Essential Physics II

英語で物理学の

エッセンス II

Lecture 10:   30-11-15

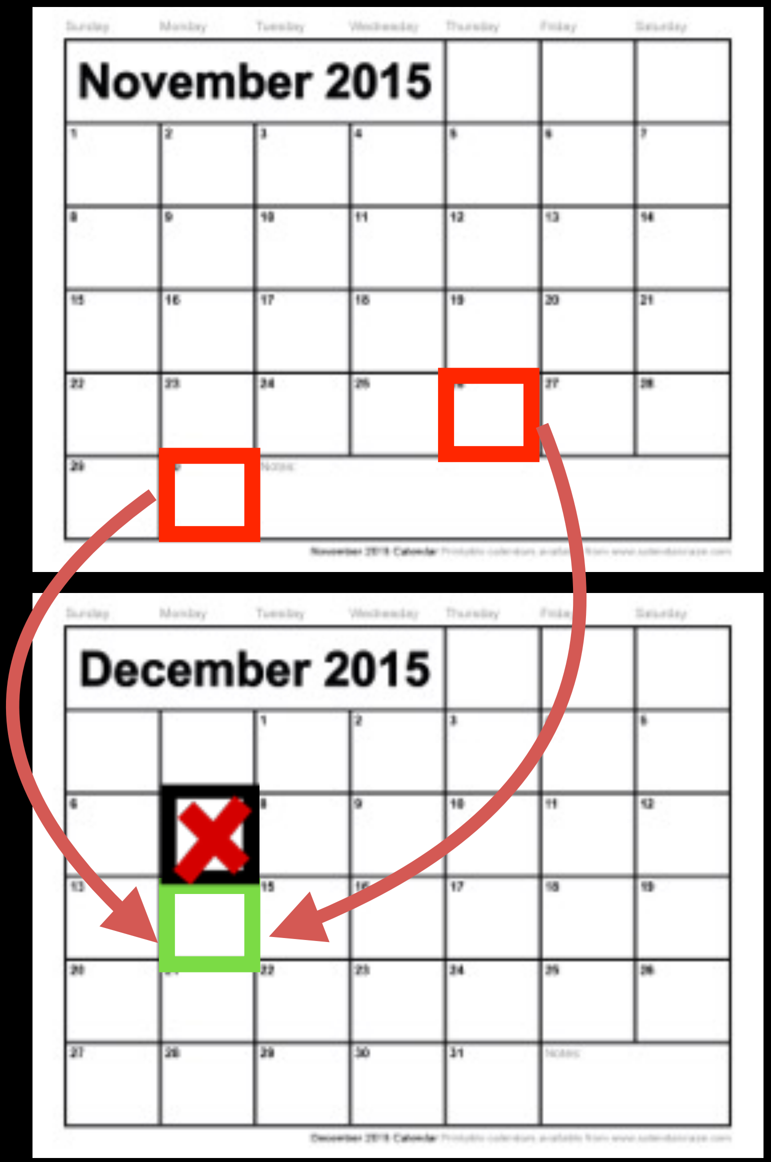
News

Schedule change:

~~Monday 7th December (１２月７日)~~ NO CLASS!

Class 11/26 homework: 12/14

This week's homework: 12/14

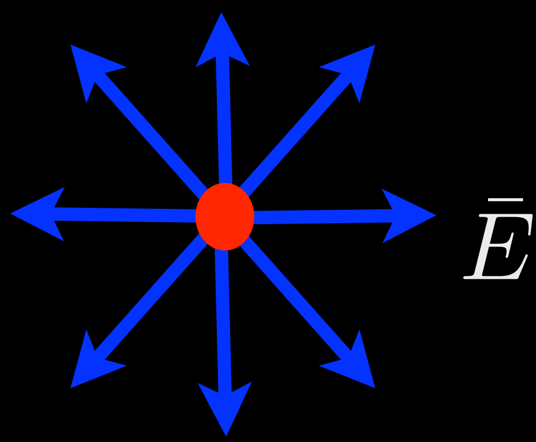


So far...

We have seen:

Electric fields, $\bar{E}$, are created by charges.

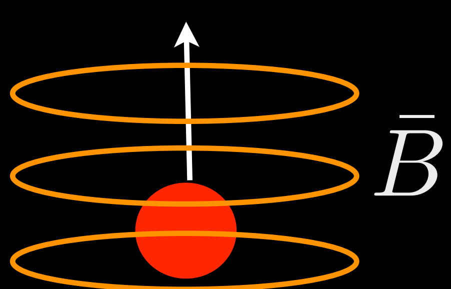
$$\bar{E} = \frac{kq}{r^2}\hat{r} \qquad \oint \bar{E} \cdot d\bar{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$



... and charges feel a force in electric fields: $\bar{F}_{12} = q\bar{E}$

Magnetic fields, $\bar{B}$, are created by moving charges.

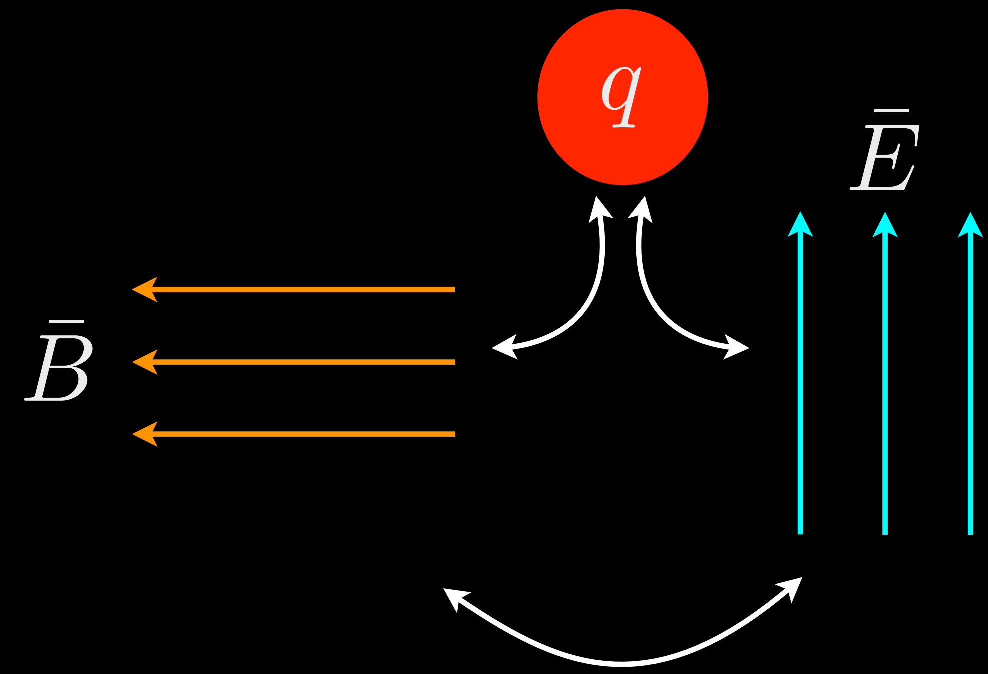
$$d\bar{B} = \frac{\mu_0}{4\pi}\frac{Id\bar{l} \times \hat{r}}{r^2} \qquad \oint \bar{B} \cdot d\bar{r} = \mu_0 I_{\text{encircled}}$$



... and moving charges feel a force in magnetic fields: $\bar{F} = q\bar{v} \times \bar{B}$

Question...

This tells us how charges interact with electric and magnetic fields



But how do electric fields and magnetic fields interact with each other?

Induced Currents

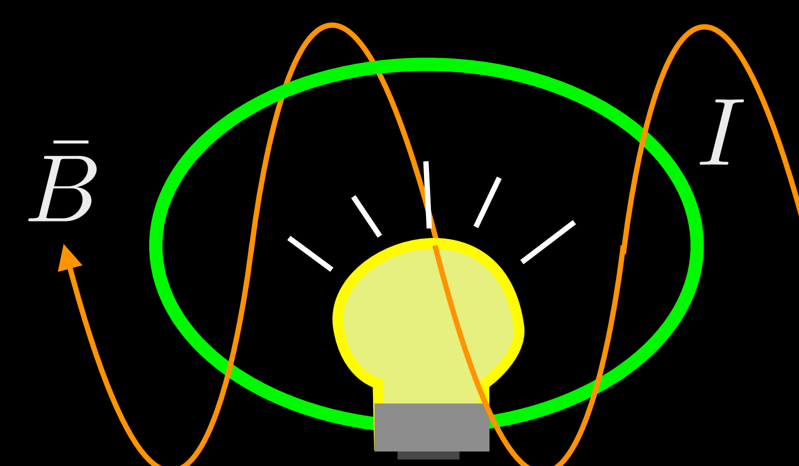
In 1831...

Michael Faraday

Joseph Henry

discovered:

electric currents, I, arise (start) in circuits in a changing magnetic field.

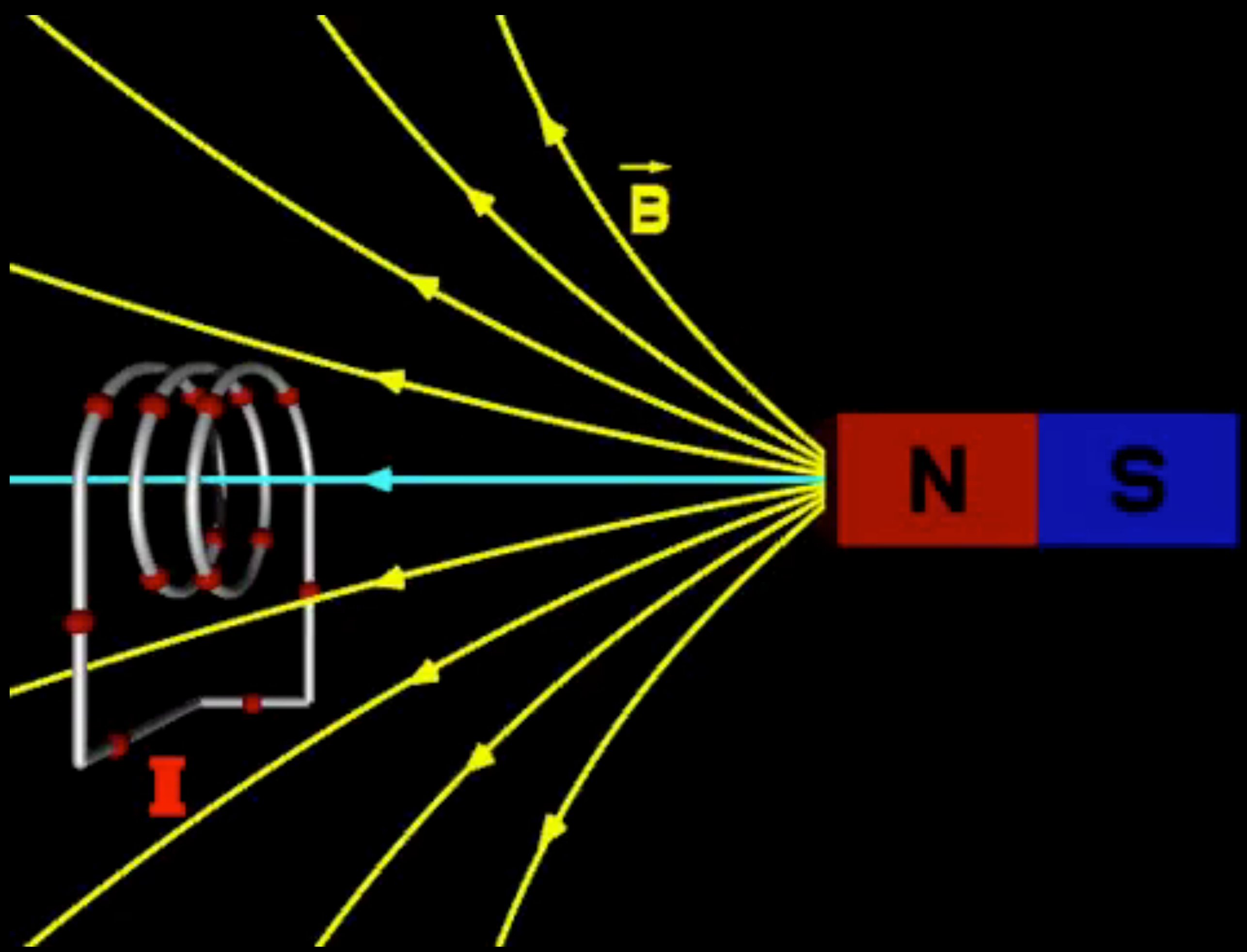


This current is the induced current.

Induced Currents

4 simple experiments:

Experiment 1: moving a magnet near a coiled (loops) circuit

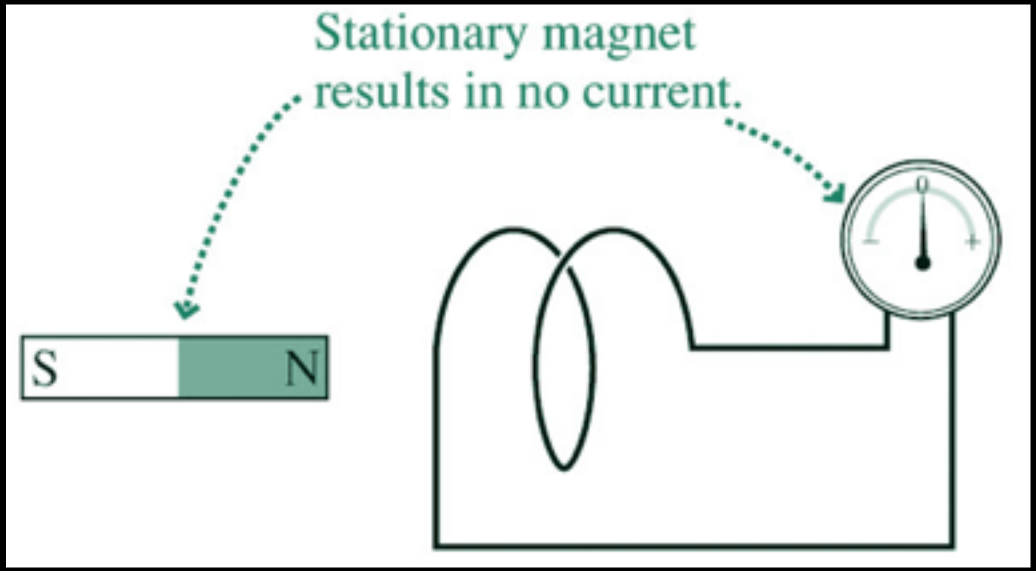


- circuit has no battery (or power source)
- move the magnet toward circuit and a current flows.
- move the magnet away from circuit and an opposite current flows.

Induced Currents

4 simple experiments:

Experiment 1: moving a magnet near a coiled (loops) circuit



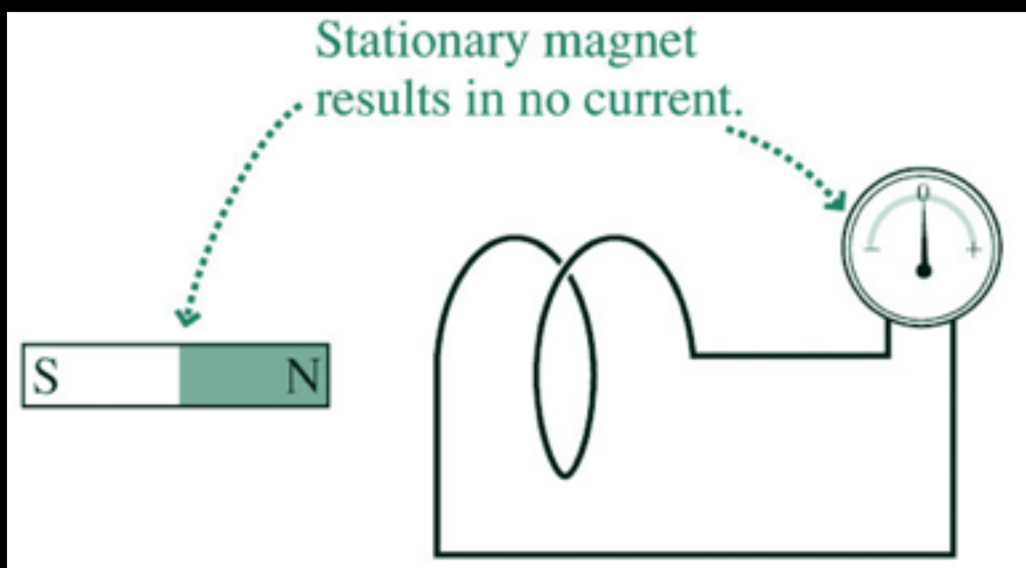
No movement = no current

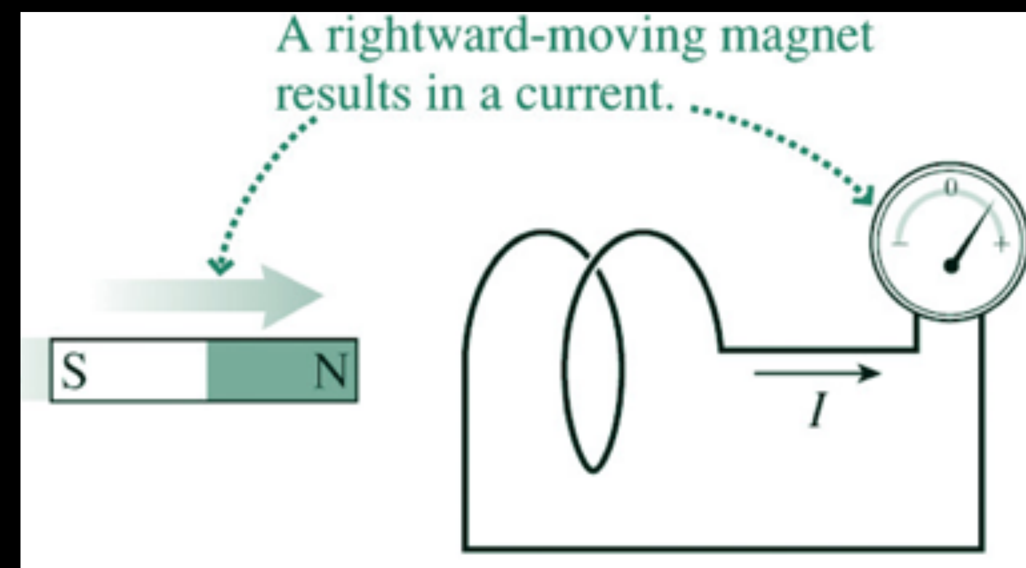
$$v = 0 \qquad I = 0$$

Induced Currents

4 simple experiments:

Experiment 1: moving a magnet near a coiled (loops) circuit





Move the magnet and a current starts in the loop.

