Essential Physics I

英語で物理学の
エッセンス I

Lecture 9:   20-06-16

News

EXTRA lecture, THURSDAY

6/23/2016

6月23日

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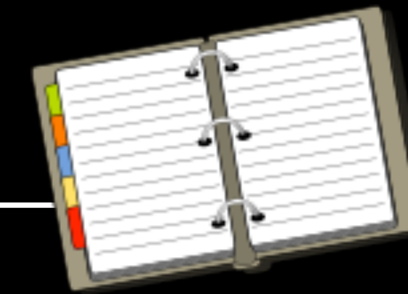
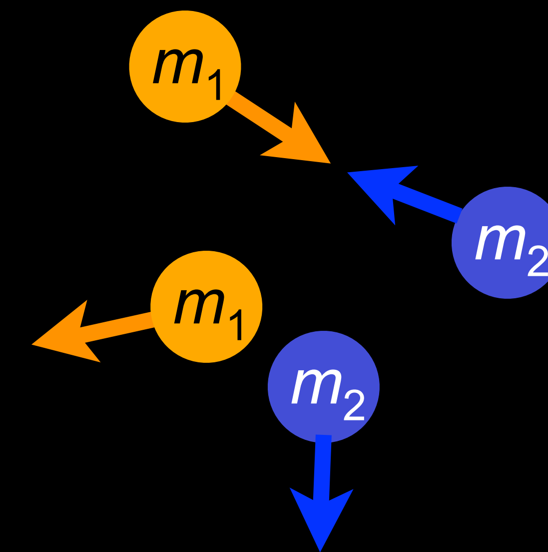
Last lecture: review



Conservation of momentum: $\bar{p} = m\bar{v}$



$$\bar{p}_{\text{before}} = \bar{p}_{\text{after}}$$

$$m_1\bar{v}_{1,i} + m_2\bar{v}_{2,i} = m_1\bar{v}_{1,f} + m_2\bar{v}_{2,f}$$

Elastic collision: + conservation of K

$$\frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2 = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2$$

Totally inelastic: objects stick together

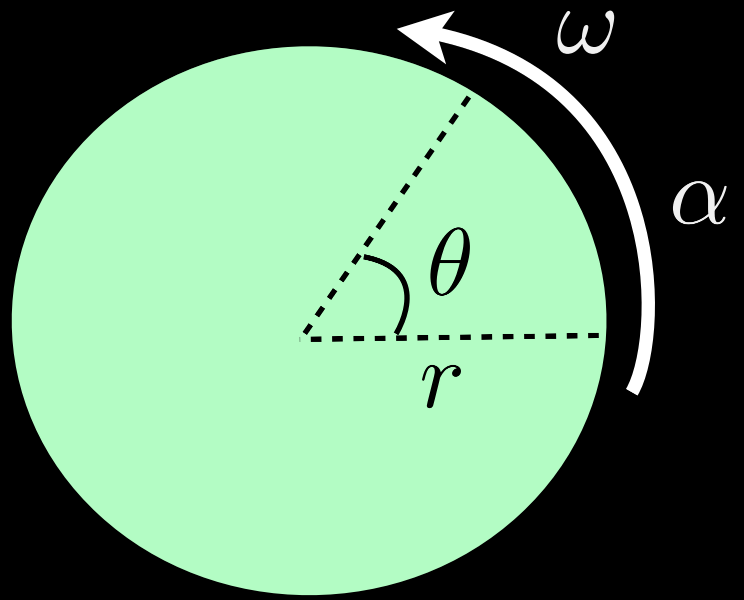
$$m_1\bar{v}_1 + m_2\bar{v}_2 = (m_1 + m_2)\bar{v}_f$$

Last lecture: review

Angular position: θ

Angular velocity: $w = \frac{d\theta}{dt}$

Angular acceleration: $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$



$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Equations for constant angular acceleration

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\tau = I\alpha$$

$$I = \Sigma m_i r_i^2$$

$$I = \int r^2 dm$$

Rotational inertia: measure how hard it is to rotate

Last lecture: review

A turbine blade rotates with angular velocity $\omega(t) = 2.0 - 2.1t^2$

What is the angular acceleration at t = 9.1s

(a) $-86.0\,\mathrm{rad/s^2}$

(b) $-19.1\,\mathrm{rad/s^2}$

(c) $-38.2\,\mathrm{rad/s^2}$

(d) $-36.2\,\mathrm{rad/s^2}$

(e) $-172\,\mathrm{rad/s^2}$

Last lecture: review

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What is the angular acceleration at t = 9.1s

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(c) $-38.2\,\text{rad/s}^2$

(d) $-36.2\,\text{rad/s}^2$

(e) $-172\,\text{rad/s}^2$

$$\alpha = \frac{d\omega}{dt} = -4.2t$$

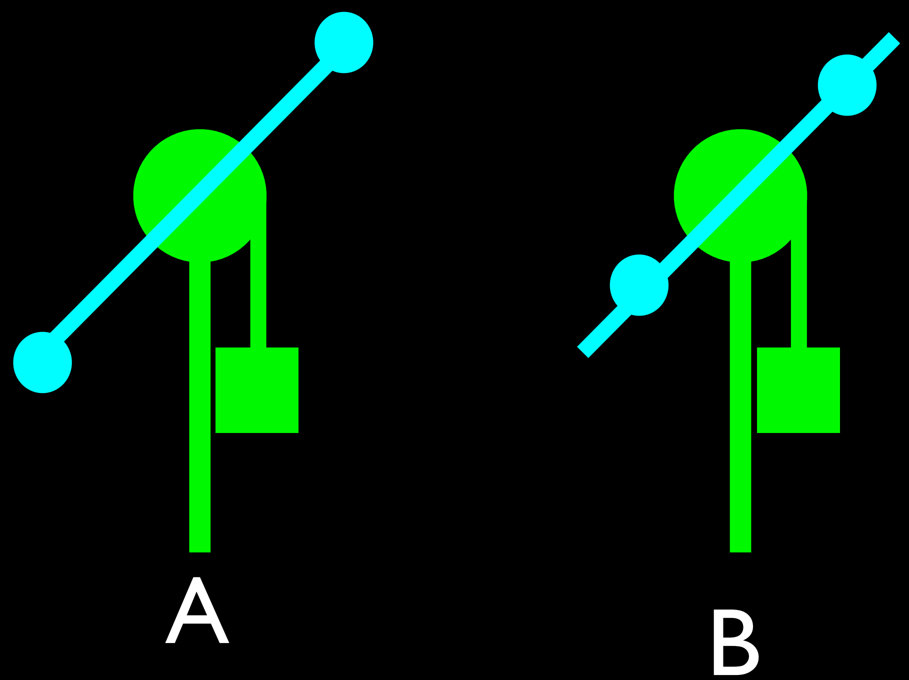
$$= -4.2(9.1)$$

$$= -38.2\,\text{rad/s}^2$$

Last lecture: review

2 rotating systems differ only by the position of 2 movable masses on the axis of rotation.

Which block lands first?



'B' has smaller rotational inertia:

$$I = \Sigma m_i r_i^2$$

(a) A

(b) B

Higher acceleration for same torque: $\tau = I\alpha$

(c) Both at same time

Last lecture: review

A dumb-bell shape had 2 equal masses connected by a rod of negligible (~ zero) mass and length r.

If I_1 is the rotational inertia of the object for an axis through the rod centre, and I_2 is the rotational inertia passing through one mass, then...

(a) $I_1 = I_2$

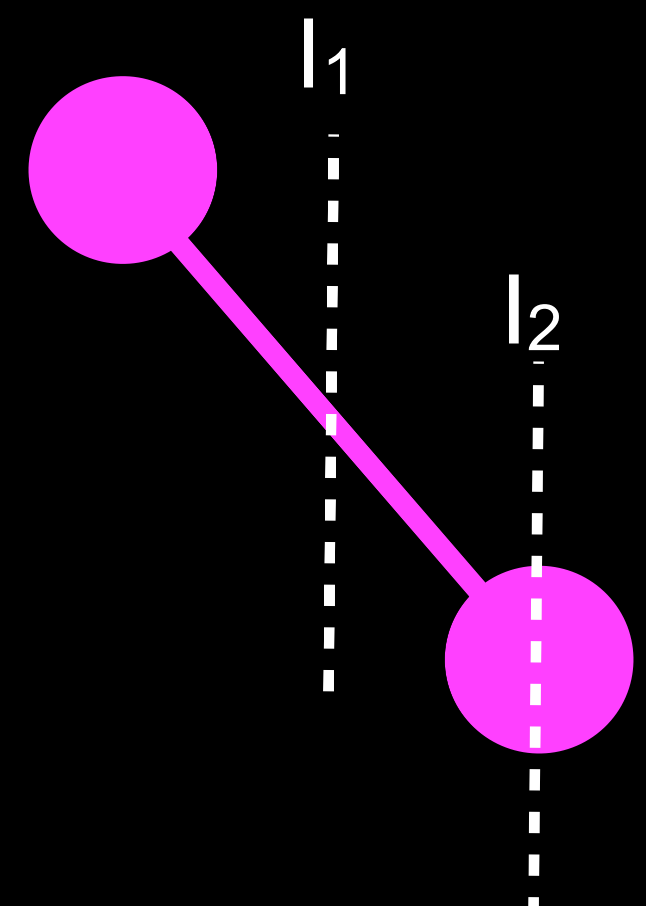
(b) $I_1 > I_2$

(c) $I_1 < I_2$

Parallel axis theorem:

$$I = I_{\rm cm} + Md^2$$

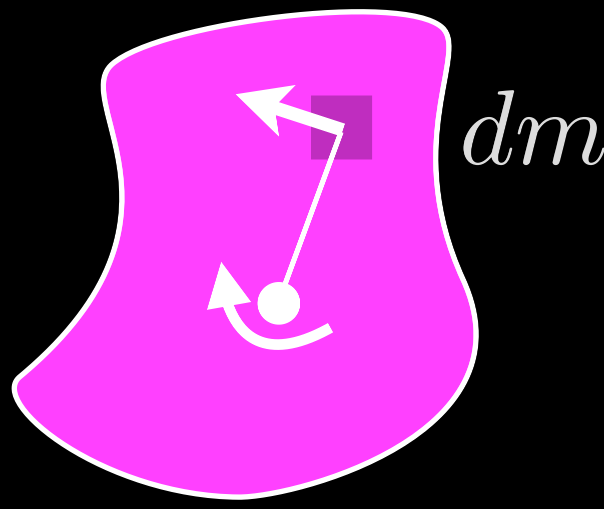
$$I_2 = I_1 + Md^2$$



Rotational Energy

$$K_{\text{rot}} = \int dK$$

sum of kinetic energy of all parts, dm



$v = r\omega$

$$= \int \frac{1}{2}(dm)(wr)^2 \quad = \frac{1}{2}w^2 \int r^2 dm$$

I

$$K_{\text{rot}} = \frac{1}{2}Iw^2$$

Rotational kinetic energy

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta \quad = \Delta K_{\text{rot}} \quad = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

Rotational work-energy theorem

Rotational Energy

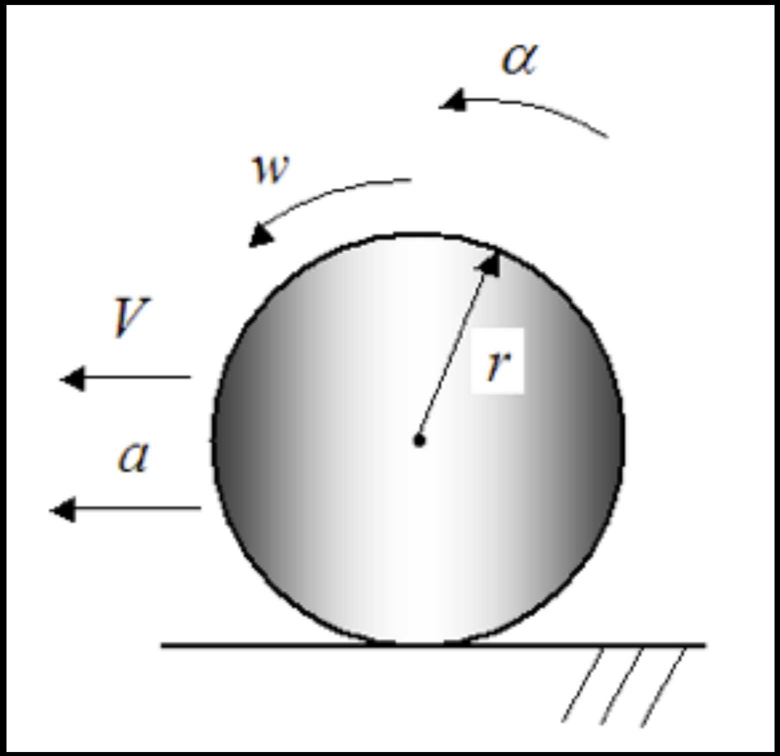
What happens when we roll?

The object has both translational motion:

$$x, v, a$$

and rotational motion:

$$\theta, \omega, \alpha$$



Total energy: $$K_{\text{total}} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

translation K rotational K

Rotational Energy

Quiz

A 150-g baseball (uniform solid sphere, radius 3.7cm) is thrown at 33 m/s, spinning at 42 rad/s.

What fraction of its kinetic energy is rotational?

(1) 0.089 %

(2) 50 %

(3) 0.1%

(4) 100%

$$I = \frac{2}{5} M R^2$$

Rotational Energy

Quiz

A 150-g baseball (uniform solid sphere, radius 3.7cm) is thrown at 33 m/s, spinning at 42 rad/s.

What fraction of its kinetic energy is rotational?

$$K_{\text{tot}} = K_{\text{translation}} + K_{\text{rotation}}$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$I = \frac{2}{5}MR^2$$

$$\frac{K_{\text{rotational}}}{K_{\text{tot}}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2}$$

$$= \frac{\frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2}{\frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2} = 8.86 \times 10^{-4} = 0.089\%$$

Rotation

Translational and rotational motion are connected

For half-rotation:

$$x = s = \pi R$$

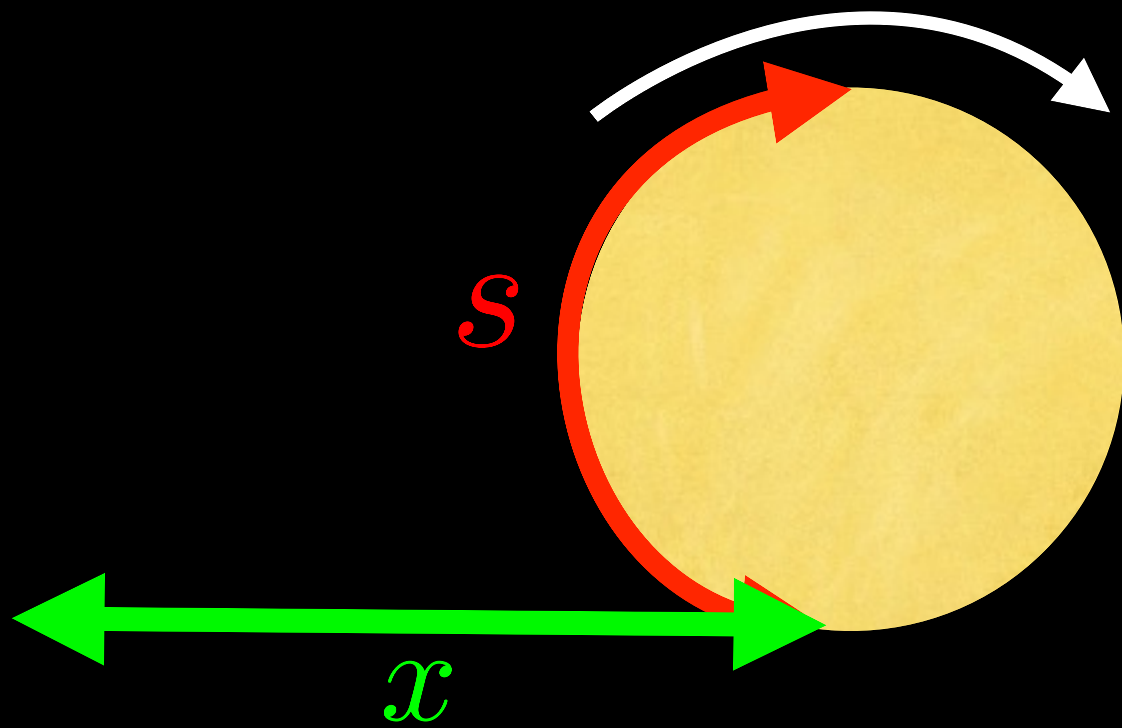
$$v = \frac{\Delta x}{\Delta t} = \frac{\pi R}{\Delta t}$$

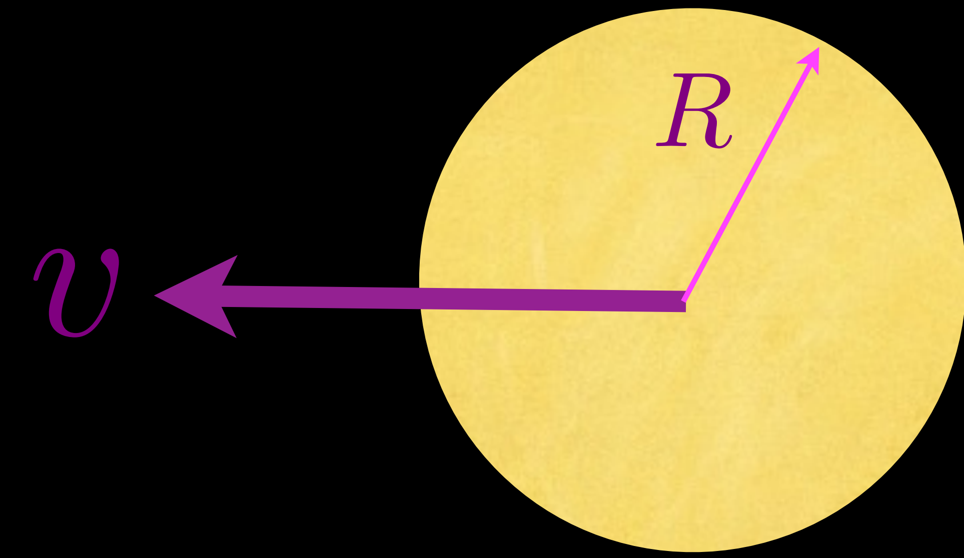
$$\omega = \frac{\Delta \theta}{\Delta t} = \frac{\pi}{\Delta t}$$

