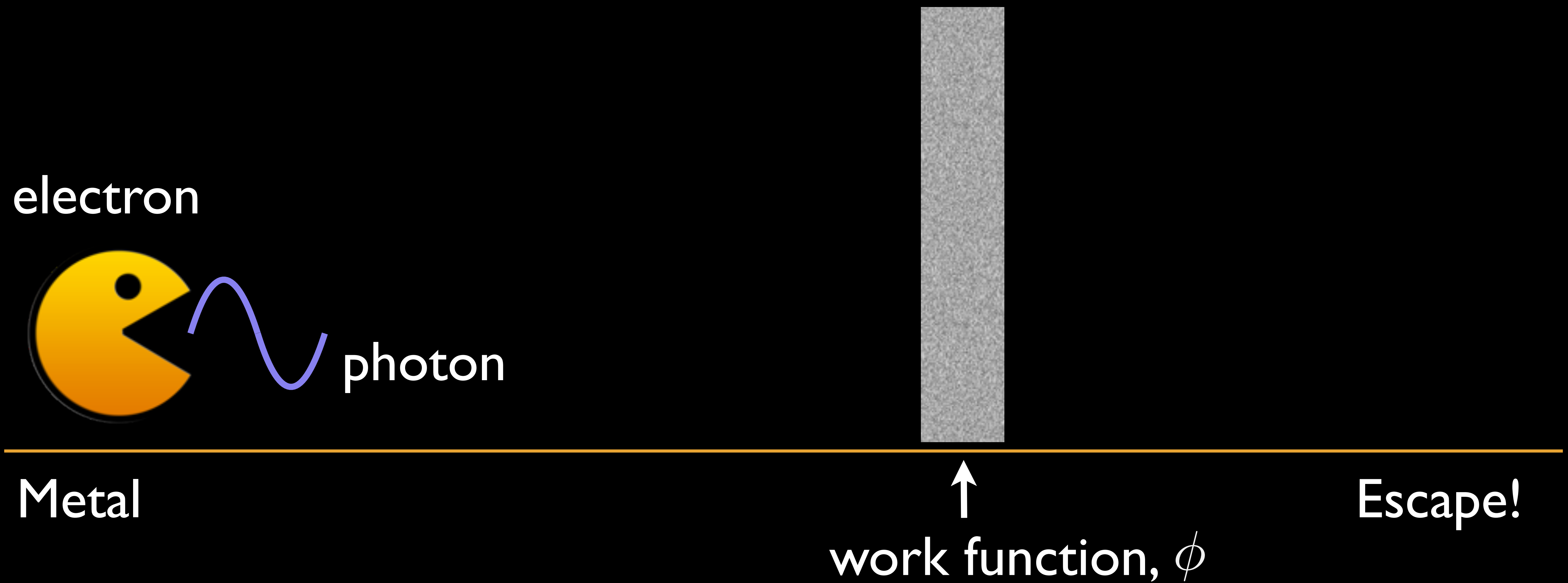


high frequency photon absorbed

Energy > work function: $E > \phi$

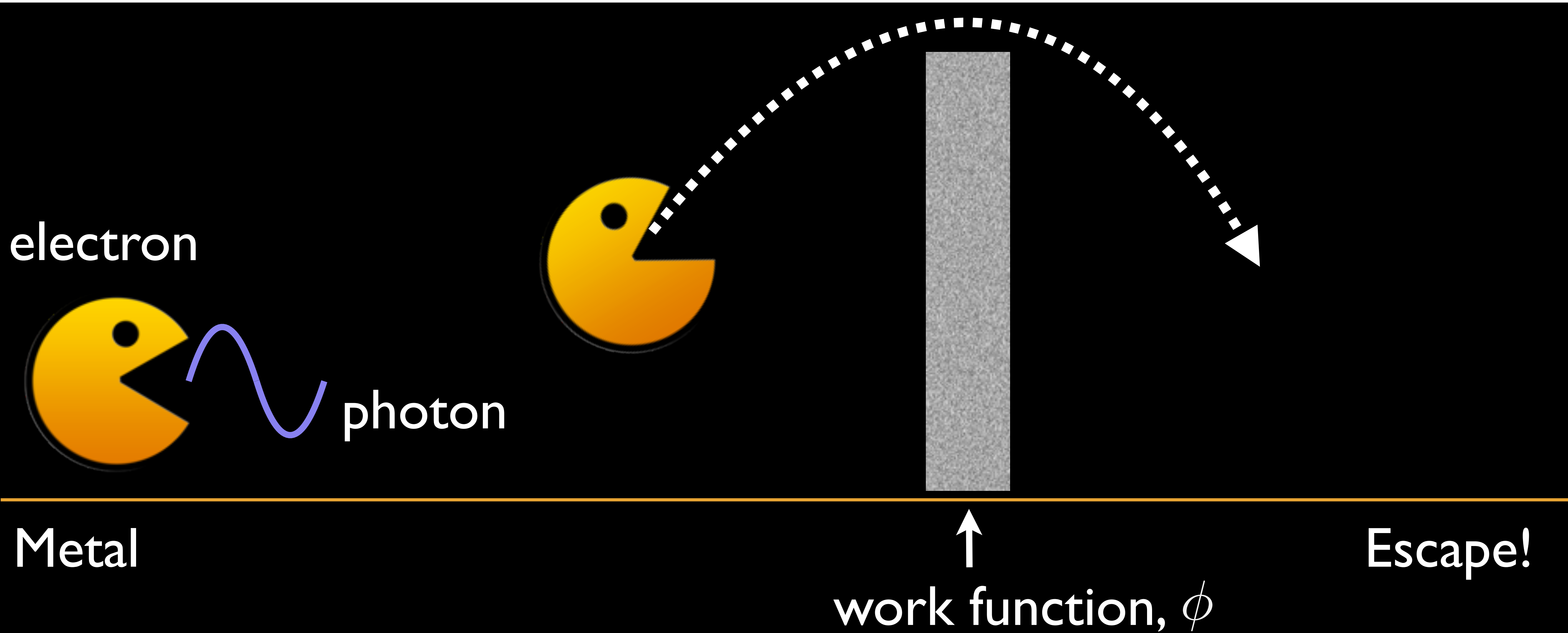
Escapes and moves ('spare' energy becomes kinetic energy)

