Essential Physics I

英語で物理学の

エッセンス I

Lecture 8:   06-06-16

News

No lecture next week!

JUNE 2016						
SUN	MON	TUE	WED	THU	FRI	SAT
			1	2	3	4
5	6	7	8	9	10	11
12	⊗ 13	14	15	16	17	18
19	✓ 20	21	22	✓ 23	24	25
26	27	28	29	30		

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⊗ 13/6/2016

６月１３日

BUT!

There IS a lecture

✓ 6/20/2016

6月20日

AND

✓ 6/23/2016

6月23日

@18:00

News

AND

There IS a lecture

✓ 4/7/2016

７月４日

2016 JULY

SUN	MON	TUE	WED	THU	FRI	SAT
					1	2
3	✓	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

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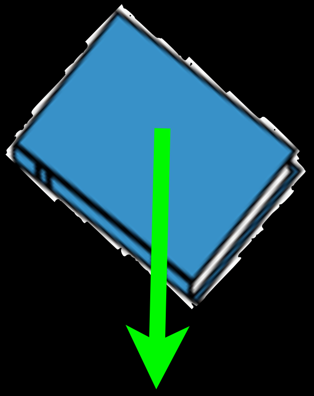
Last lecture: review

Potential energy:

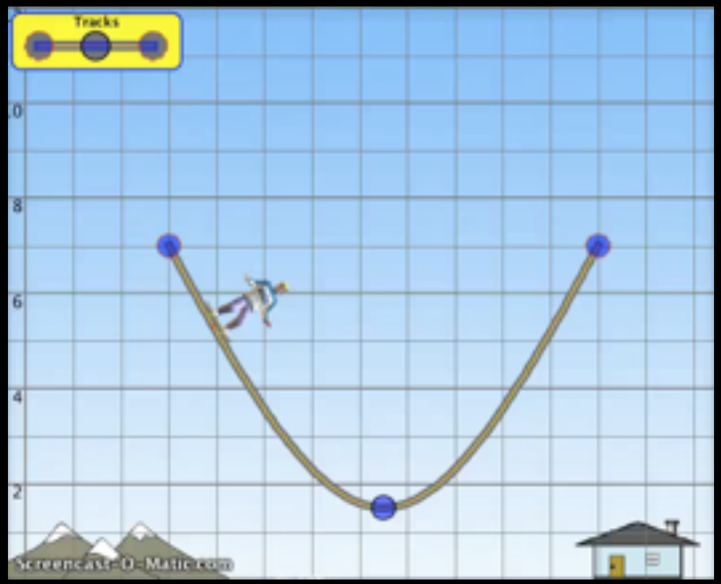
Gravity: $\Delta U = -mg\Delta y$

Spring: $U = \frac{1}{2}kx^2$

= stored energy we can get back as kinetic energy



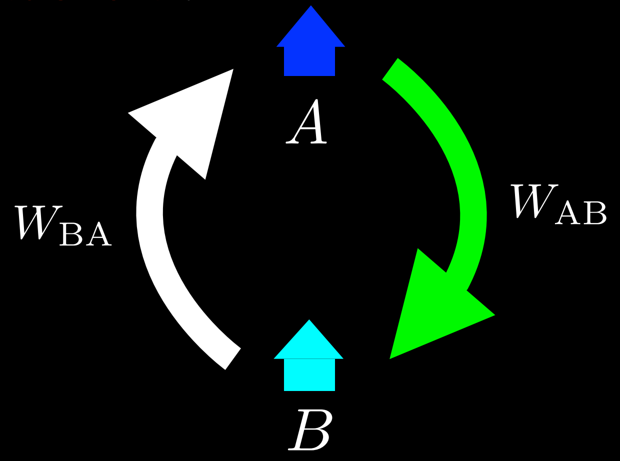
$\Delta U \rightarrow K$



If a force stores energy to give back later, it is conservative:

total work around a closed path (start = end) is 0

$$\oint \bar{F} \cdot d\bar{r} = 0$$

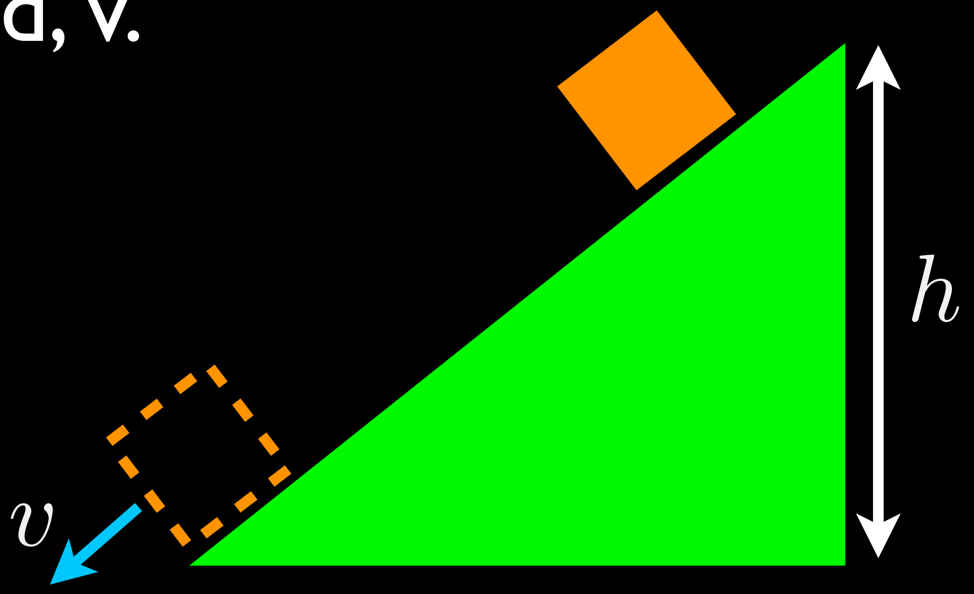


Last lecture: review

A block of mass m starts from rest ($v_i = 0$) on a ramp at height, h.

It slides down the ramp and reaches a speed, v.

What height is needed to reached the speed 2v?



(a) h

(b) 1.41h

(c) 2h

(d) 3h

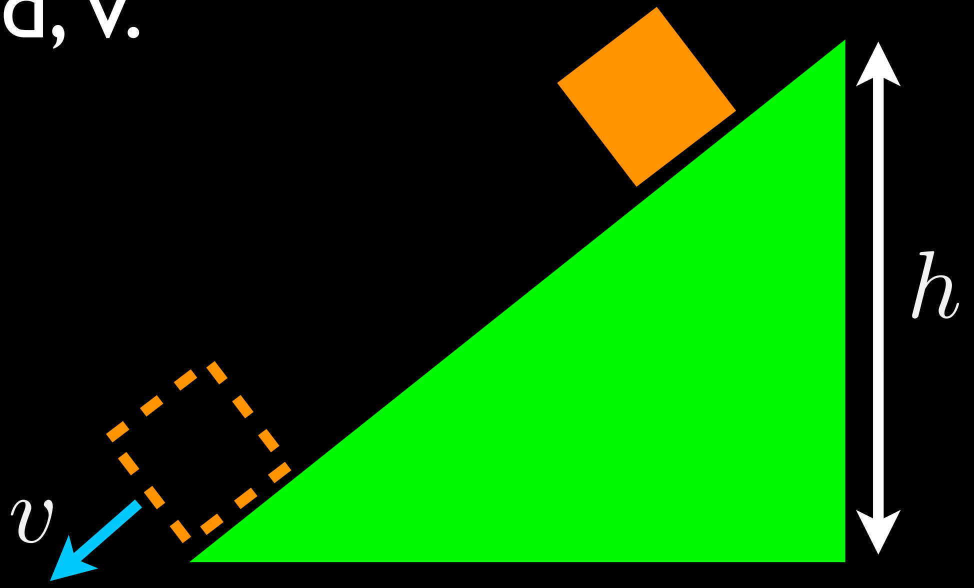
[✓ (e) 4h]

(f) 6h

Last lecture: review

A block of mass m starts from rest ($v_i = 0$) on a ramp at height, h.