Therefore: $v = wR$

translational velocity

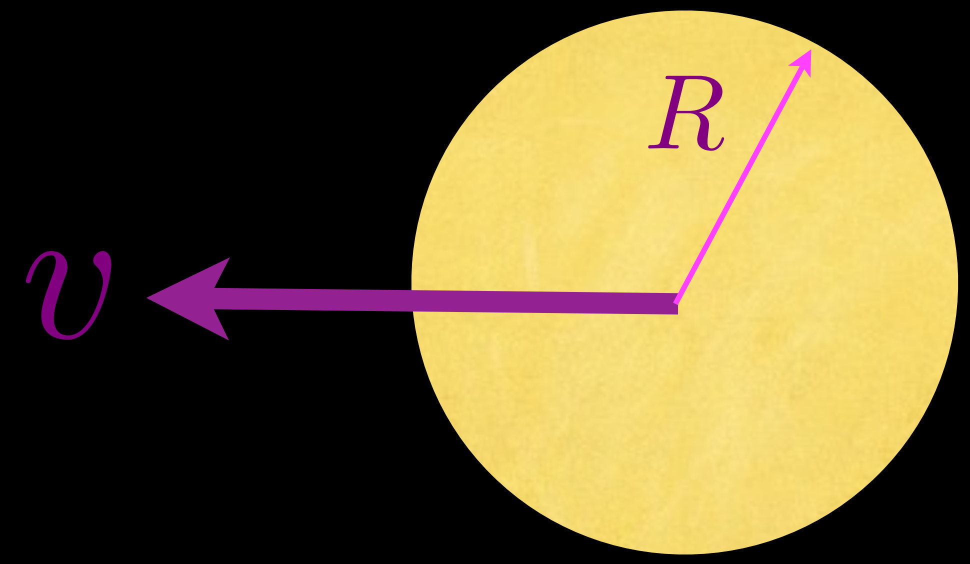
radius of object





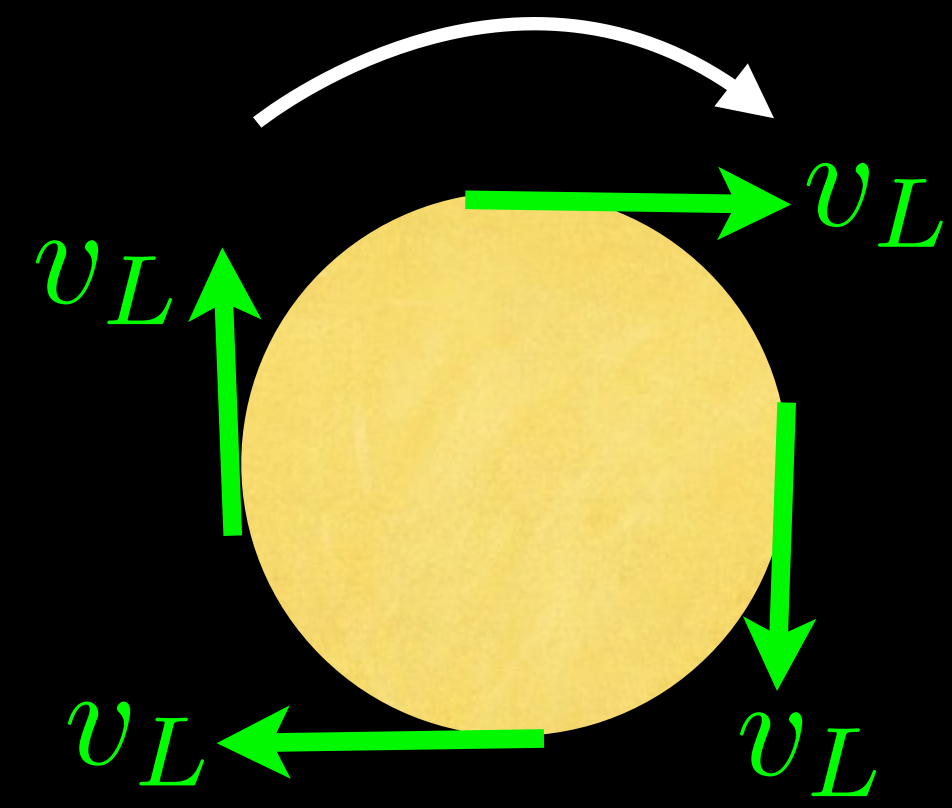
Rotation

$$v = wR$$



translational velocity of whole object of radius R

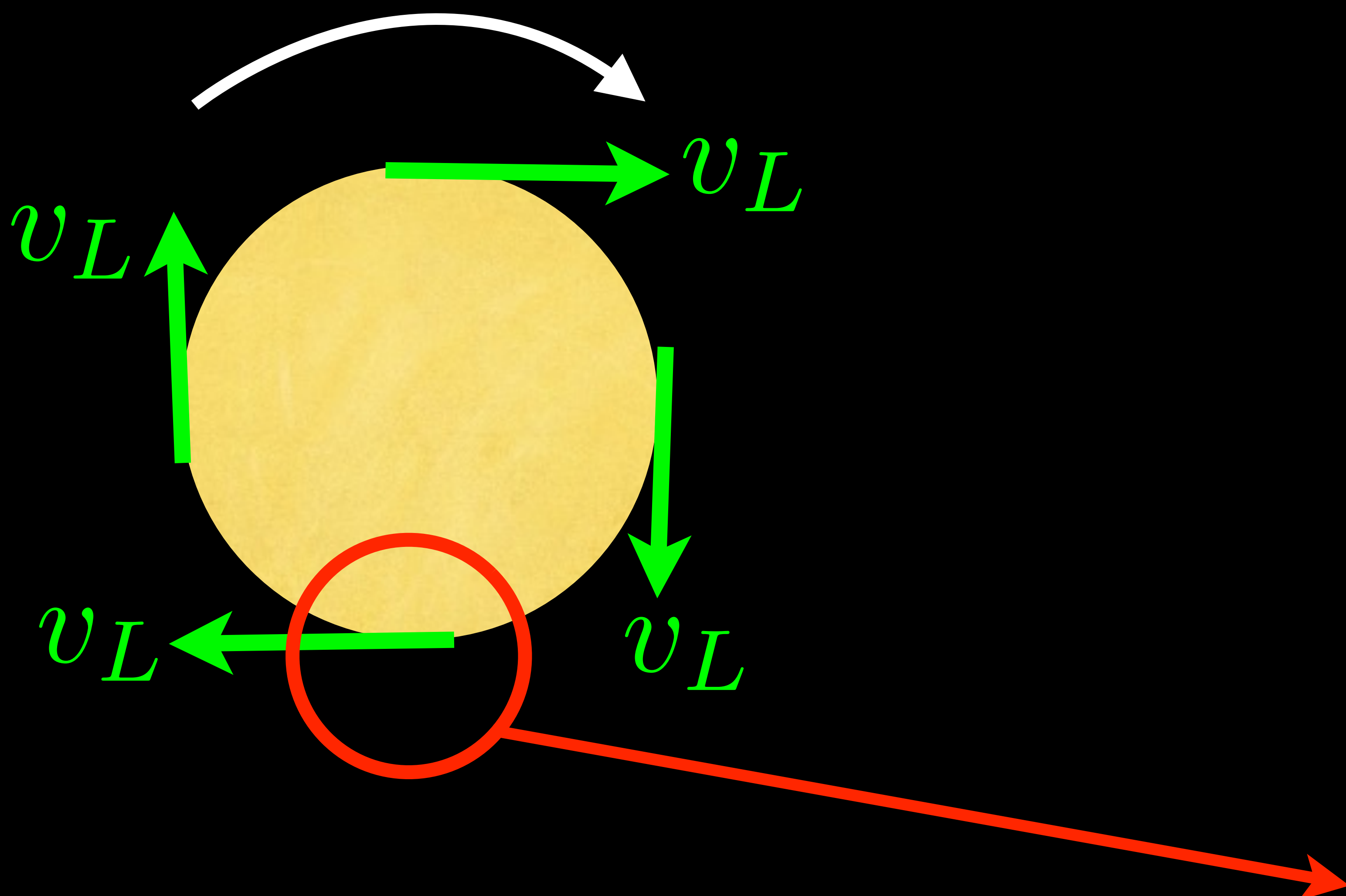
$$v = \omega r$$

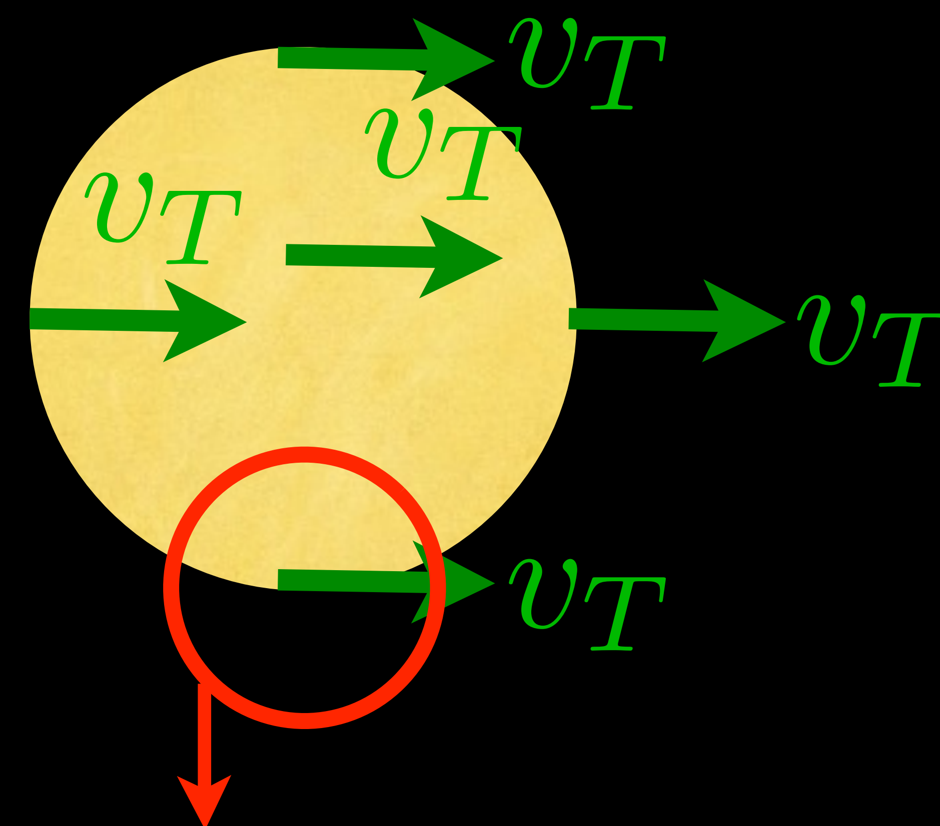


linear velocity of point at radius r

Rotation

But.... how does an object roll?

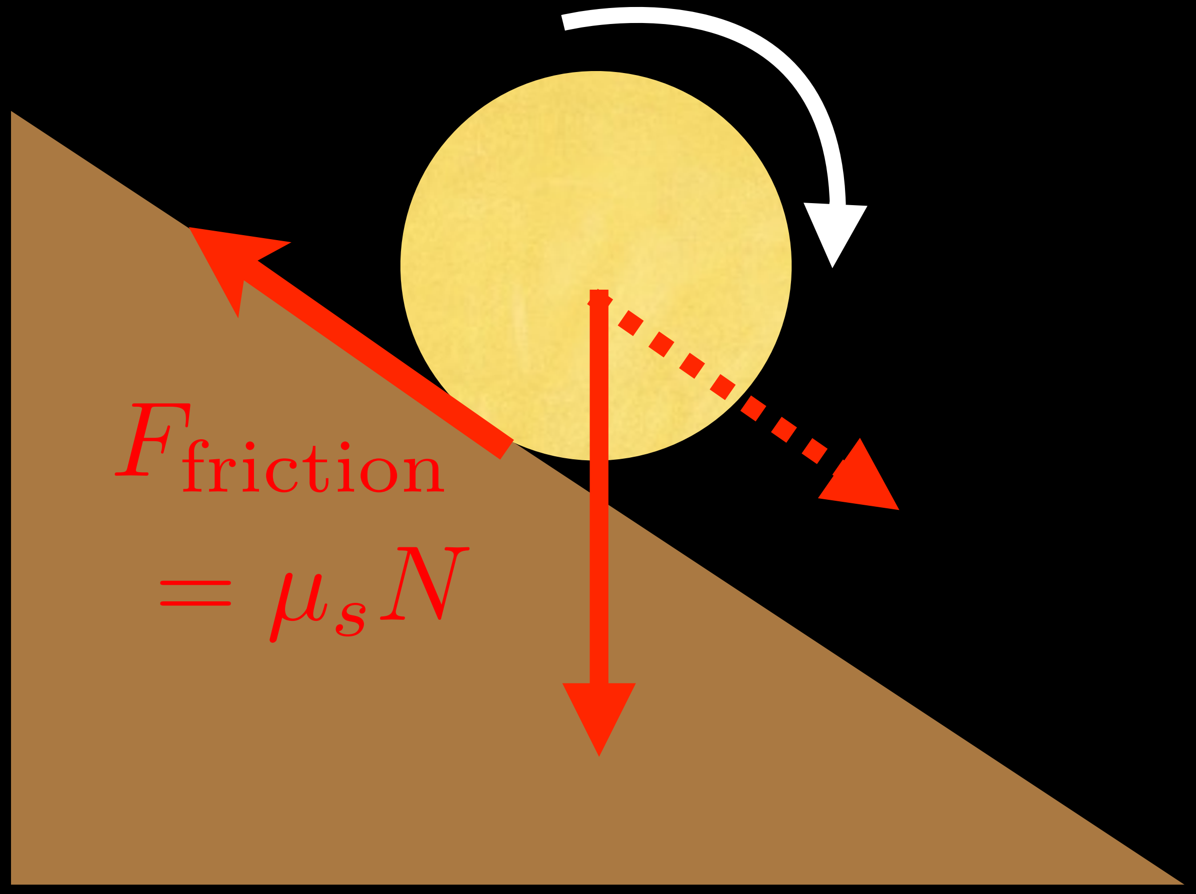




Why don't the linear velocities cancel?

The translational velocity cancels the linear velocity at the bottom.

Rotation



If $\bar{v} = 0$ at the bottom,

why doesn't it slide down a hill?

Gravity should pull it down

Static friction stops an object sliding and makes it roll.

On ice, it would slide.

Since the bottom point is at rest, no work is done by the friction. Mechanical energy is conserved.

Rotation

Quiz

A solid 2.4 kg sphere is rolling at 5.0 m/s. Find:

(a) Its translational kinetic energy

(b) Its rotational kinetic energy

(1) 30 J, 30 J

(2) 30 J, 12 J

(3) 12 J, 5 J

(4) 0 J, 30 J

$$I = \frac{2}{5}MR^2$$

Rotation

Quiz

A solid 2.4 kg sphere is rolling at 5.0 m/s. Find:

(a) Its translational kinetic energy

(b) Its rotational kinetic energy

$$K_T = \frac{1}{2}mv^2 = \frac{1}{2}(2.5\,\mathrm{kg})(5.0\,\mathrm{m/s})^2 \;= 30\,J$$

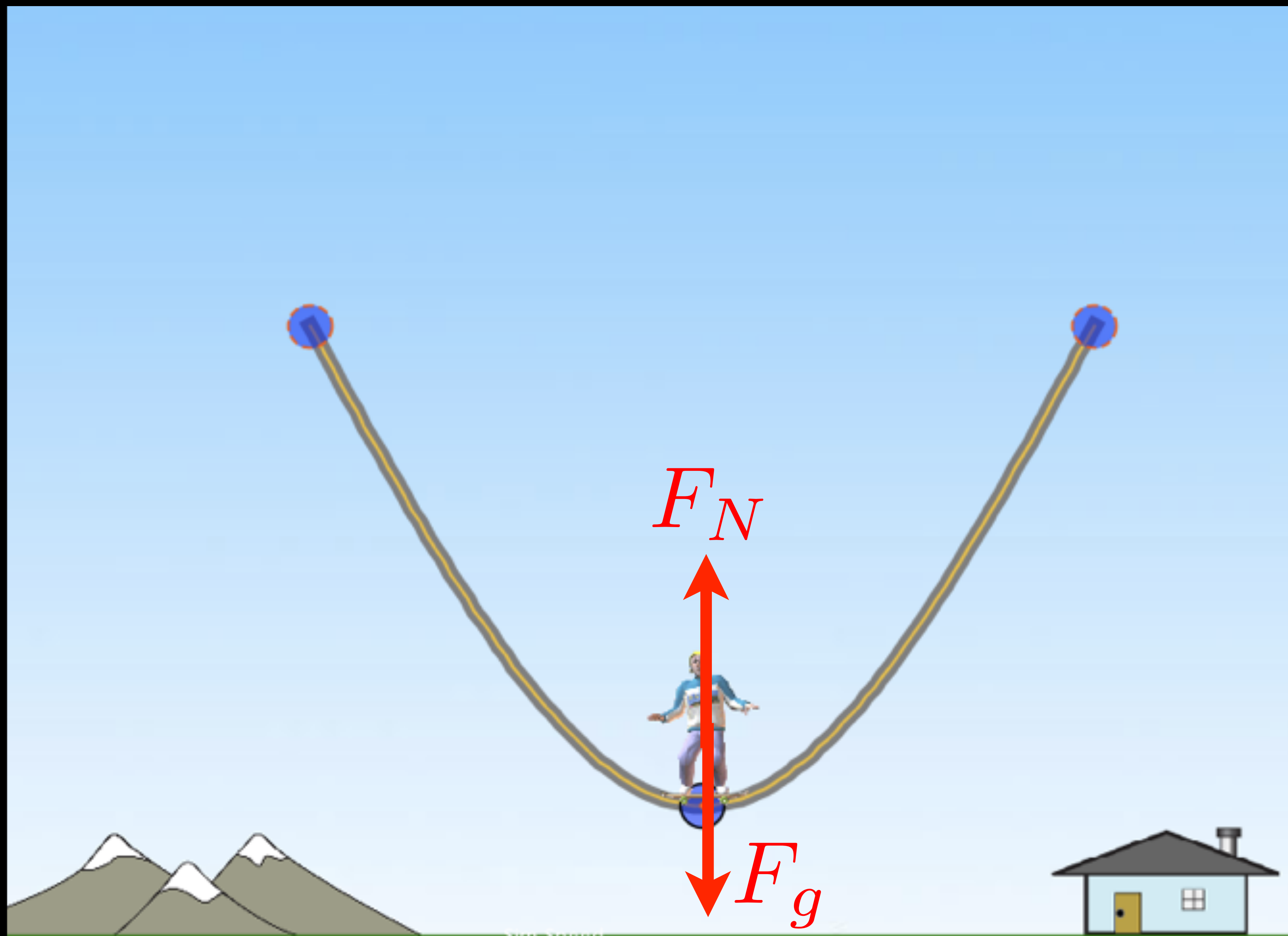
$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{5}mv^2 \quad = \frac{2}{5}K_T$$

$$= \frac{2}{5}(30\,J) \quad = 12\,J$$

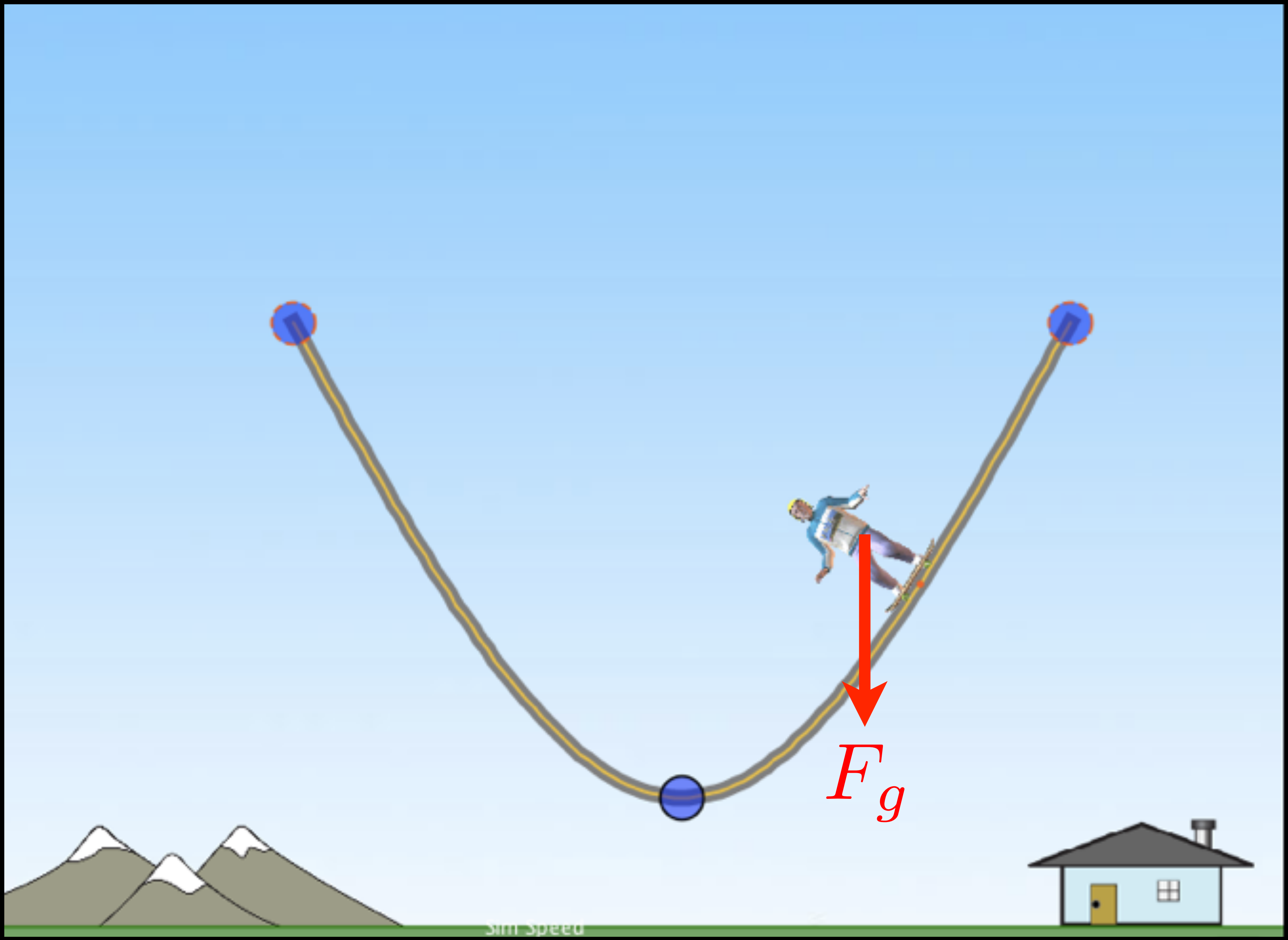
Oscillations

Oscillations



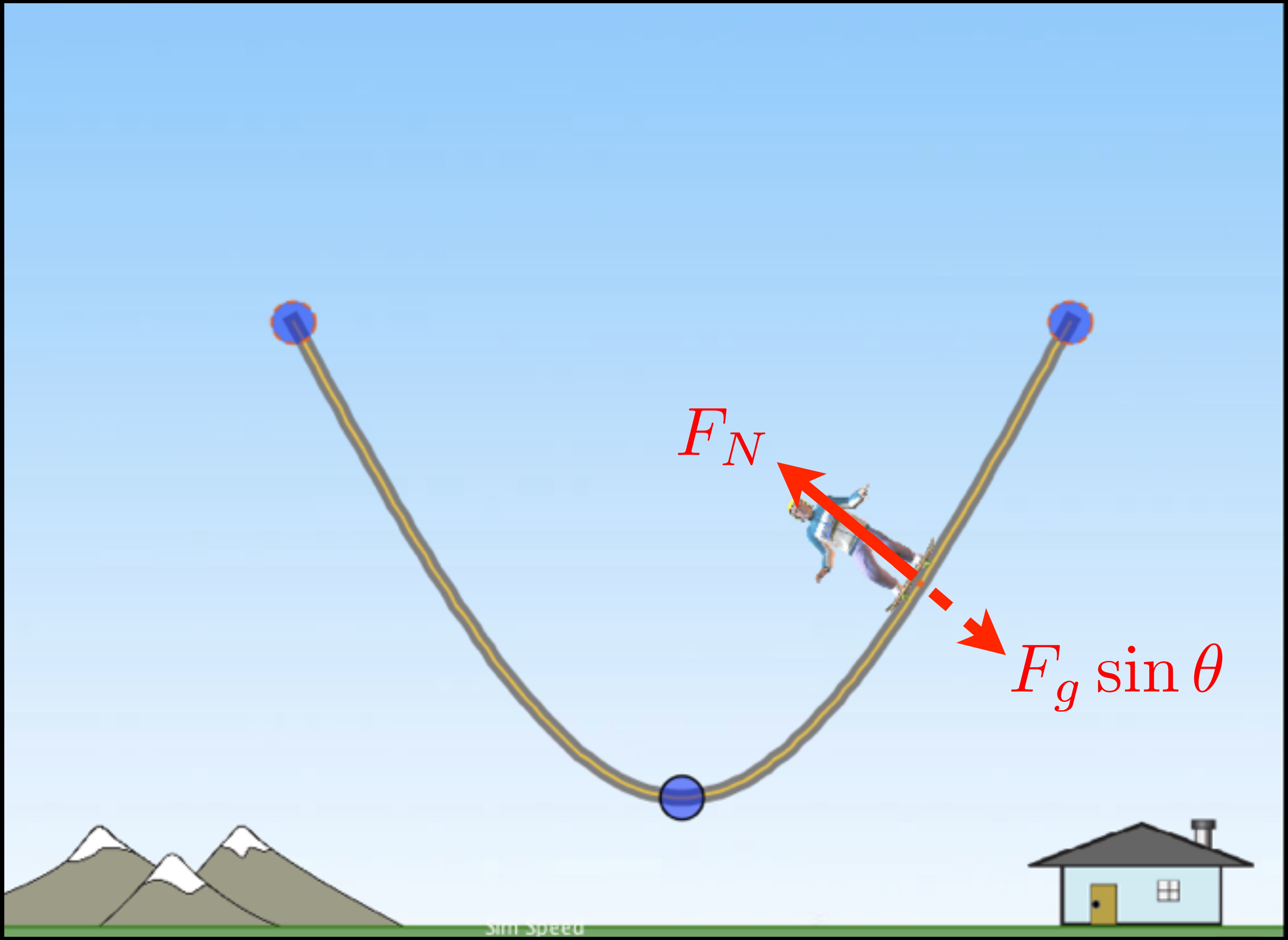
Stable equilibrium: $F_{\text{net}} = 0$