Photoelectric Effect



Even higher frequency photon absorbed

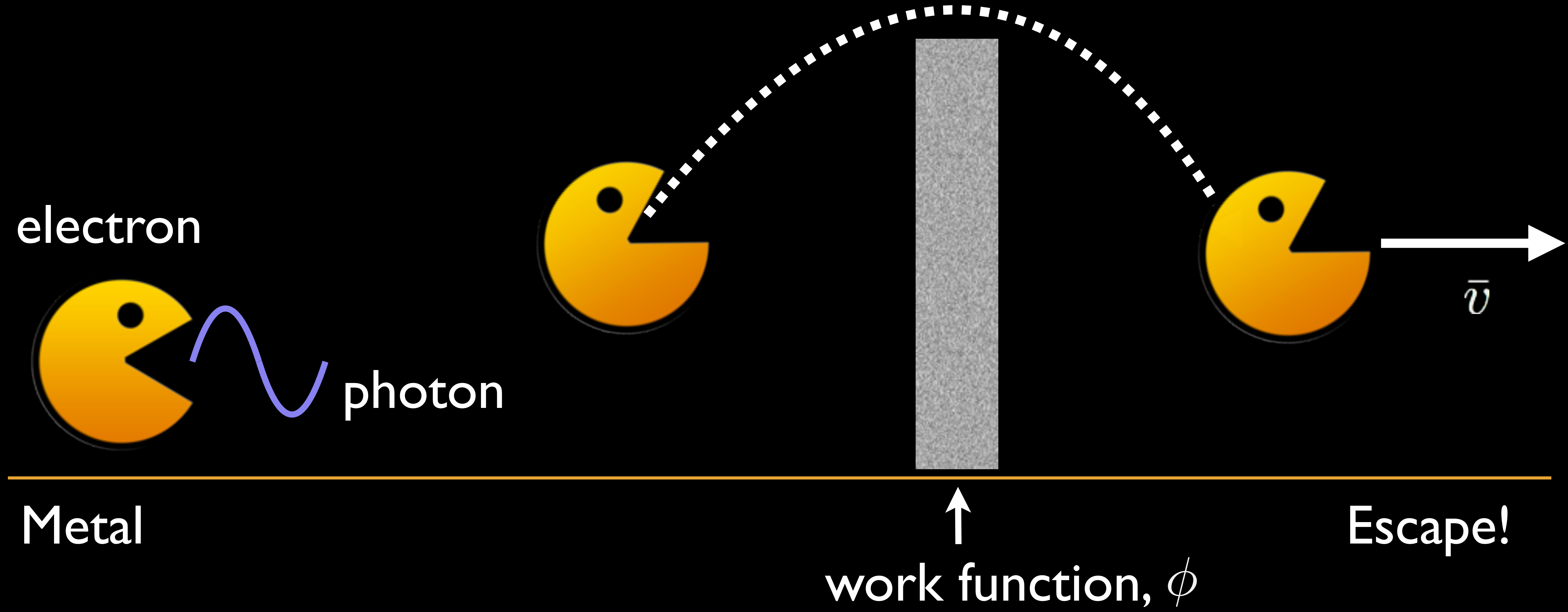
Photoelectric Effect



Even higher frequency photon absorbed

Energy > work function: $E > \phi$

Photoelectric Effect



Even higher frequency photon absorbed

Energy > work function: $E > \phi$

Escapes and moves with higher speed (higher K energy)

Photoelectric Effect

Table 34.1 Work Functions

Element (Symbol)	ϕ (eV)
Silver (Ag)	4.26
Aluminum (Al)	4.28
Cesium (Cs)	2.14
Copper (Cu)	4.65
Potassium (K)	2.30
Sodium (Na)	2.75
Nickel (Ni)	5.15
Silicon (Si)	4.85

Photoelectric Effect

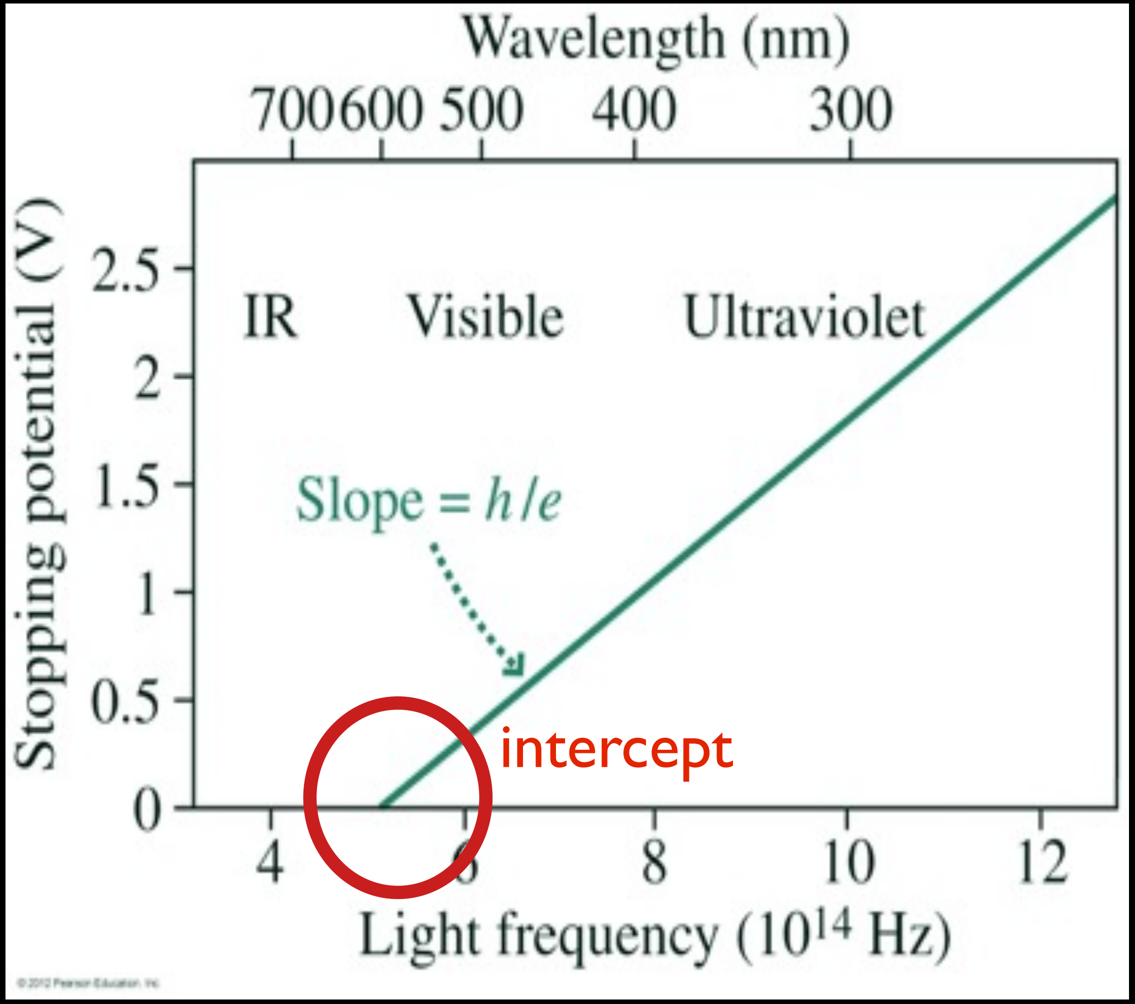
Quiz

If you re-plot the graph for a material with a different work function, ϕ, what will happen to the gradient and x-axis intercept?

(A) Gradient changes, intercept unchanged

(B) Both gradient and intercept change

(C) Gradient unchanged, intercept changes



(D) Gradient and intercept unchanged.

Photoelectric Effect

Quiz

When light of $\lambda = 300\mathrm{nm}$ is incident on potassium, the emitted electrons have a $K_{\mathrm{max}} = 2.03\mathrm{eV}$.

What is the work function for potassium? $1\mathrm{eV} = 1.6 \times 10^{-19}\mathrm{J}$

(A) $2.03\mathrm{eV}$

(B) $4.1\mathrm{eV}$

(C) $2.1\mathrm{eV}$

(D) $3.5\mathrm{eV}$

$$K_{\mathrm{max}} = hf - \phi = h\frac{c}{\lambda} - \phi$$

$$\phi = h\frac{c}{\lambda} - K_{\mathrm{max}}$$

$$= 6.63 \times 10^{-34}\mathrm{J\cdot s}\frac{2.99 \times 10^{8}\mathrm{m/s}}{300 \times 10^{-9}\mathrm{m}} - (2.03\mathrm{eV})(1.6 \times 10^{-19}\mathrm{J})$$