It slides down the ramp and reaches a speed, v.



What height is needed to reached the speed 2v?

Energy conservation: $\Delta U_{\text{top}} = K_{\text{bottom}}$

$$mgh = \frac{1}{2}mv^2 \longrightarrow h_1 = \frac{1}{2}\frac{v_1^2}{g}$$

$$h_2 = \frac{1}{2}\frac{(2v_1)^2}{g}$$

$$\frac{h_1}{h_2} = \frac{1}{4} \longrightarrow h_2 = 4h_1$$

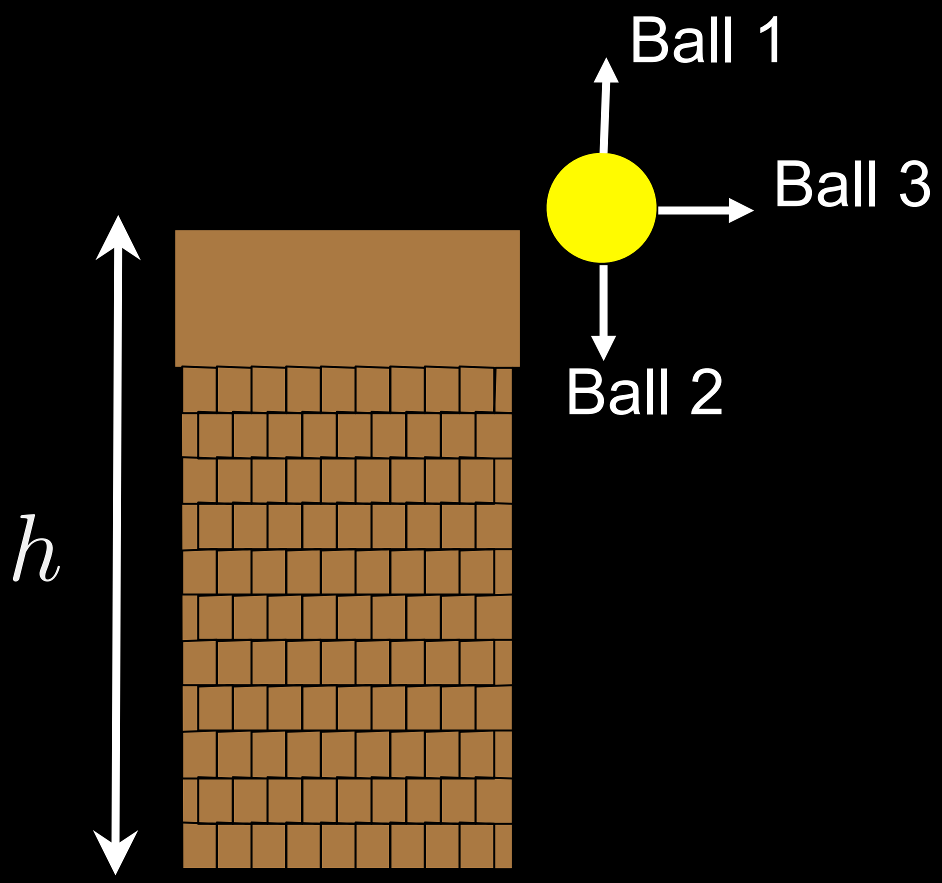
Last lecture: review

3 balls of *equal mass* are fired at *equal speeds* from the *same height*.

Ball 1 is fired up, ball 2 down and ball 3 horizontally.

Rank their speeds from big to small before they hit the ground.

(ignore friction / drag)



(a) $v_1 > v_2 > v_3$

(b) $v_3 > v_2 > v_1$

[✓ (c) $v_1 = v_2 = v_3$]

(d) $v_2 > v_1 > v_3$

(e) $v_1 = v_2 > v_3$

Last lecture: review

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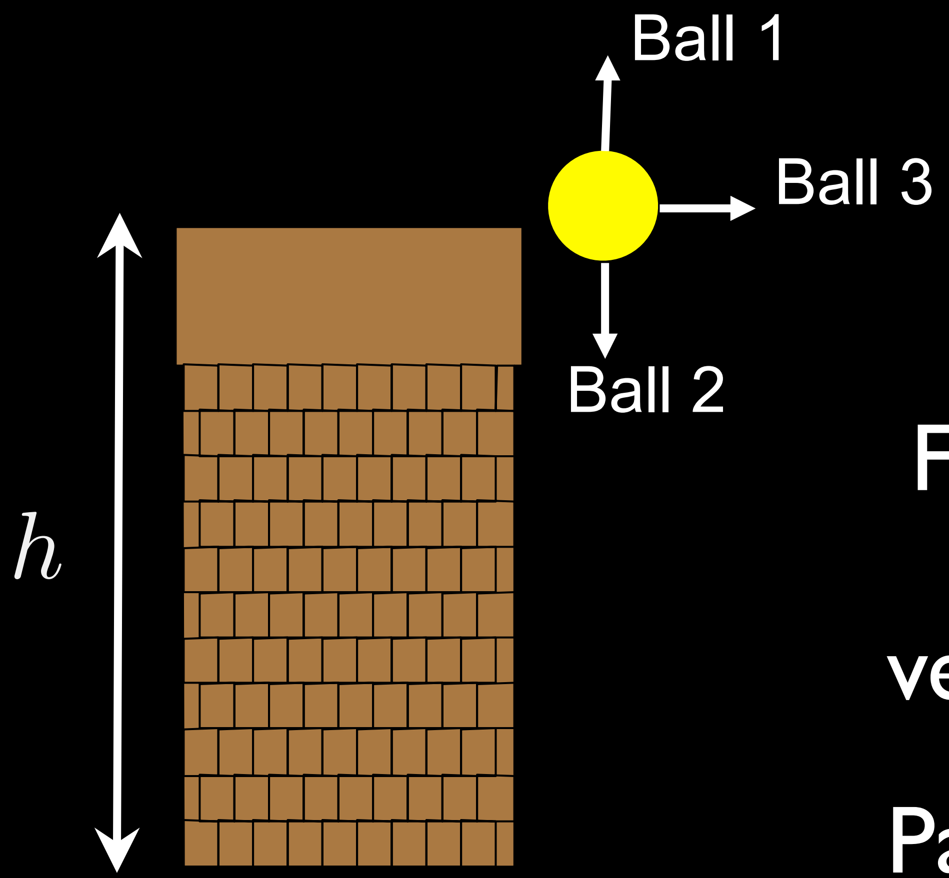
Ball 1 is fired up, ball 2 down and ball 3 horizontally.

Rank their speeds from big to small before they hit the ground.