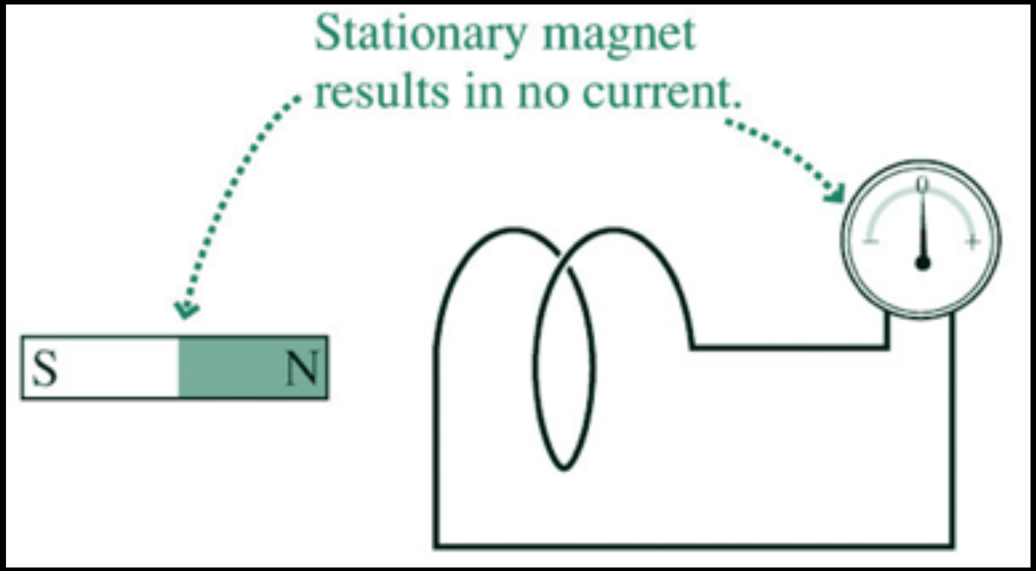
$v \neq 0$ $\qquad$ $I \neq 0$

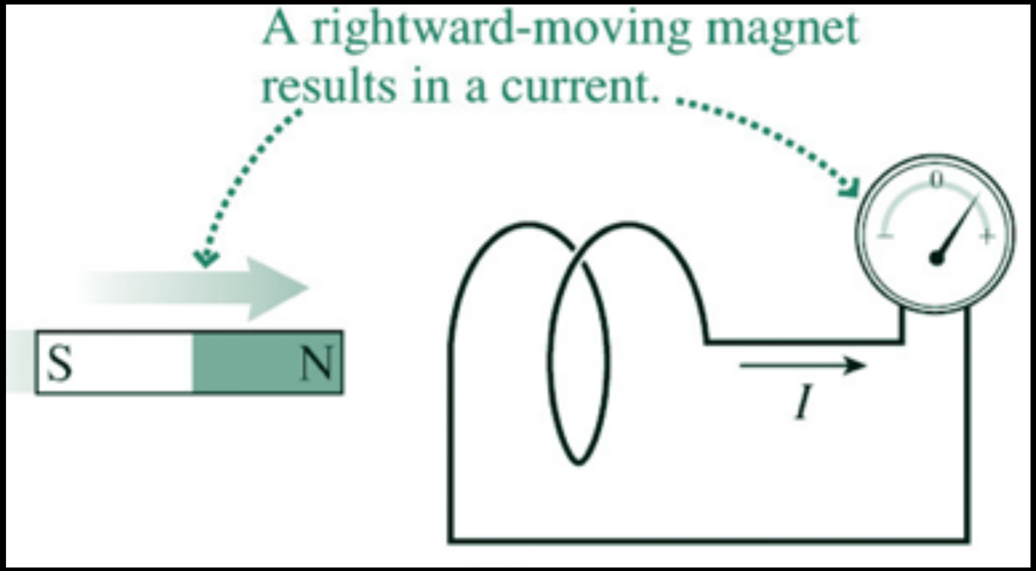
(but no battery!)

Induced Currents

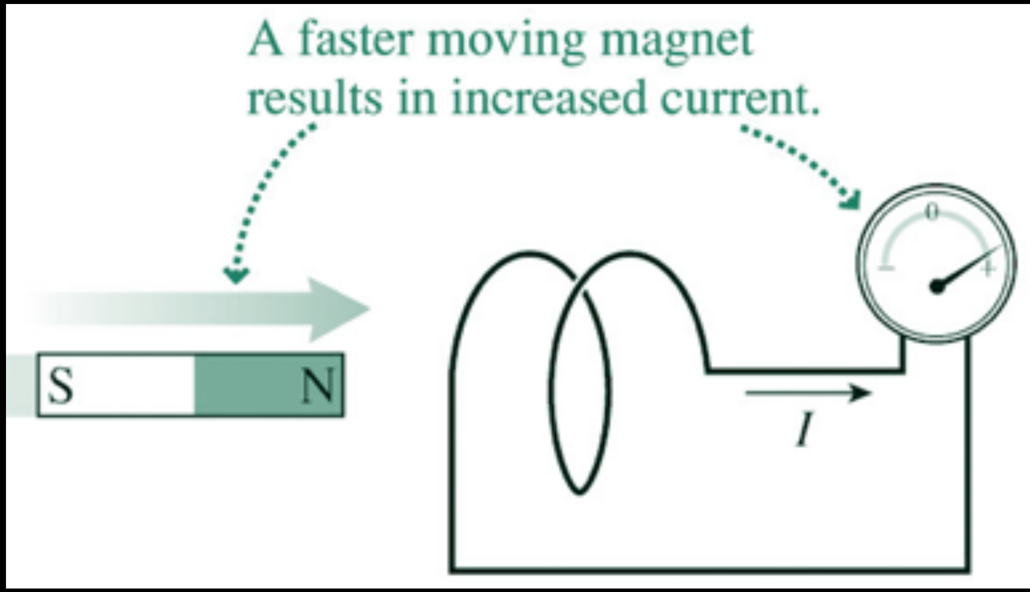
4 simple experiments:

Experiment 1: moving a magnet near a coiled (loops) circuit





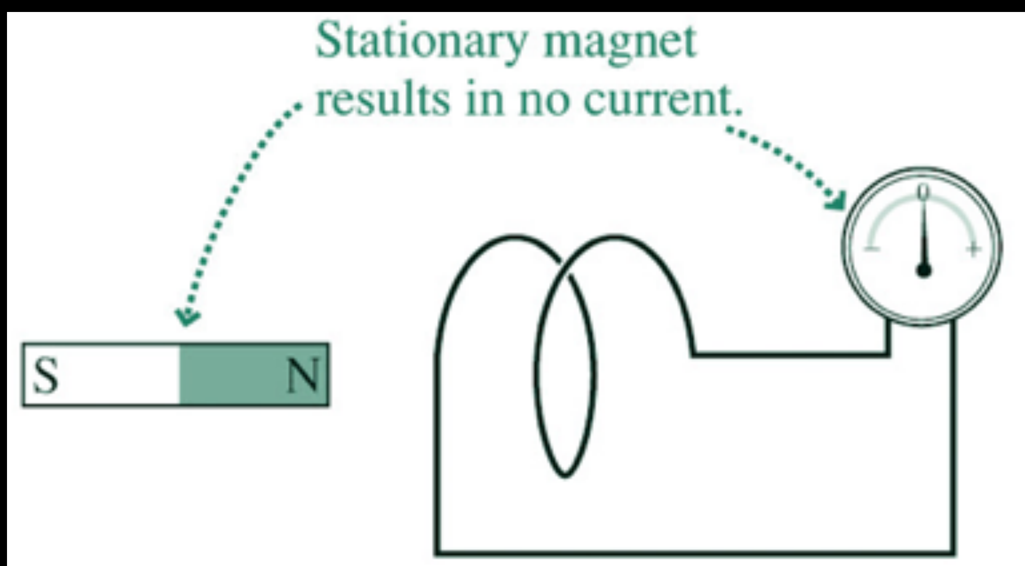
Move the magnet faster and current increases.

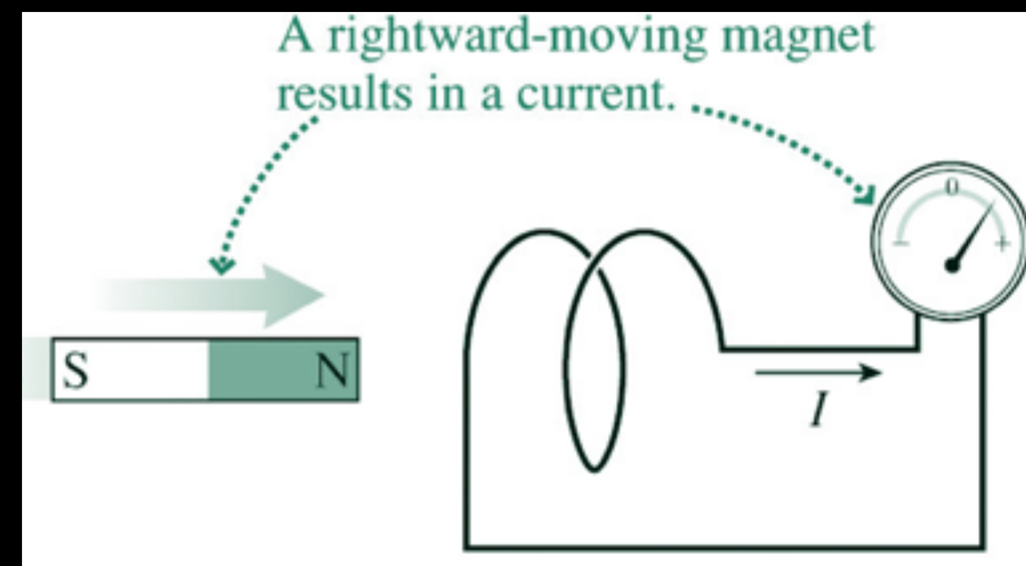


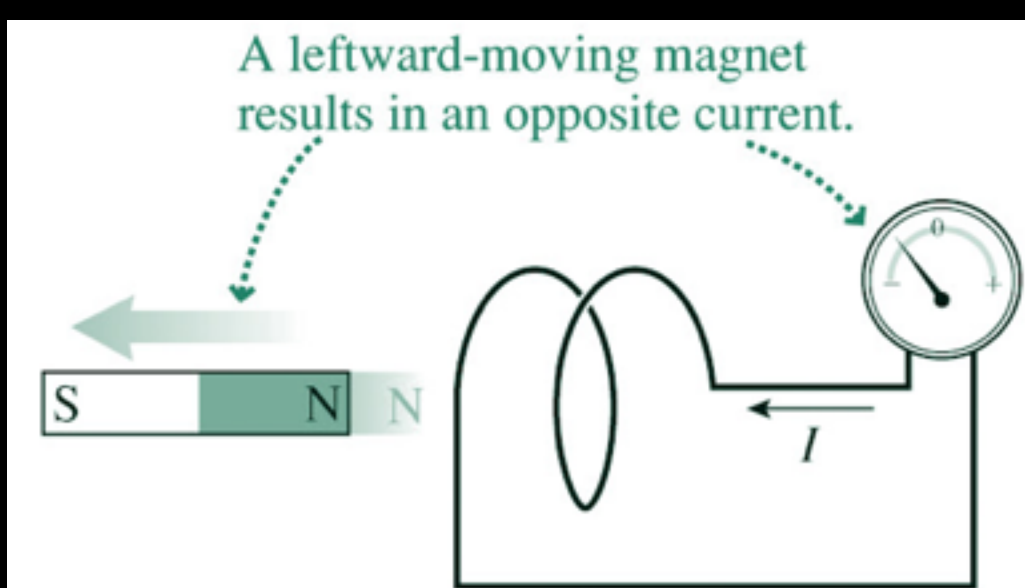
Induced Currents

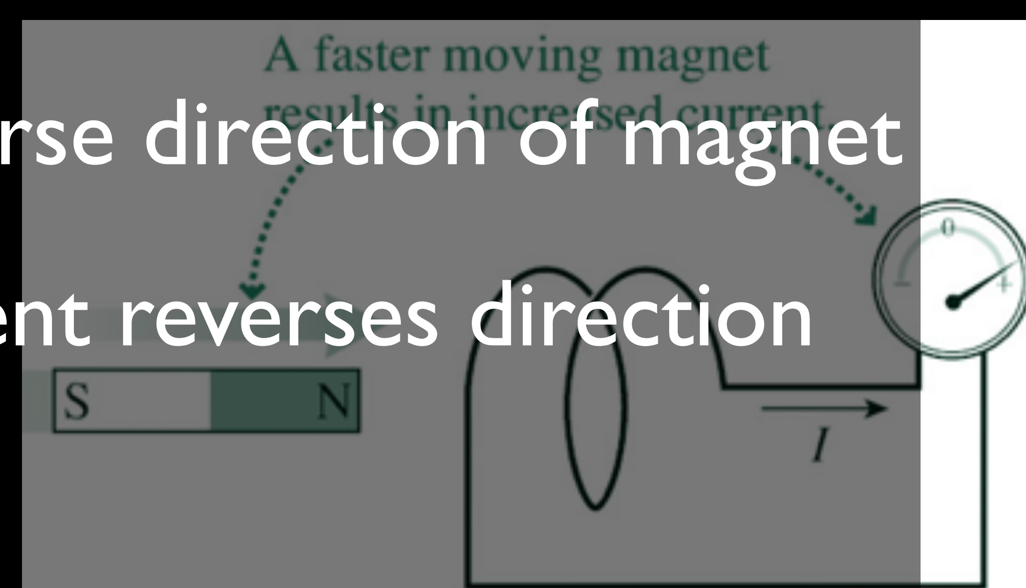
4 simple experiments:

Experiment 1: moving a magnet near a coiled (loops) circuit









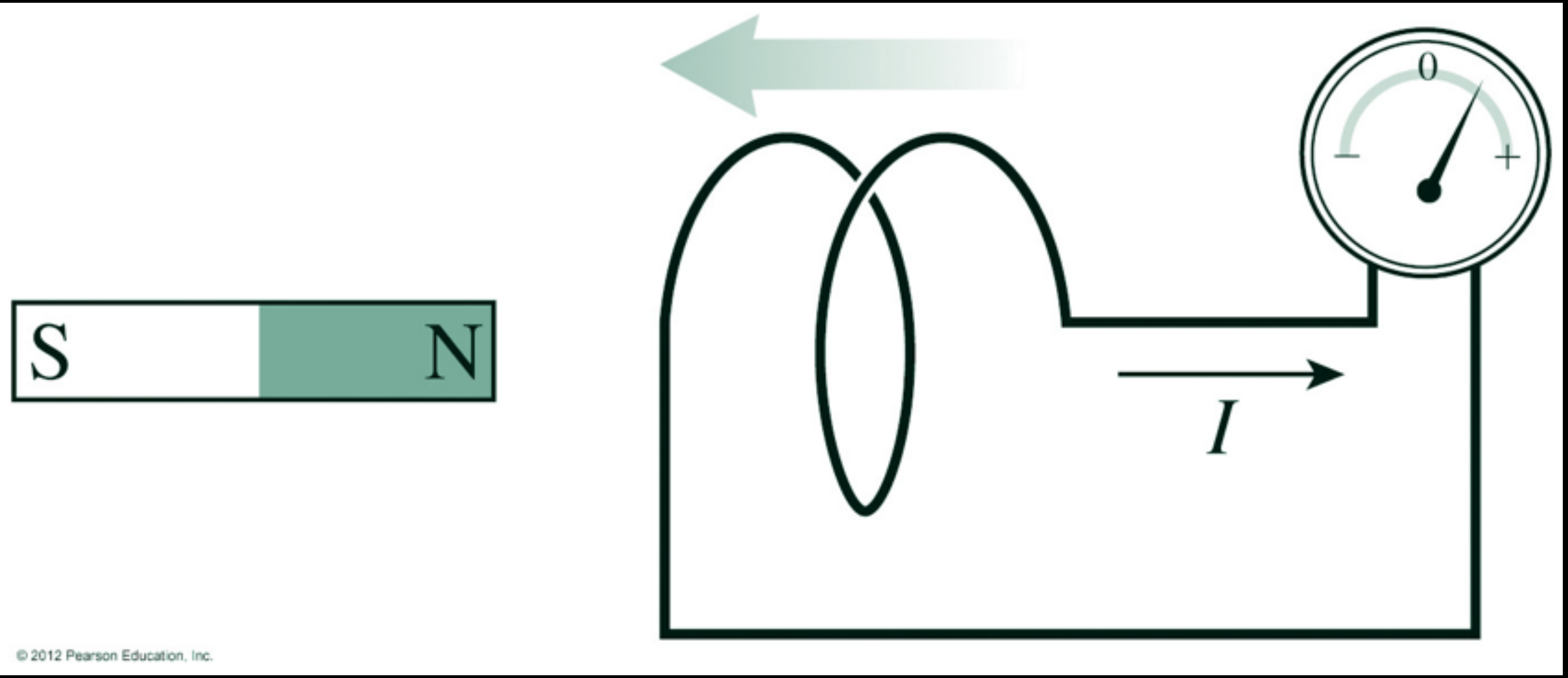
Reverse direction of magnet

current reverses direction

Induced Currents

4 simple experiments:

Experiment 2:   moving a coiled circuit near a magnet



It doesn’t matter if the magnet moves or the circuit.