$$= 3.36 \times 10^{-19}\mathrm{J} = 2.1\mathrm{eV}$$

The problem with atoms

Problem 1: Atom model

Atom: Electrons orbit the nucleus

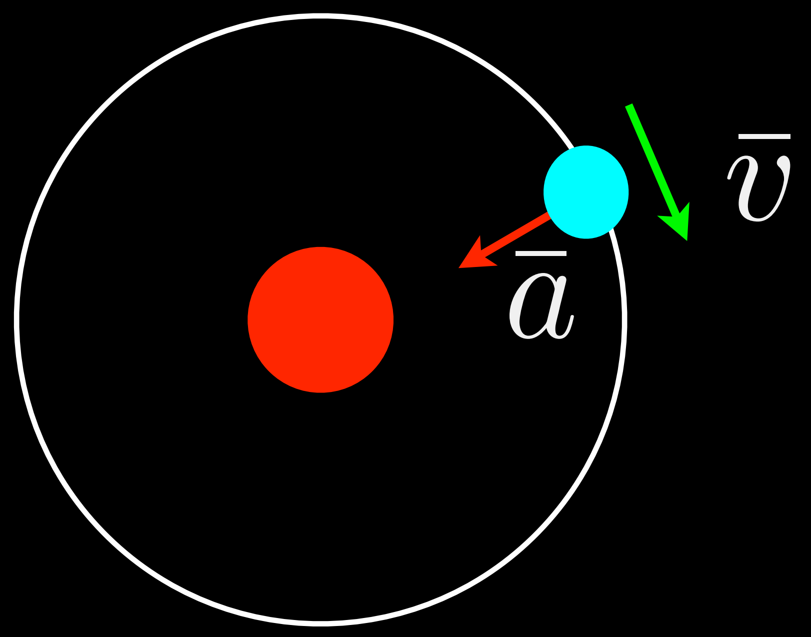
accelerating charge

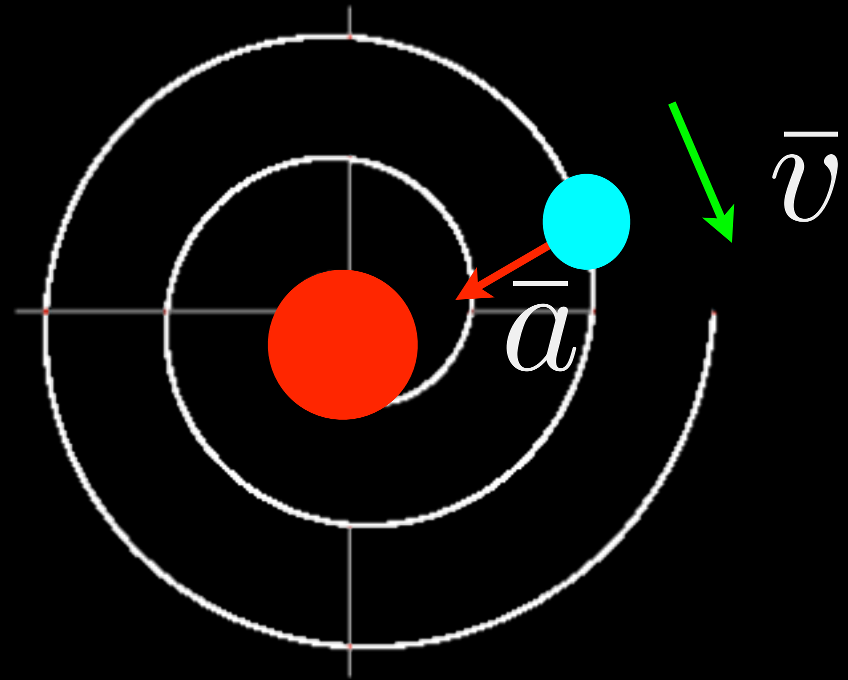
changing electric field

Produce an EM wave

Electron will radiate away all energy!

Spiral towards atom's centre

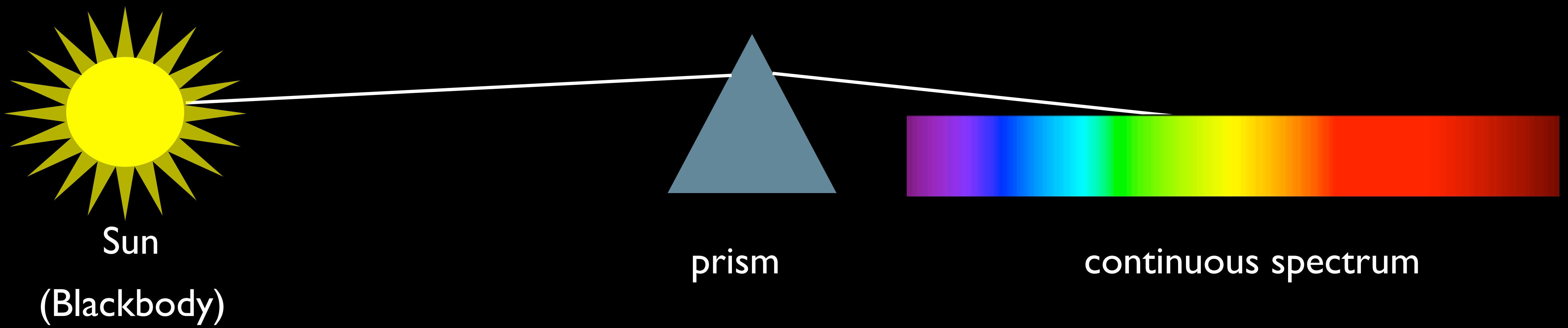




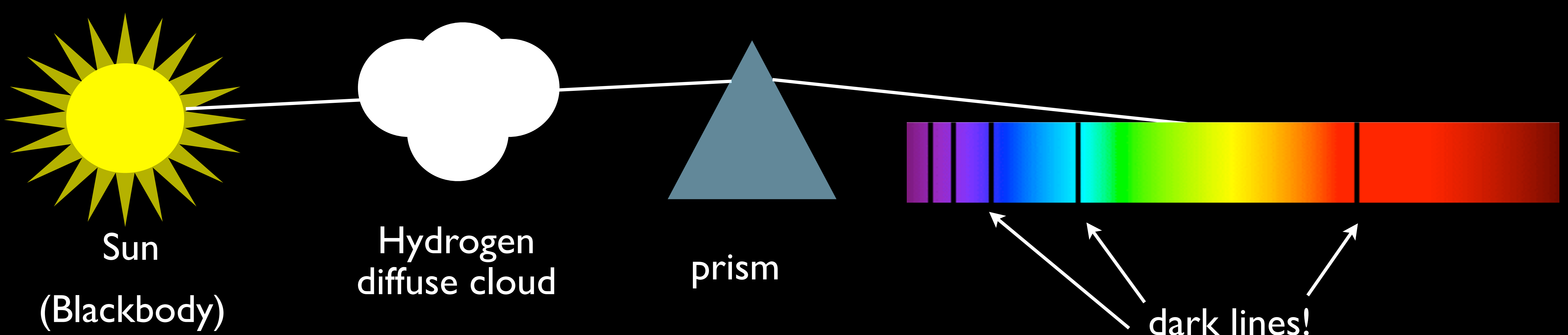
The problem with atoms

Problem 2: The Hydrogen Spectrum

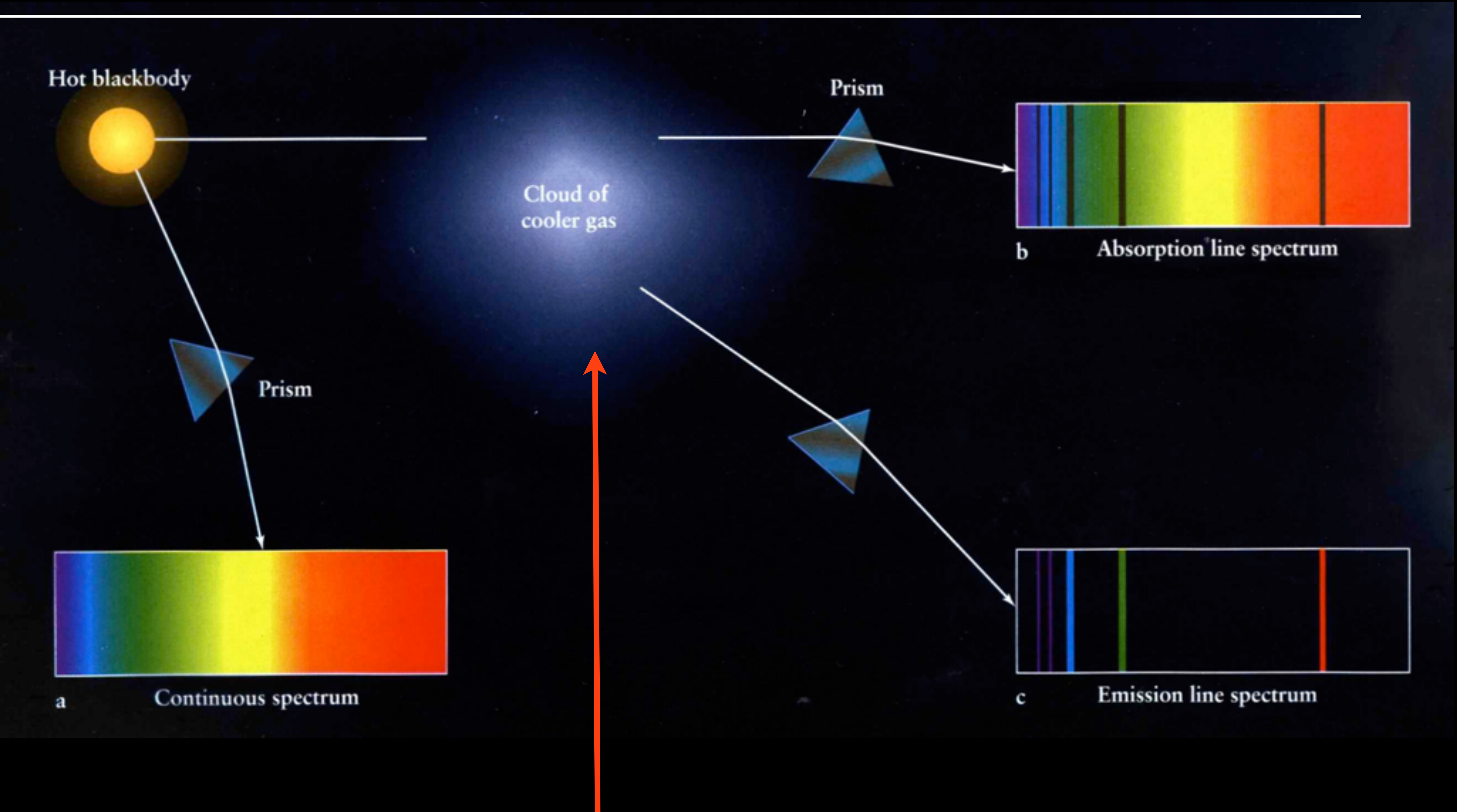
Expected:



Seen:

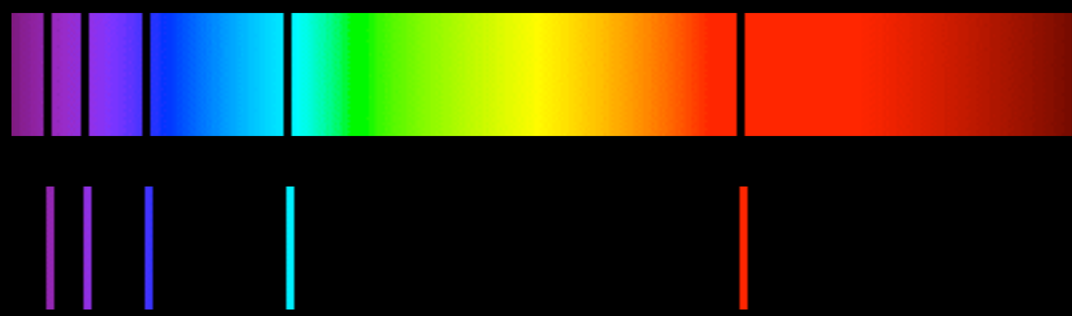


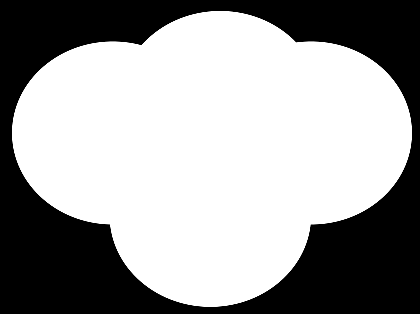
The problem with atoms



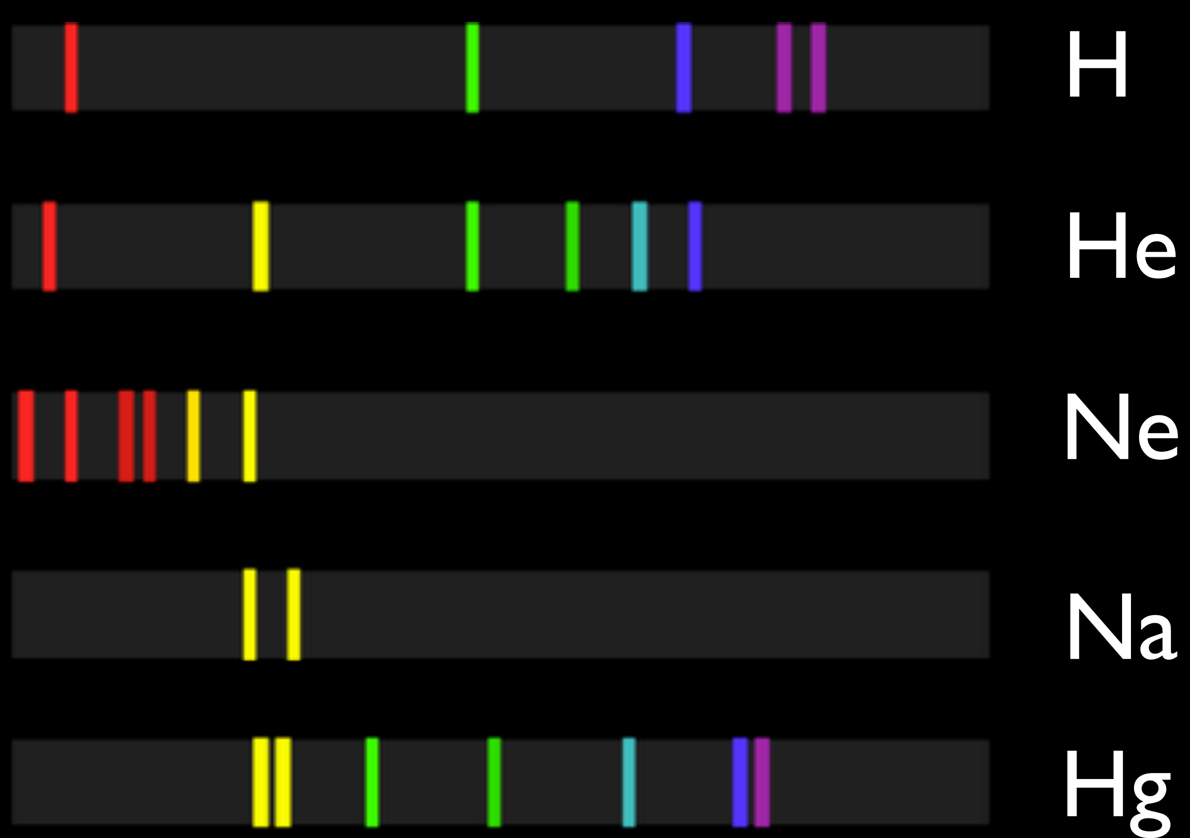
What is happening?

The problem with atoms





Diffuse (low density) gases absorb / emit specific frequencies.



Which frequencies (or 'lines') depend on the element

The problem with atoms

In 1884, Johann Balmer found that for Hydrogen:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

1st 4 lines in visible spectrum

$$n = 3, 4, 5, 6...$$

$R_H = 1.097 \times 10^7 \mathrm{m}^{-1}$ Rydberg constant

generalised form

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right)$$

$$n_1 = n_2 + 1, n_2 + 2, ...$$

all lines in Hydrogen spectrum

$n_2 = 1$	Lyman series	IR
$n_2 = 2$	Balmer series	visible
$n_2 = 3$	Paschen series	UV

The problem with atoms

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1st 4 lines in visible spectrum

$$n = 3, 4, 5, 6 ...$$

$$R_H = 1$$

But why do these lines exist?

generalised form

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right)$$

$$n_1 = n_2 + 1, n_2 + 2, ...$$

all lines in Hydrogen spectrum

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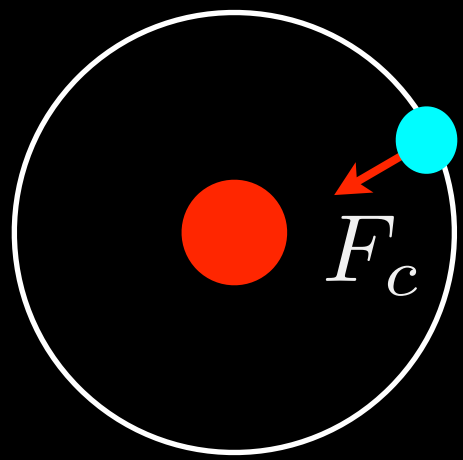
The problem with atoms