Initial energy, ball 1:

$$E_{\text{ball1}} = U_{\text{ball1}} + K_{\text{ball1}} = mgh + \frac{1}{2}mv^2$$

$$= E_{\text{ball2}} = E_{\text{ball3}} = E_{\text{initial}}$$

Final energy: $E_{\text{final}} = E_{\text{initial}} = \frac{1}{2}mv^2$

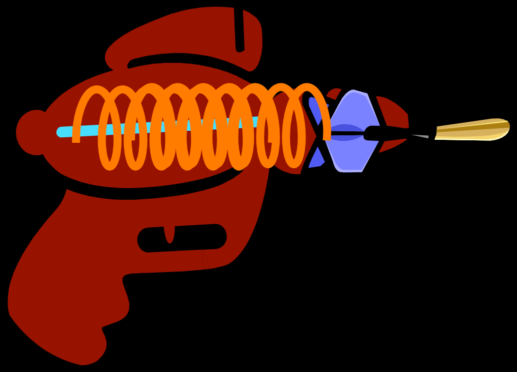
velocity is the same for all balls.

Path doesn't matter.

Last lecture: review

A spring-loaded gun shoots a ball with speed 4 m/s.

If the spring is compressed twice as far, the ball's speed will be....



(a) 16 m/s

[✓ (b) 8 m/s]

(c) 4 m/s

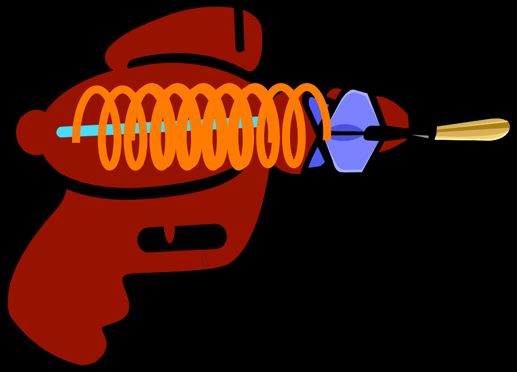
(d) 2 m/s

(e) 1 m/s

Last lecture: review

A spring-loaded gun shoots a ball with speed 4 m/s.

If the spring is compressed twice as far, the ball's speed will be....



Energy conservation: $U_{\text{spring}} = K_{\text{ball}}$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv_1^2$$

$$\frac{1}{2}k(2x)^2 = \frac{1}{2}mv_2^2$$

$$\left(\frac{v_1}{v_2}\right)^2 = \frac{1}{4}$$

$$v_2 = v_1 \times 2 = 4 \times 2 = 8\,\text{m/s}$$

This lecture:

Momentum

Momentum

In lecture 4:

Newton's 2nd law: $\bar{F}_{\text{net}} = \frac{d\bar{p}}{dt} = m\bar{a}$

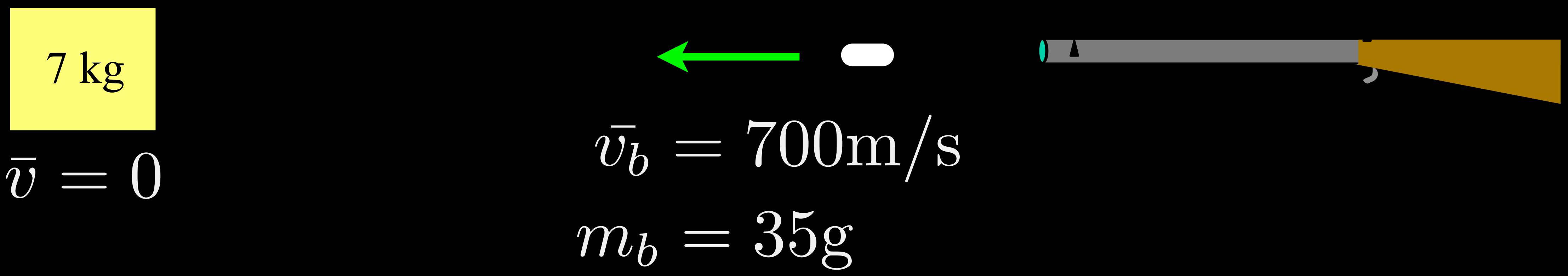
where the momentum, $\bar{p} = m\bar{v}$

If $\bar{F}_{\text{net}} = 0$, $\boxed{\bar{p} = \text{constant}}$ Conservation of linear momentum

"If there is no net external force on a system,
the total momentum is constant"

Momentum

Example



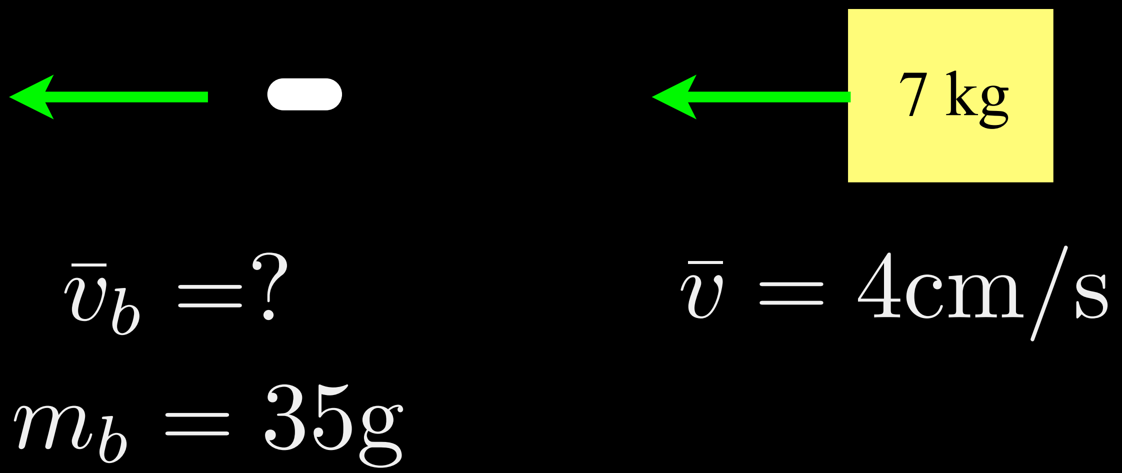
$\bar{v} = 0$

$\bar{v}_b = 700 \mathrm{m/s}$

$m_b = 35 \mathrm{g}$

A gun fires a bullet into butter.

The bullet travels through the butter, slows and pushes the butter to the right on a frictionless surface.



$\bar{v}_b = ?$

$m_b = 35 \mathrm{g}$

$\bar{v} = 4 \mathrm{cm/s}$

What is the final speed of the bullet?