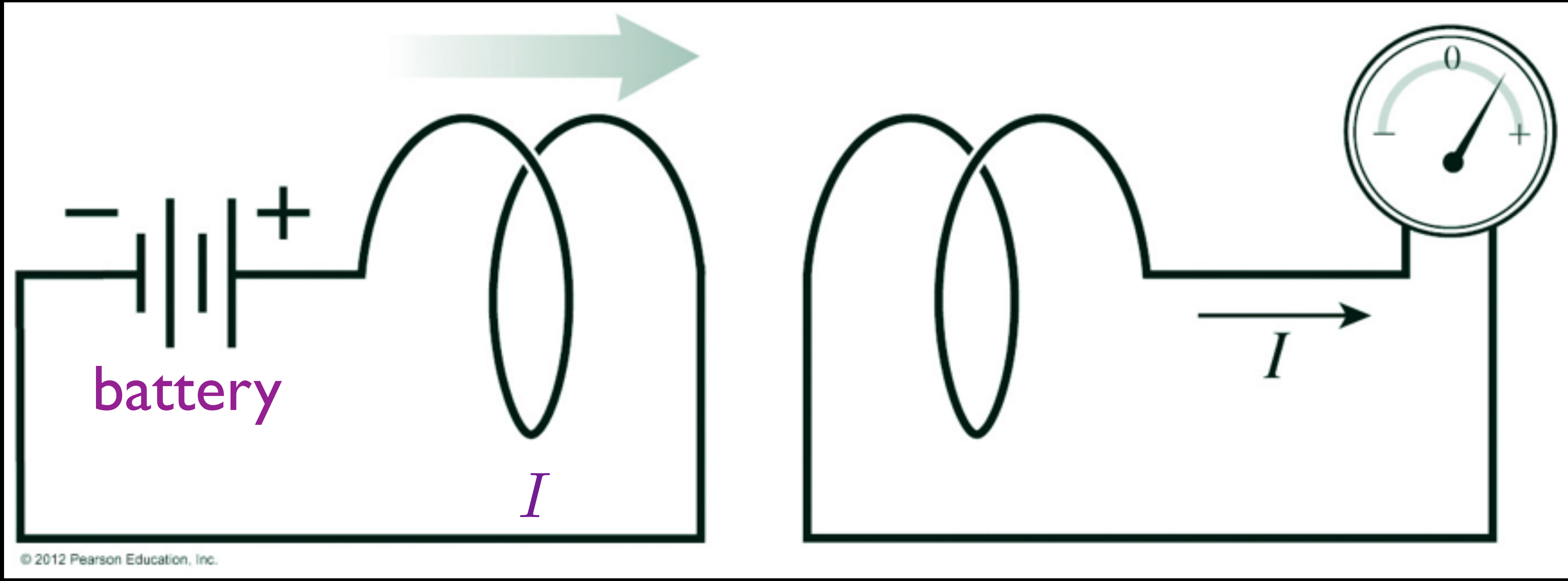
Only relative motion is important.

Induced Currents

4 simple experiments:

Experiment 3: moving a current near a circuit

Replace magnet with 2nd circuit



Steady current, I creates $\bar{B}$ field.

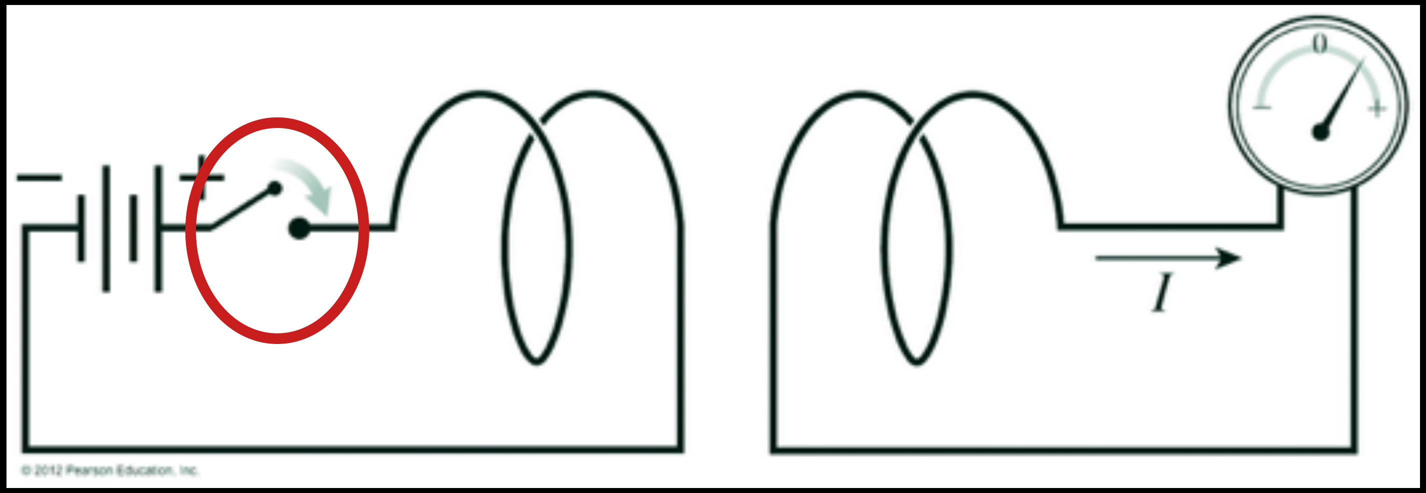
Circuit moves

Induced current starts

Induced Currents

4 simple experiments:

Experiment 4: varying current in one circuit



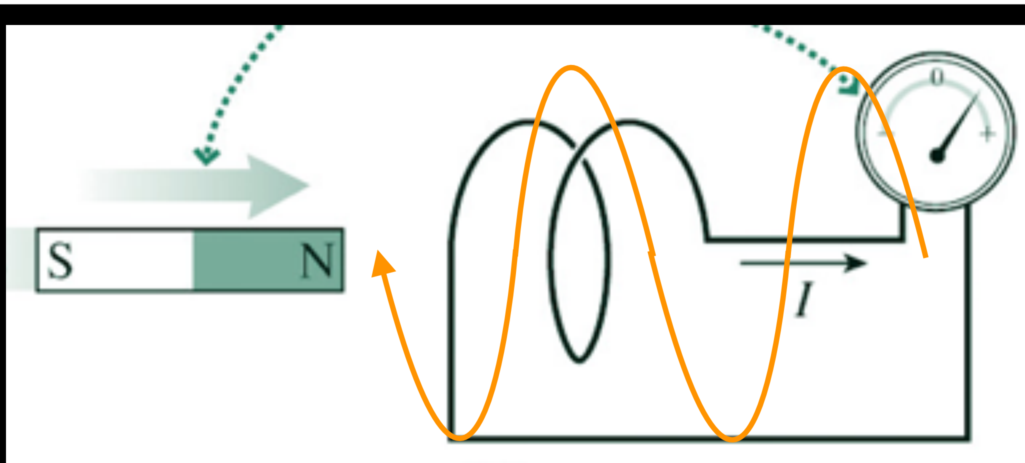
When neither circuits moves; no induced current

But, open switch in left circuit → current drops $I \to 0$ $B \to 0$ → Brief (short time) current induced in right circuit

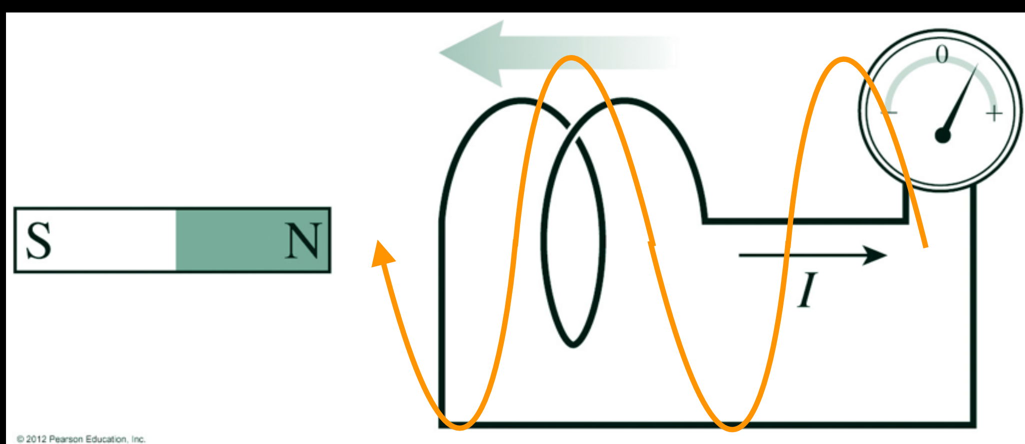
Close switch in left circuit → current increases $0 \to I$ $0 \to B$ → Brief current induced in right circuit (opposite direction)

Induced Currents

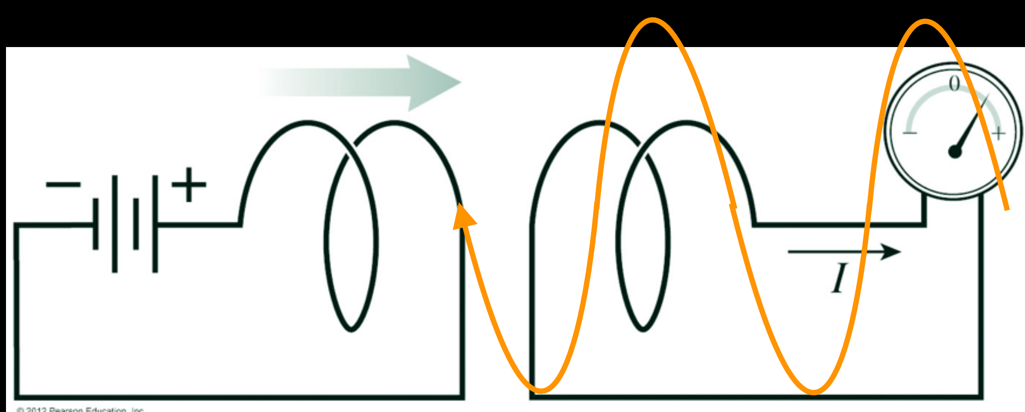
$\bar{B}$ changes because ...



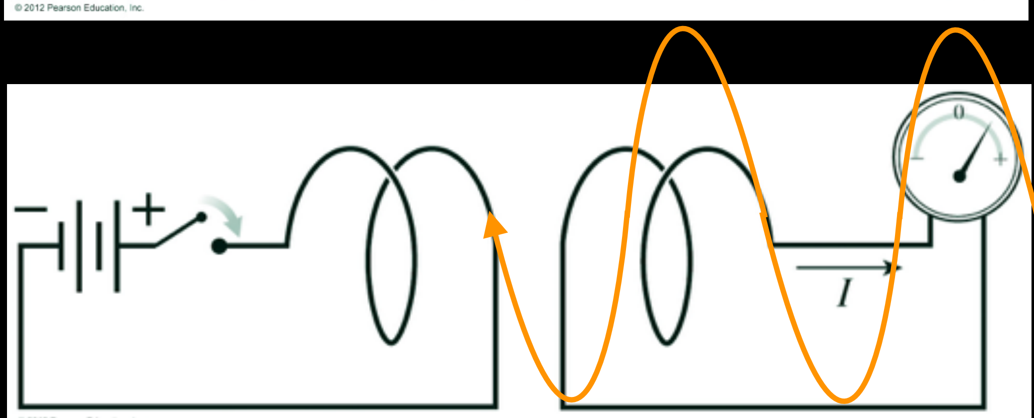
magnet moves towards circuit



circuit moves towards magnet

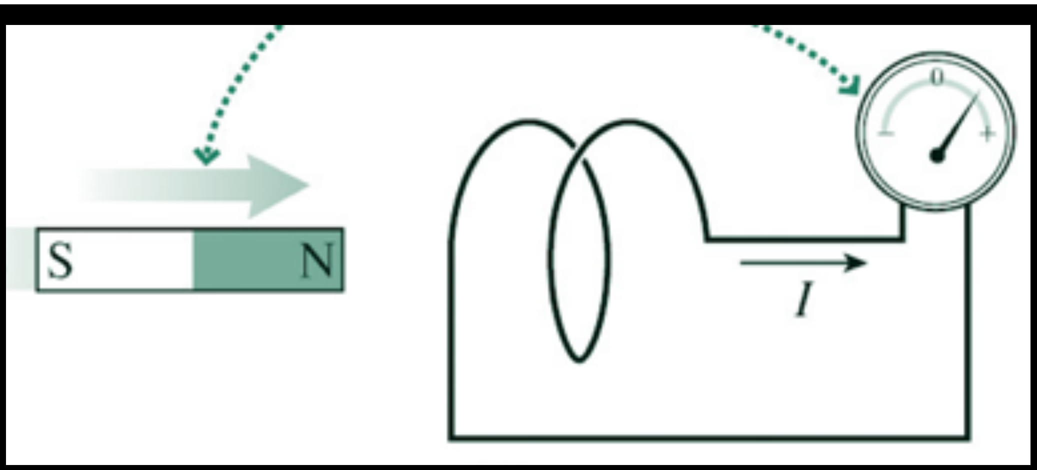


circuit creating magnetic field moves

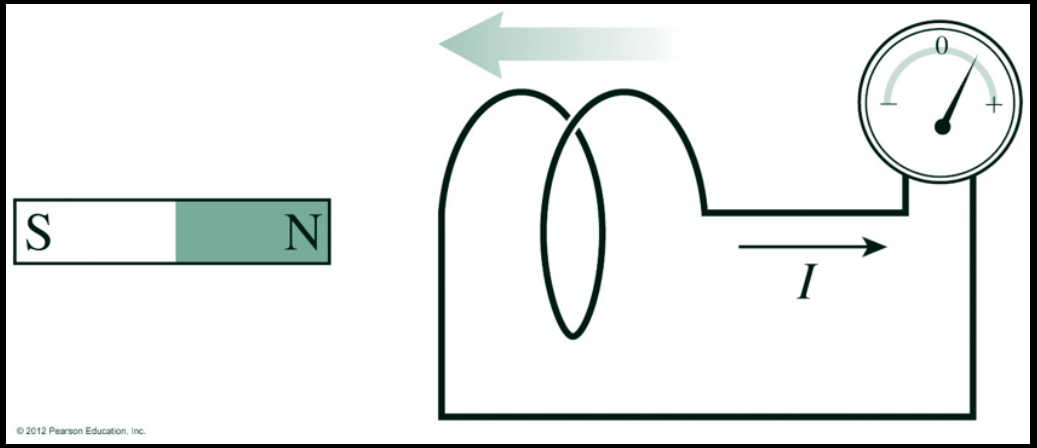


magnetic field increases / decreases

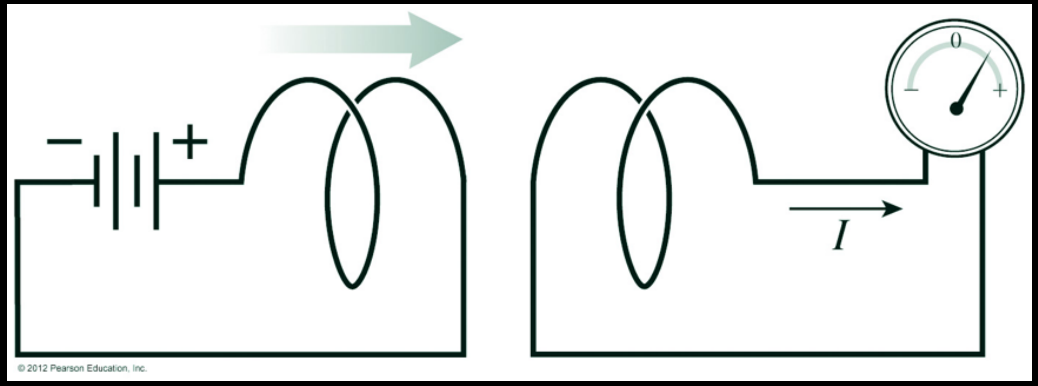
Induced Currents



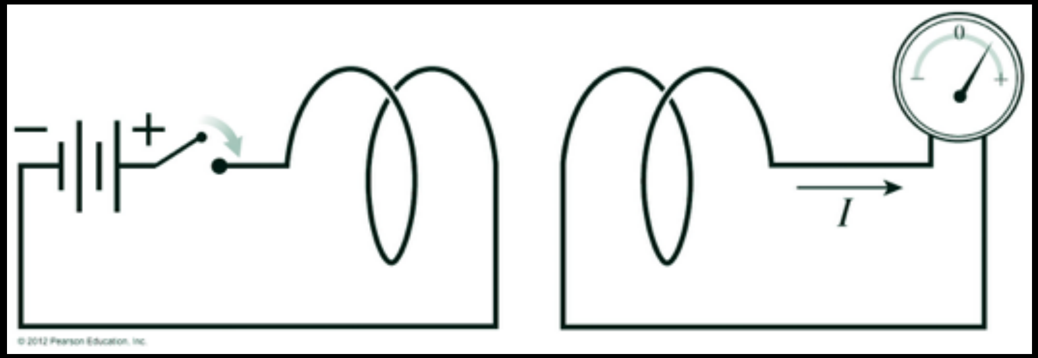
Why $\bar{B}$ changes is not important.



A current appears in a circuit feeling a changing magnetic field.



This is electromagnetic induction.



Induced Currents

Quiz

Which will cause an induced current in a coil of wire?

(A) A magnet resting near the coil.

(B) The constant field of the Earth passing through the coil.

(C) A magnet being moved into or out of the coil.

(D) A wire carrying a constant current near the coil.

The EMF

Because the magnitude of current depends on the wire material, when describing induced current we talk about EMF.

EMF, $\mathcal{E}$: electromotive force

Potential difference (ΔV) needed to cause a current to flow.

$$\mathcal{E} = -\Delta V_{\text{circuit}} = \oint \bar{E} \cdot d\bar{r} \quad = IR \quad \text{(Ohm's Law)}$$

lecture 7   around circuit

material's resistance; how easy it is to create current

The EMF can be: e.g. a battery

e.g. the induced EMF from induction

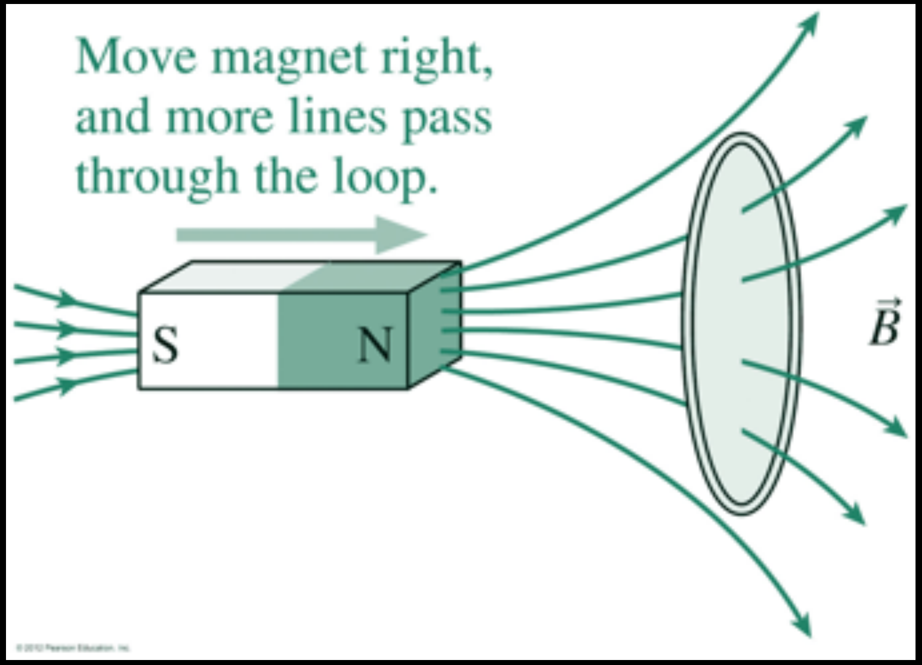
Changing $\bar{B}$ field creates an EMF.