Oscillations



If we move a small distance from a stable equilibrium

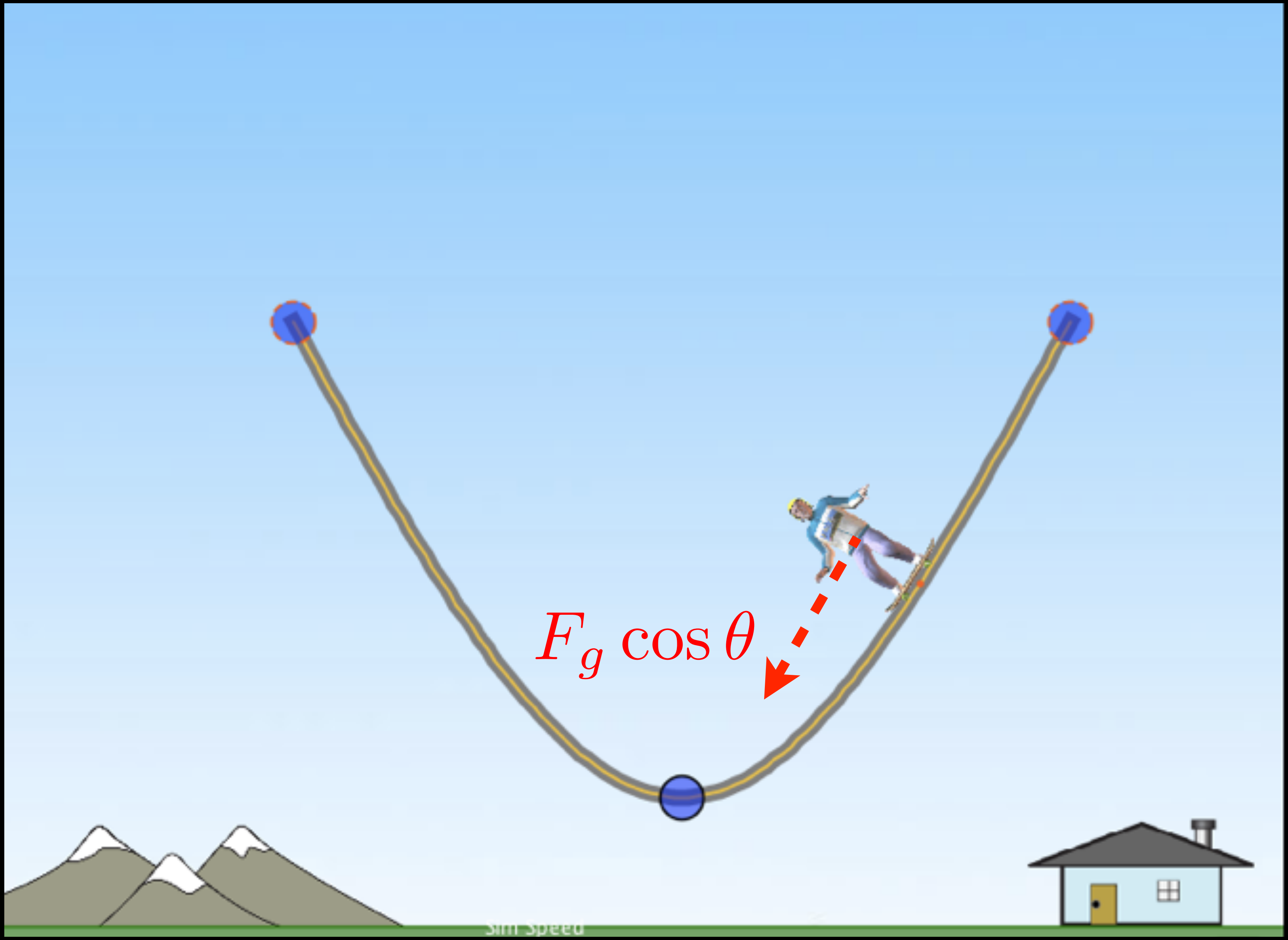
Oscillations



If we move a small distance from a stable equilibrium

Perpendicular forces cancel

Oscillations

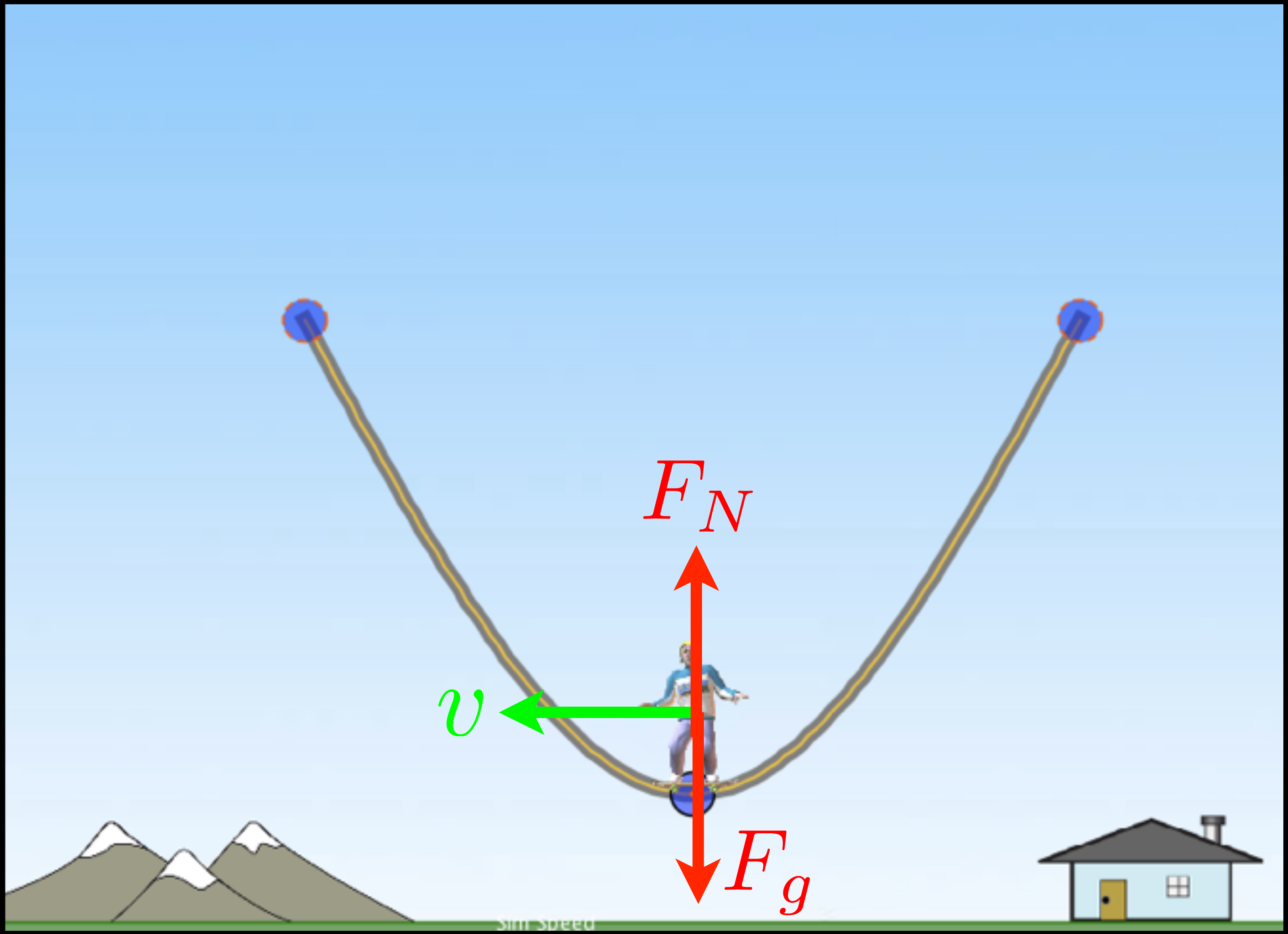


If we move a small distance from a stable equilibrium

Perpendicular forces cancel

Restoring force towards stable equilibrium

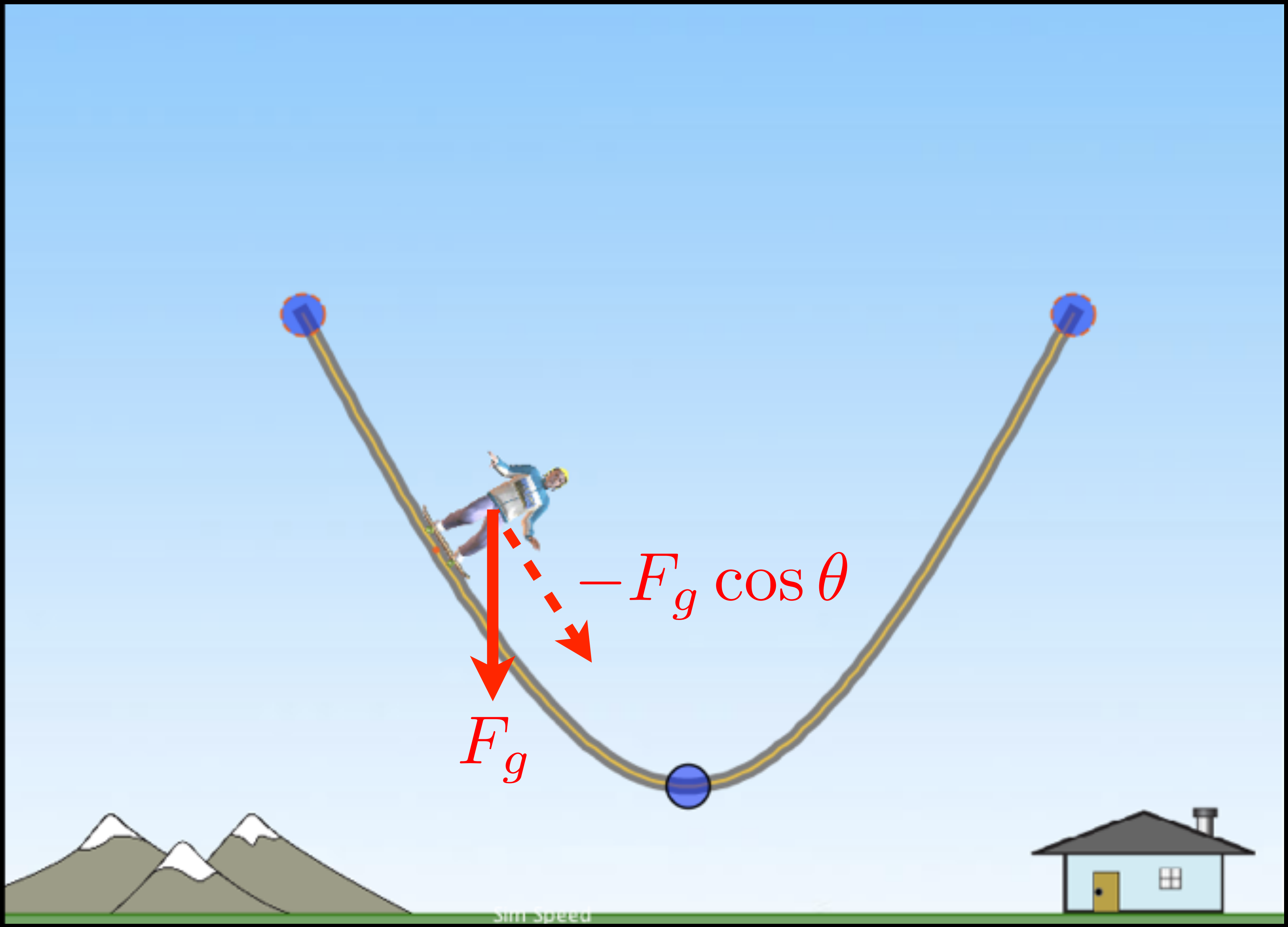
Oscillations



Back at equilibrium: $F_{\text{net}} = 0$

But velocity causes skater to keep moving.