The Bohr model

1 Electrons move in circular orbits, held by Coulomb electric force.



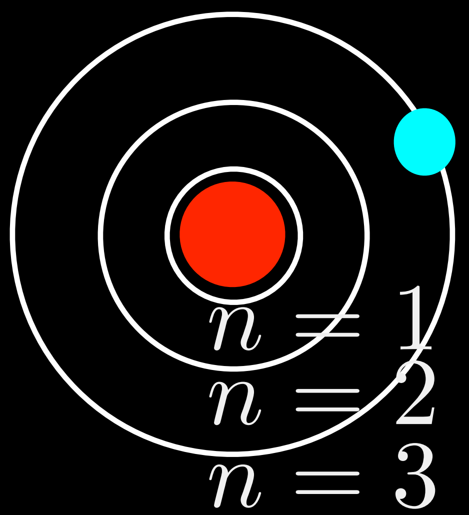
2 Only certain orbits are allowed:

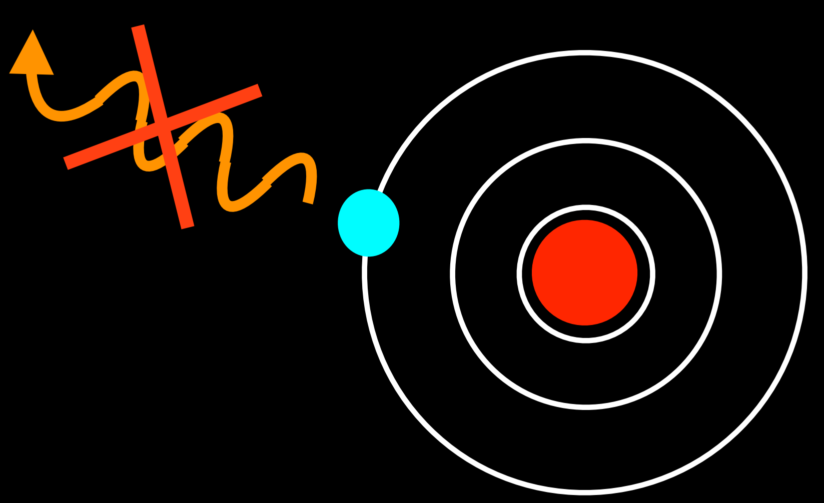
$$L = mvr = n\frac{h}{2\pi} = n\hbar \qquad n = 1, 2, 3\ldots$$

angular momentum

integer multiples

$$\hbar = \frac{h}{2\pi}$$

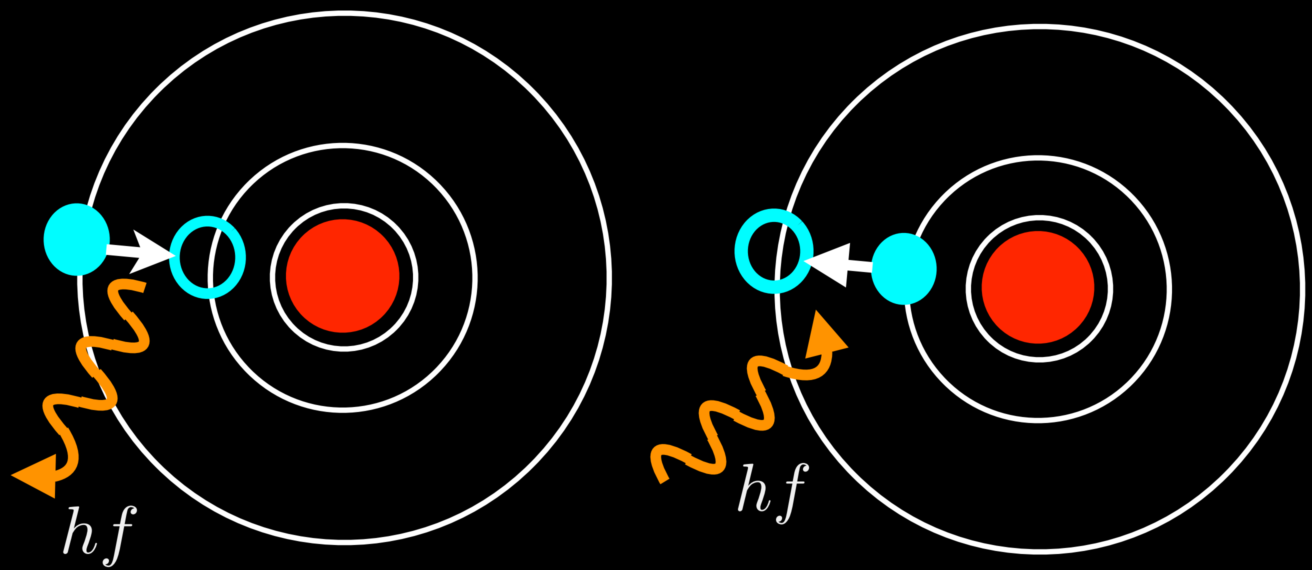




3 Electrons do not radiate in orbit

The problem with atoms

4 Moving between orbits causes photon to be emitted or absorbed.



The problem with atoms

What orbits are allowed?

Balance forces: $-\frac{mv^2}{r} = \frac{kq_pq_e}{r^2}$

centripetal force | Coulomb force

$= -\frac{ke^2}{r^2}$

$(q_e = -q_p = e)$

Rearrange: $mv^2r = ke^2$

From: $mvr = n\hbar$ ➡ $v = \frac{n\hbar}{mr}$ ➡ $r = \frac{n^2\hbar^2}{mke^2}$

Define: $a_0 = \frac{\hbar^2}{mke^2} = 0.0529\text{nm}$ Bohr radius

➡ $r = n^2a_0$ allowed orbits

The problem with atoms

Total energy: $E = KE + U(r)$

kinetic $\frac{1}{2}mv^2$

point source potential $U(r) = \frac{ke^2}{r}$

(lecture 7)

$$E = \frac{1}{2}mv^2 - \frac{ke^2}{r}$$

From last slide

$$mv^2r = ke^2$$

$$mvr = n\hbar$$

$$v = \frac{ke^2}{n\hbar}$$

and

$$r = \frac{n^2\hbar^2}{mke^2}$$

$$E = -\frac{k^2e^4m}{2\hbar^2n^2}$$

$$E = -\frac{ke^2}{2a_0}\left(\frac{1}{n^2}\right)$$

The problem with atoms

$$E = -\frac{ke^2}{2a_0}\left(\frac{1}{n^2}\right)$$ energy levels, Bohr atom

for $n = 1$: $E_1 = -2.18 \times 10^{-19}\mathrm{J} = -13.6\mathrm{eV}$

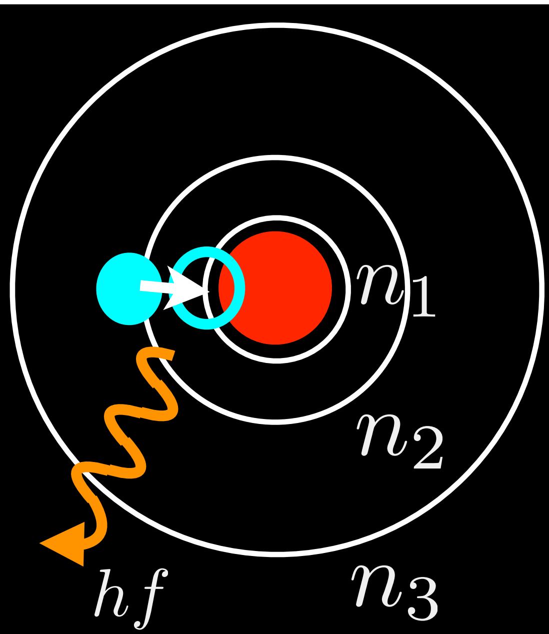
Simpler to write: $E = -\frac{13.6\mathrm{eV}}{n^2}$ $n = 1, 2, 3, \ldots$

$n = 1$ is the ground state.

$n > 1$ are the excited states.

The problem with atoms

So...



When an electron changes orbits, it emits/absorbs a photon.