Magnetic flux

For electromagnetic induction, the surface is the inside of the circuit.

$\bar{B}$ is stronger closer to the magnet.

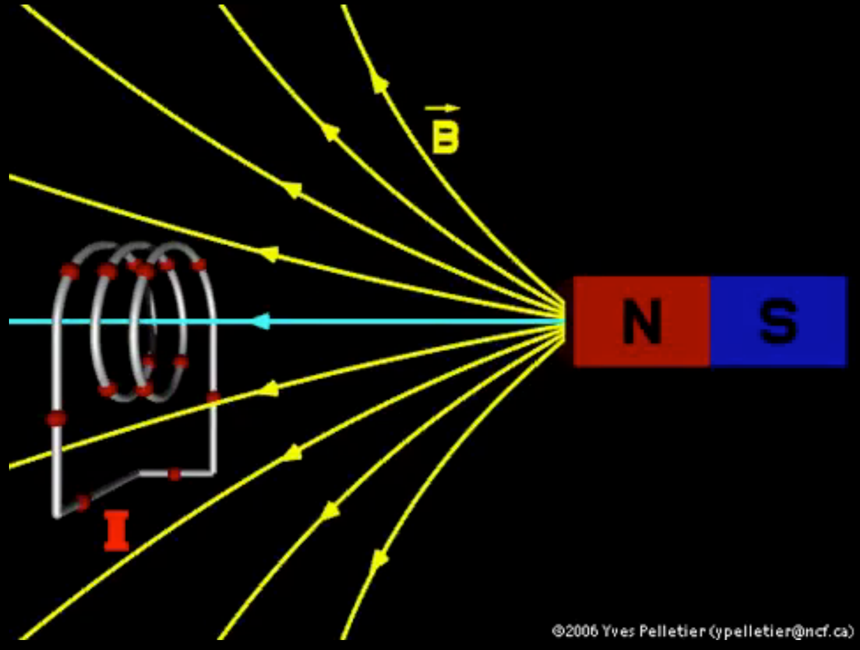
Flux increases through circuit when magnet is close.



For a flat surface and uniform field:

$$\Phi_B = \bar{B} \cdot \bar{A} = BA\cos\theta$$

Unit: Tm^2 or Wb (weber)



Magnetic flux

A circular loop with 5.0 cm diameter makes an angle of 30° to uniform magnetic field, $|\bar{B}| = 80\text{mT}$.

What is magnetic flux, Φ_B ?

(A) $1.6 \times 10^{-4}\text{Wb}$

(B) $1.7 \times 10^{-3}\text{Wb}$

(C) $1.4 \times 10^{-4}\text{Wb}$

(D) 1.3Wb

$$\Phi_B = \bar{B} \cdot \bar{A} = BA\cos\theta$$

$$= (80 \times 10^{-3})\pi(2.5 \times 10^{-2})^2 \cos(30^\circ)$$

Magnetic flux

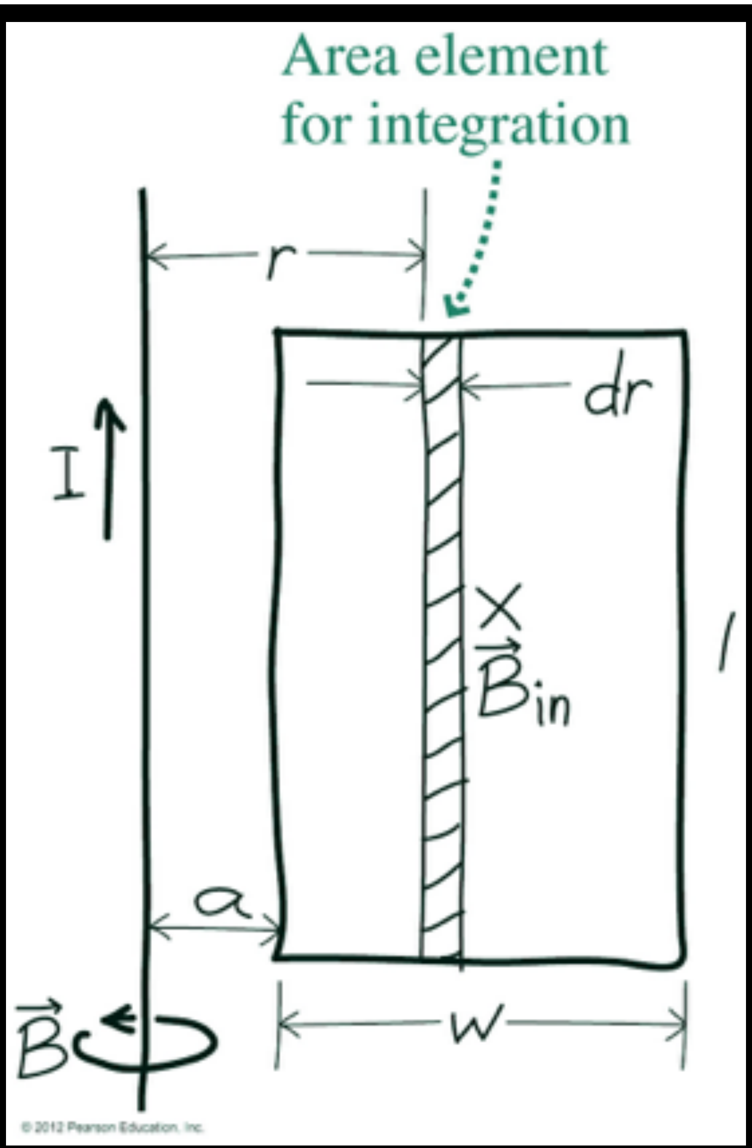
Magnetic flux through loop next to current carrying wire.

$\bar{B}$ field encircles wire.

Points into the page in the loop: $\bar{B} \cdot d\bar{A} = BdA$

Field around a wire: $B = \frac{\mu_0 I}{2\pi r}$
(lecture 8)

varies with distance



$$\Phi_B = \int BdA = \int_a^{a+W} \frac{\mu_0 I}{2\pi r} l dr = \frac{\mu_0 I l}{2\pi} \int_a^{a+w} \frac{dr}{r}$$

$$= \frac{\mu_0 I l}{2\pi} \ln\left(\frac{a+w}{a}\right)$$

Faraday's Law

Using magnetic flux, Φ_B, and the EMF, $\mathcal{E}$, we can write:

Faraday's law

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

change in magnetic flux through circuit area

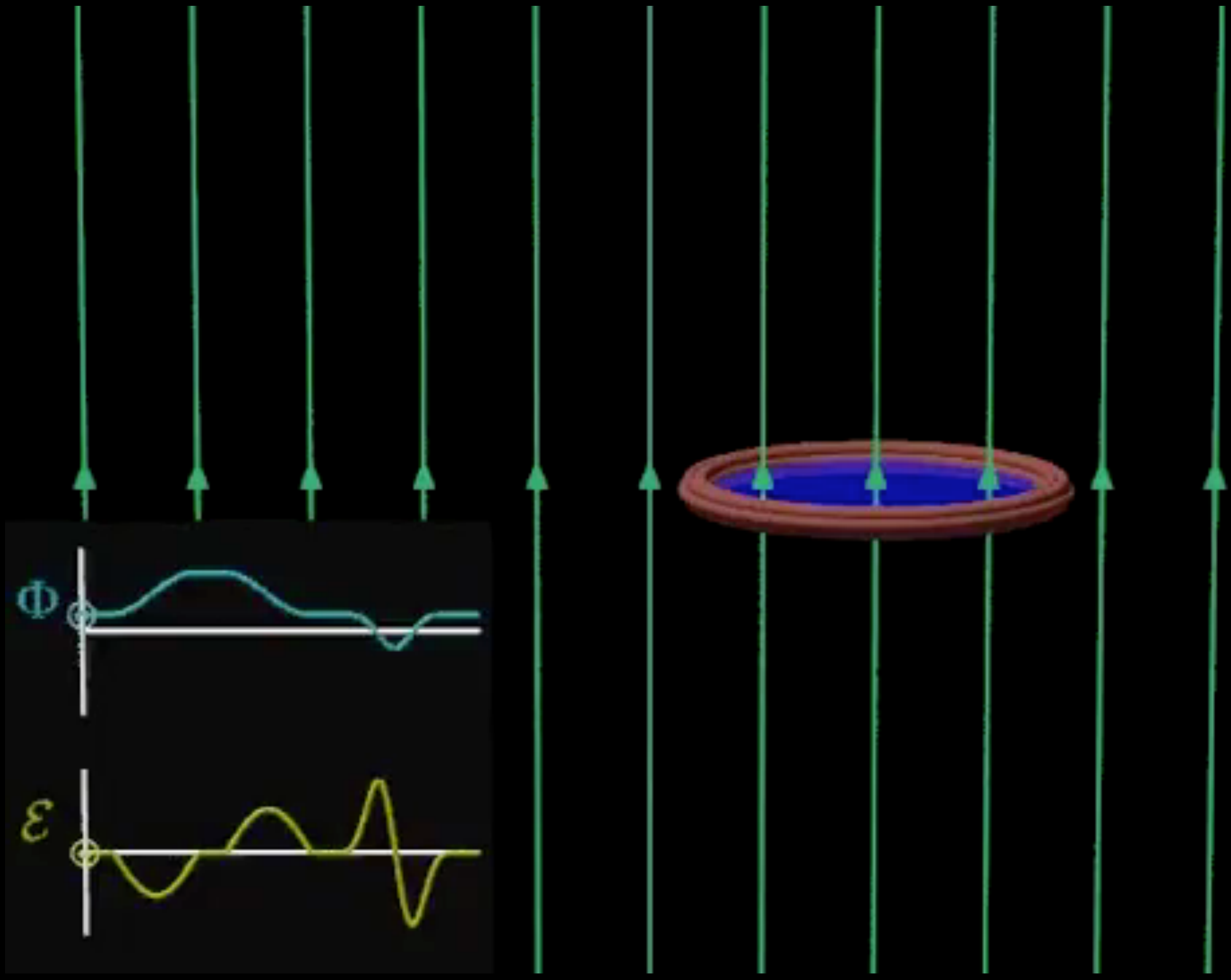
induced EMF

The induced EMF opposes the flux change.

Describes electromagnetic induction in circuits.

(more general form later)

Faraday's Law



$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \bar{B} \cdot \bar{A} = BA\cos\theta$$

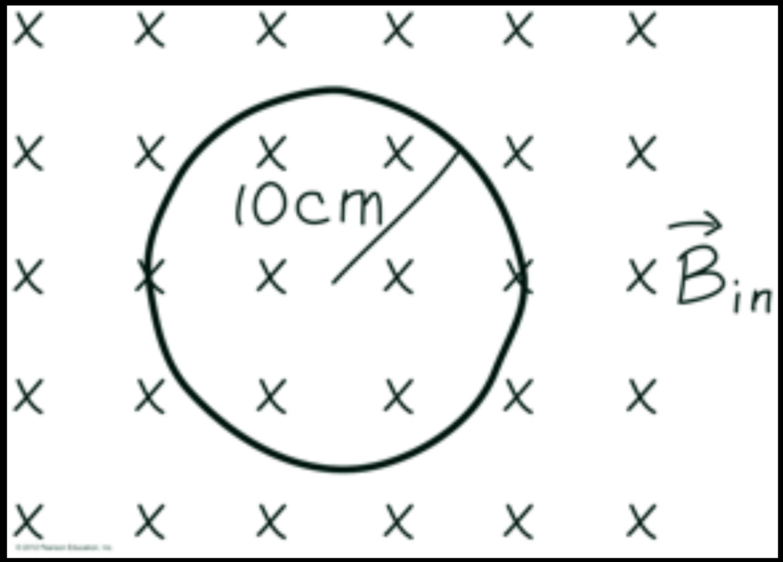
Faraday's Law

Example 1: changing $\bar{B}$

Loop of radius r = 10 cm, resistance R = 2Ω

Uniform $\bar{B}$ increasing at $0.1\mathrm{T/s}$

Find magnitude of induced current, I .



Ring is perpendicular to field: $\Phi_B = BA = B\pi r^2$

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(B\pi r^2) = \pi r^2 \frac{dB}{dt} = \pi(0.1)^2 0.1$$

radius constant

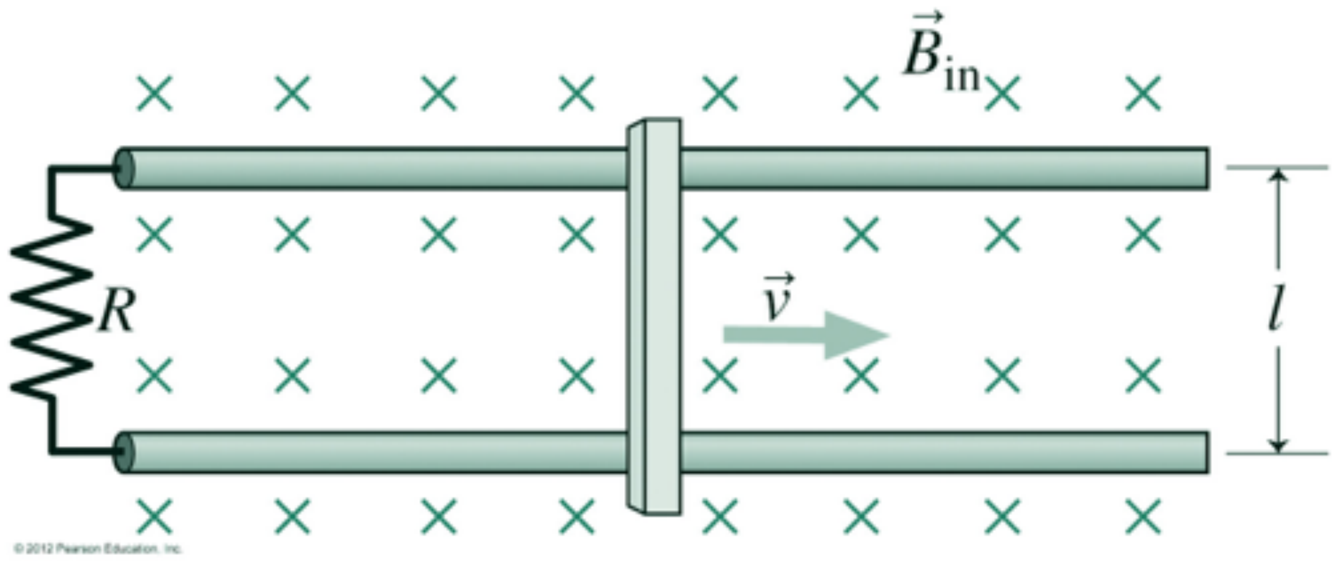
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -3.14\mathrm{mV}$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{3.14\mathrm{mV}}{2.0\Omega} = 1.6\mathrm{mA}$$

Faraday's Law

Example 2: changing area

Sliding conducting bar changes circuit area.



Constant $\bar{B}: \ \Phi_B = BA = Blx$

$$\frac{d\Phi_B}{dt} = Bl\frac{dx}{dt} = Blv$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{Blv}{R}$$

Faraday's Law

Quiz

A loop of area $240\mathrm{cm}^2$ and resistance R = 12Ω carried an induced current $I = 320\mathrm{mA}$.

What rate is the magnetic field changing? $\frac{dB}{dt}$

(A) $160\mathrm{T/s}$

(B) $0.67\mathrm{T/s}$

(C) $0.016\mathrm{T/s}$

(D) $3.84\mathrm{T/s}$

$$I = \frac{|\mathcal{E}|}{R} = \frac{|-d\Phi_B/dt|}{R}$$

$$= \frac{|-d(BA)/dt|}{R} = A\frac{|dB/dt|}{R}$$

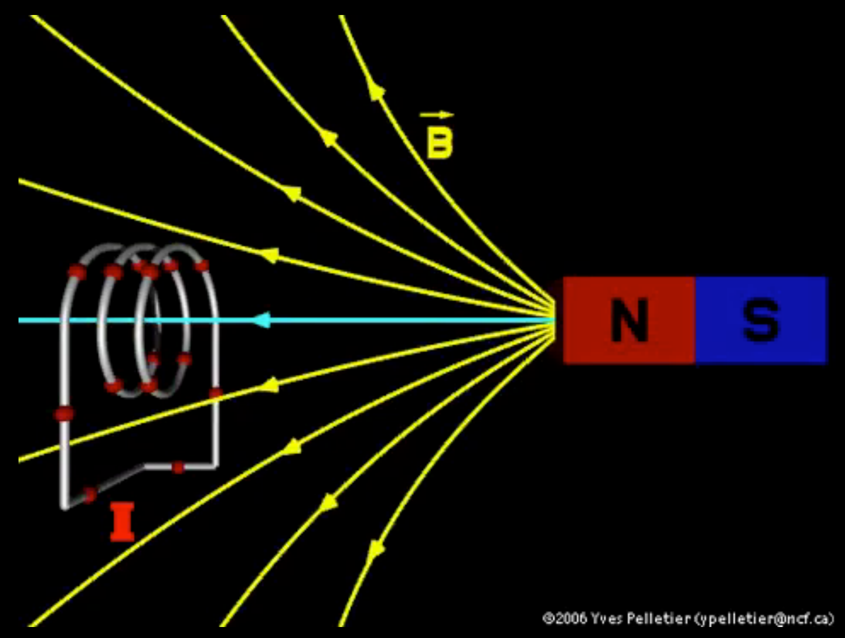
$$\left|\frac{dB}{dt}\right| = \frac{IR}{A} = \frac{(0.32\mathrm{A})(12\Omega)}{240 \times 10^{-4}\mathrm{m}^2} = 160\mathrm{T/s}$$

Lenz's Law

Magnet moves

Induced current

Loop heats



Where did the energy come from?

work in moving the magnet

But, movement at constant speed doesn't take work

... unless there is a force opposing the movement.

The induced current must cause an opposing magnetic force.

Gives the direction of the current.