Momentum

Example

Total momentum before:

$$\bar{p}_{\text{before}} = m\bar{v} + m_b\bar{v}_b$$

butter bullet

$\bar{v} = 0$

$\bar{v}_b = 700\text{m/s}$

$m_b = 35\text{g}$

Momentum

Example

Total momentum before:

$$\bar{p}_{\text{before}} = m\bar{v} + m_b\bar{v}_b$$

$$= (7\text{kg})(0) + (0.035\text{kg})(700\text{m/s})$$

$$= 24.5\text{kgm/s}$$

$$\bar{v} = 0$$

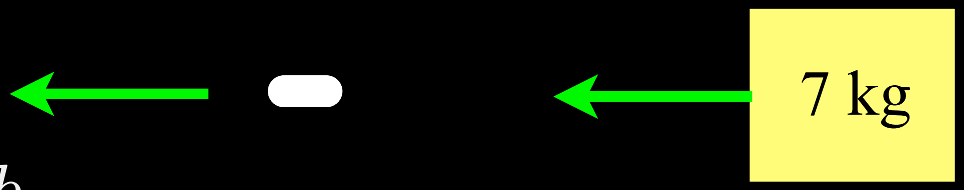
$$\bar{v}_b = 700\text{m/s}$$

$$m_b = 35\text{g}$$

Total momentum after:

$$\bar{p}_{\text{after}} = (7\,\text{kg})(0.04\,\text{m/s}) + (0.035\,\text{kg})\bar{v}_b$$

$$= 0.28 + 0.035v_b$$



$$\bar{v}_b =?$$

$$m_b = 35\text{g}$$

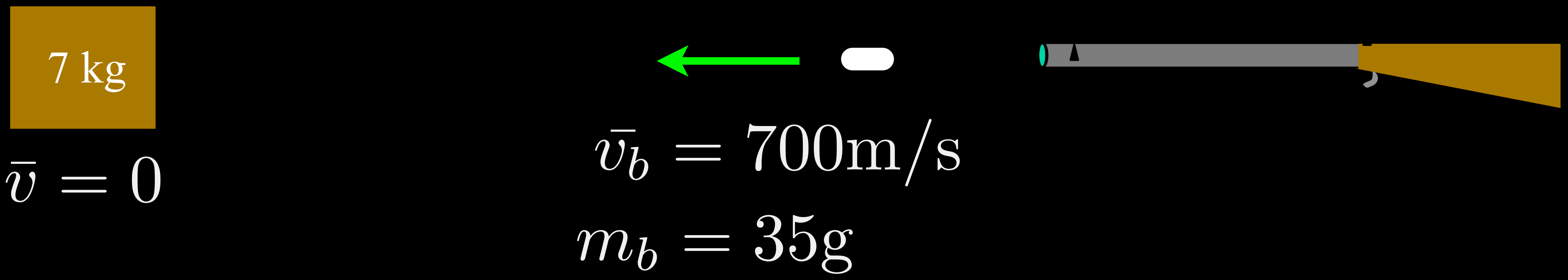
$$\bar{v} = 4\text{cm/s}$$

$$\bar{p}_{\text{before}} = \bar{p}_{\text{after}} \quad \rightarrow 24.5 = 0.28 + 0.035v_b$$

$$v_b = 692\text{m/s}$$

Momentum

Quiz



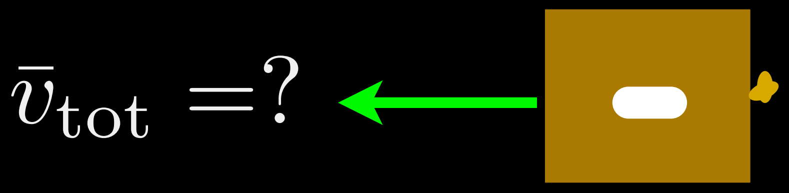
$\bar{v} = 0$

$\bar{v}_b = 700\text{m/s}$

$m_b = 35\text{g}$

Same problem, but butter is now wood.

The bullet stops in the wood and they move together on a frictionless surface.

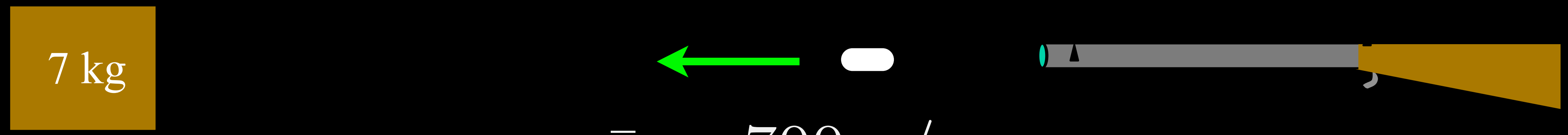


$\bar{v}_{\text{tot}} = ?$

[✓ (a) 3.48 m/s] (b) 4 cm/s (c) 700 m/s (d) 3.5 m/s

Momentum

Quiz

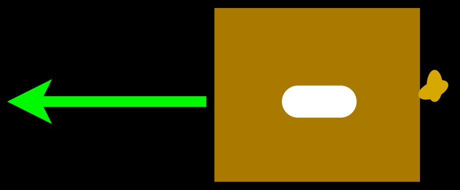


$\bar{v} = 0$

$\bar{v_b} = 700\text{m/s}$

$m_b = 35\text{g}$

$\bar{v}_{\text{tot}} = ?$



$$\bar{p}_{\text{before}} = m_{\text{b}}\bar{v}_b + m_{\text{butter}}\cancelto{0}{\bar{v}_{\text{butter}}} = m_{\text{b}}\bar{v}_{\text{b}}$$

$$= (0.035\,\text{kg}) \times (700\,\text{m/s})$$

$$\bar{p}_{\text{before}} = \bar{p}_{\text{after}}$$

$$\bar{p}_{\text{after}} = m_{\text{tot}}\bar{v}_{\text{tot}} = (7\,\text{kg} + 0.035\,\text{kg})\bar{v}_{\text{tot}}$$

$$v_{\text{tot}} = \frac{24.5}{7.035} = 3.48\,\text{m/s}$$

Momentum

Quiz

A popcorn kernal at rest in a hot pan bursts into two pieces, with masses 91-mg and 64-mg.

The more massive piece moves horizontally at 47 cm/s. What is the velocity of the 2nd piece?

(a) 33 cm/s

(b) 67 cm/s

[✓ (c) -67 cm/s]

(d) -33 cm/s

Momentum

Quiz

A popcorn kernal at rest in a hot pan bursts into two pieces, with masses 91-mg and 64-mg.

The more massive piece moves horizontally at 47 cm/s. What is the velocity of the 2nd piece?

$$\begin{aligned}\bar{p}_{\text{before}} &= m_{\text{tot}}\bar{v}_{\text{tot}} \\ &= (91\,\text{mg} + 64\,\text{mg}) \times 0\end{aligned}$$

$$\begin{aligned}\bar{p}_{\text{after}} &= m_1\bar{v}_1 + m_2\bar{v}_2 \\ &= (91\,\text{mg})(47\,\text{cm/s}) + (64\,\text{mg})\bar{v_2}\end{aligned}$$

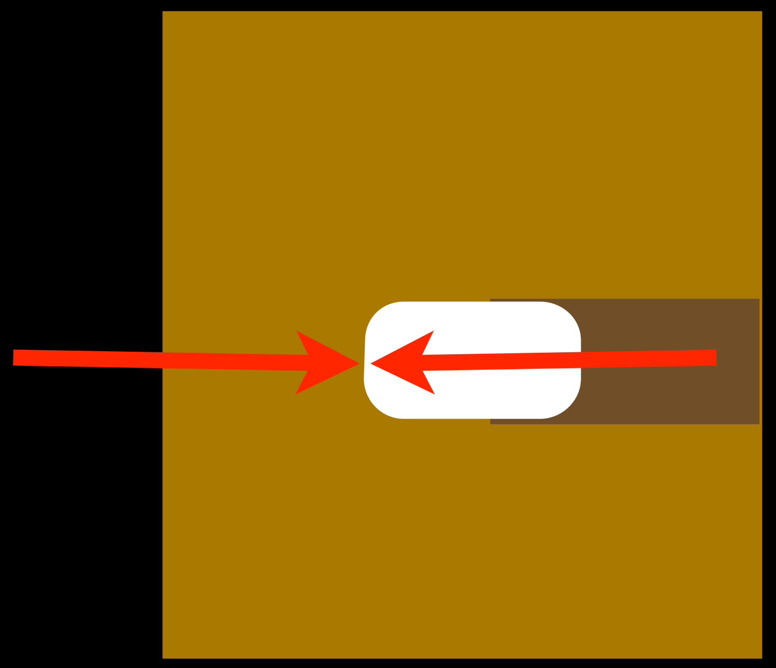
$$\bar{p}_{\text{before}} = \bar{p}_{\text{after}}$$

$$\bar{v}_2 = -\frac{(91\,\text{mg})(47\,\text{cm/s})}{64\,\text{mg}} \qquad = -67\,\text{cm/s}$$

Momentum

Why does this work?