Photon energy = difference in orbit energies

$$\Delta E = -\frac{ke^2}{2a_0}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad = \frac{ke^2}{2a_0}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

energy of the emitted photon

But photon energy is: $E = hf = h\frac{c}{\lambda}$

Therefore:

$$\frac{1}{\lambda} = \frac{ke^2}{2a_0 hc}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

The problem with atoms

$$\frac{1}{\lambda} = \frac{ke^2}{2a_0hc}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \quad \Longleftrightarrow \quad \frac{1}{\lambda} = R_H\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

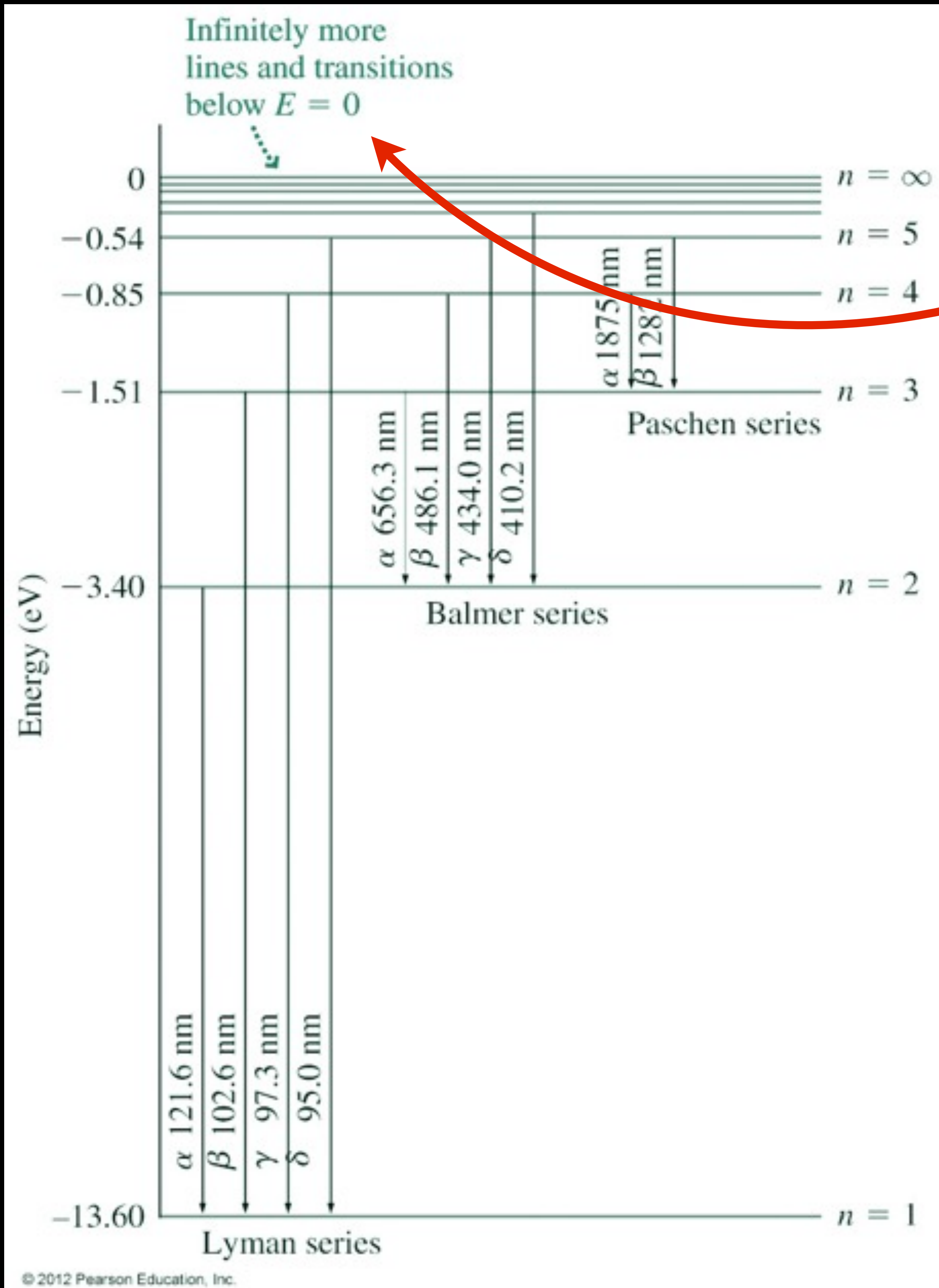
$$\frac{ke^2}{2a_0hc} \rightarrow R_H$$

Balmer spectrum!

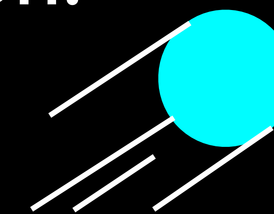
$$\frac{ke^2}{2a_0hc} = 1.09 \times 10^7 \mathrm{m}^{-1}$$

slight difference from measured value because assumed proton did not move.

The problem with atoms



Above $E = 0$, electron is no longer bound to the proton.



Removing an electron is ionisation.

To remove an electron from the ground state of Hydrogen:

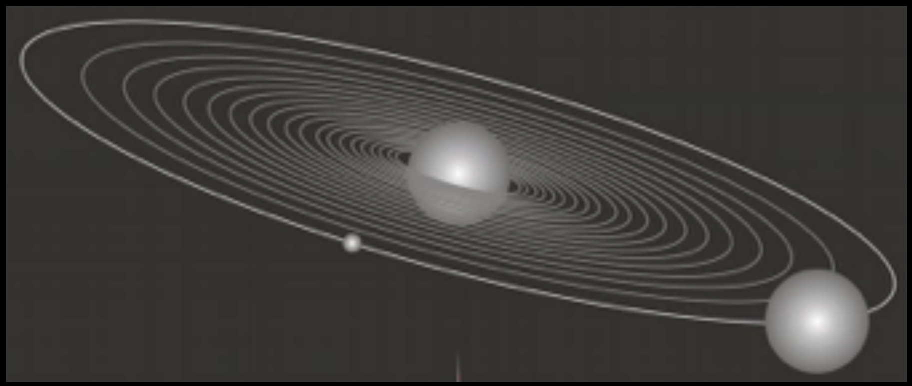
$$E = 13.6\text{eV}$$

Ionisation energy

The problem with atoms

Example

Found in diffuse gas in space, Rydberg atoms have electrons in a highly excited state.



What is the diameter of the hydrogen atom in the n = 273 state?

What wavelength of photon would be emitted in the n = 273 to n = 272 transition?

$$r = n^2 a_0 \Rightarrow d = 2(273)^2 a_0 = 7.9\mu\text{m}$$

75,000 x Hydrogen ground state atom

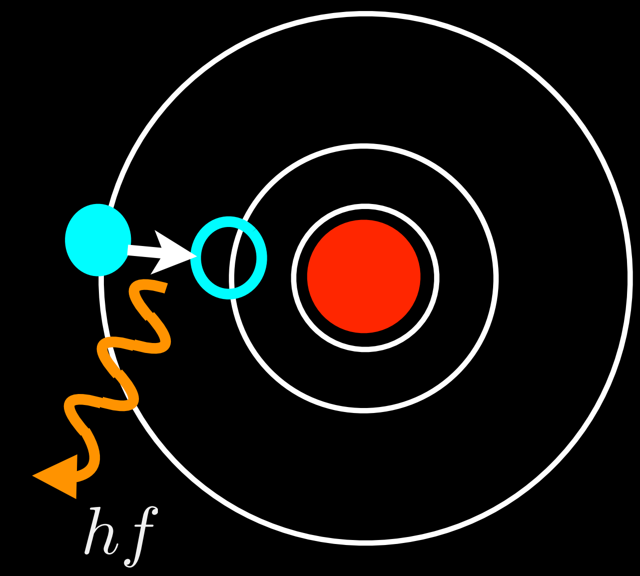
~ size of red blood cell

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \Rightarrow \lambda = \left[R_H\left(\frac{1}{272^2} - \frac{1}{273^2}\right)\right]^{-1} = 92\text{cm}$$

radio wave!

The problem with atoms

<u>Limitation of the Bohr Model</u>



VERY successful model for Hydrogen-like atoms.
(1 electron atoms, or with 1 less bound electron)

... not as good for more complicated atoms.

Does not say *when* a orbit change will occur

... or intensity of spectral lines

Mix of classical and non-classical ideas.