Lenz's Law

We knew this from Faraday's Law: $\mathcal{E} = -\frac{d\Phi_B}{dt}$

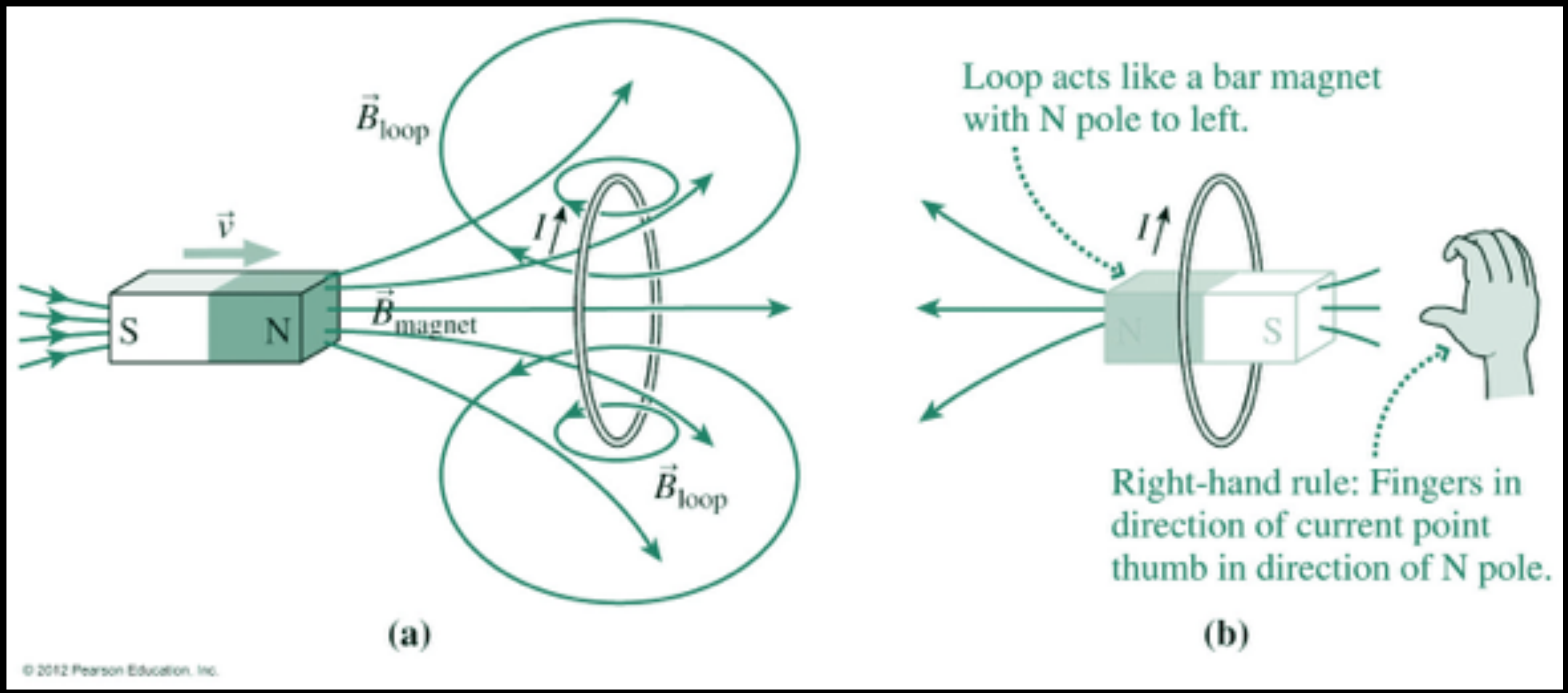
The induced EMF opposes the flux change.

But it's easier to find current direction from conservation of energy.

Ask: What direction will make it hard to move the magnet?

Lenz's Law

When magnet is pushed into loop:

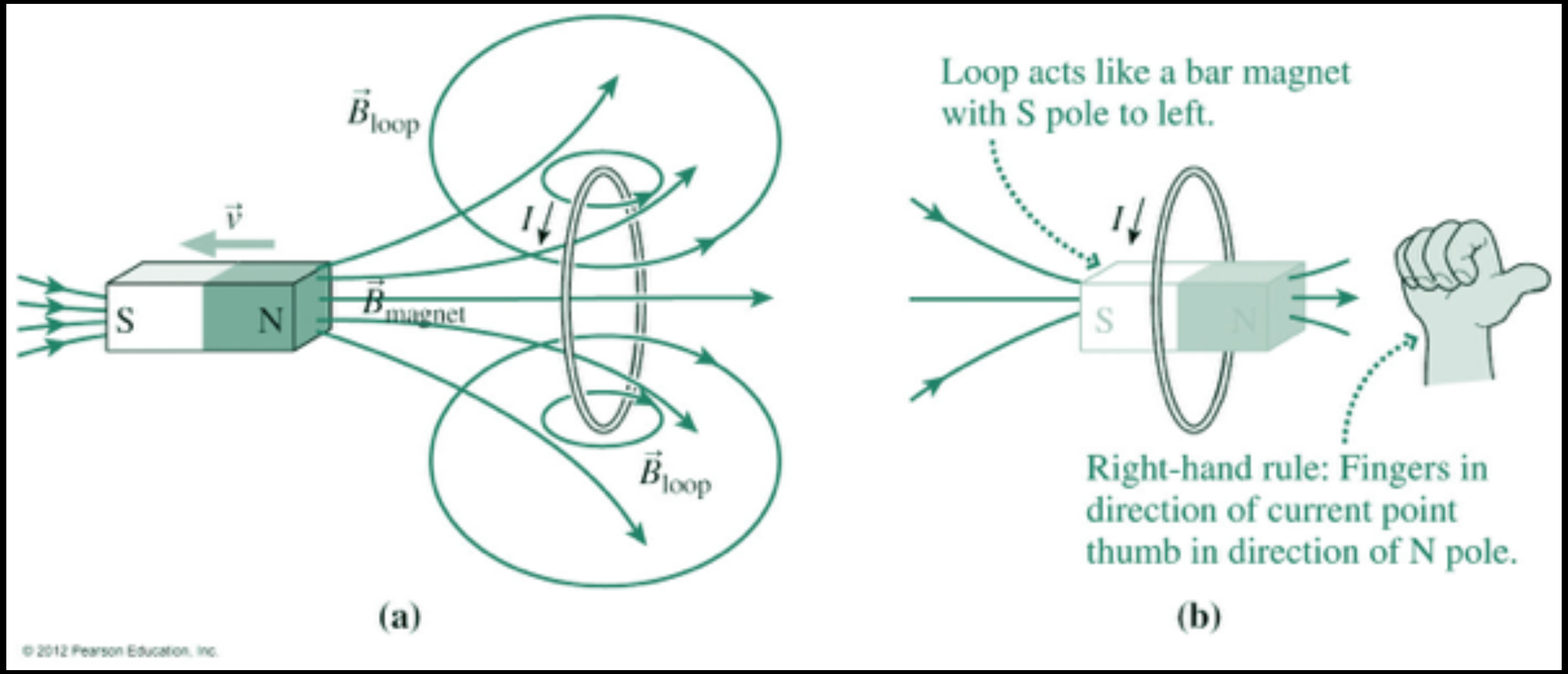


Want loop to make N pole:

$\bar{B}$ points left

Right-hand rule gives current direction.

When magnet is pushed out of loop:



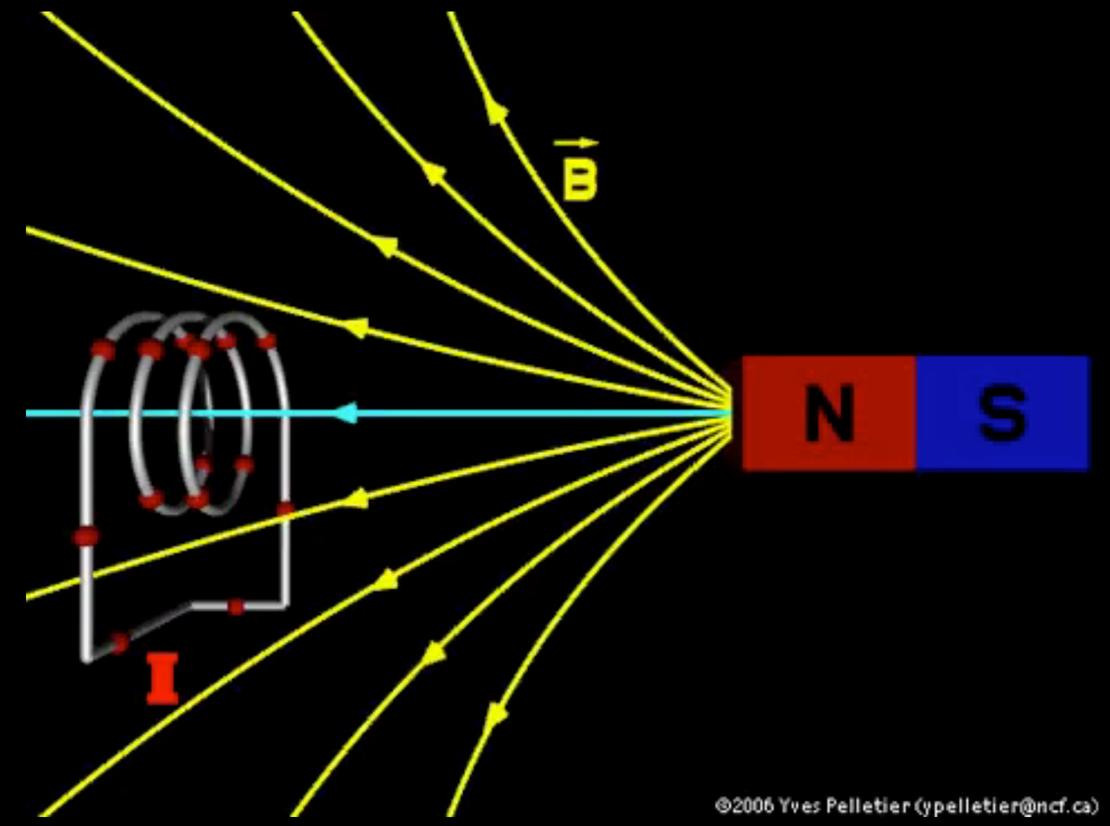
Want loop to make S pole:

$\bar{B}$ points right

Right-hand rule gives current direction.

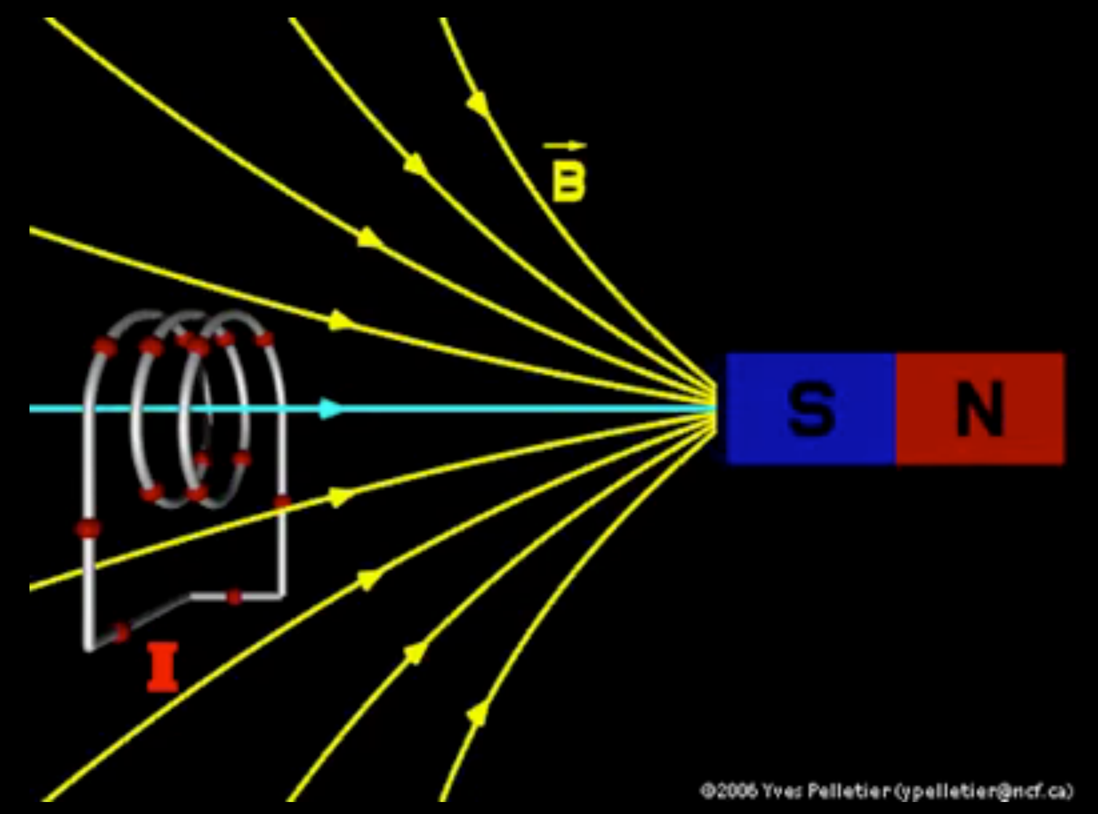
Lenz's Law

Current direction to oppose magnet motion.



Lenz's Law:

The direction of an induced EMF or current is such that the magnetic field created by the induced current opposes the change in magnetic flux that created the current.



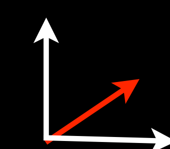
(current direction opposes motion)

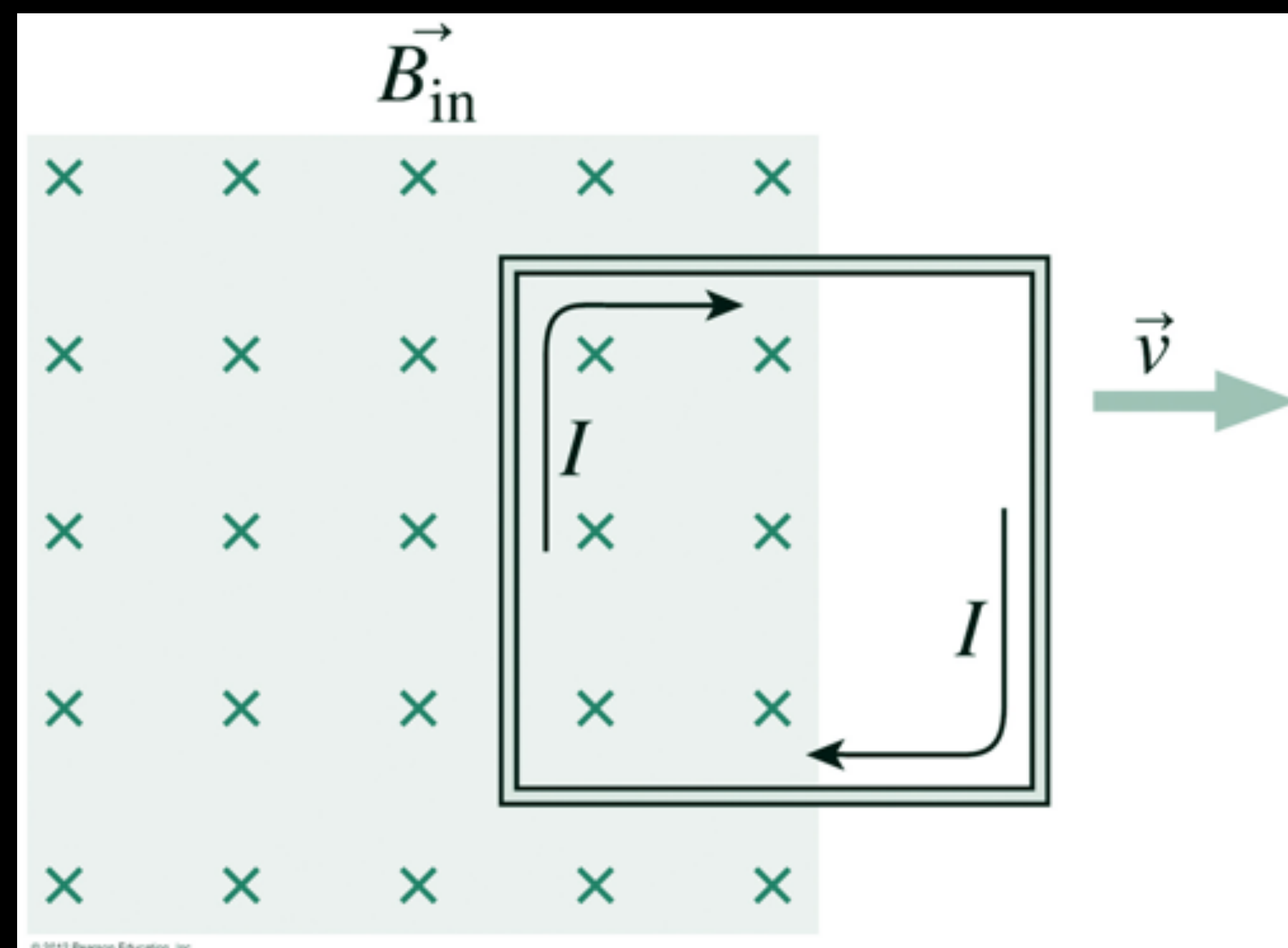
Lenz's Law

What will be the direction of the current in the loop when it enters the field, coming from the left?

(A) clockwise

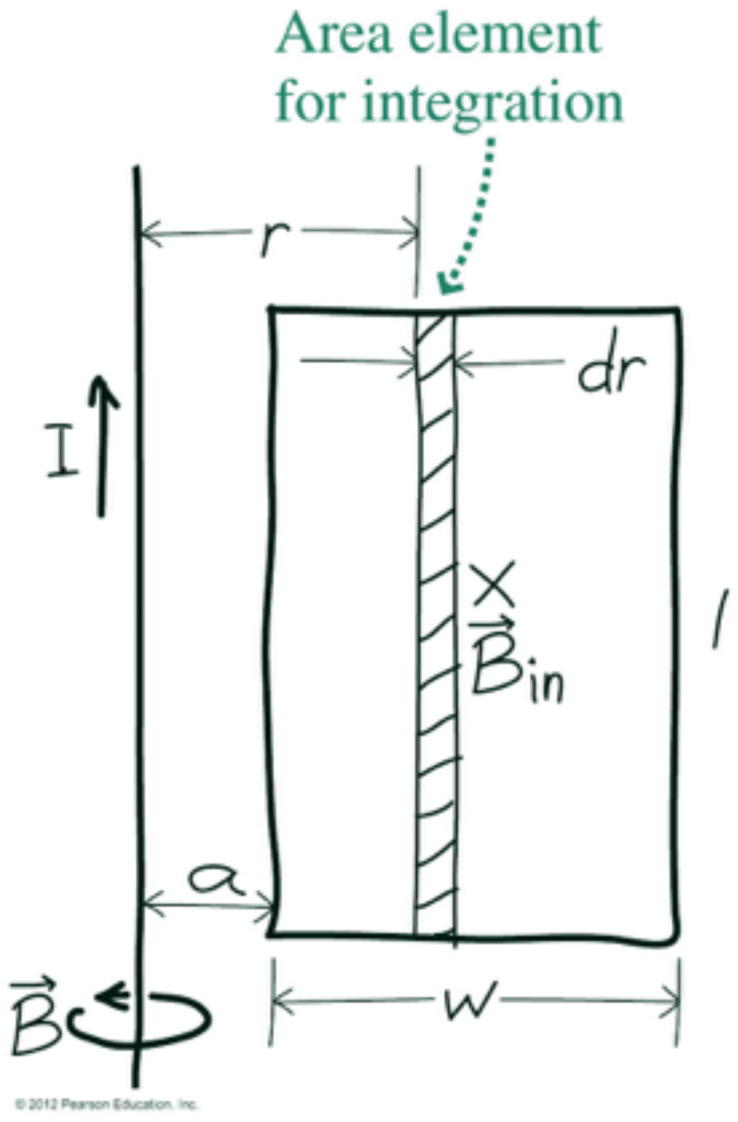
(B) anti-clockwise

$\bar{B}$  into page



Lenz's Law

Rectangular loop near a long wire.