What about the other forces in the system?



Because of Newton's 3rd law, internal forces are equal and opposite.

Therefore, the change in momentum is equal and opposite.

So $\Delta p = 0$ and momentum is conserved (if no external forces)

Momentum

An apple is dropped from a height. After t seconds, its velocity is v.

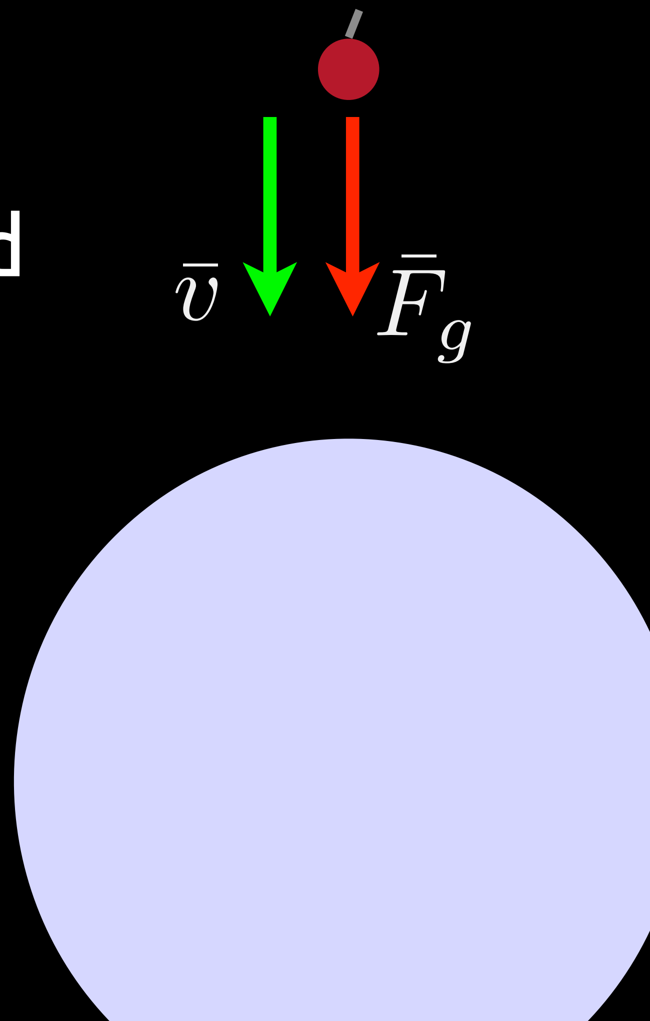
Which of the following is true?

(a) the apple's momentum is conserved

(b) the Earth's momentum is conserved

(c) the apple + the Earth's momentum is conserved

(d) all the above



Momentum

An apple is dropped from a height. After t seconds, its velocity is v.

Which of the following is true?

(a) the ~~apple's momentum is conserved~~

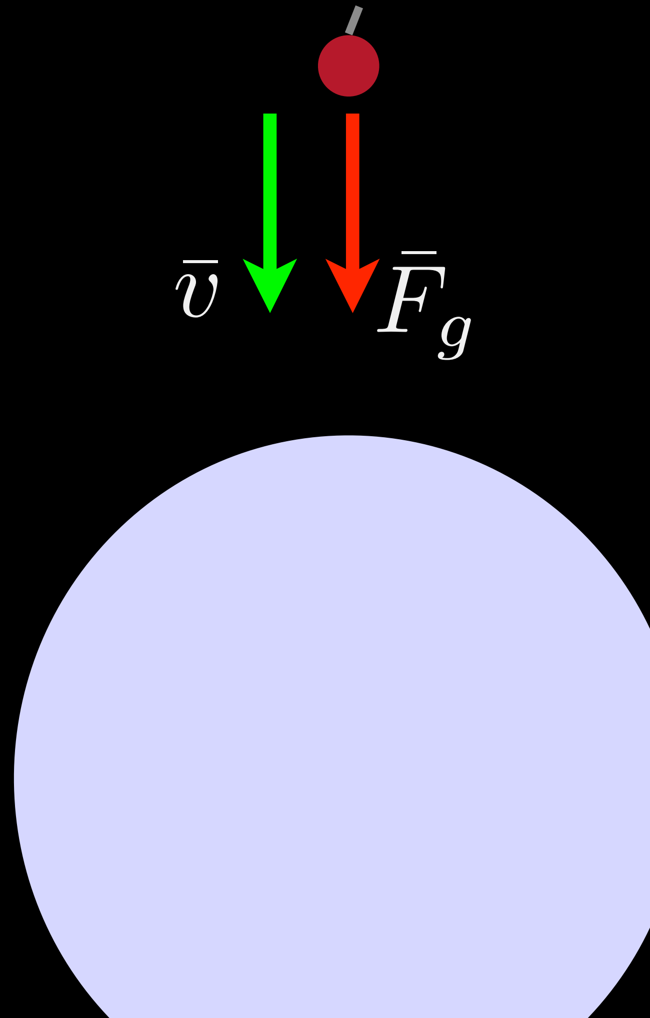
initial

$$\bar{p}_{\text{apple,i}} = 0.0$$

final

$$\bar{p}_{\text{apple,f}} = mv$$

$$\bar{p}_{\text{apple,i}} \neq \bar{p}_{\text{apple,f}}$$



Gravity is an external force on the apple.

Therefore, the apple's momentum alone is not conserved.

Momentum

An apple is dropped from a height. After t seconds, its velocity is v.

Which of the following is true?