The problem with atoms

Quiz

Which spectral line of the Paschen series ($n_2 = 3$) has $\lambda = 1282\text{nm}$?

$$R_H = 0.01097\text{nm}^{-1}$$

(A) 6

(B) 5

(C) 4

(D) 2

$$n_2 = 3 \qquad n_1 = 4, 5, 6, ...$$

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right)$$

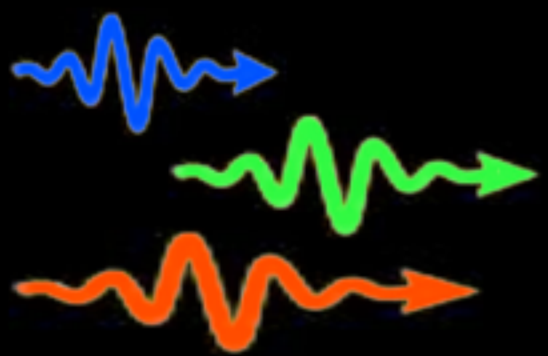
$$\lambda = 1282\text{nm}$$

$$n_1 = \sqrt{\frac{9 \times 14.1}{14.1 - 9}} = 5$$

Particles or waves?

THIS lecture:

Light is a particle of energy called a photon



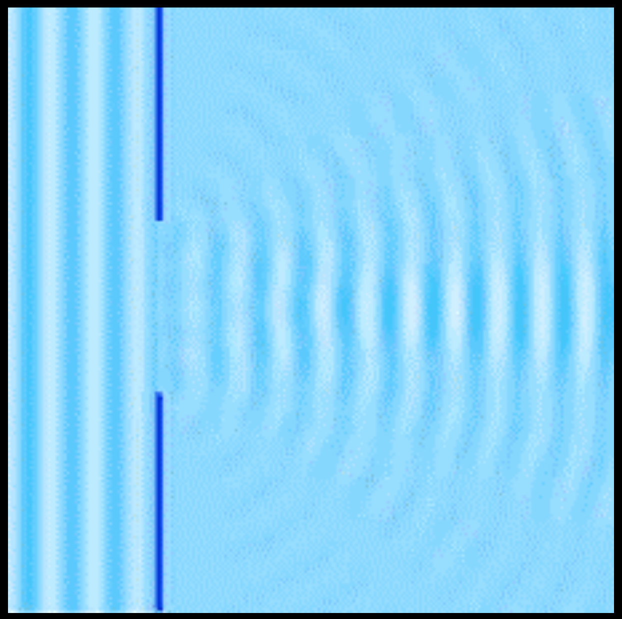
To explain blackbody radiation, photoelectric effect and the atomic structure...

... it MUST be a discrete package, not a continuous wave.

But LAST lecture

Light is an electromagnetic WAVE.

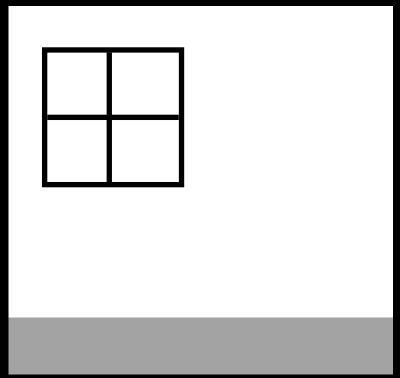
It undergoes diffraction and interference.



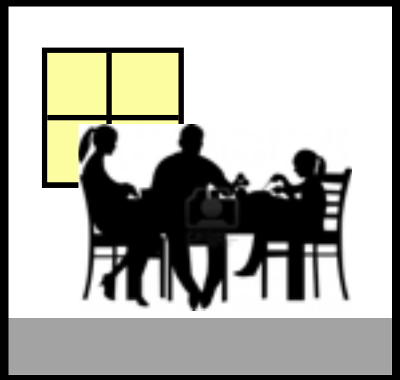
... it MUST be a continuous wave.

Is light a WAVE or a PARTICLE?

Analogy...



Imagine an empty room...



In the morning .... it's a dining room

In the evening .... it's a ballroom

Is the room a dining room or a ballroom? It depends how you use it.

Is light a particle or a wave? It depends how you measure it.

(analogy from the book 'Uncle Albert and the Quantum Quest')

Particles or waves?

Diffraction experiment

measures wave properties ➡ light is a wave

Photoelectric effect

measures particle properties ➡ light is a particle

➡ Wave-particle duality

Bohr's correspondence principal:

When the size of the quanta is very very small compared to what you measure ➡ classical physics ➡ light is a wave.

e.g. 1000-W radio beam; contains many photons / length with $E = hf$

➡ continuous wave

1000-W X-ray beam; photon energy much higher.

➡ quantised particles

Matter Waves

1923, Louis de Broglie (pronounced "de Broy"):

If light can be both particle and wave....

can matter be both particle and wave too?

For both particles and waves:

$$\lambda = \frac{h}{p}$$

(de Broglie wavelength)

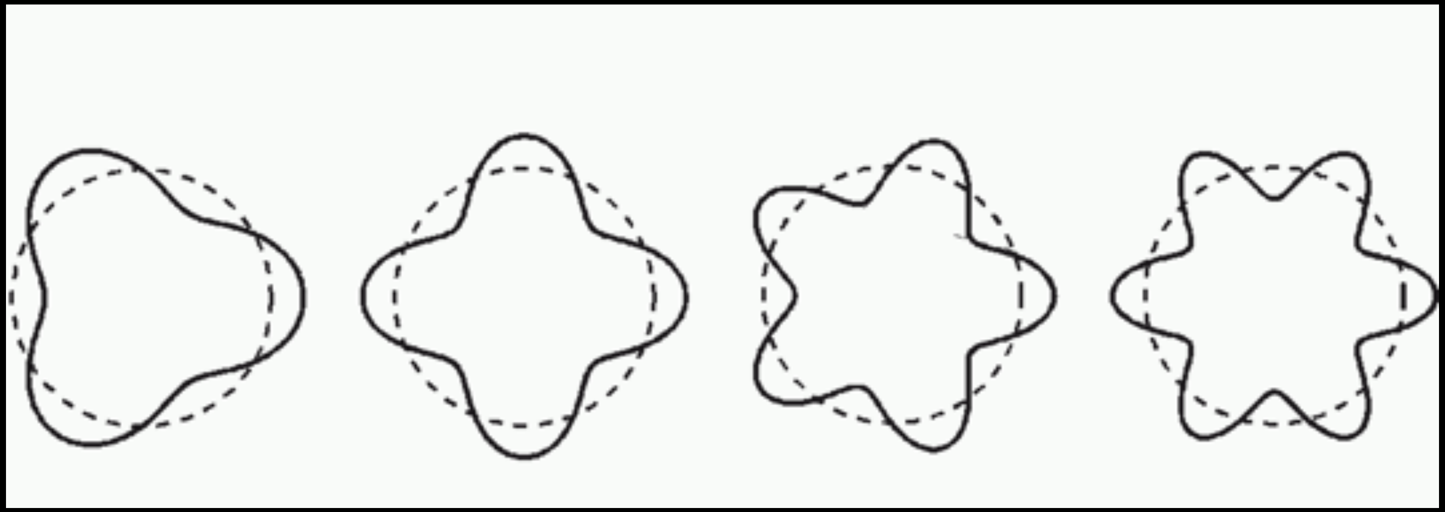
momentum mv

Matter Waves

Apply this to the electron

electron is a wave

atomic orbits are electron standing waves



n full wavelengths around circumference of orbit:

$$n\lambda = n\left(\frac{h}{p}\right) = 2\pi r$$

$$\times \frac{mv}{2\pi}$$

$$mvr = \frac{nh}{2\pi} = n\hbar$$

Bohr's quantisation condition

Matter Waves

Example

Find the de Broglie wavelength of a 150-g baseball pitches at 45 m/s

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{0.150 \times 45}$$

$$\simeq 10^{-34}\mathrm{m}$$

tiny compared to pitch!

particle effect dominates

Find the de Broglie wavelength of an electron with velocity 1 Mm/s

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1 \times 10^{-30} \times 1 \times 10^{6}} \simeq 0.7\mathrm{nm}$$

several times size of atom

wave effect dominates

Matter Waves

Quiz

An electron with kinetic energy of 1000 eV has a de Broglie wavelength of 0.04 nm.