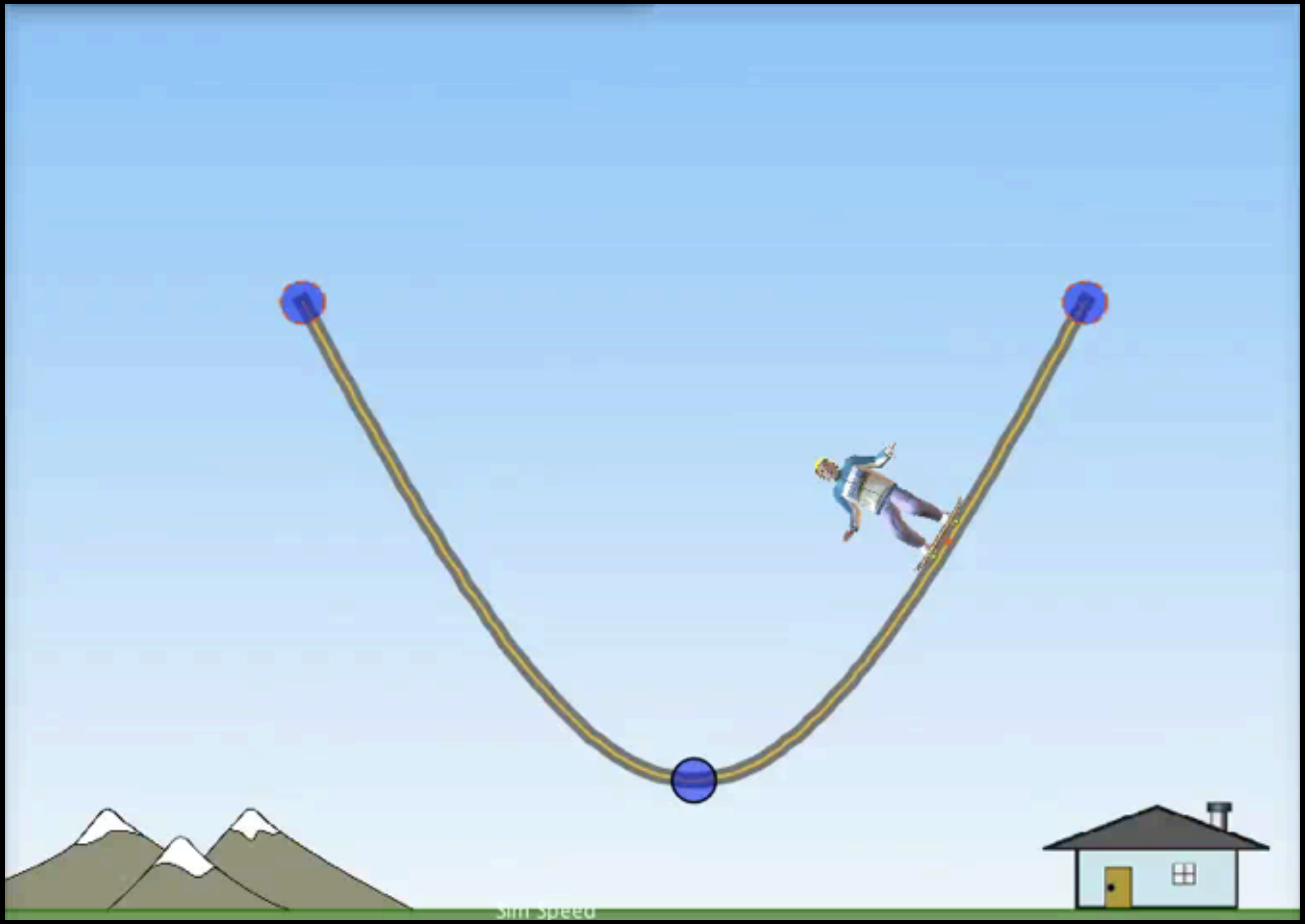
Oscillations



Skater overshoots equilibrium position

Restoring force towards stable equilibrium

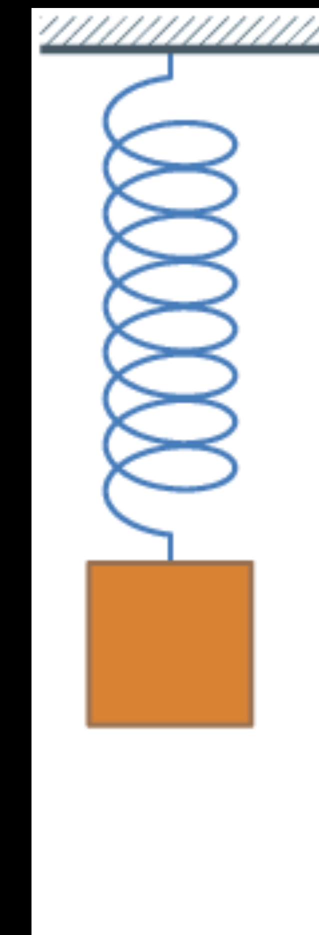
Oscillations

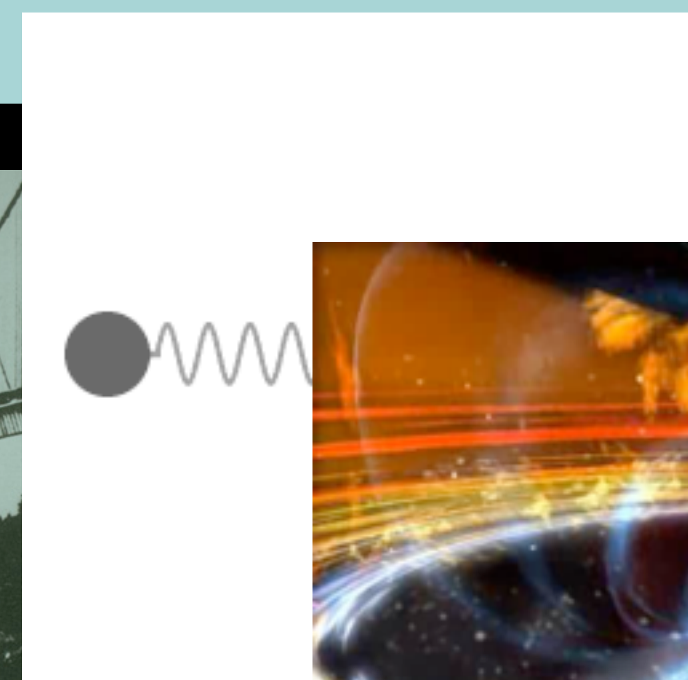


Skater oscillates about the stable equilibrium

Oscillations

Oscillation is very common

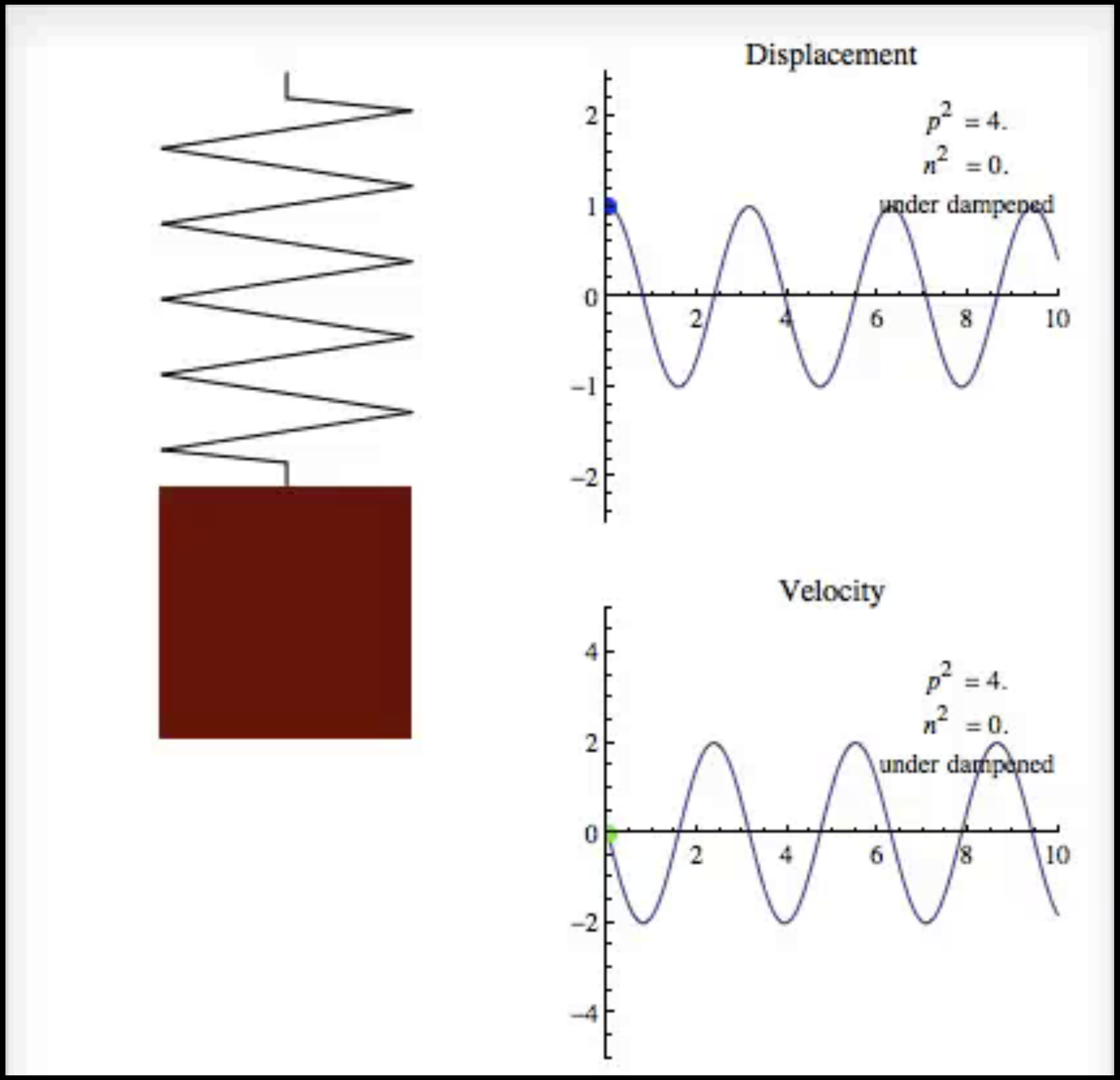


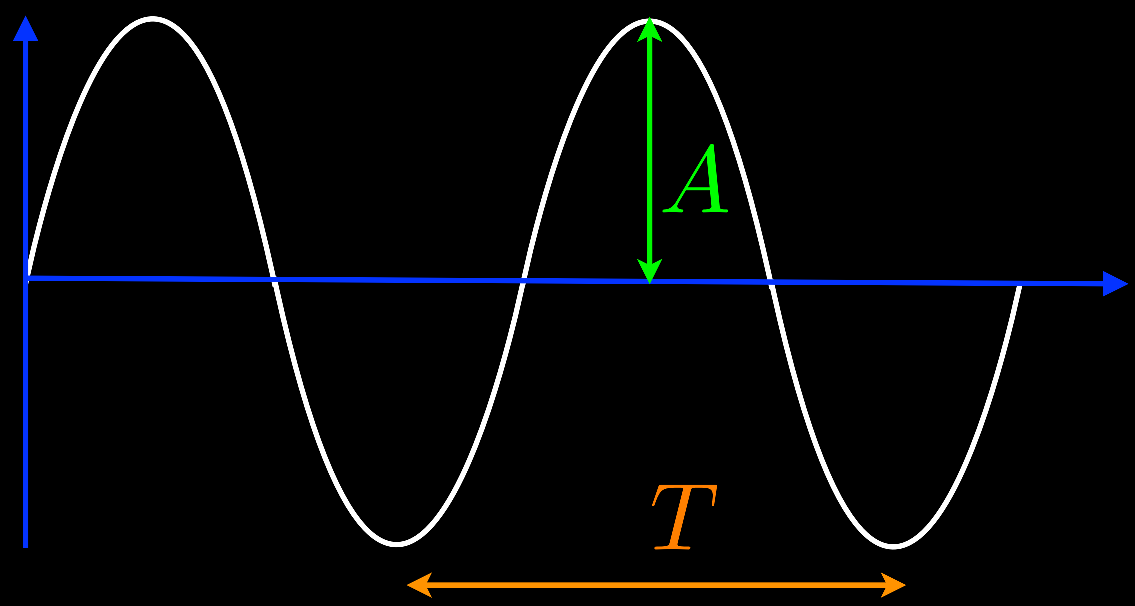


stars

Oscillations

What are the equations for oscillations?





A amplitude
(Max. displacement from equilibrium)

f frequency (Hz) T period

$$f = \frac{1}{T}$$

Oscillations

A violin string oscillates at 440 Hz. What is its period?

(a) $2.0 \times 10^{-3} s$

(b) $2.27 \times 10^{-3} s$

(c) $3.27 \times 10^{-3} s$

(d) $4.40 \times 10^{-3} s$

$$T = \frac{1}{f} = \frac{1}{440\,\mathrm{Hz}}$$

$$= 2.27 \times 10^{-3}\,\mathrm{s}$$

Oscillations

The top of a skyscraper sways back and forth, completing 9 oscillation cycles in 1 minute.

Find the period and frequency of its motion.

(a) $f = 0.0025\text{Hz}, T = 0.00003\text{s}$

(b) $f = 6.7\text{Hz}, T = 0.15\text{s}$

(c) $f = 9.0\text{Hz}, T = 0.1\text{s}$

(d) $f = 0.15\text{Hz}, T = 6.7\text{s}$

Oscillations

Quiz

The top of a skyscraper sways back and forth, completing 9 oscillation cycles in 1 minute.

Find the period and frequency of its motion.

(a) $f = 0.0025\text{Hz}, T = 0.00003\text{s}$

(b) $f = 6.7\text{Hz}, T = 0.15\text{s}$

(c) $f = 9.0\text{Hz}, T = 0.1\text{s}$

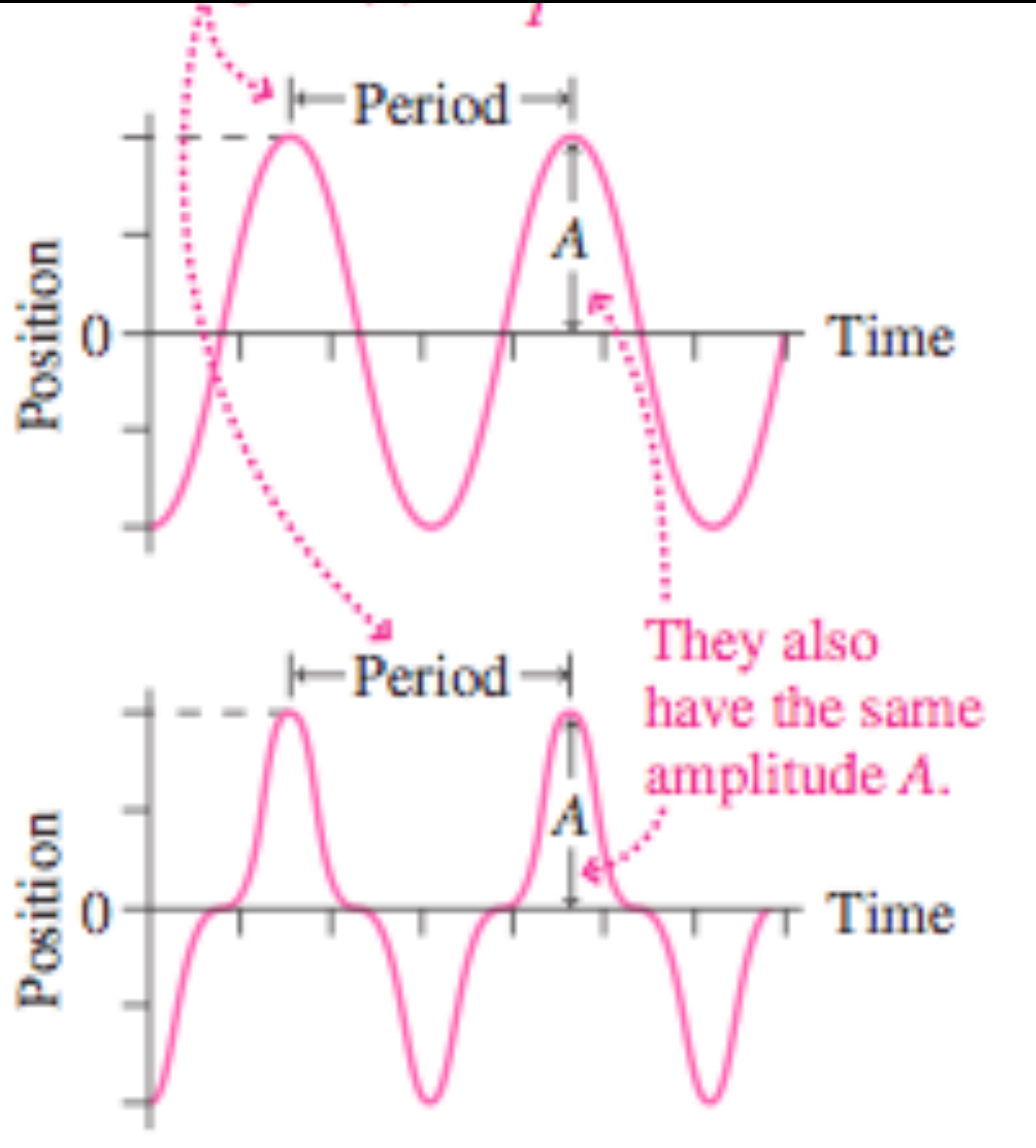
(d) $f = 0.15\text{Hz}, T = 6.7\text{s}$

$$f = \frac{9}{60\,\text{s}} = 0.15\,\text{Hz}$$

$$T = \frac{1}{f} = 6.7\,\text{s}$$

Oscillations

Is this enough? No!



2 oscillations:

Same period, T

Same amplitude, A

But different motion!

Why?

The restoring force (returning to equilibrium) is different

Simple Harmonic Motion

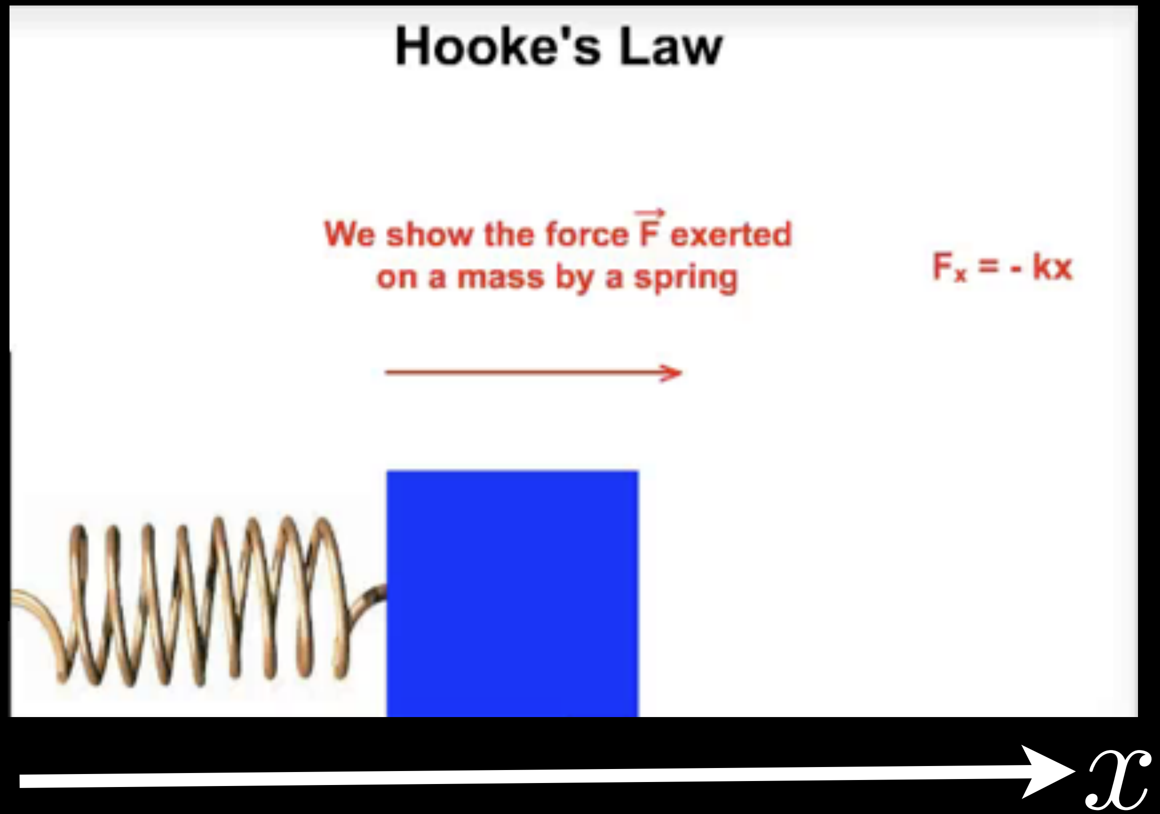
When the restoring force is proportional to displacement ($F \propto \Delta x$)

⟶ simple harmonic motion (SHM)

e.g. spring: $F = -kx$

Newton's 2nd law: $F = ma$

$$m\frac{d^2x}{dt^2} = -kx \qquad \frac{dv}{dt}$$



Simple Harmonic Motion

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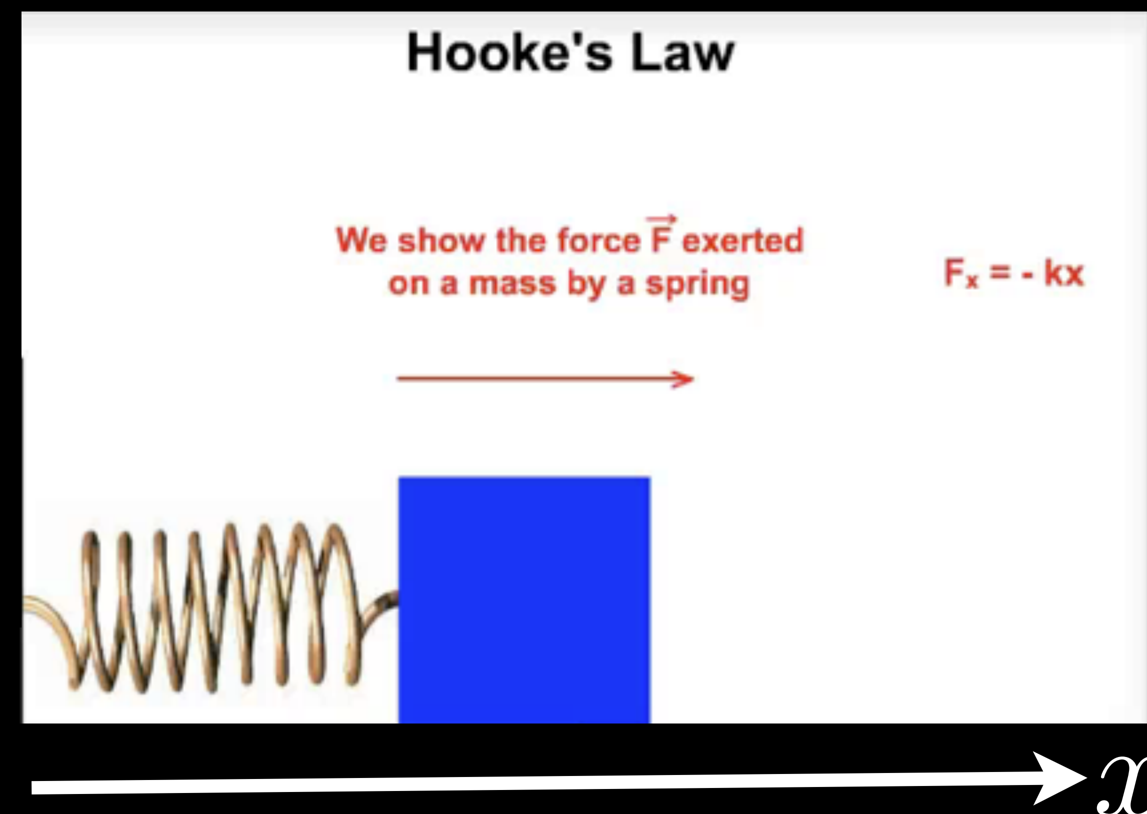
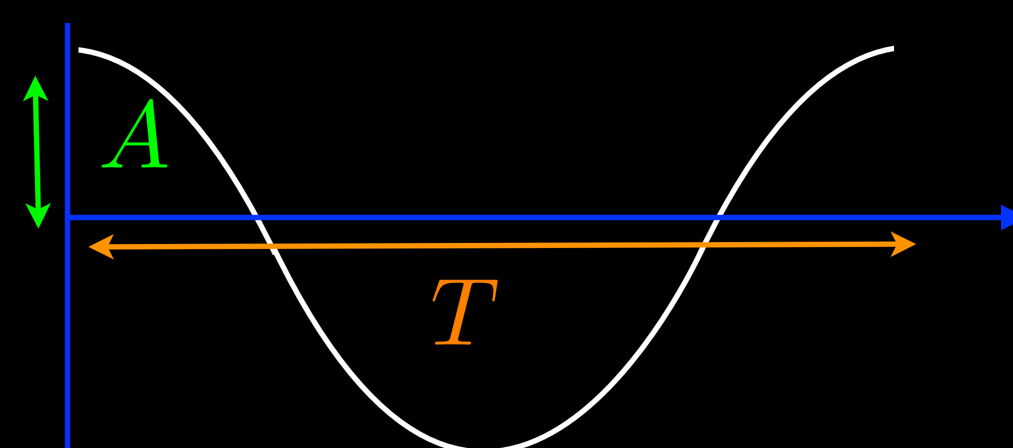


Newton's 2nd law: $F = ma$

$$m\frac{d^2x}{dt^2} = -kx$$

Trial solution: $x(t) = A\cos\omega t$



when $t = T$: $\omega T = 2\pi$ ⟶ $T = \dfrac{2\pi}{w}$

Simple Harmonic Motion

Is this solution right?

$$\frac{dx}{dt} = \frac{d}{dt}(A\cos\omega t) = -A\omega\sin\omega t$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}(-A\omega\sin\omega t) = -A\omega^2\cos\omega t$$

From: $m\frac{d^2x}{dt^2} = -kx$

$$m\left(-A\omega^2\cos\omega t\right) \stackrel{?}{=} -k(A\cos\omega t)$$

True if $w = \sqrt{\frac{k}{m}}$ angular frequency for SHM

Simple Harmonic Motion

From: $T = \frac{2\pi}{w}$ and $w = \sqrt{\frac{k}{m}}$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and:

$$f = \frac{w}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Does this make sense?