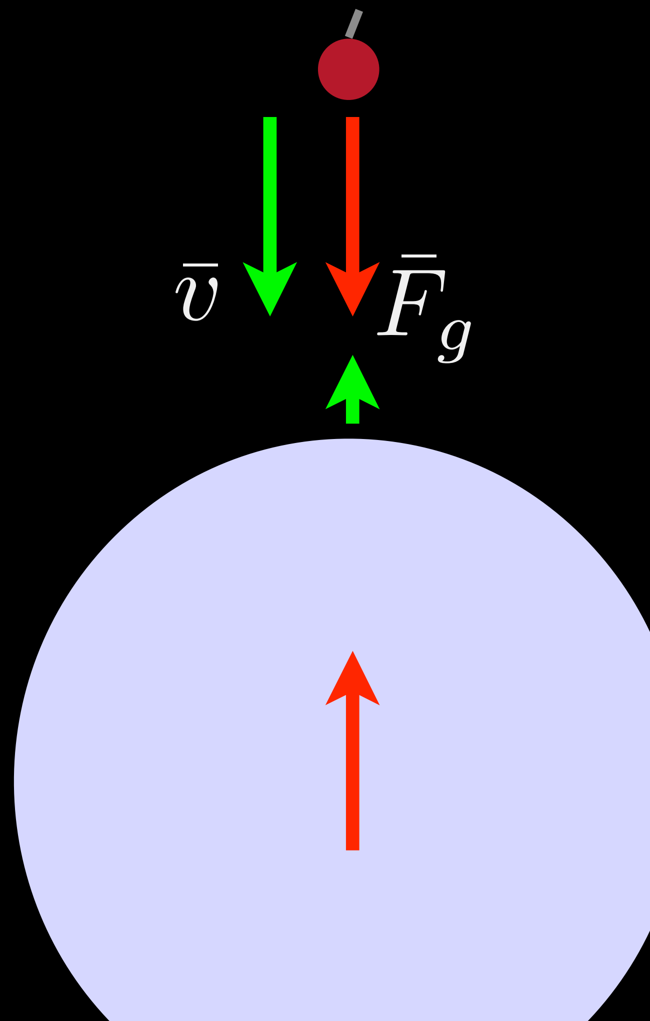
(b) the ~~Earth's momentum is~~ conserved

From Newton's 3rd law, the apple also exerts a force on the Earth,

causing a (tiny!) velocity change.

Therefore: $\bar{p}_{\mathrm{Earth,i}} \neq \bar{p}_{\mathrm{Earth,f}}$

The Earth's momentum alone is not conserved.



Momentum

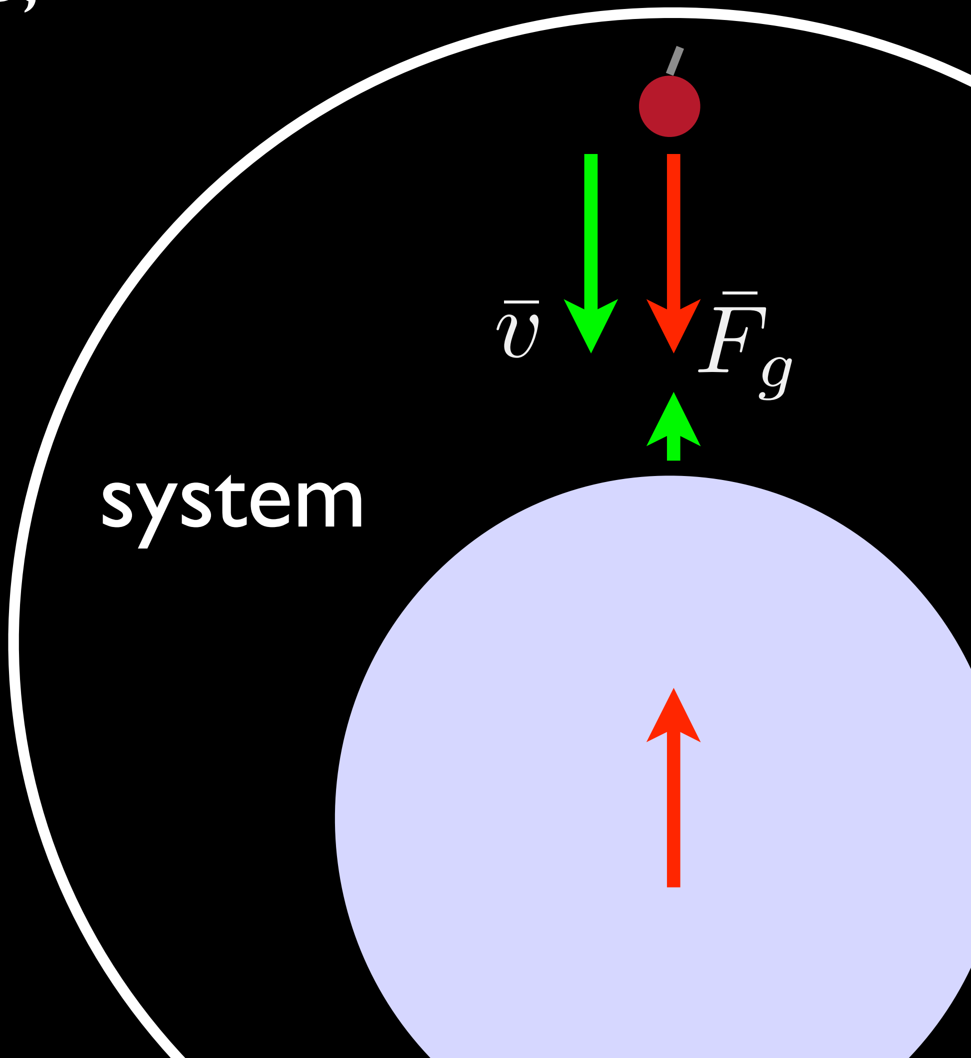
An apple is dropped from a height. After t seconds, its velocity is v.

Which of the following is true?

(c) the apple + the Earth's momentum is conserved

If our system includes both Earth and apple,

gravity is an internal force.



$$\bar{p}_{\text{total}} = M_{\text{Earth}} v_{\text{Earth}} + m_{\text{apple}} v_{\text{apple}}$$

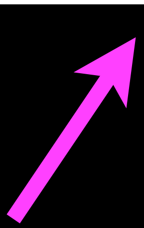
$$= 0$$

$$\bar{p}_{\text{total,i}} = \bar{p}_{\text{total,f}}$$

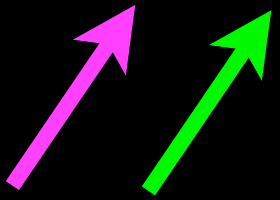
Momentum

Facts

Momentum is a vector: $\bar{p}$



$\bar{v}$ and $\bar{p}$ point in the same direction.



SI unit: $\mathrm{kgm/s}$

Momentum is conserved (like energy) if there is no external force

A net force ($\bar{F}_{\mathrm{net}} \neq 0$) is required to change a body's momentum

Momentum is proportional to mass and velocity: $\bar{p} = m\bar{v}$

Therefore, a big and slow object can have the same momentum as a small and fast object