If the energy is increased by x 10 the wavelength will be...:

(A) increased by a factor of 10

(B) decreased by a factor of 1/10

(C) decreased by a factor of 100

(D) decreased by a factor of $1/\sqrt{10}$

$$E_2 = 10E_1$$

$$E = \frac{1}{2}mv^2$$

$$v_2 = \sqrt{10}v_1$$

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\lambda = \frac{1}{\sqrt{10}}\frac{h}{mv}$$

Down the rabbit hole

Curious facts about quantised particles

Uncertainty Principal

It is impossible to know both position and momentum of a particle

error in position — error in momentum

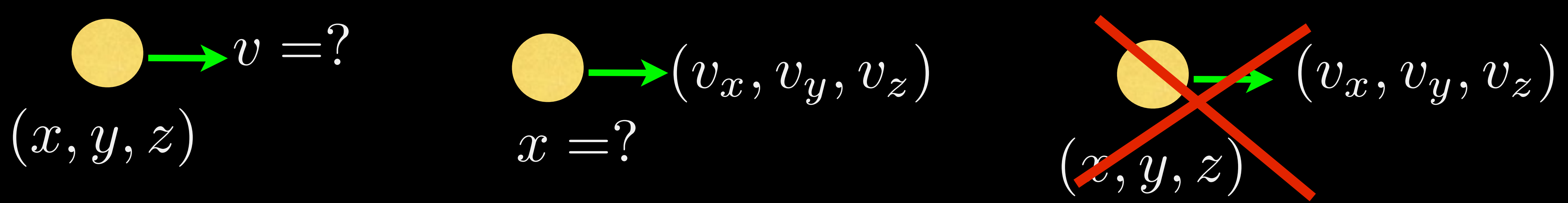
$$\Delta x \Delta p \geq \hbar$$

uncertainty principal

Either you know position very accurately $\Delta x \to 0$

OR momentum $\Delta p \to 0$

but not both



You can know where something is, but not how fast it's going

Uncertainty Principal

Why?

Use a single light photon to observe an electron.

Short wavelength photon

↓

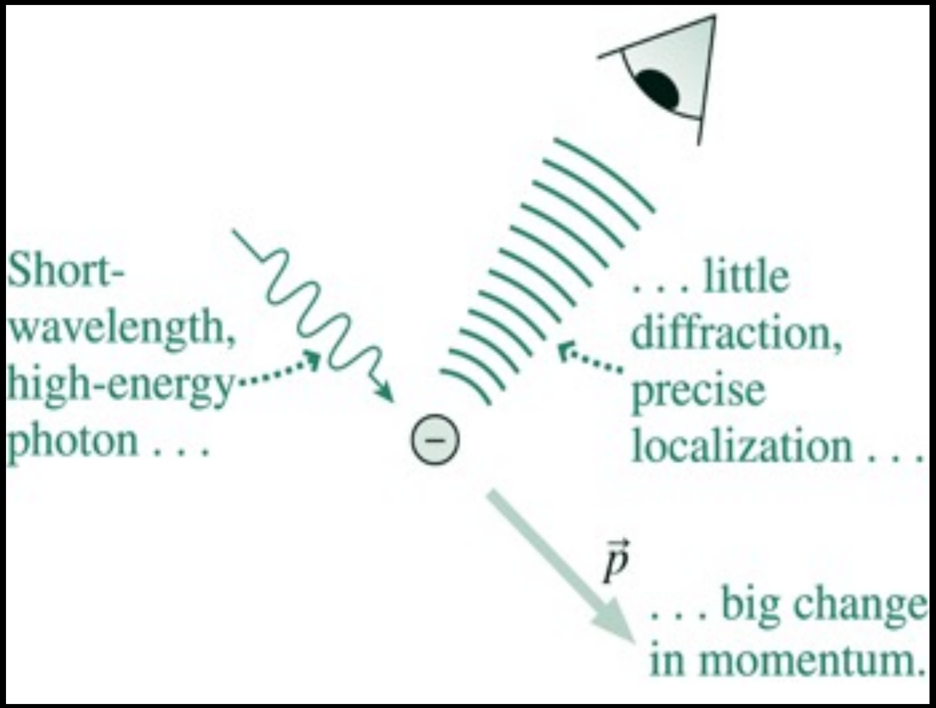
precise location of electron ✓

↓

high energy photon changes electron's momentum

↓

accuracy of electron's momentum decreases ✗



Uncertainty Principal

Why?

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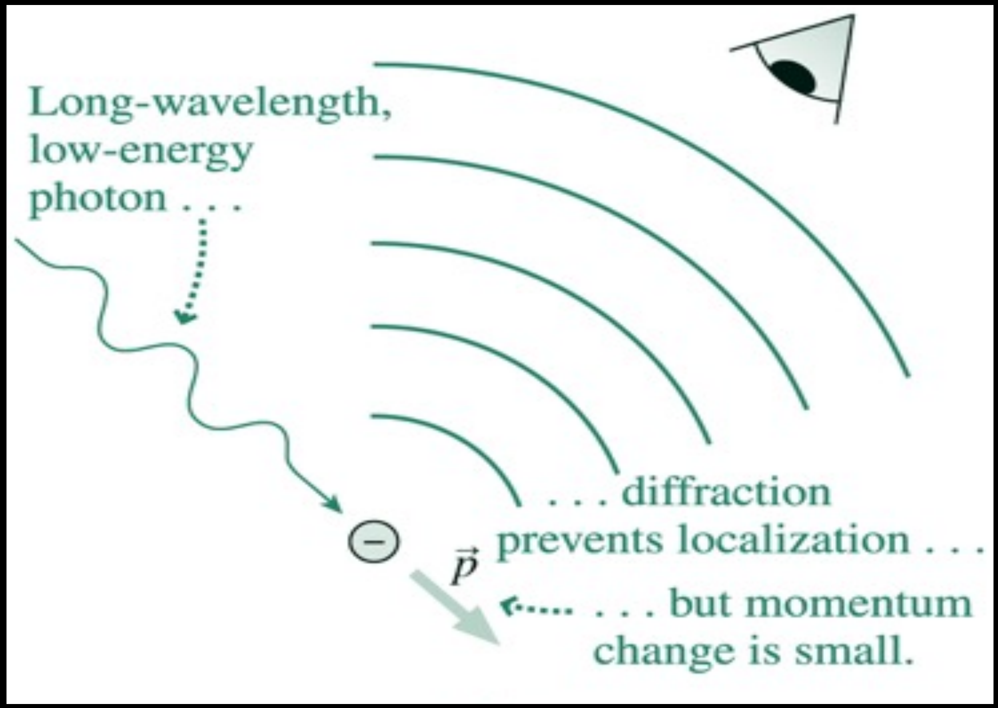

long wavelength photon

⬇

low energy photon does not disturb electron: momentum known ✓

⬇

Diffraction of long wavelength photon ➡ limit resolution

⬇

accuracy of electron's position decreases ✗

Uncertainty Principal

Example

A beam of Al atoms have a velocity known within 0.2 m/s.

How accurately can they be positioned?

$$\Delta x \Delta p \geq \hbar$$

$$m = 26.98u$$

$$u = 1.66 \times 10^{-27}\text{kg/u}$$ universal mass unit (u)

$$\Delta p = m\Delta v = (26.98u)(1.66 \times 10^{-27})(0.20\text{m/s})$$

$$= 9 \times 10^{-27}\text{kg} \cdot \text{m/s}$$

$$\Delta x = \frac{\hbar}{\Delta p} = 12\text{nm}$$

Uncertainty Principal

Quiz

An electron is moving at 50 Mm/s, accurate to $\pm 10\%$

What is the minimum uncertainty in its position?

$$m_e = 9.11 \times 10^{-31}\mathrm{kg}$$

(A) 73pm

(B) 146pm

(C) 23pm

(D) 12pm

$$\Delta p = m\Delta v$$

uncertainty in v: $\Delta v = (0.20)(50 \times 10^6\mathrm{m/s})$

$$\Delta x \geq \frac{\hbar}{\Delta p}$$

$$= \frac{(6.63 \times 10^{-34}\mathrm{J \cdot s})/2\pi}{(9.11 \times 10^{-31}\mathrm{kg})(0.20)(50 \times 10^6\mathrm{m/s})}$$

Uncertainty Principal

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$$= 12\text{pm}$$