$a = 1.0\text{cm}$

$w = 3.5\text{cm}$

$l = 6.0\text{cm}$

$R = 50\text{m}\Omega$

$$\Phi_B = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{a+w}{a}\right)$$

Current in long wire increases at 25A/s.

$\mu_0 = 4\pi \times 10^{-7}\text{N/A}^2$

What is the induced current in the loop?

(A) $18.0\mu\text{A}$

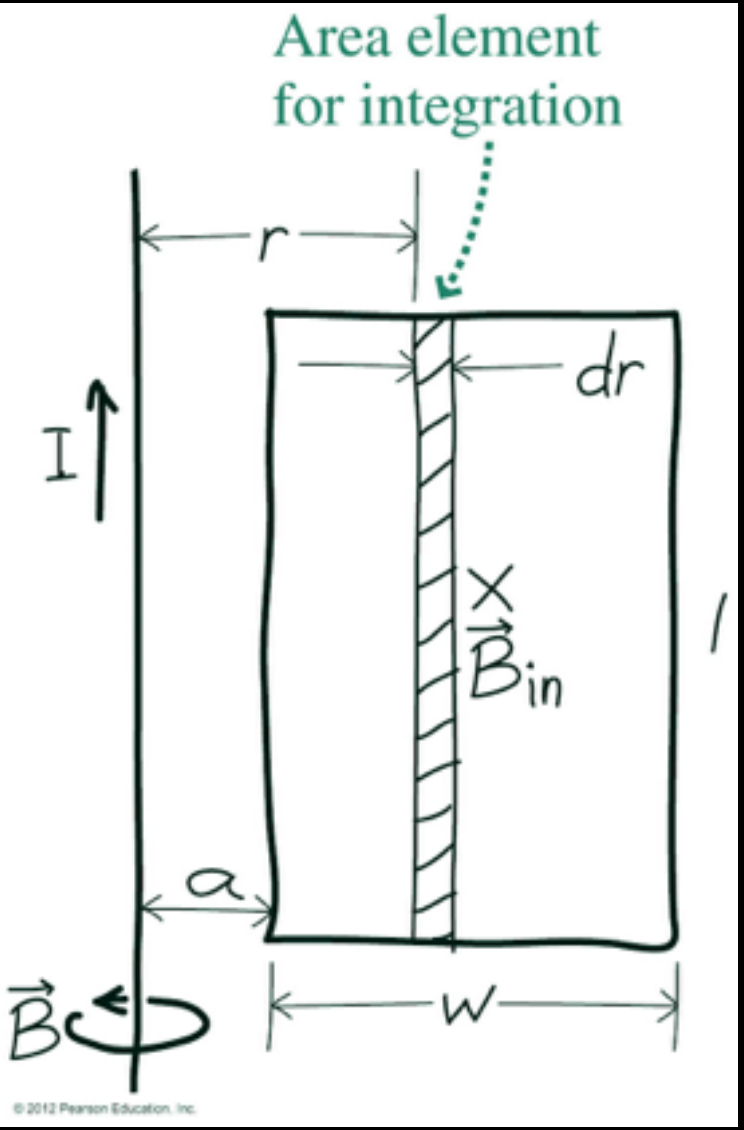
(C) $0.45\mu\text{A}$

(B) $0.9\mu\text{A}$

(D) $9.0\mu\text{A}$

Lenz's Law

Rectangular loop near a long wire.



$$a = 1.0\text{cm}$$
$$w = 3.5\text{cm}$$
$$l = 6.0\text{cm}$$
$$R = 50\text{m}\Omega$$

$$\Phi_B = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{a+w}{a}\right)$$

Current in long wire increases at 25A/s.

$$I = \frac{|\mathcal{E}|}{R} = \frac{|-d\Phi_B/dt|}{R} = \frac{\mu_0 l (dI/dt)}{2\pi R} \ln\left(\frac{a+w}{a}\right)$$

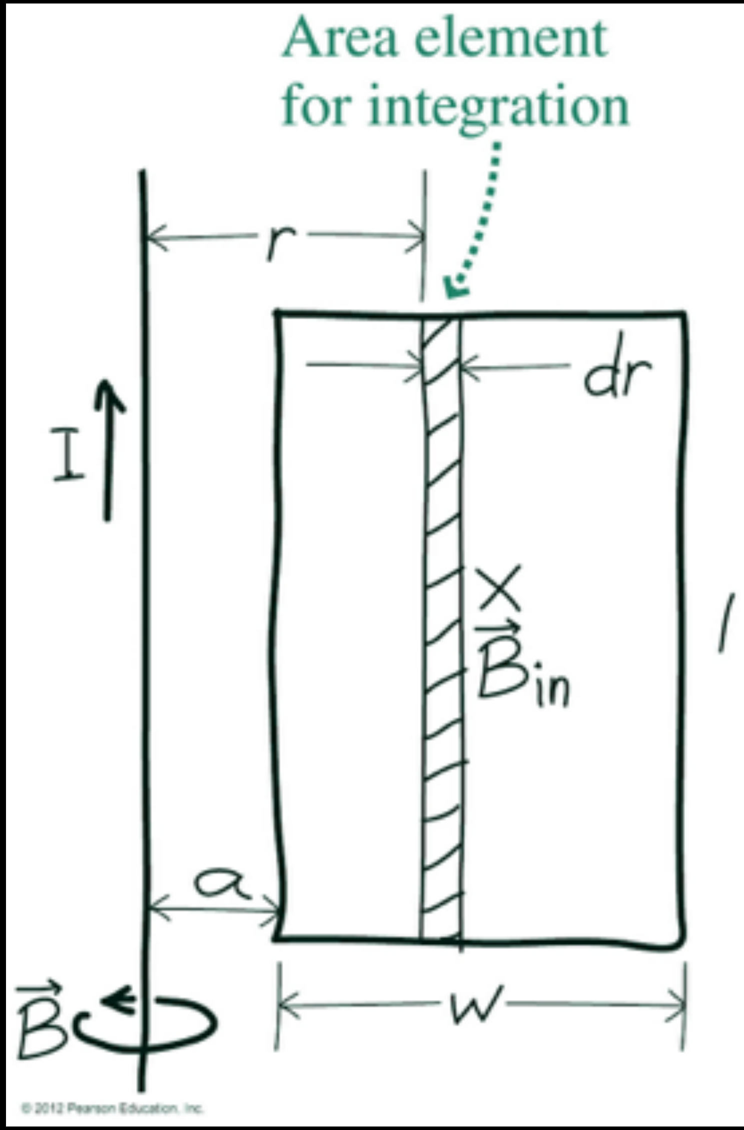
$$= \frac{(4\pi \times 10^{-7}\text{N/A}^2)(6.0\text{cm})(25\text{A/s})}{2\pi(50\text{m}\Omega)} \ln\left(\frac{4.5\text{cm}}{1.0\text{cm}}\right) = 9.0\mu A$$

Lenz's Law

Rectangular loop near a long wire.

Current in long wire increases at $25\mathrm{A/s}$.

$$a = 1.0\mathrm{cm}$$
$$w = 3.5\mathrm{cm}$$
$$l = 6.0\mathrm{cm}$$
$$R = 50\mathrm{m}\Omega$$



What is the direction of the induced current?

(A) clockwise

(B) anti-clockwise

Electric Generators

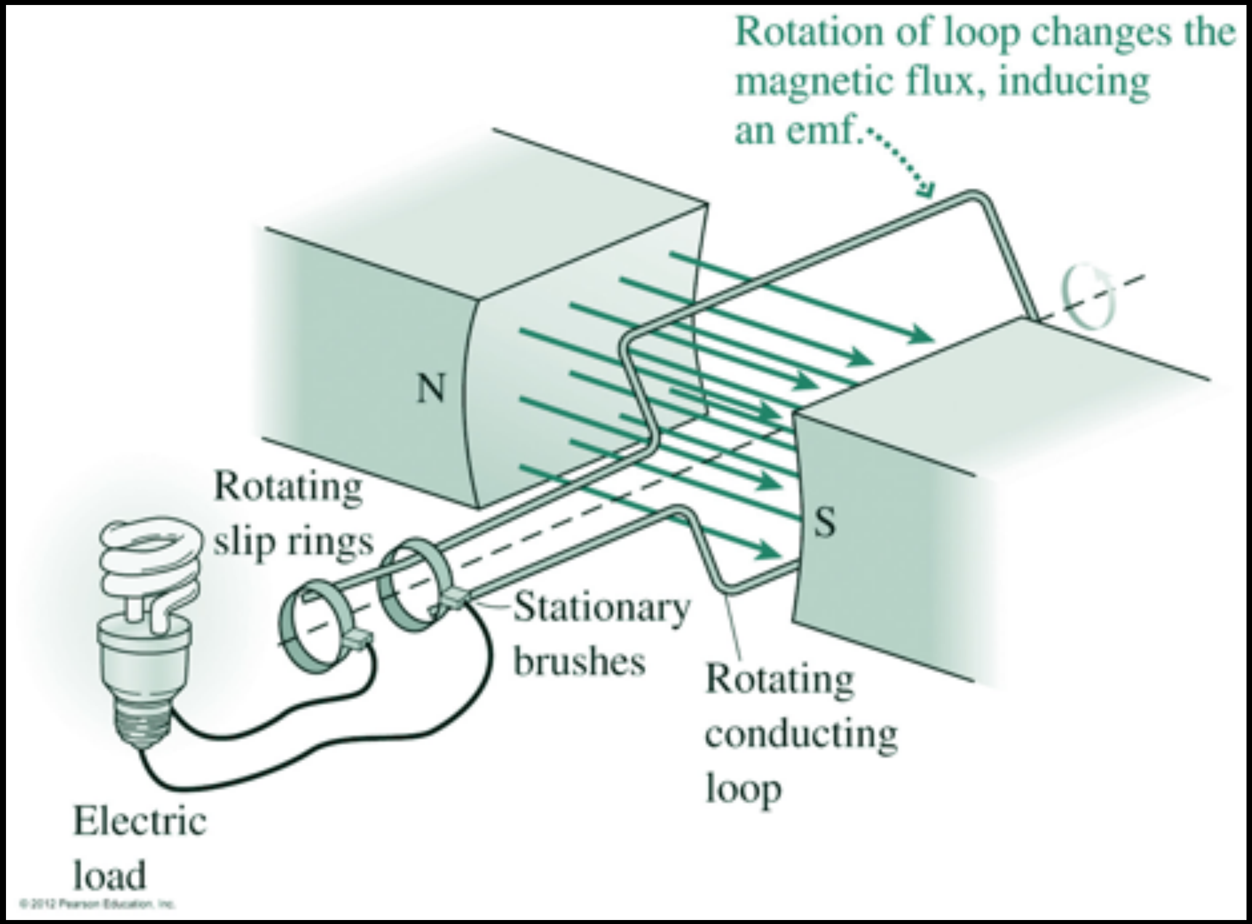
Mechanical energy rotates loop

θ changes $\rightarrow \Phi_B$ changes

$(\Phi_B = BA\cos\theta)$

Creates alternating current

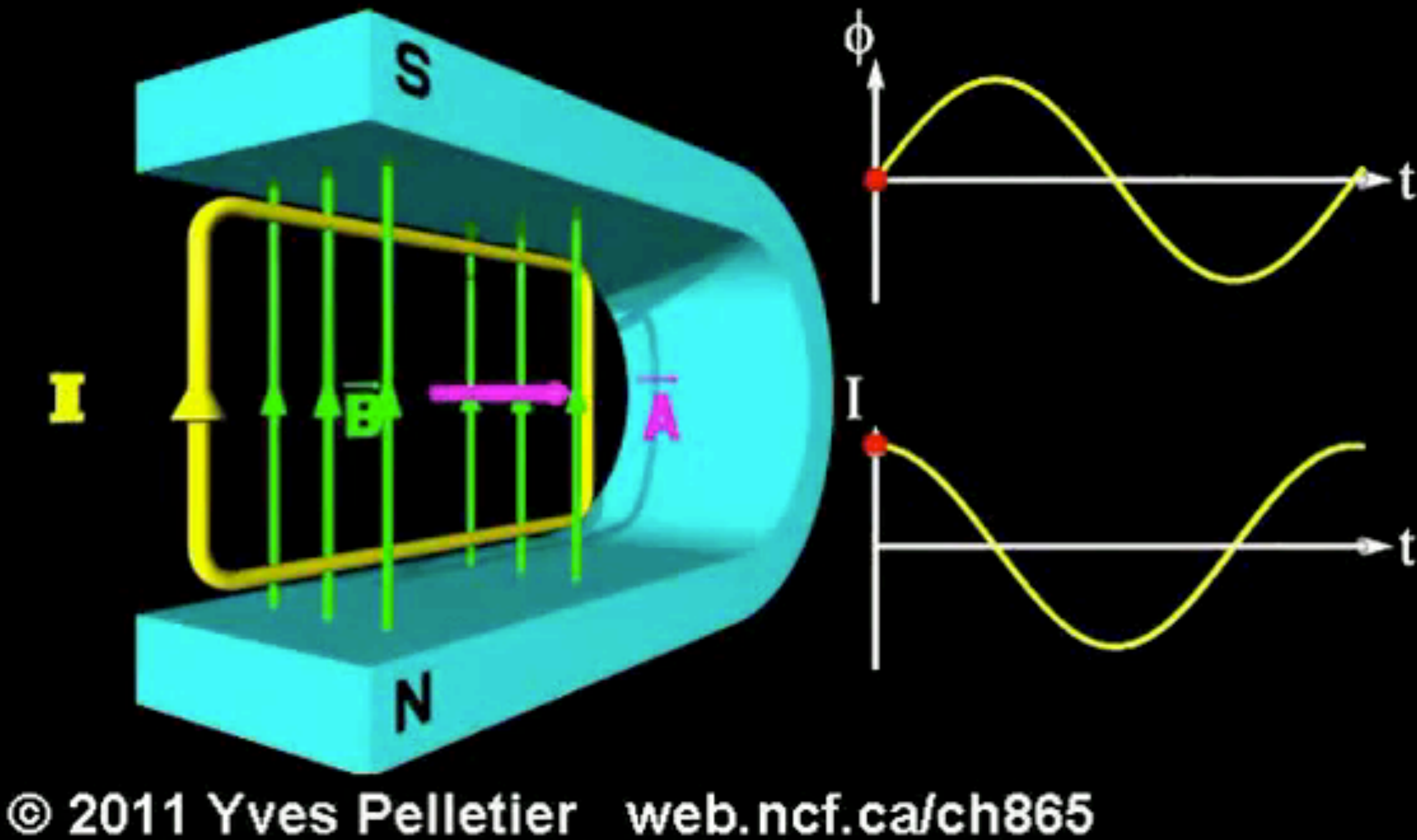
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = IR$$



Lenz's law makes it hard to turn the loop.

Must burn fossil fuels (e.g. coal) or use nuclear fission to power the generator.

Electric Generators



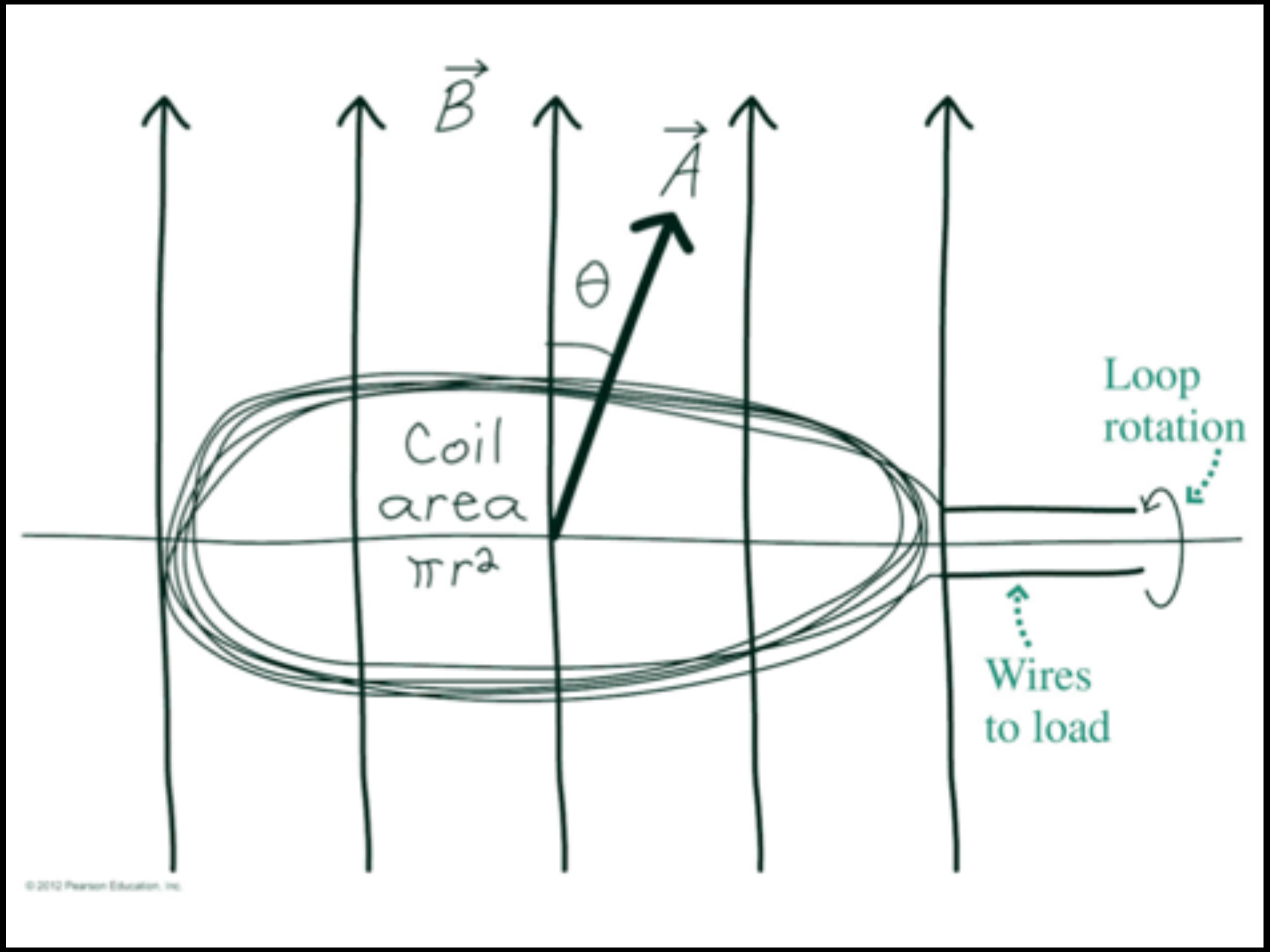
Electric Generators

An electrical generator has a 100-turn loop with diameter 50 cm.