Momentum

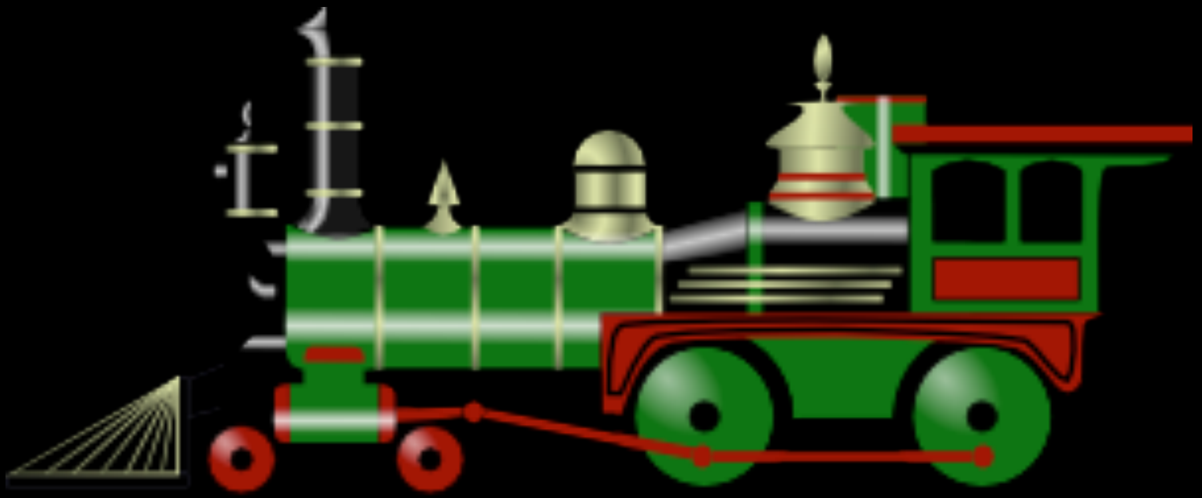
Equivalent momentum:

Car: $m = 1800\ \text{kg}; \quad v = 80\ \text{m}/\text{s}$

$p = 1.44 \cdot 10^5\ \text{kg} \cdot \text{m}/\text{s}$

Bus: $m = 9000\ \text{kg}; \quad v = 16\ \text{m}/\text{s}$

$p = 1.44 \cdot 10^5\ \text{kg} \cdot \text{m}/\text{s}$



Train: $m = 3.6 \cdot 10^4\ \text{kg}; \quad v = 4\ \text{m}/\text{s}$

$p = 1.44 \cdot 10^5\ \text{kg} \cdot \text{m}/\text{s}$

The same force would be needed to stop all three. $\bar{F}_{\text{net}} = \dfrac{d\bar{p}}{dt}$

Momentum

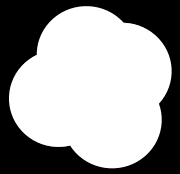
Since $\bar{p}$ is a vector, 2D problems can be solved with components:

Example:

A 60-kg skater, at rest on a frictionless ice, tosses a 12-kg snowball with velocity $\bar{v} = 53.0\bar{i} + 14.0\bar{j}\mathrm{m/s}$. Find the skater's velocity.

$\bar{p}_\mathrm{i} = 0.0$ $\bar{p}_\mathrm{f} = m_1\bar{v}_1 + m_2\bar{v}_2$

snowball



skater

$\bar{v_2} = ?$

Momentum

Since $\bar{p}$ is a vector, 2D problems can be solved with components:

Example:

A 60-kg skater, at rest on a frictionless ice, tosses a 12-kg snowball with velocity $\bar{v} = 53.0\bar{i} + 14.0\bar{j}\mathrm{m/s}$. Find the skater's velocity.

$\bar{p}_{\mathrm{i}} = 0.0 \qquad \bar{p}_{\mathrm{f}} = m_1\bar{v}_1 + m_2\bar{v}_2$

Conservation of momentum: $\bar{v}_2 = -\frac{m_1}{m_2}\bar{v}_1$

i: $v_{2,i} = -\frac{m_1}{m_2}v_{1,\mathrm{i}} = -\frac{12\mathrm{kg}}{60\mathrm{kg}}53.0\bar{i}\mathrm{m/s})$

Momentum

Since $\bar{p}$ is a vector, 2D problems can be solved with components:

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$\bar{p}_\mathrm{i} = 0.0 \qquad \bar{p}_\mathrm{f} = m_1\bar{v}_1 + m_2\bar{v}_2$

Conservation of momentum: $\bar{v}_2 = -\frac{m_1}{m_2}\bar{v}_1$

i: $v_{2,i} = -\frac{m_1}{m_2}v_{1,\mathrm{i}} = -\frac{12\mathrm{kg}}{60\mathrm{kg}}(53.0\bar{i}\,\mathrm{m/s})$

j: $v_{2,j} = -\frac{m_1}{m_2}v_{1,j} = -\frac{12\mathrm{kg}}{60\mathrm{kg}}(14.0\bar{j}\,\mathrm{m/s})$

$\bar{v}_2 = ?$

Momentum

Since $\bar{p}$ is a vector, 2D problems can be solved with components:

Example:

A 60-kg skater, at rest on a frictionless ice, tosses a 12-kg snowball with velocity $\bar{v} = 53.0\bar{i} + 14.0\bar{j}\,\mathrm{m/s}$. Find the skater's velocity.

$\bar{p}_{\mathrm{i}} = 0.0 \qquad \bar{p}_{\mathrm{f}} = m_1\bar{v}_1 + m_2\bar{v}_2$

Conservation of momentum: $\bar{v}_2 = -\frac{m_1}{m_2}\bar{v}_1$

$\bar{v}_2 = ?$

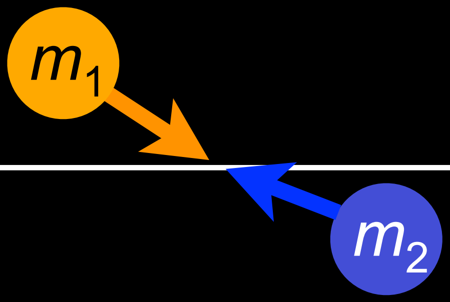
i: $v_{2,i} = -\frac{m_1}{m_2}v_{1,\mathrm{i}} = -\frac{12\mathrm{kg}}{60\mathrm{kg}}(53.0\bar{i}\,\mathrm{m/s})$

j: $v_{2,j} = -\frac{m_1}{m_2}v_{1,j} = -\frac{12\mathrm{kg}}{60\mathrm{kg}}(14.0\bar{j}\,\mathrm{m/s})$

$\bar{v}_2 = -10.6\bar{i} - 2.8\bar{j}\,\mathrm{m/s}$

Collisions

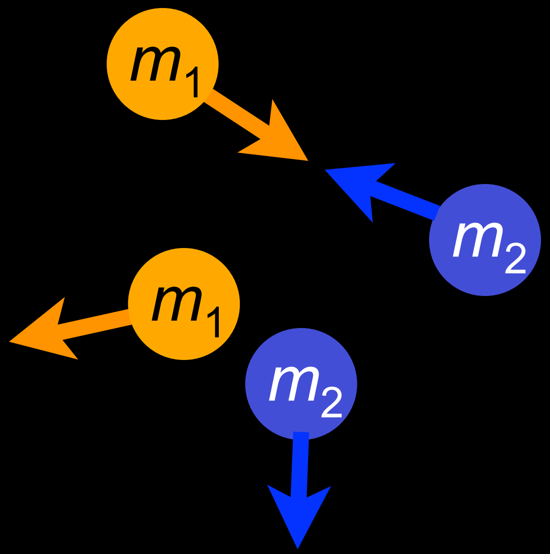
Collisions



Three types of collisions:

(1) Elastic

K and p are conserved. Nothing sticks together.