Simple Harmonic Motion

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

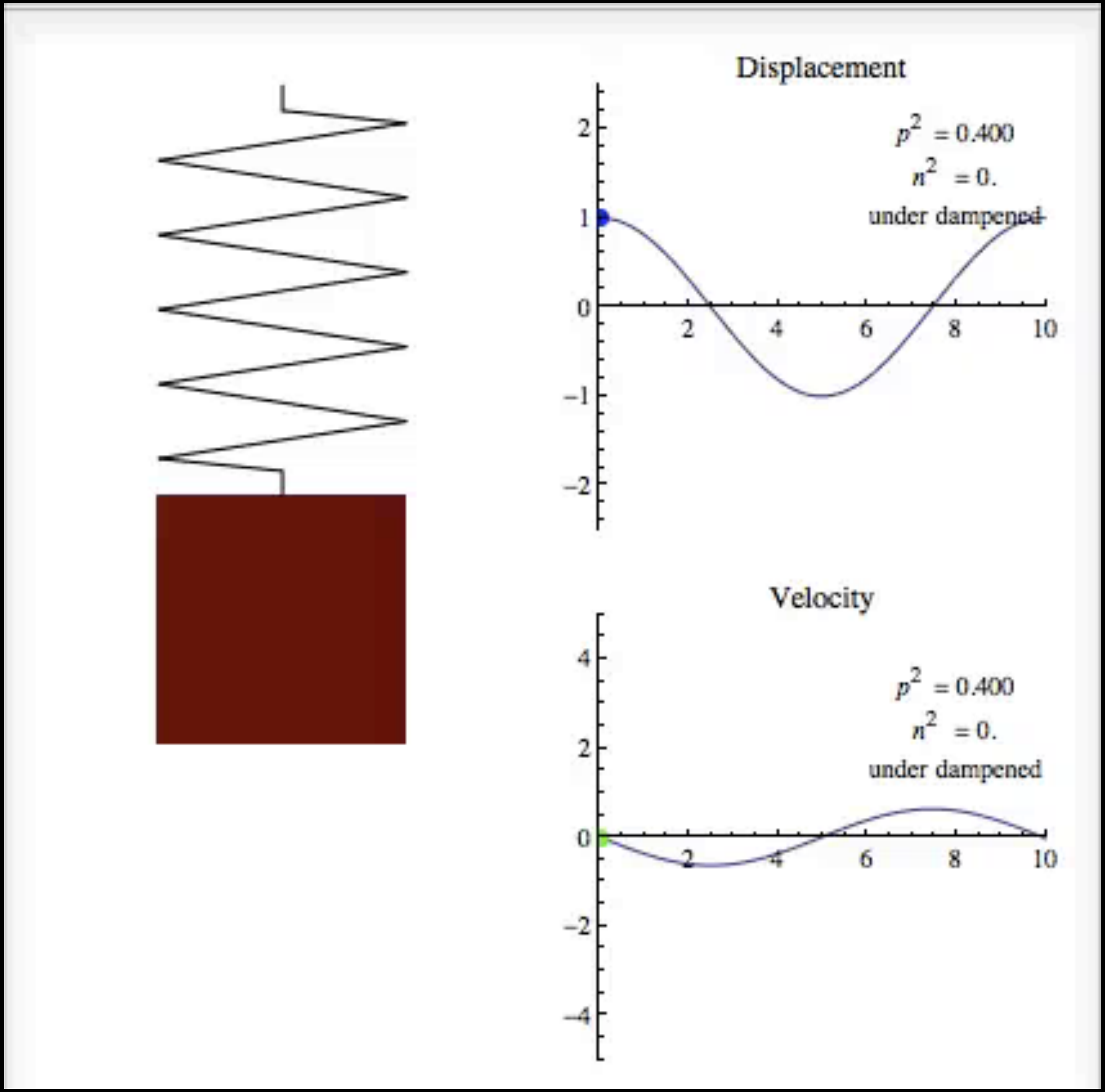
If mass increases,
it is harder to accelerate

↓

slower oscillations

↓

longer T



$m = 10$ $k = 4$

Simple Harmonic Motion

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

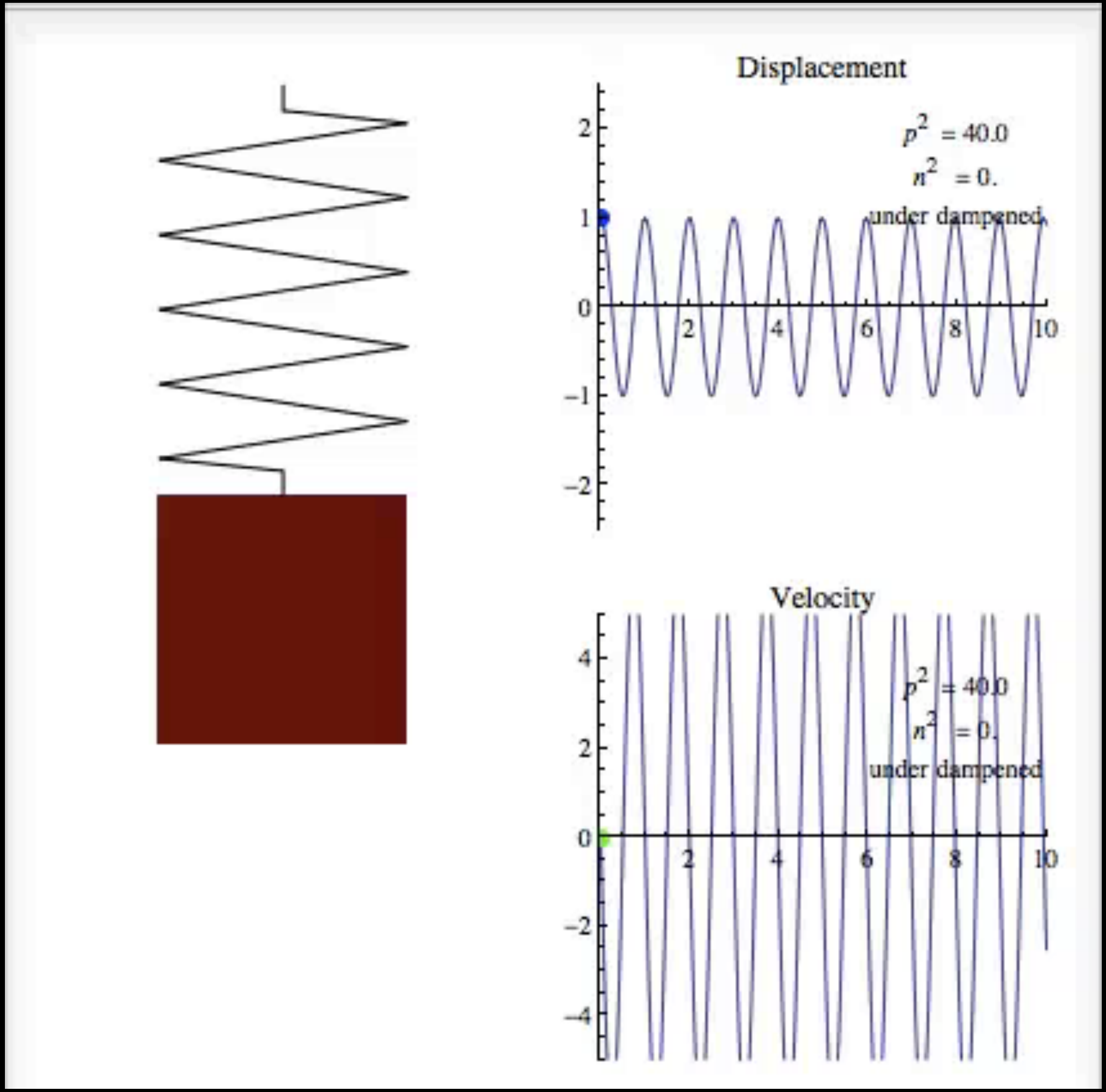
If k increases,
the spring is stiffer,
produces greater force

↓

faster oscillations

↓

shorter T



$m = 1$ $\quad$ $k = 40$

Simple Harmonic Motion

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Frequency **does not** depend on amplitude

because $F \propto \Delta x$ (SHM)

(Amplitude $\uparrow$ $\longrightarrow$ Force $\uparrow$ so frequency = constant)

Simple Harmonic Motion

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Frequency does not depend on amplitude

because $F \propto \Delta x$ (SHM)

When the restoring force, F, is not proportional to Δx

frequency does depend on amplitude.

In many systems, $F \propto \Delta x$ breaks when Δx becomes large.

Therefore, SHM is usually for small oscillations.

Simple Harmonic Motion Example

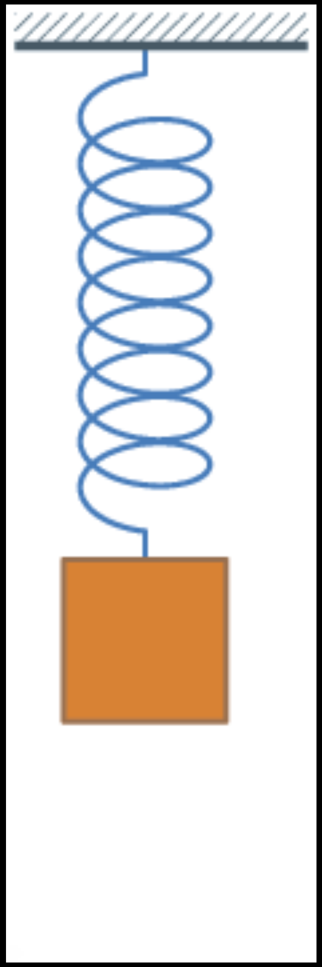
A 200-g mass is attached to a spring of constant k = 5.6 N/m and set into oscillation with amplitude A = 25 cm. Find:

(a) frequency, f

(b) period, T

(c) maximum velocity

(d) maximum force in the spring



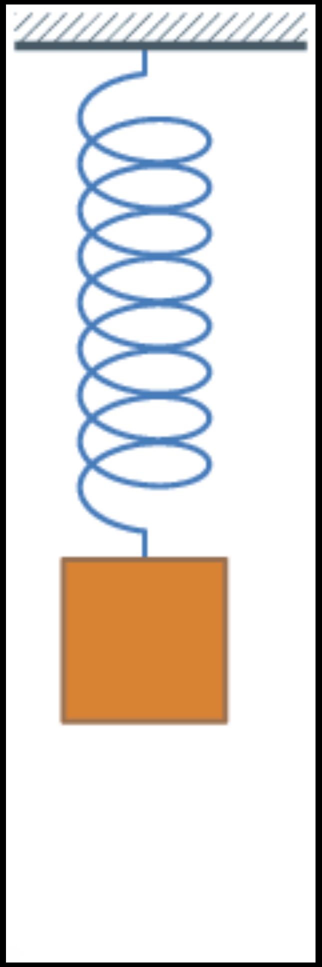
Simple Harmonic Motion

Example

A 200-g mass is attached to a spring of constant k = 5.6 N/m and set into oscillation with amplitude A = 25 cm. Find:

(a) frequency, f

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad = \frac{1}{2\pi}\sqrt{\frac{5.6\mathrm{N/m}}{0.2\mathrm{kg}}} = 0.84\mathrm{Hz}$$



(b) period, T

$$T = \frac{1}{f} \quad = 1.2\mathrm{s}$$

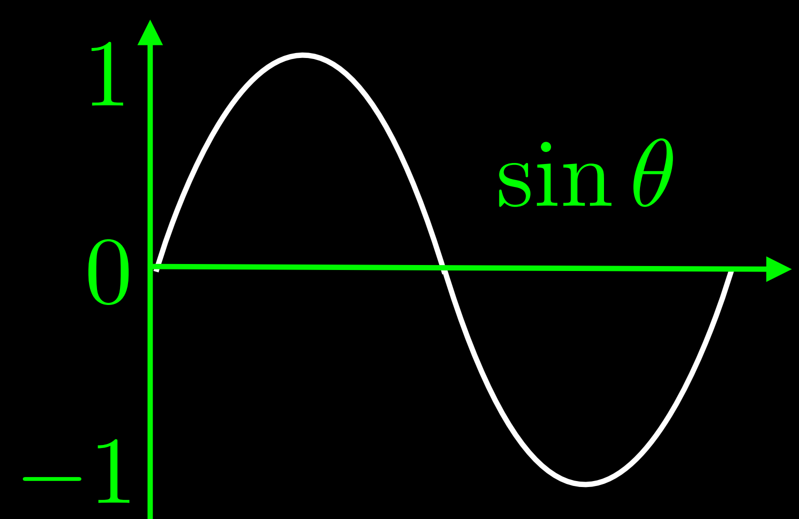
Simple Harmonic Motion

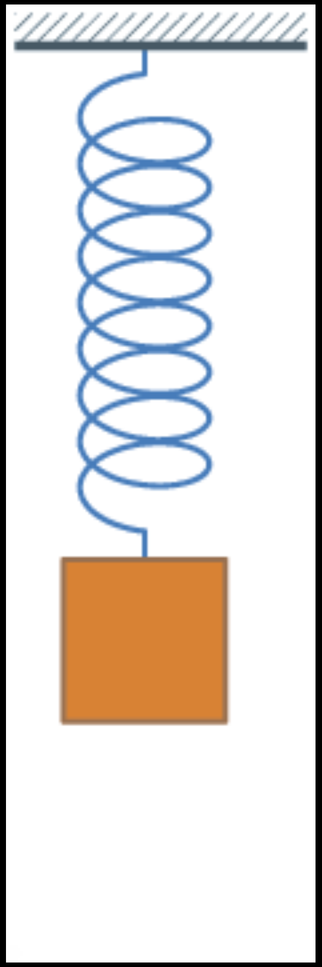
Example

A 200-g mass is attached to a spring of constant k = 5.6 N/m and set into oscillation with amplitude A = 25 cm. Find:

(c) maximum velocity

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}(A\cos\omega t) = -\omega A \sin\omega t$$





maximum when: $\sin\omega t = -1.0$

Therefore: $v_{\mathrm{max}} = \omega A$

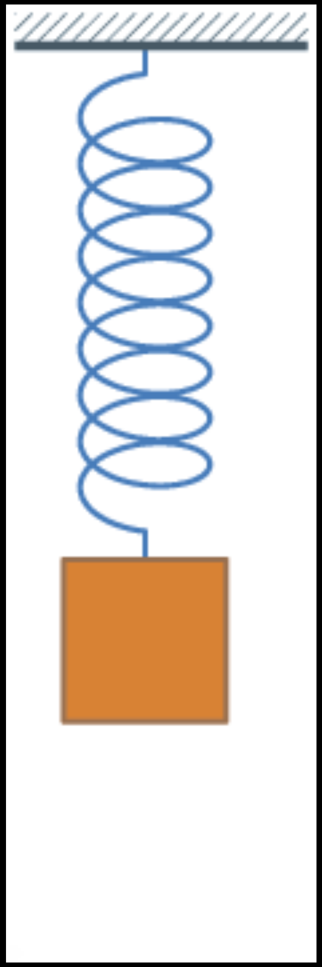
since $w = \sqrt{\frac{k}{m}} = 5.28\mathrm{s}^{-1}$ $\quad v_{\mathrm{max}} = (5.28\mathrm{s}^{-1})(0.25\mathrm{m}) = 1.3\mathrm{m/s}$

Simple Harmonic Motion

Example

A 200-g mass is attached to a spring of constant k = 5.6 N/m and set into oscillation with amplitude A = 25 cm. Find:

(d) maximum force in the spring



$$F_{\max} = ma_{\max}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(-\omega A \sin\omega t) = -\omega^2 A \cos\omega t$$

maximum when: $\cos\omega t = -1$

$$a_{\max} = \omega^2 A$$

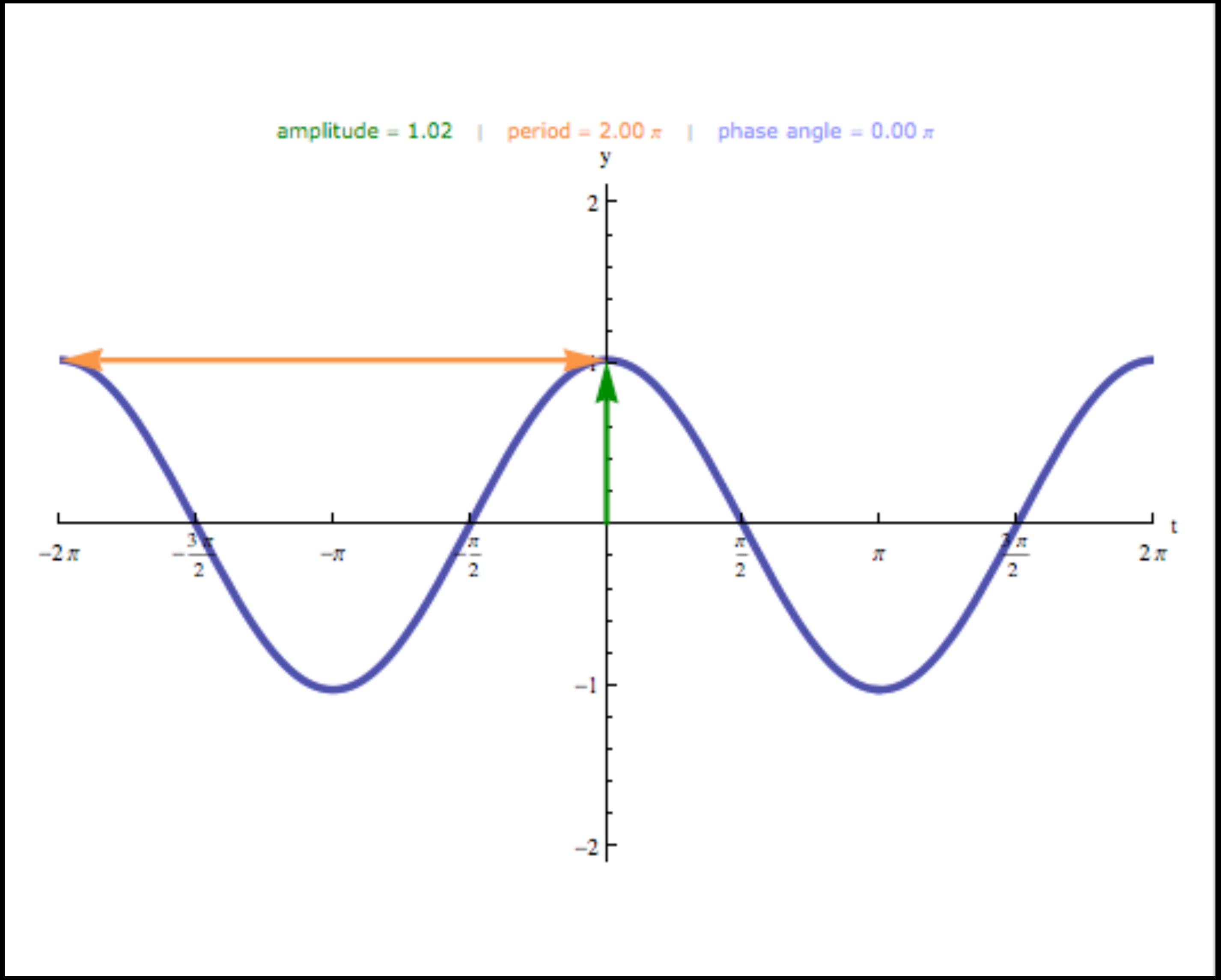
$$F_{\max} = m\omega^2 A = (0.2\mathrm{kg})(5.29\mathrm{s}^{-1})^2(0.25\mathrm{m}) = 1.4\mathrm{N}$$