Rotated at frequency, $f = 60\text{rev/s}$ to produce 60 Hz current.

Find $\bar{B}$ for peak output voltage of 170 V

(Produces actual peak of 120 V: normal in USA)



$$\Phi_{1\text{turn}} = \bar{B} \cdot \bar{A} = BA\cos\theta$$
$$= B\pi r^2 \cos\theta$$

Constant rotation: $\omega = 2\pi f \rightarrow \theta = 2\pi f t$

$$\Phi_{\text{total}} = NB\pi r^2 \cos(2\pi f t)$$ (N = 100 turns)

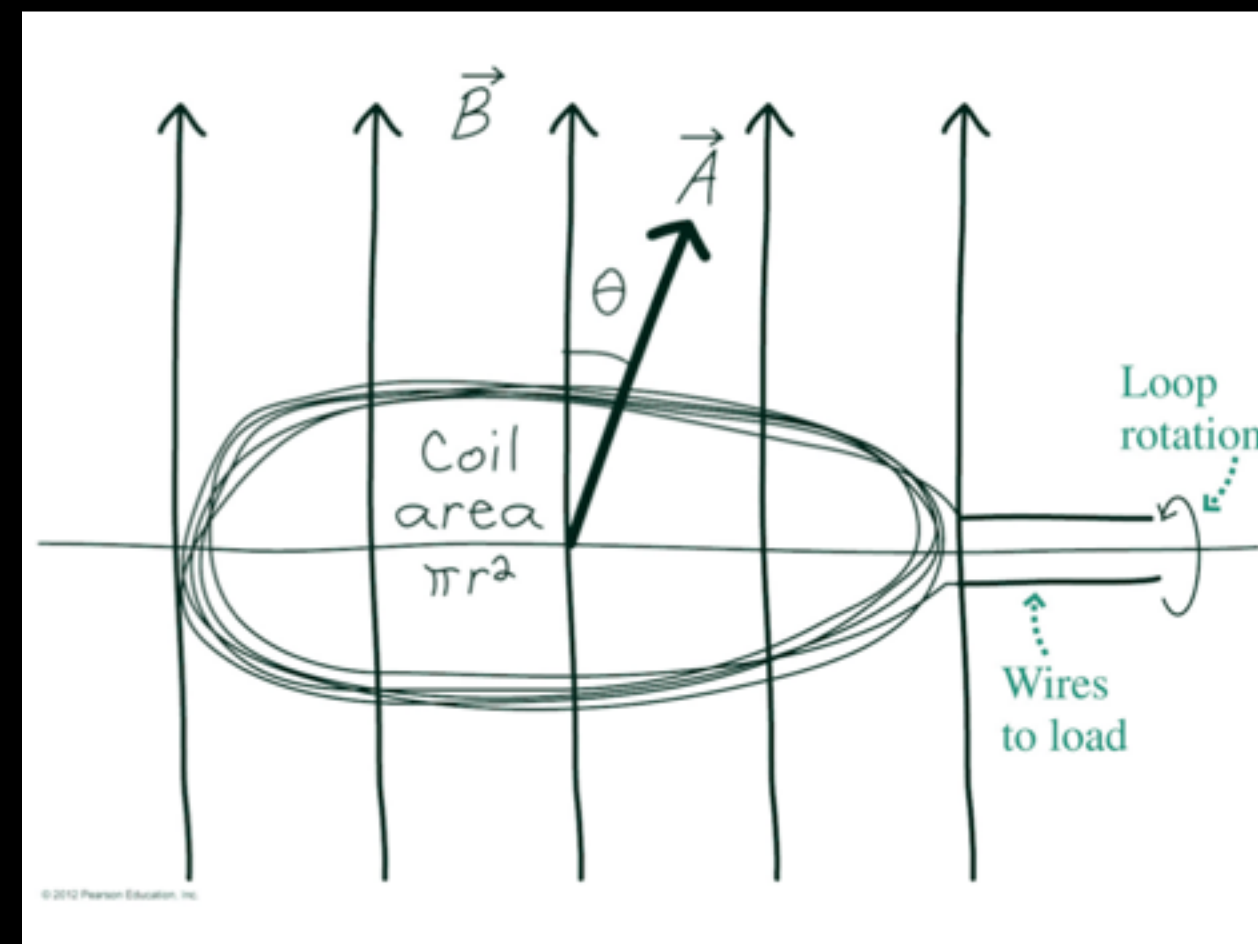
Electric Generators

An electrical generator has a 100-turn loop with diameter 50 cm.

Rotated at frequency, $f = 60\text{rev/s}$ to produce 60 Hz current.

Find $\bar{B}$ for peak output voltage of 170V

(Produces actual peak of 120V: normal in USA)



$$\mathcal{E} = -\frac{d\Phi_{\text{total}}}{dt}$$

$$= -NB\pi r^2 \frac{d}{dt}[\cos(2\pi f t)]$$

$$= -NB\pi r^2[-2\pi f \sin(2\pi f t)]$$

Peak value: $\mathcal{E}_{\max} = 2\pi^2 r^2 NBf \; = 170\text{V}$ $\quad B = 23\text{mT}$

Electric Generators

If power, $P = I^2 R$

and you lower the electrical resistance (R) while turning the generator at constant speed, how will turning change?

(A) It will get harder

(B) It will get easier

(C) No change

Constant speed = constant change in θ

= constant $\frac{d\Phi_B}{dt}$

= constant EMF, $\mathcal{E}$

$\mathcal{E} = IR$

if R drops, I must increase.

Therefore power increases

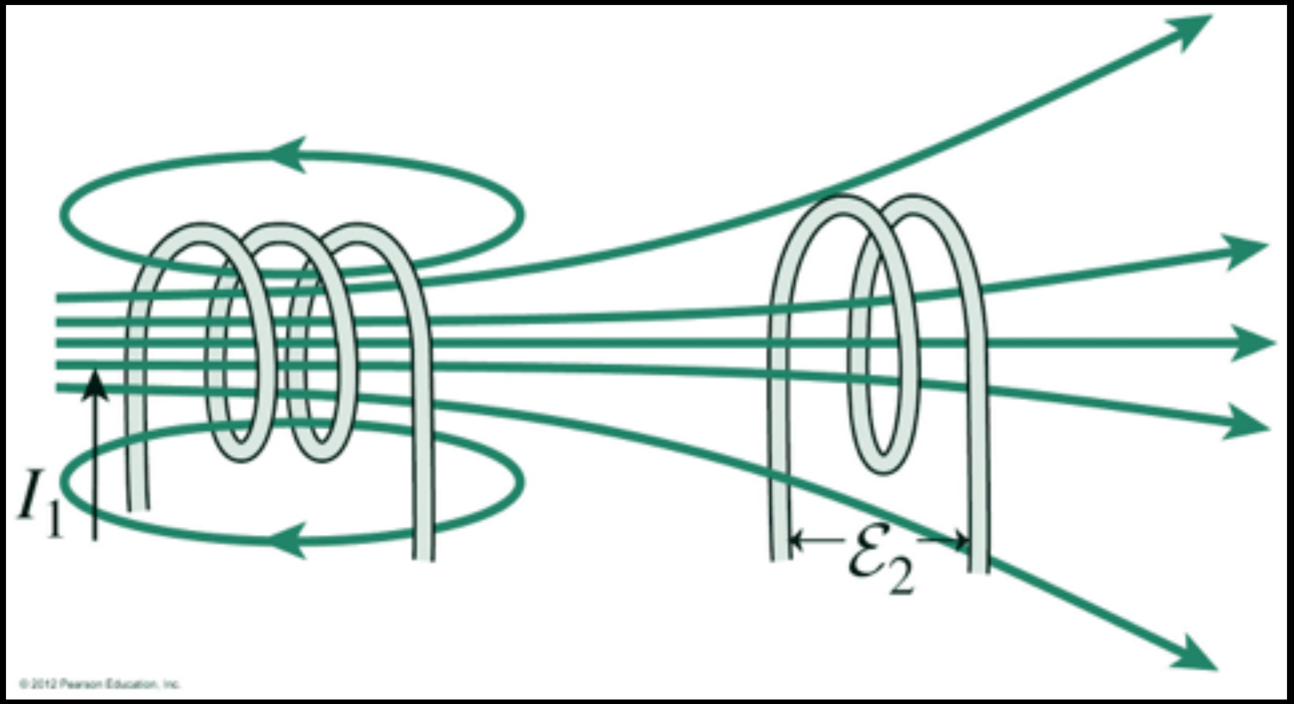
Mutual Inductance

As before (experiment 4)

A changing I in the left-hand coil

Creates a changing flux in right-hand coil

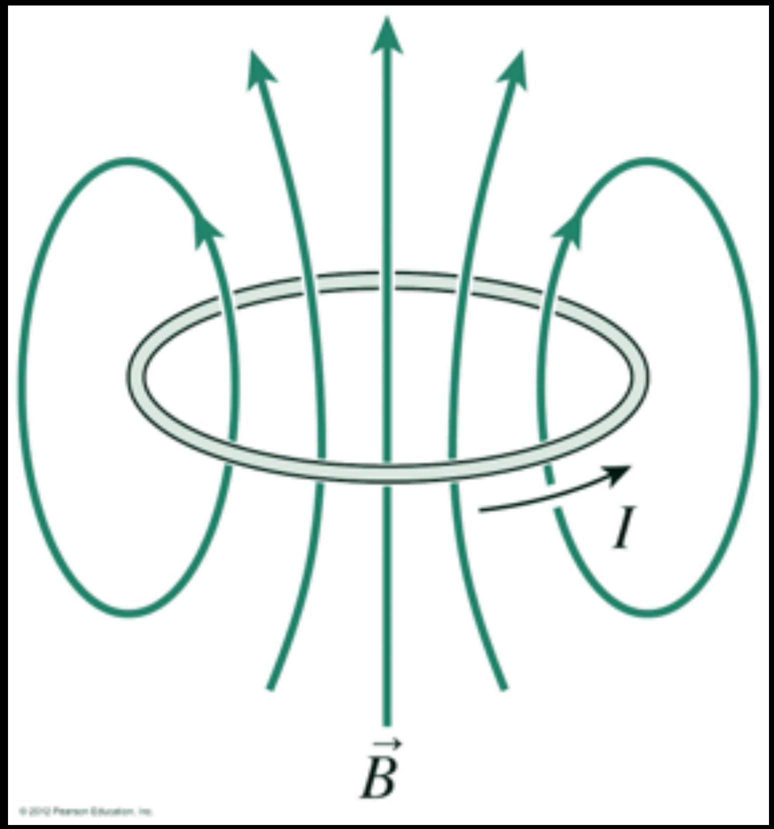
Induces a current in the right-hand coil



This is called mutual inductance.

The strength of the induced current depends on the flux that passes through the second coil.

Self Inductance



But the flux from a changing current also passes through its own circuit.

This induces an EMF that opposes the current change.

This makes it harder to increase the current.

This is called self inductance.

Self inductance is often a problem in circuits, but sometimes it is needed.

An inductor is designed to show self inductance. e.g. used to select specific frequencies by resisting change.

Self Inductance

Self inductance is defined as: $$L = \frac{\Phi_B}{I}$$ $[\mathrm{Tm}^2/\mathrm{A}]$ or H (henry)

Differentiating: $$\frac{d\Phi_B}{dt} = L\frac{dI}{dt}$$

Giving Faraday's law: $$\mathcal{E}_L = -L\frac{dI}{dt}$$ (inductor EMF)

Since the inductor EMF opposes the current change, it is often called the back EMF.

If $\frac{dI}{dt}$ is very large, the back EMF is very large.

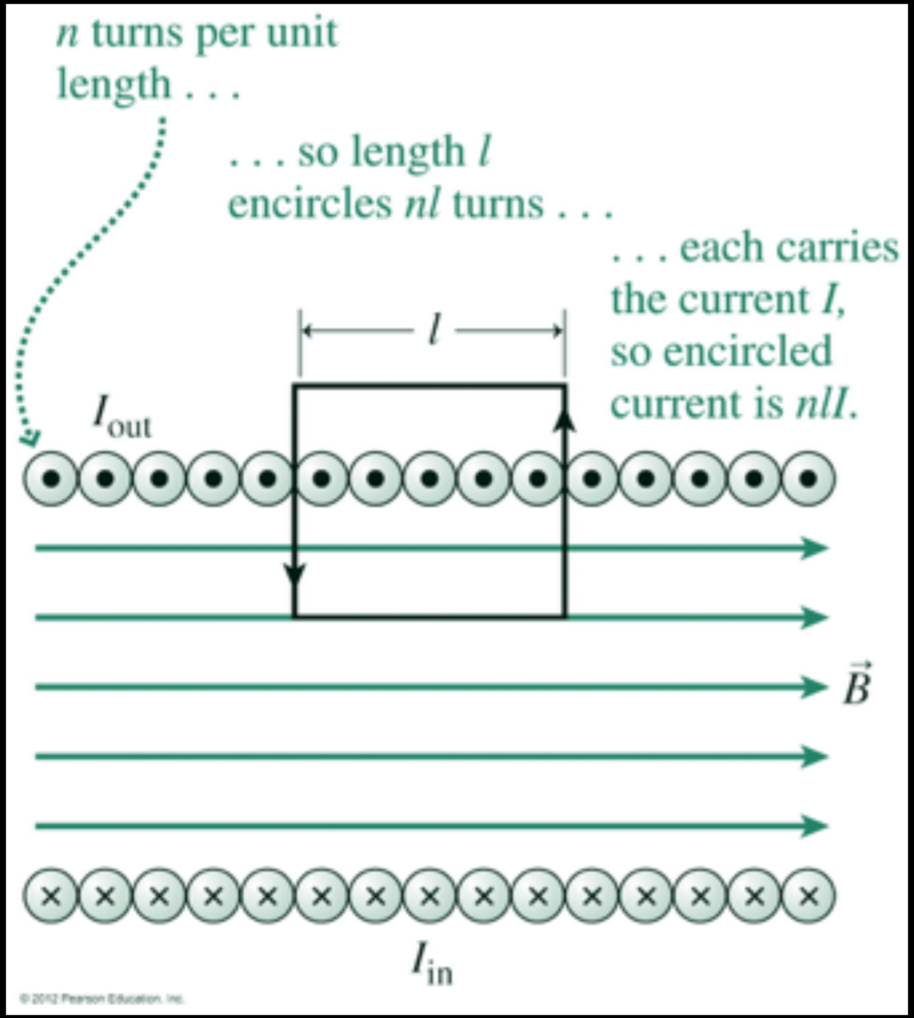
This can be very dangerous.

Self Inductance

A solenoid: cross-sectional area A

length l

n turns per unit length



What is self-inductance, L?

For a solenoid: $B = \mu_0 n I$

Field perpendicular to coils: $\Phi_{1\text{turn}} = BA$

$$\Phi_B = nlBA$$

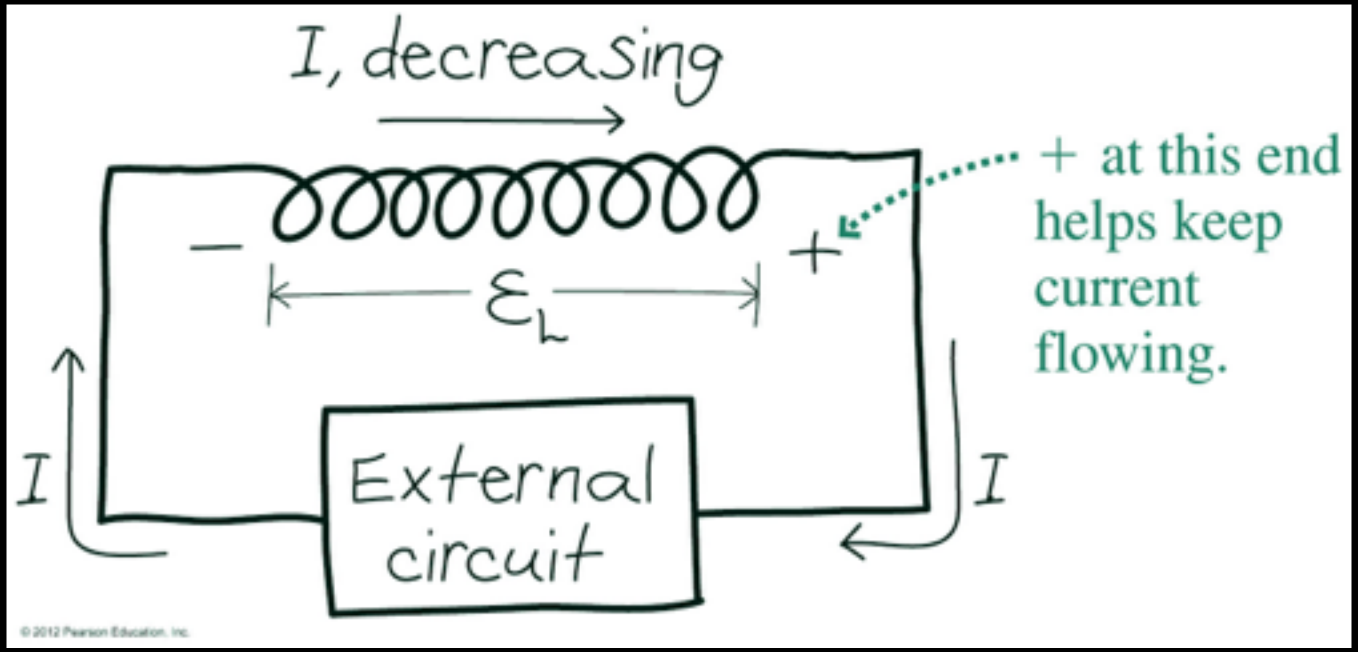
$$= (nl)(\mu_0 n I)A = \mu_0 n^2 A l I$$

$$L = \frac{\Phi_B}{I} = \mu_0 n^2 A l$$ (inductance in a solenoid)

Self Inductance

A 5.0 A current is flowing in a 2.0 H inductor.