Simple Harmonic Motion

$x(t) = A\cos\omega t$ is not the only solution to $m\frac{d^2x}{dt^2} = -kx$

We can choose t = 0 anywhere in the oscillation cycle

$\omega t + 0$



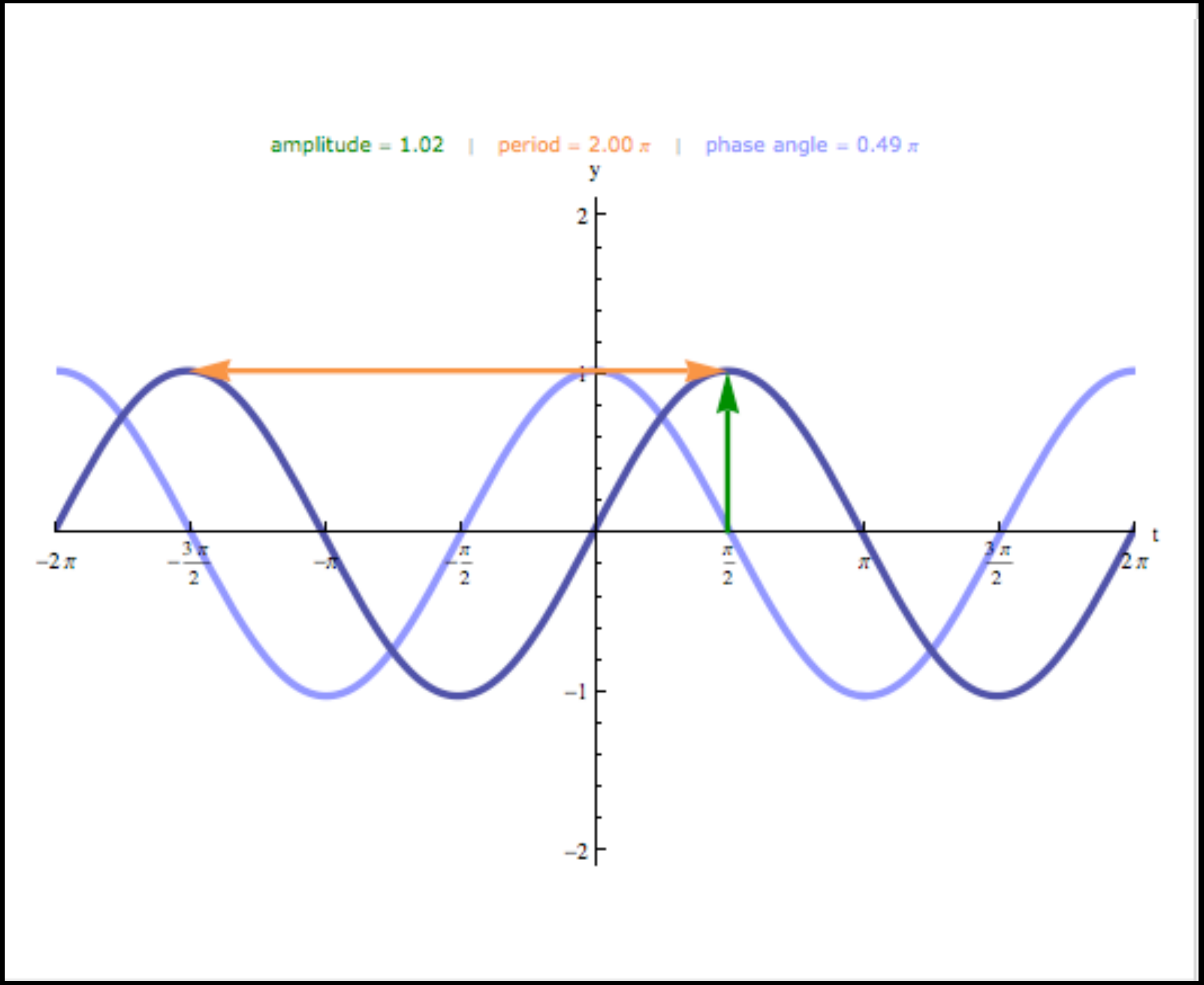
Simple Harmonic Motion

$x(t) = A\cos\omega t$ is not the only solution to $m\frac{d^2x}{dt^2} = -kx$

We can choose t = 0 anywhere in the oscillation cycle

$\omega t + 0$

$\omega t - \frac{\pi}{2}$



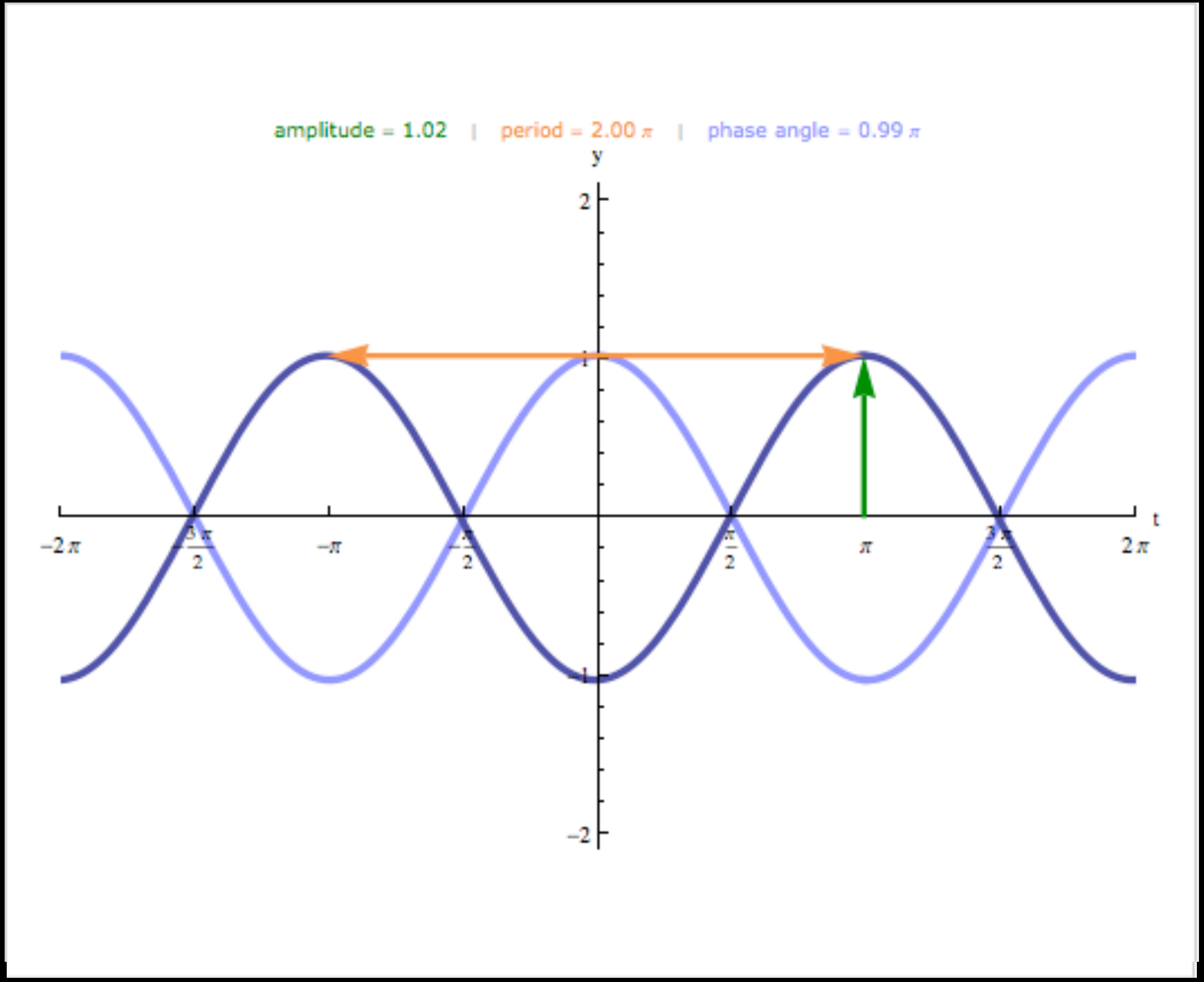
Simple Harmonic Motion

$x(t) = A\cos\omega t$ is not the only solution to $m\dfrac{d^2x}{dt^2} = -kx$

We can choose t = 0 anywhere in the oscillation cycle

$\omega t + 0$

$\omega t - \pi$



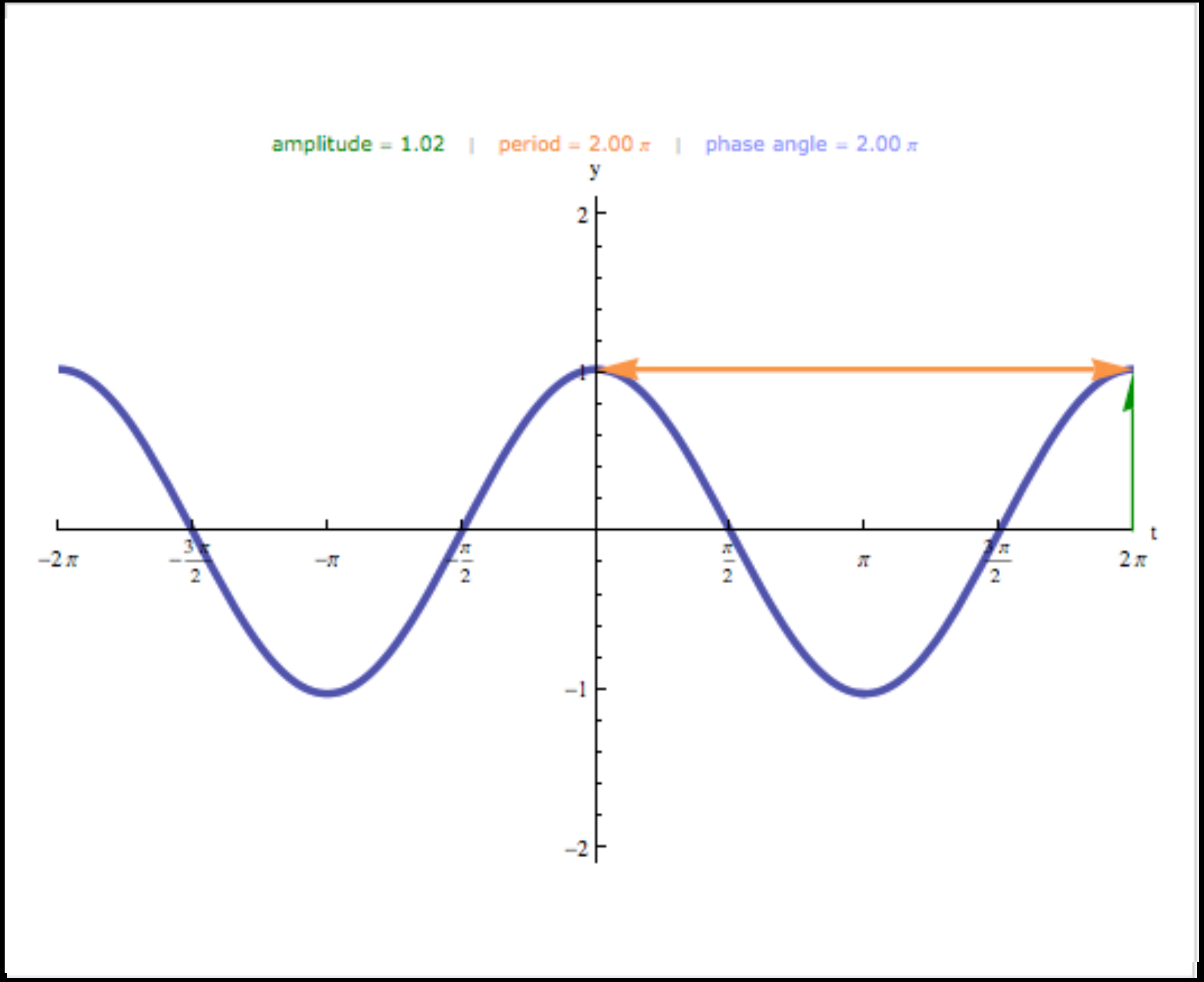
Simple Harmonic Motion

$x(t) = A\cos\omega t$ is not the only solution to $m\frac{d^2x}{dt^2} = -kx$

We can choose t = 0 anywhere in the oscillation cycle

$\omega t + 0$

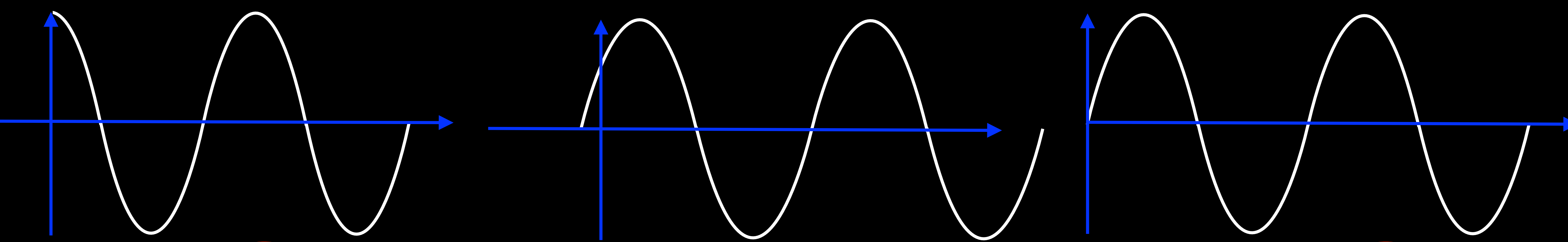
$\omega t - 2\pi$



Simple Harmonic Motion

$x(t) = A\cos\omega t$ is not the only solution to $m\dfrac{d^2x}{dt^2} = -kx$

We can choose t = 0 anywhere in the oscillation cycle



$\omega t + 0$ $\qquad$ $\omega t - \dfrac{\pi}{4}$ $\qquad$ $\omega t - \dfrac{\pi}{2}$

ϕ phase constant

ϕ shifts the curve left or right, but does not change A or f.

$$x(t) = A\cos(\omega t + \phi)$$

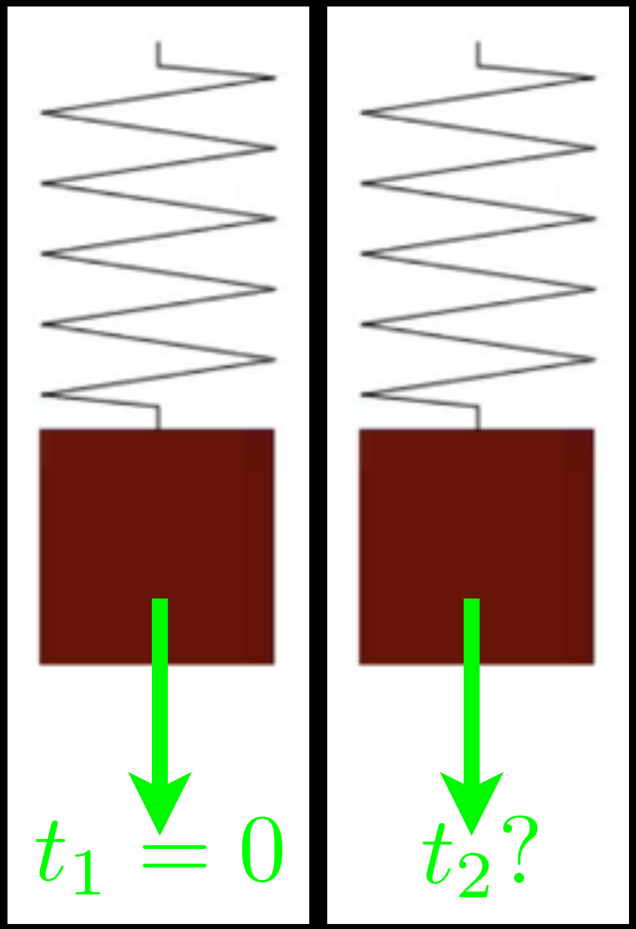
Simple Harmonic Motion

Example

2 identical mass-spring systems consist of 430-g masses on springs of constant k = 2.2 N/m.

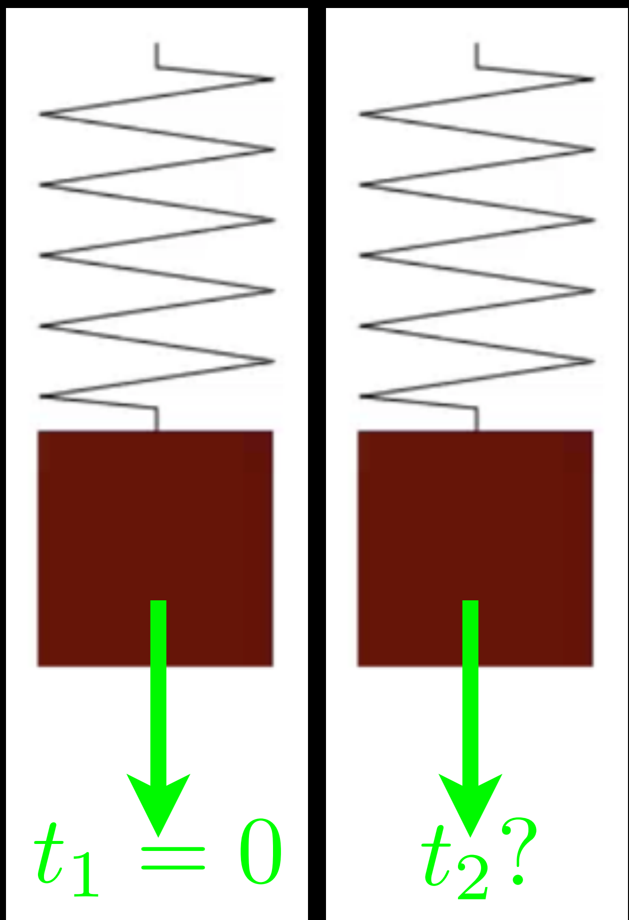
Both are displaced from equilibrium, the 1st released at t = 0.

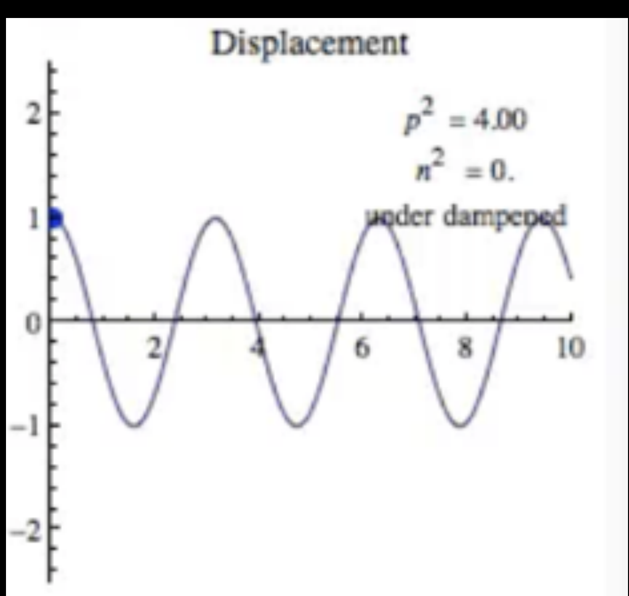
When should the second be released so their oscillations differ in phase by $\pi/2$?



Simple Harmonic Motion

Example





since: $\omega = \sqrt{\frac{k}{m}}$, $\omega_1 = \omega_2$

$$x_1(t) = A\cos(\omega t_1 + \phi_1) \quad x_2(t) = A\cos(\omega t_2 + \phi_2)$$

Springs start at max. displacement:

$$\cos(\omega t_1 + \phi_1) = \cos(\omega t_2 + \phi_2) = 1$$

Since $\cos(0) = 1$:

$$\omega t_1 + \phi_1 = \omega t_2 + \phi_2 = 0$$

0

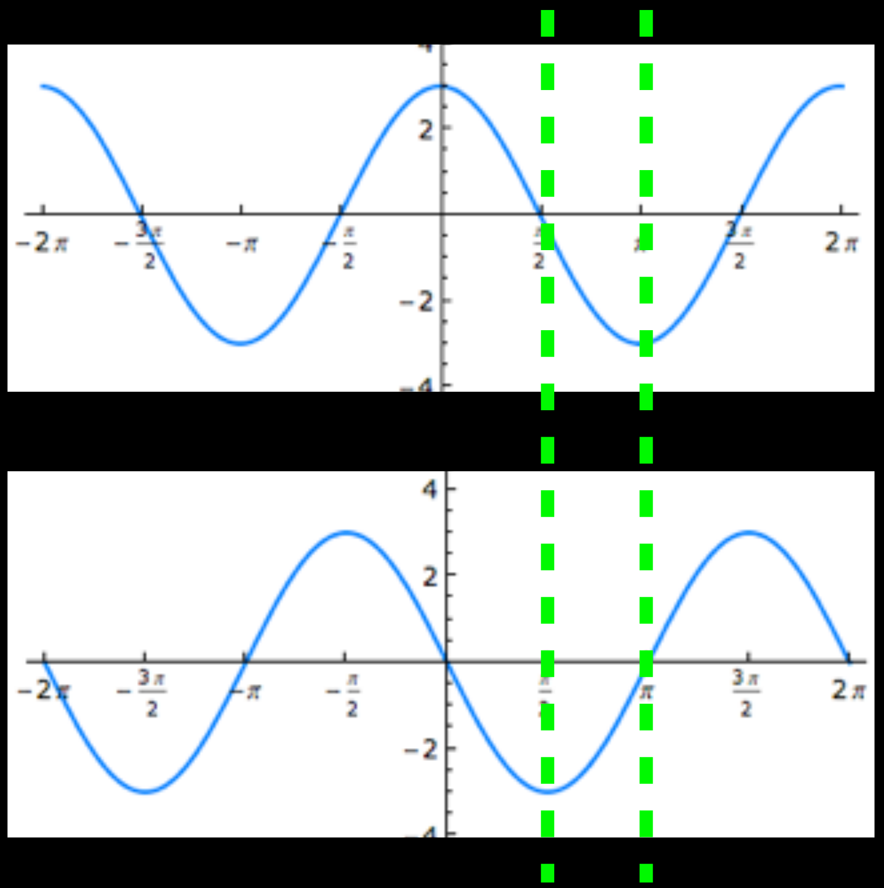
$$\phi_1 = 0 \quad t_2 = -\frac{\phi_2}{\omega}$$

$$\Delta\phi = \frac{\pi}{2} = \phi_1 - \phi_2 = -\phi_2$$

$$t_2 = \frac{\pi}{2\omega} = \frac{\pi}{2\sqrt{k/m}} = 0.70\text{s}$$

Simple Harmonic Motion

Velocity $v(t) = \frac{dx}{dt} = \frac{d}{dt}(A\cos\omega t) = -\omega A \sin\omega t$



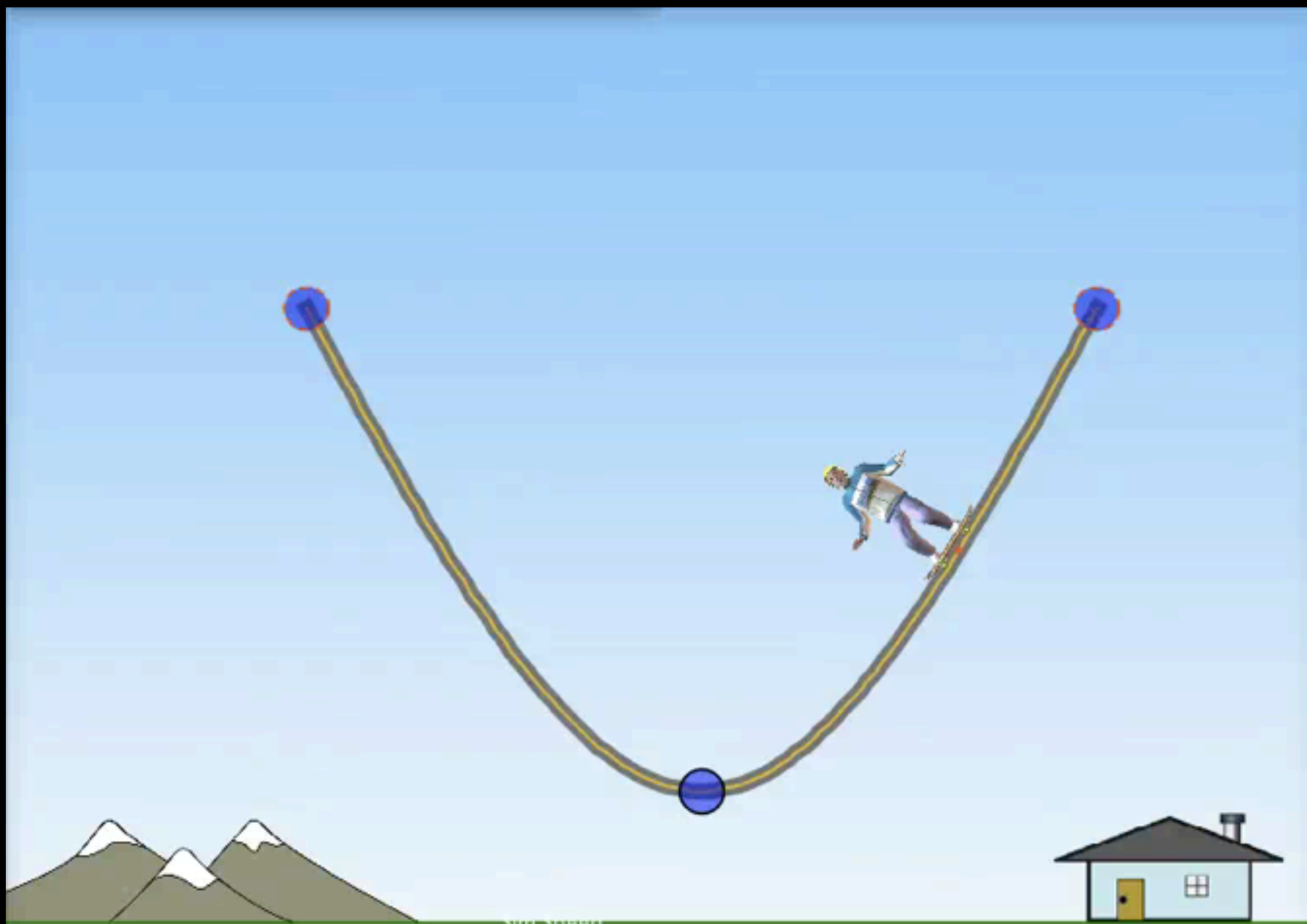
position $x(t) = A\cos\omega t$

velocity $v(t) = -\omega A \sin\omega t$

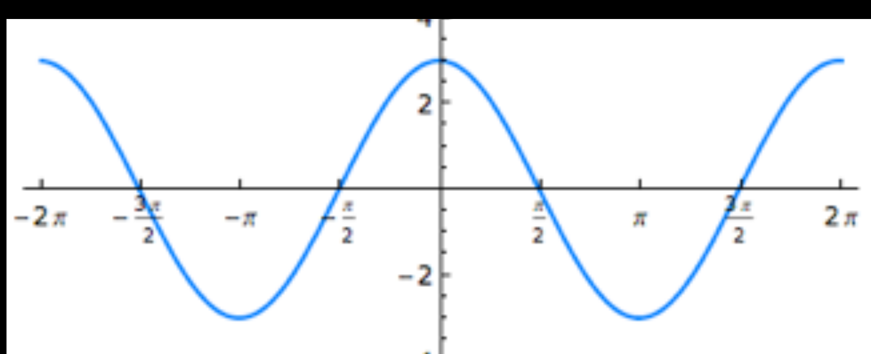
max & min displacement, x, when v = 0

max & min velocity, v, when x = 0

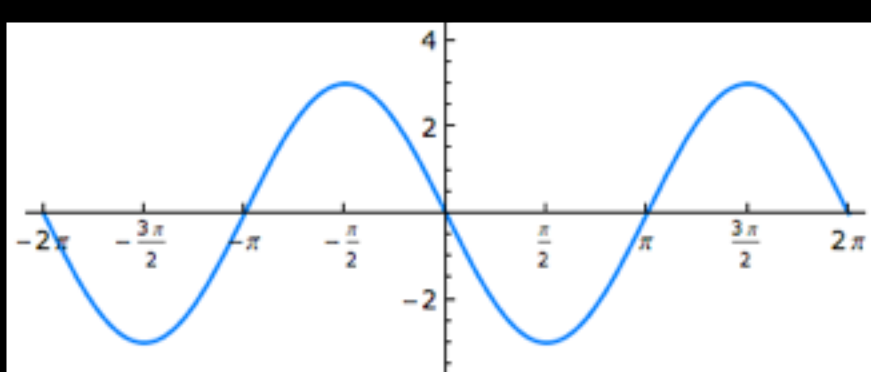
Simple Harmonic Motion



We have already seen this for oscillations



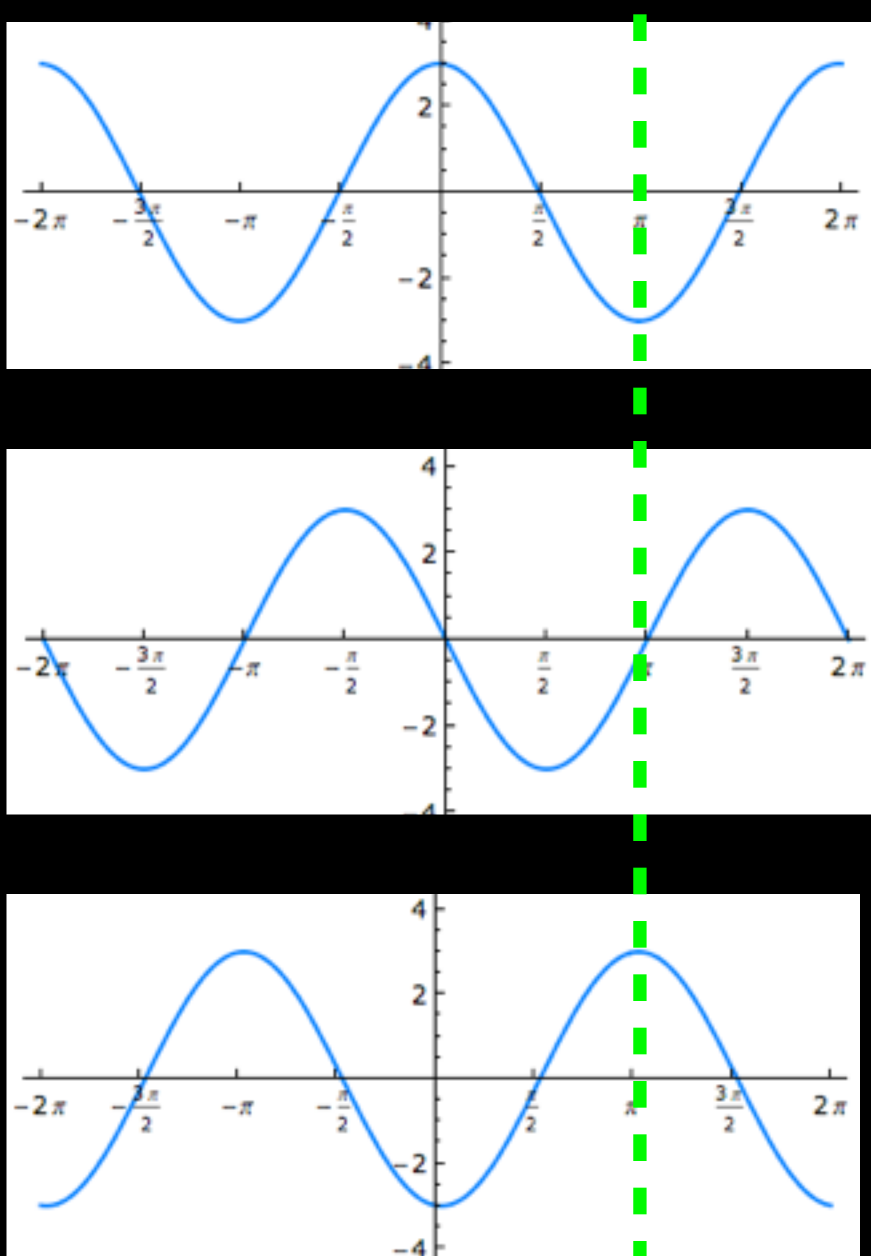
position $$x(t) = A \cos \omega t$$



velocity $$v(t) = -\omega A \sin \omega t$$

Simple Harmonic Motion

Acceleration $a(t) = \frac{dv}{dt} = \frac{d}{dt}(-\omega A \sin \omega t) = -\omega^2 A \cos \omega t$



position $x(t) = A \cos \omega t$

velocity $v(t) = -\omega A \sin \omega t$

acceleration $a(t) = -\omega^2 A \cos \omega t$

max & min acceleration, a, when v = 0

Simple Harmonic Motion

position

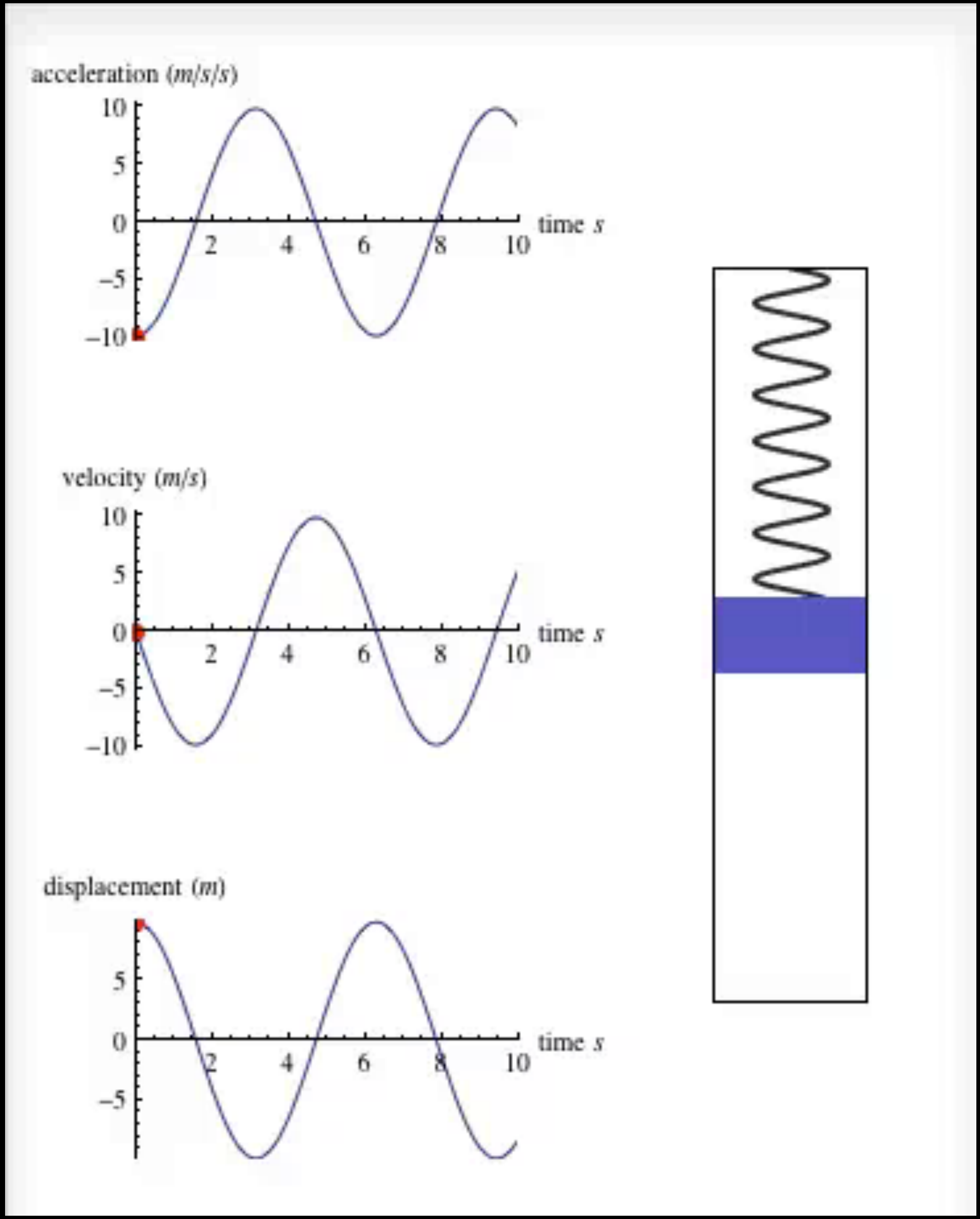
$$x(t) = A\cos\omega t$$

velocity

$$v(t) = -\omega A\sin\omega t$$

acceleration

$$a(t) = -\omega^2 A\cos\omega t$$



Simple Harmonic Motion

Quiz

A particle undergoes SHM with max speed 1.4m/s and max acceleration 3.1m/s^2 .

Find (a) angular frequency (b) period (c) amplitude

(a) $\omega = 0.45\text{rad/s}, T = 13.9\text{s}, A = 6.89\text{m}$

(b) $\omega = 2.21\text{rad/s}, T = 2.8\text{s}, A = 0.63\text{m}$

(c) $\omega = 4.34\text{rad/s}, T = 1.45\text{s}, A = 0.16\text{m}$

(d) $\omega = 4.34\text{rad/s}, T = 1.75\text{s}, A = 0.16\text{m}$

Simple Harmonic Motion

Quiz

A particle undergoes SHM with max speed $1.4\mathrm{m/s}$ and max acceleration $3.1\mathrm{m/s^2}$.

Find (a) angular frequency (b) period (c) amplitude

$$v_{\max} = \omega A \qquad a_{\max} = \omega^2 A$$

(a) $$\omega = \frac{a_{\max}}{v_{\max}} = \frac{3.1\,\mathrm{m/s^2}}{1.4\,\mathrm{m/s}} = 2.21\,\mathrm{rad/s}$$

(b) $$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{2.21\,\mathrm{rad/s}} = 2.8\,\mathrm{s}$$

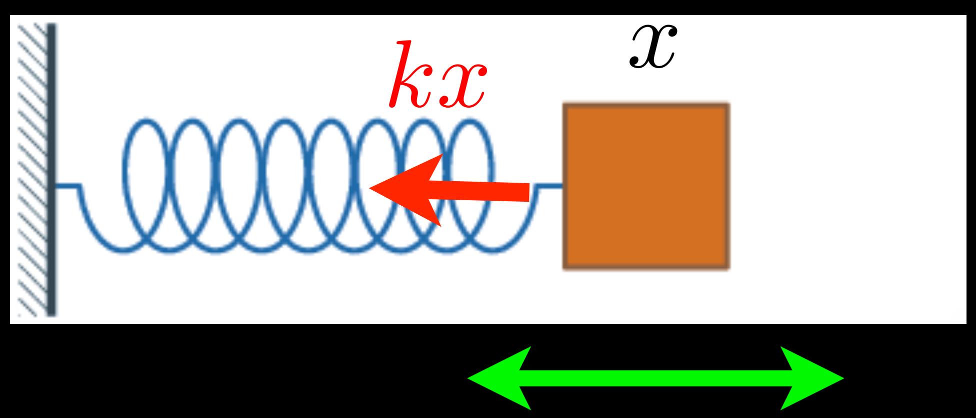
(c) $$A = \frac{a_{\max}}{\omega^2} = a_{\max}\left(\frac{v_{\max}}{a_{\max}}\right)^2 = \frac{v_{\max}^2}{a_{\max}} = \frac{(1.4\,\mathrm{m/s})^2}{3.1\,\mathrm{m/s^2}} = 0.63\,\mathrm{m}$$

Simple Harmonic Motion

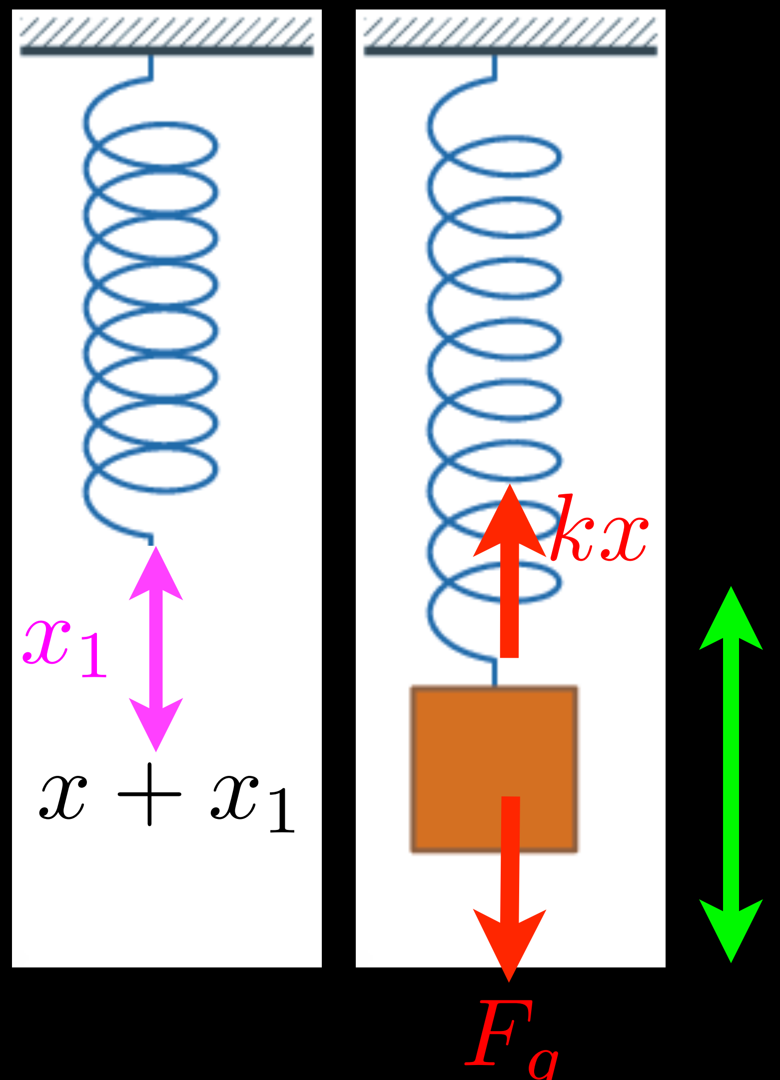
Types of SHM

(1) Spring - Mass systems

horizontal



vertical



Since F_g does not depend on Δx , it only changes position of the equilibrium.

Simple Harmonic Motion

Quiz

A mass is attached to a vertical spring, which oscillates.

At the high point of the oscillation, the spring is in the original, unstretched equilibrium position it had before the mass was attached.

The low point is 5.8 cm below this.