The current is reduced to zero in 1.0 ms.



Find the inductor EMF.

$$\mathcal{E}_L = -L\frac{dI}{dt} = -(2.0\text{H})\left(\frac{-5.0\text{A}}{1.0\text{ms}}\right) = 10,000\text{V}$$

Potentially lethal!

Positive EMF: EMF increase in the same direction as the current.

Tries to keep current flowing.

Self Inductance

Quiz

A 2.0 A current is flowing in a 20 H inductor.

A switch opens, stopping the current in 1.0 ms.

What is the inducted EMF?

(A) $2.0\mathrm{kV}$

(B) $20\mathrm{kV}$

(C) $40\mathrm{kV}$

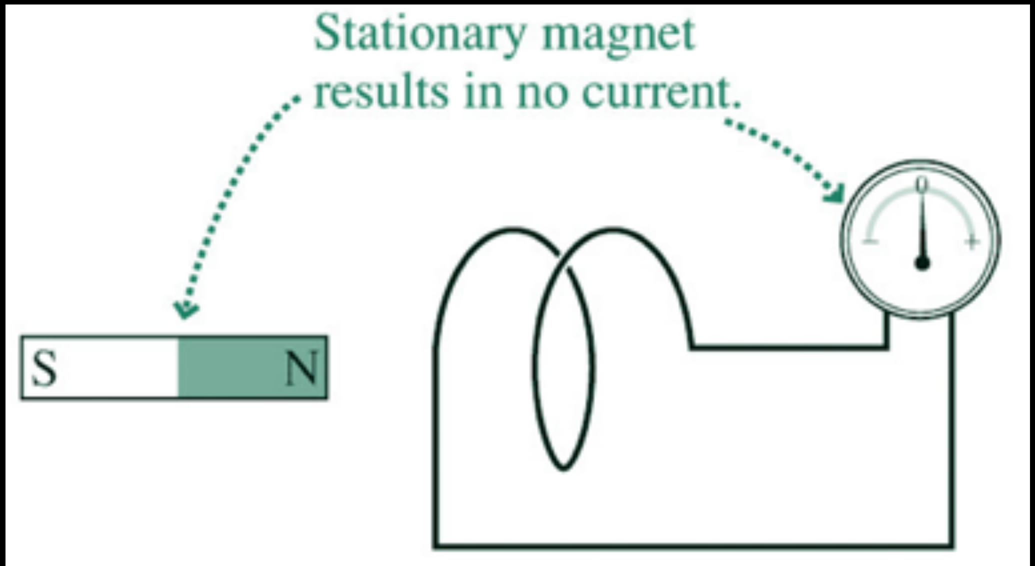
(D) $30.0\mathrm{kV}$

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

$$= -(20\mathrm{H})(2.0\times 10^3\mathrm{A/s}) = 40\mathrm{kV}$$

Induced Electric Fields

The induced EMF acts on static charges to produce a current.



Therefore, it is an electric field.

From earlier: $\mathcal{E} = \oint \bar{E} \cdot d\bar{r}$ work per unit charge gained around circuit

Faraday's Law:

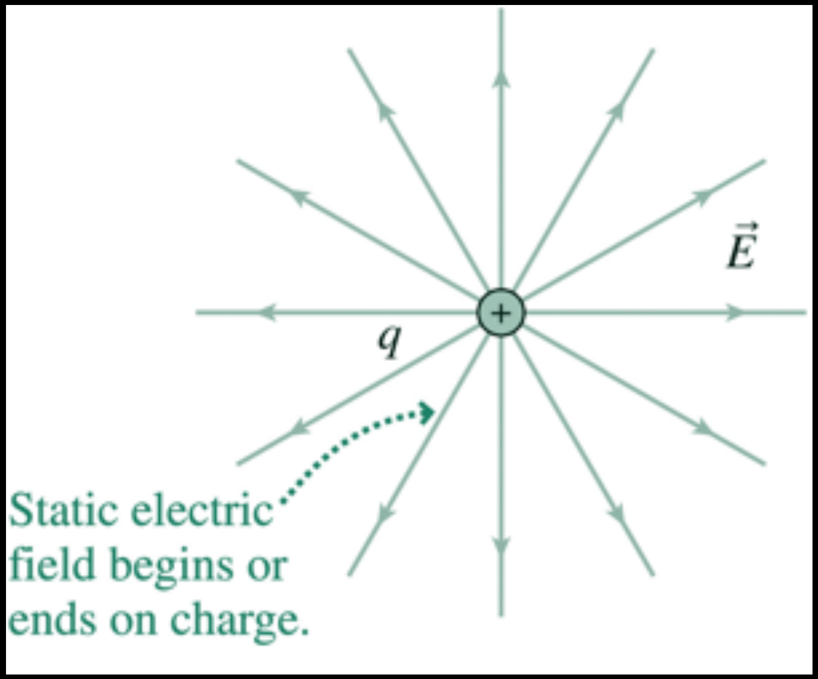
$$\oint \bar{E} \cdot d\bar{r} = -\frac{d\Phi_B}{dt}$$

works for any closed loop

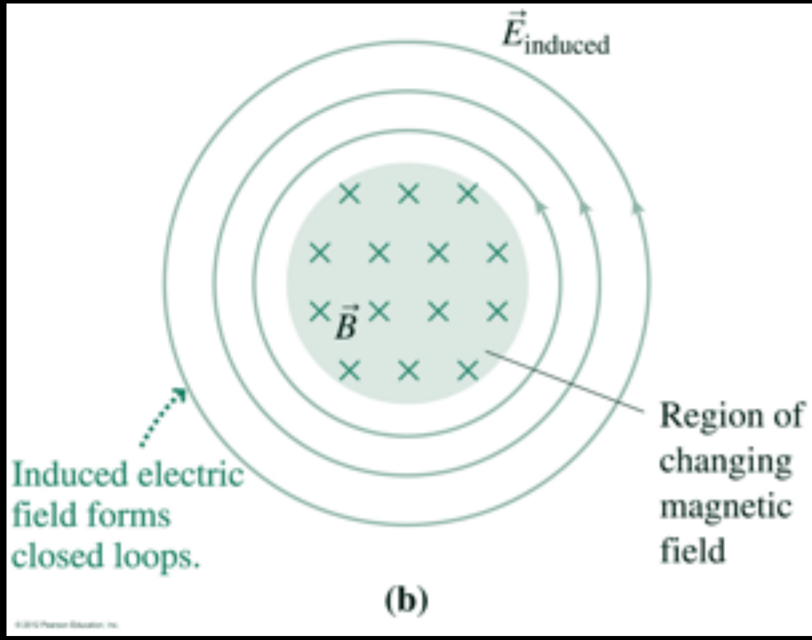
Induced Electric Fields

Faraday's law tells us that electric fields have 2 sources:

Static charges



Changing magnetic fields



Field lines for induced electric fields have no start or end:

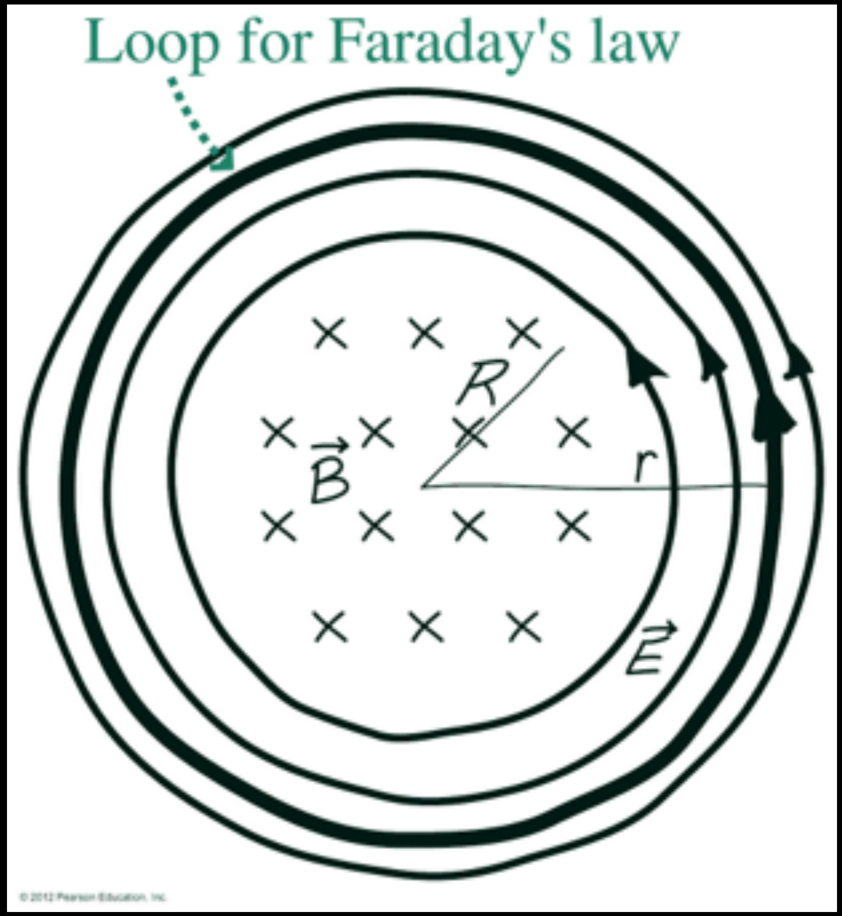
form closed loops encircling regions of changing magnetic field.

Induced Electric Fields

Solenoid: current increasing

field strength: $B = bt$

constant



Find the induced electric field outside the solenoid at a distance r from the centre.

(Note, very similar method to Ampere's law)

Faraday: $\oint \bar{E} \cdot d\bar{r} = -\frac{d\Phi_B}{dt}$

$2\pi r E$

$$\Phi_B = BA = bt\pi R^2$$

$$\frac{d\Phi_B}{dt} = \pi R^2 b$$

$$E = -\frac{R^2 b}{2r}$$

Induced Electric Fields

The induced electric field 12 cm from the axis of a 10 cm radius solenoid is 45 V/m.

What is the rate of change of the solenoid's magnetic field?

(A) $0.75\mathrm{T/ms}$

(B) $0.6\mathrm{T/ms}$

(C) $0.06\mathrm{T/ms}$

(D) $1.1\mathrm{T/ms}$

$$\oint \bar{E}\cdot d\bar{r} = -\frac{d\Phi_B}{dt}$$

$$2\pi r E$$

$$\Phi_B = BA = \pi R^2 B$$

$$\frac{d\Phi_B}{dt} = \pi R^2 \left|\frac{dB}{dt}\right|$$

$$\left|\frac{dB}{dt}\right| = \frac{2r|E|}{R^2} = \frac{2(12\mathrm{cm})(45\mathrm{V/m})}{(10\mathrm{cm})^2}$$

Diamagnetism

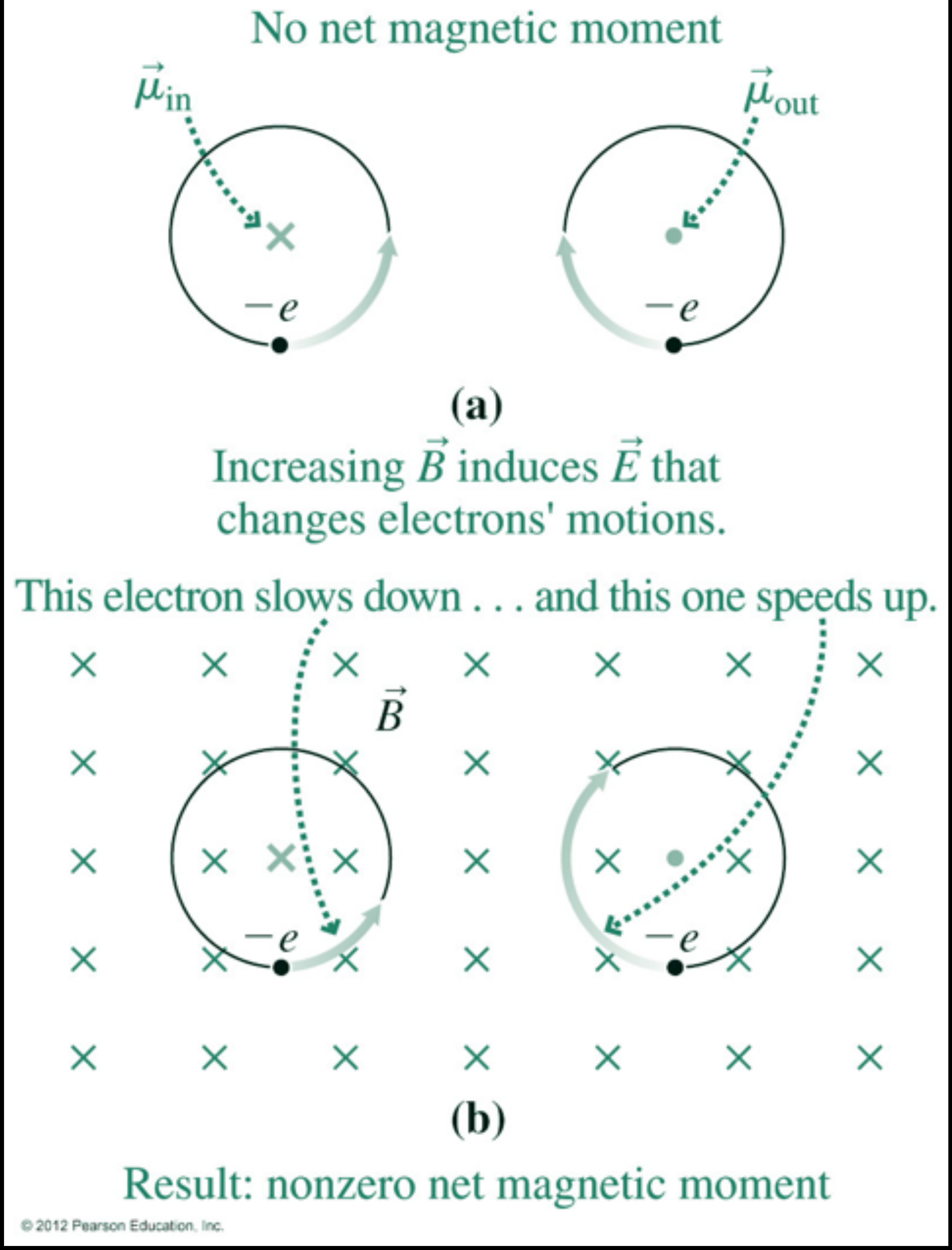
A simple model for diamagnetism:

In a non-ferromagnetic material, atomic current loops point in different (random) directions.

With no magnetic field, the dipoles cancel.

In a changing magnetic field, the induced current causes one current direction to increase, the other to decrease.

Non-zero net magnetic moment opposing the field.



Diamagnetism

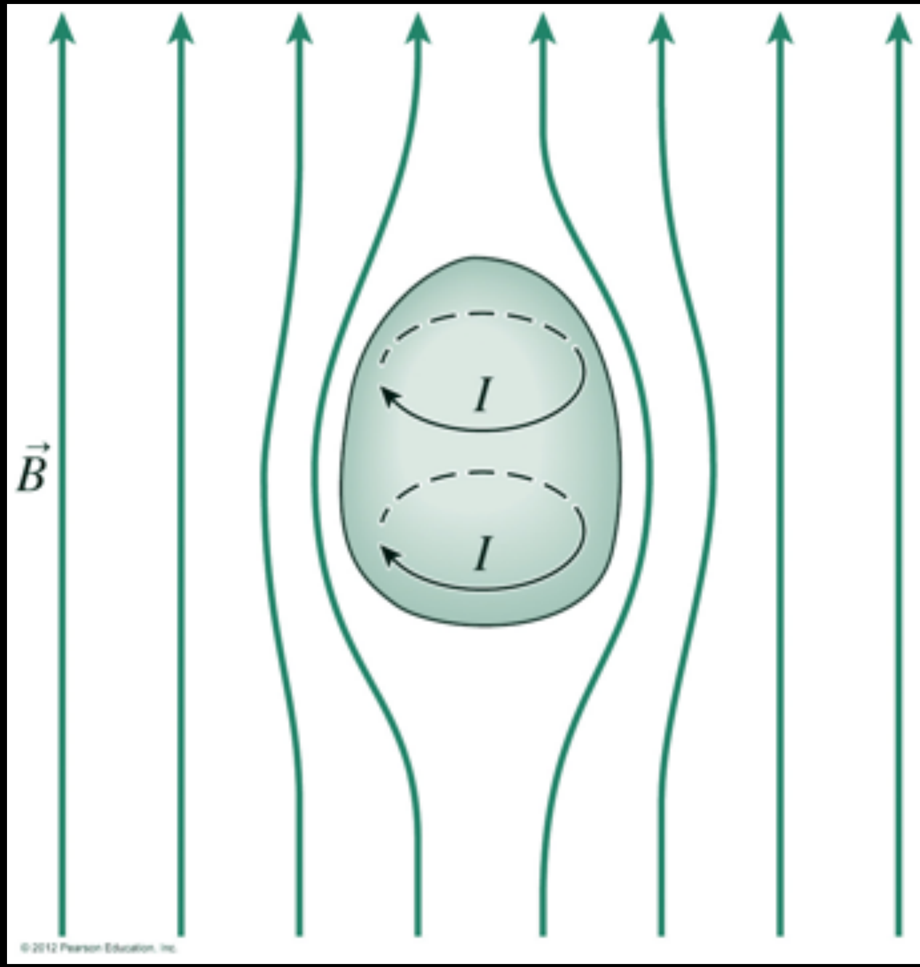
A superconductor is a perfect diamagnet.

The induced currents completely cancels the applied magnetic field.

Inside the superconductor: $B = 0$

The magnetic field is excluded from the superconductor.

This is called the Meissner effect.



The repulsive force between the superconductor and magnet causes magnetic levitation.

Diamagnetism