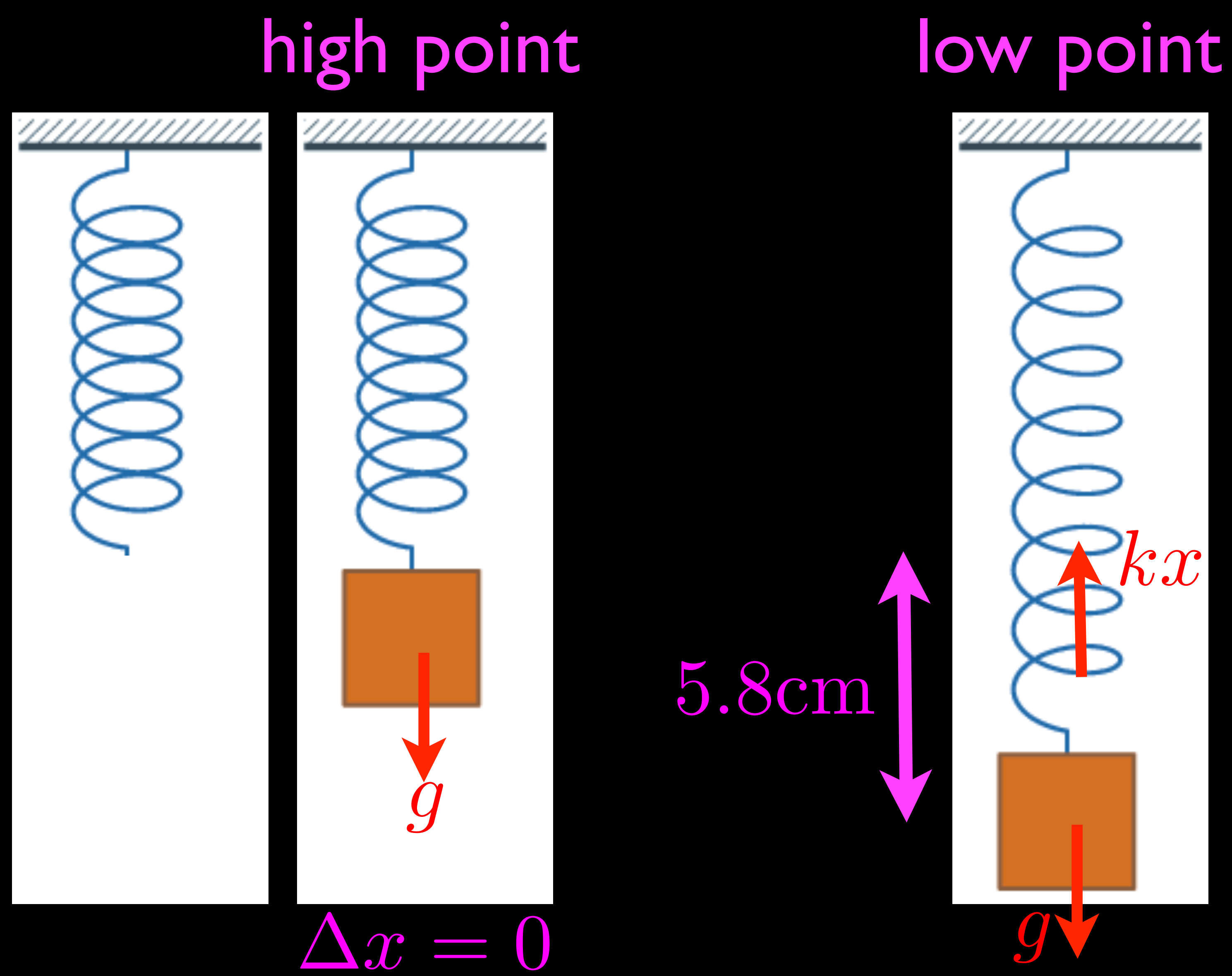
Find the period, T

(a) 0.34 s

(b) 0.48 s

(c) 0.018 s

(d) 0.037 s



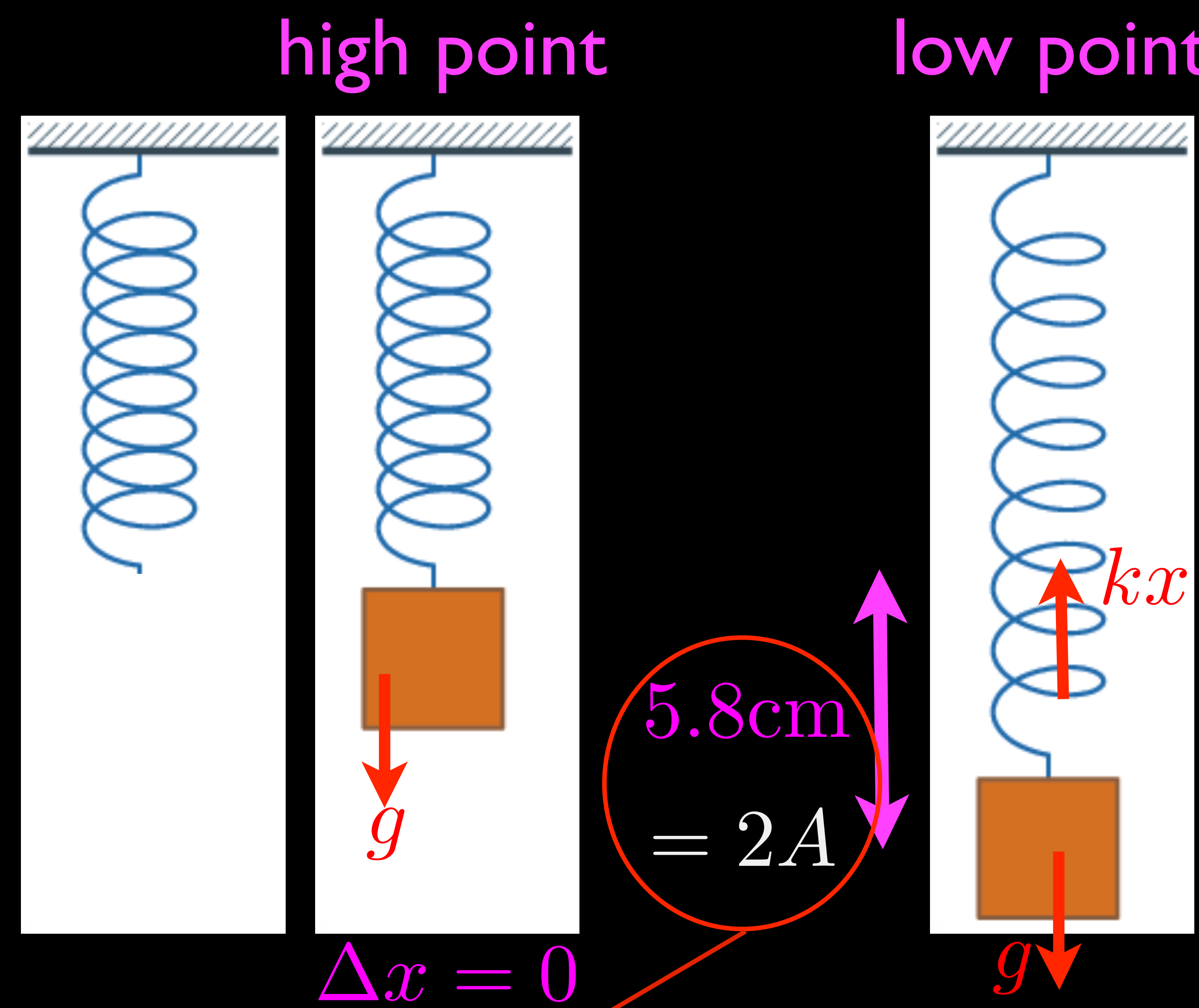
Simple Harmonic Motion

At highest point:

$\Delta x = 0$ no spring force: $a = g$

Max acceleration (since $a_{\max}$ is at $x_{\max}$)

$$a_{\max} = g = \omega^2 A$$



$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{A}{g}}$$

$$= 2\pi\sqrt{\frac{0.029\,\text{m}}{9.8\text{m/s}^2}} = 0.34\,\text{s}$$

Simple Harmonic Motion

(2) Torsional oscillator

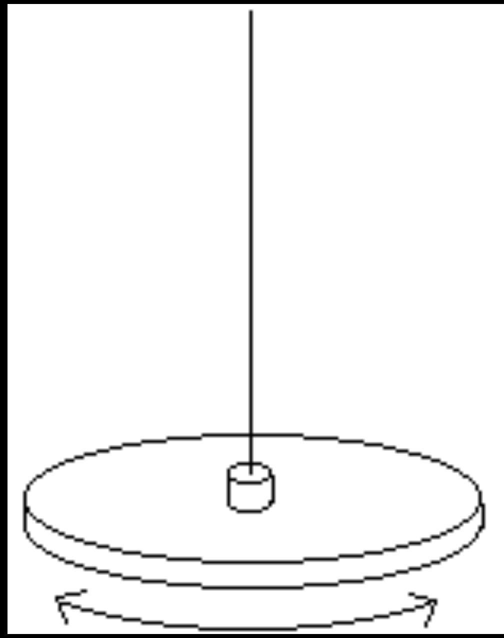
Use rotational co-ordinates:

angular displacement, θ

restoring torque, τ

torsional constant, κ

$$\tau = -\kappa\theta$$



From Newton's 2nd law for rotation, $\tau = I\alpha$:

$$I\frac{d^2\theta}{dt^2} = -\kappa\theta \quad \longleftrightarrow \quad m\frac{d^2x}{dt^2} = -kx$$

Therefore:

$$\theta(t) = A\cos\omega t\,, \quad \omega = \sqrt{\frac{\kappa}{I}}$$

Simple Harmonic Motion

Quiz

A 640-g hollow ball 21 cm in diameter is suspended by a wire and undergoing torsional oscillations at 0.78 Hz.

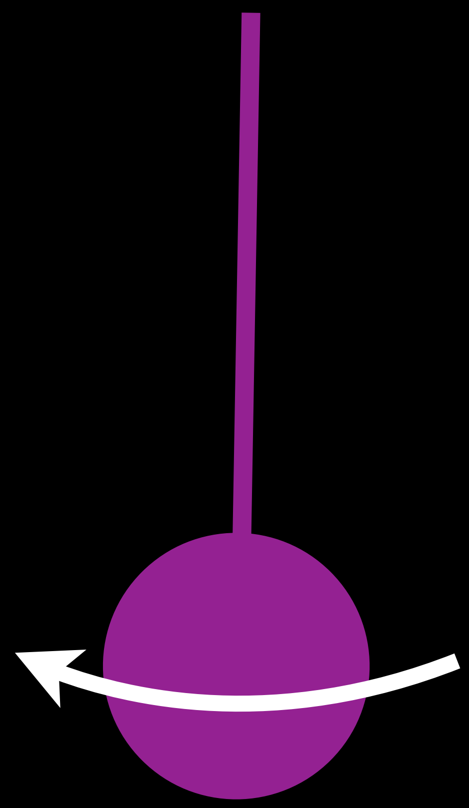
Find the torsional constant of the wire.

(a) $0.003 \mathrm{Nmrad}^2$

(b) $1.8 \mathrm{Nmrad}^2$

(c) $0.11 \mathrm{Nmrad}^2$

(d) $1.2 \mathrm{Nmrad}^2$



$$I = \frac{2MR^2}{3}$$

Simple Harmonic Motion

Quiz

A 640-g hollow ball 21 cm in diameter is suspended by a wire and undergoing torsional oscillations at 0.78 Hz.

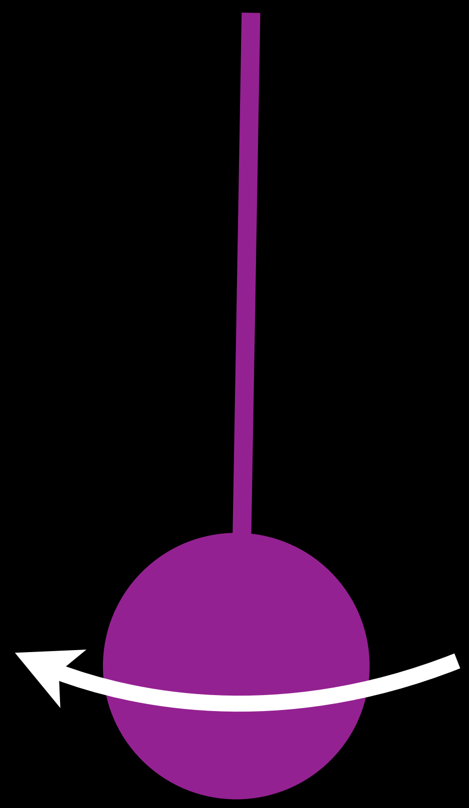
Find the torsional constant of the wire.

$$\kappa = \omega^2 I = \frac{(2\pi f)^2 2MR^2}{3}$$

$$= \frac{(2\pi f)^2 2MD^2}{12}$$

$$= \frac{(2\pi \times 0.78\,\mathrm{s}^{-1})^2 (0.64\,\mathrm{kg})(0.21\,\mathrm{m})^2}{6}$$

$$= 0.11\,\mathrm{Nmrad}^2$$

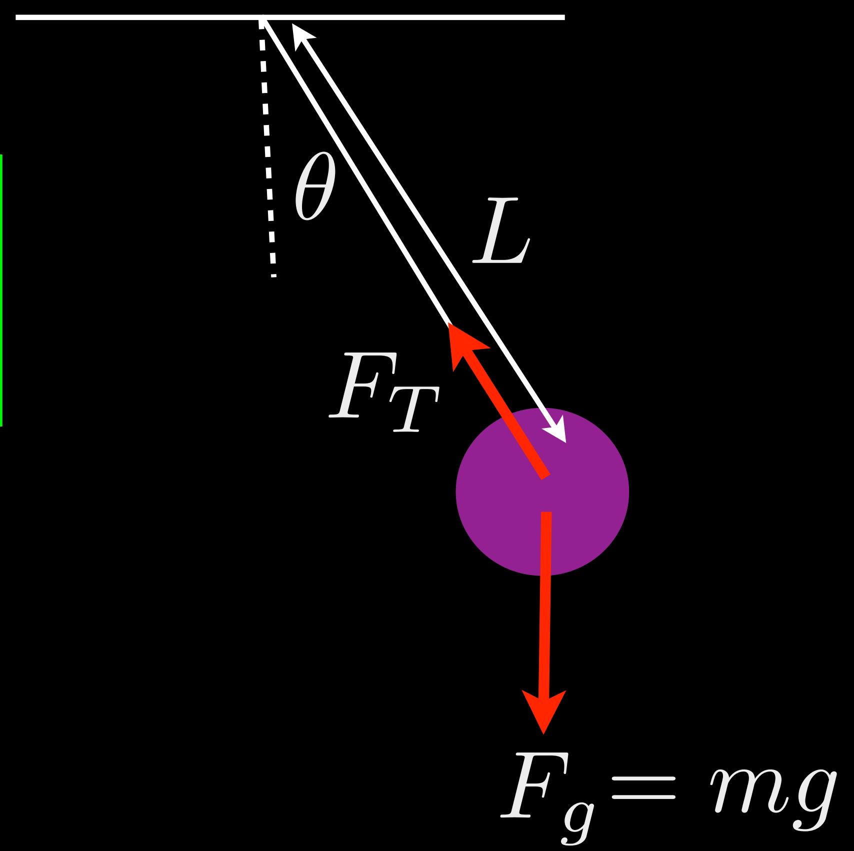


$$I = \frac{2MR^2}{3}$$

Simple Harmonic Motion

(3) Simple pendulum

string mass ≈ 0
ball's volume ≈ 0 (point mass)



$$I\frac{d^2\theta}{dt^2} = -mgL\sin\theta$$

Force not directly proportional to θ → not SHM

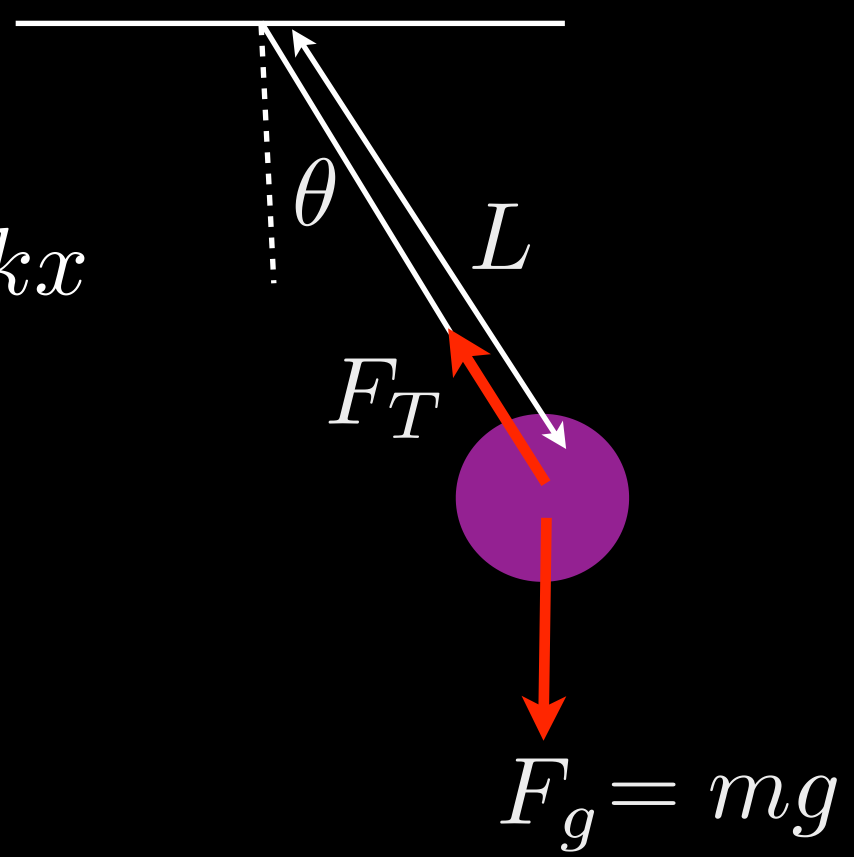
But for small oscillations $\sin\theta \approx \theta$

$$I\frac{d^2\theta}{dt^2} = -mgL\theta$$

Simple Harmonic Motion

(3) Simple pendulum

$$I\frac{d^2\theta}{dt^2} = -mgL\theta \quad \longleftrightarrow \quad m\frac{d^2x}{dt^2} = -kx$$



$$\omega = \sqrt{\frac{mgL}{I}}$$

Simple pendulum, point mass: $I = mL^2$

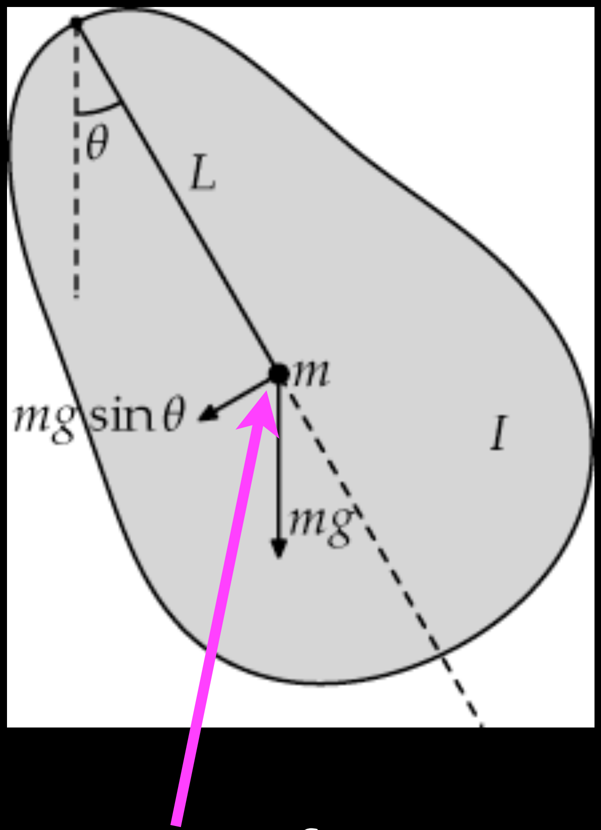
$$\omega = \sqrt{\frac{mgL}{mL^2}} = \sqrt{\frac{g}{L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

Simple Harmonic Motion

(4) Physical Pendulum

Not a point mass e.g. a leg, punching bag etc



$$\omega = \sqrt{\frac{mgL}{I}}$$ is still true

but $I \neq mL^2$

and L = distance to centre of gravity

centre of gravity

Simple Harmonic Motion

Quiz

A physics student, bored by a lecture on SHM, idly picks up his pencil (mass 9.2 g, length 17 cm) by the tip with frictionless fingers.

He allows it to swing back and forth.

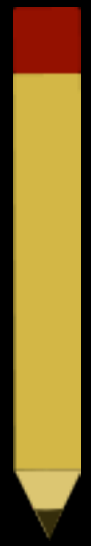
If the pencil completes 6279 full cycles during the lecture, how long is the lecture?

(a) 60 min

(b) 90 min

(c) 40 min

(d) 50 min



$$I = \frac{mR^2}{3}$$

Assume center of gravity is at the end of the pencil.

Simple Harmonic Motion

Quiz

A physics student, bored by a lecture on SHM, idly picks up his pencil (mass 9.2 g, length 17 cm) by the tip with frictionless fingers.

He allows it to swing back and forth.

$$I = \frac{mR^2}{3}$$

If the pencil completes 6279 full cycles during the lecture, how long is the lecture?

(a) 60 min

(b) 90 min

(c) 40 min

(d) 50 min

one cycle:

$$T = 2\pi\sqrt{\frac{I}{mgL}} = 2\pi\sqrt{\frac{mL^2}{3mgL}} = 2\pi\sqrt{\frac{L}{3g}}$$

$$t = (6279)\left(2\pi\sqrt{\frac{0.17\,\mathrm{m}}{3(9.81\,\mathrm{m/s^2}}}\right) = 3\times10^3\,\mathrm{s}$$

$$= 50\,\mathrm{min}$$