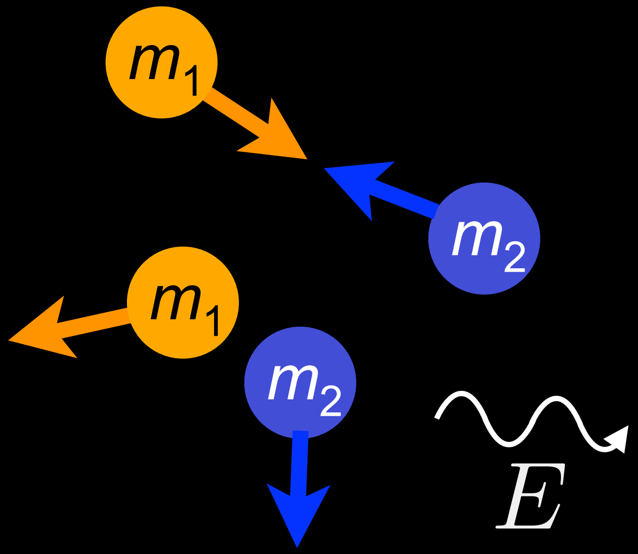


p: $$m_1\bar{v}_{1,i} + m_2\bar{v}_{2,i} = m_1\bar{v}_{1,f} + m_2\bar{v}_{2,f}$$

K: $$\frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2 = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2$$

(2) Inelastic

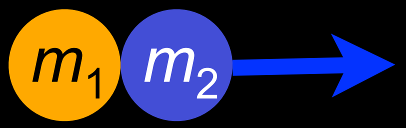
Only p is conserved. Nothing sticks together.



p: $$m_1\bar{v}_{1,i} + m_2\bar{v}_{2,i} = m_1\bar{v}_{1,f} + m_2\bar{v}_{2,f}$$

(3) Totally inelastic

Only p is conserved. Objects sticks together.



p: $$m_1\bar{v}_1 + m_2\bar{v}_2 = (m_1 + m_2)\bar{v}_f$$

Collisions

Quiz

In real life, most collisions are:

(a) Elastic

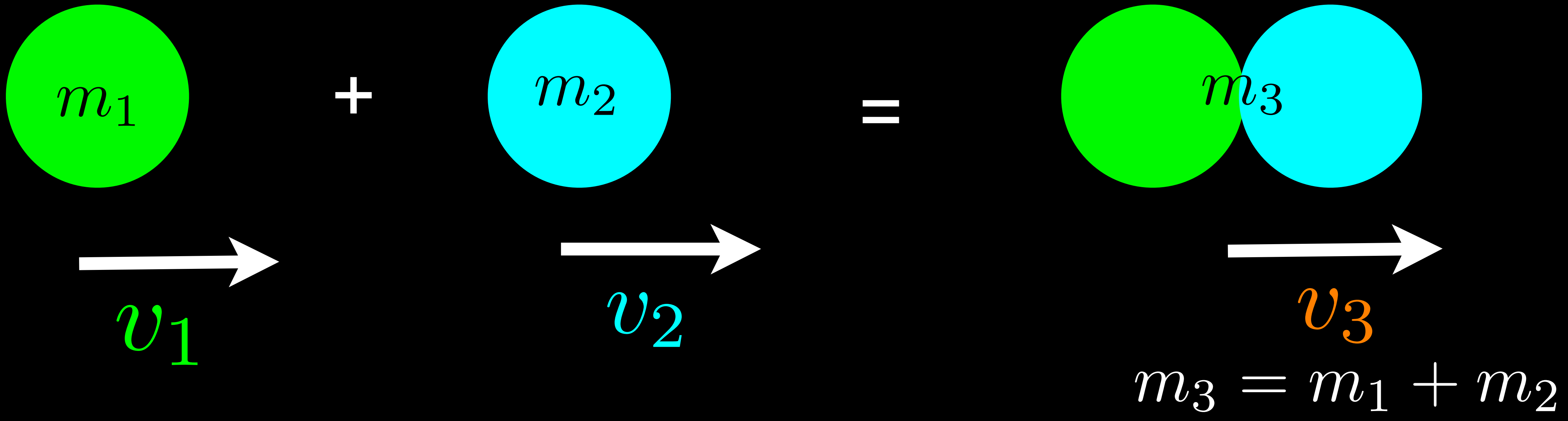
(K and p are conserved)

[✓ (b) Inelastic]

(p is conserved)

Collisions

Totally inelastic: 1D



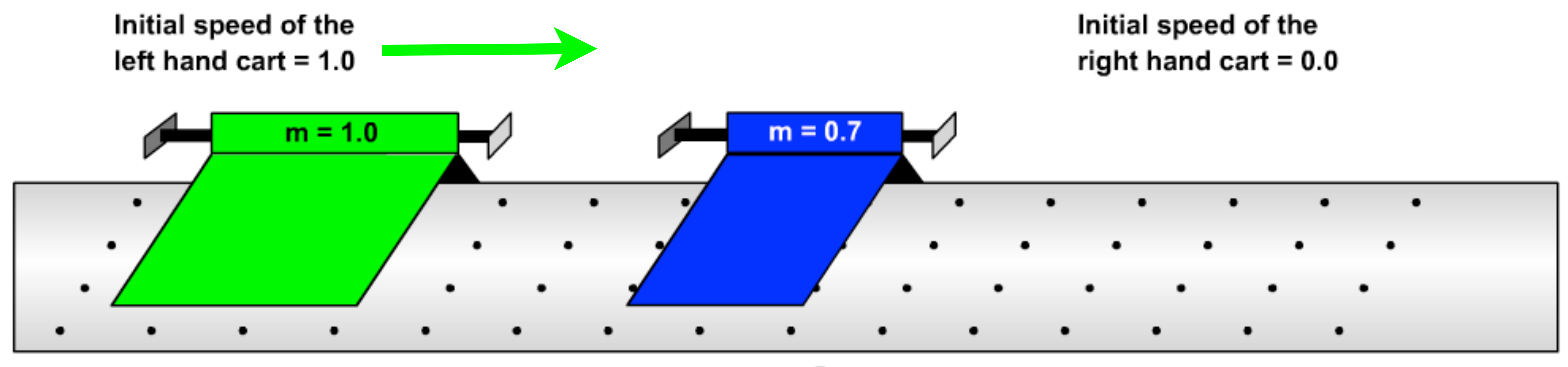
Conservation of momentum:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_3$$

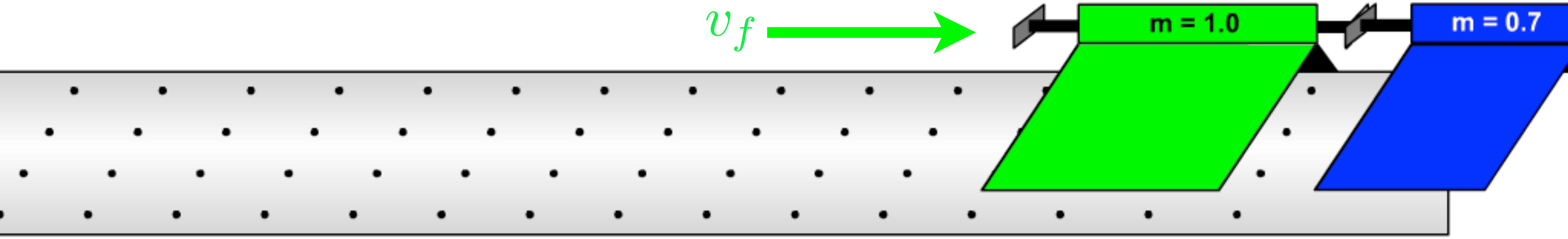
Collisions

Quiz

Inelastic collision 1D:



2 carts on frictionless track collide and stick together.

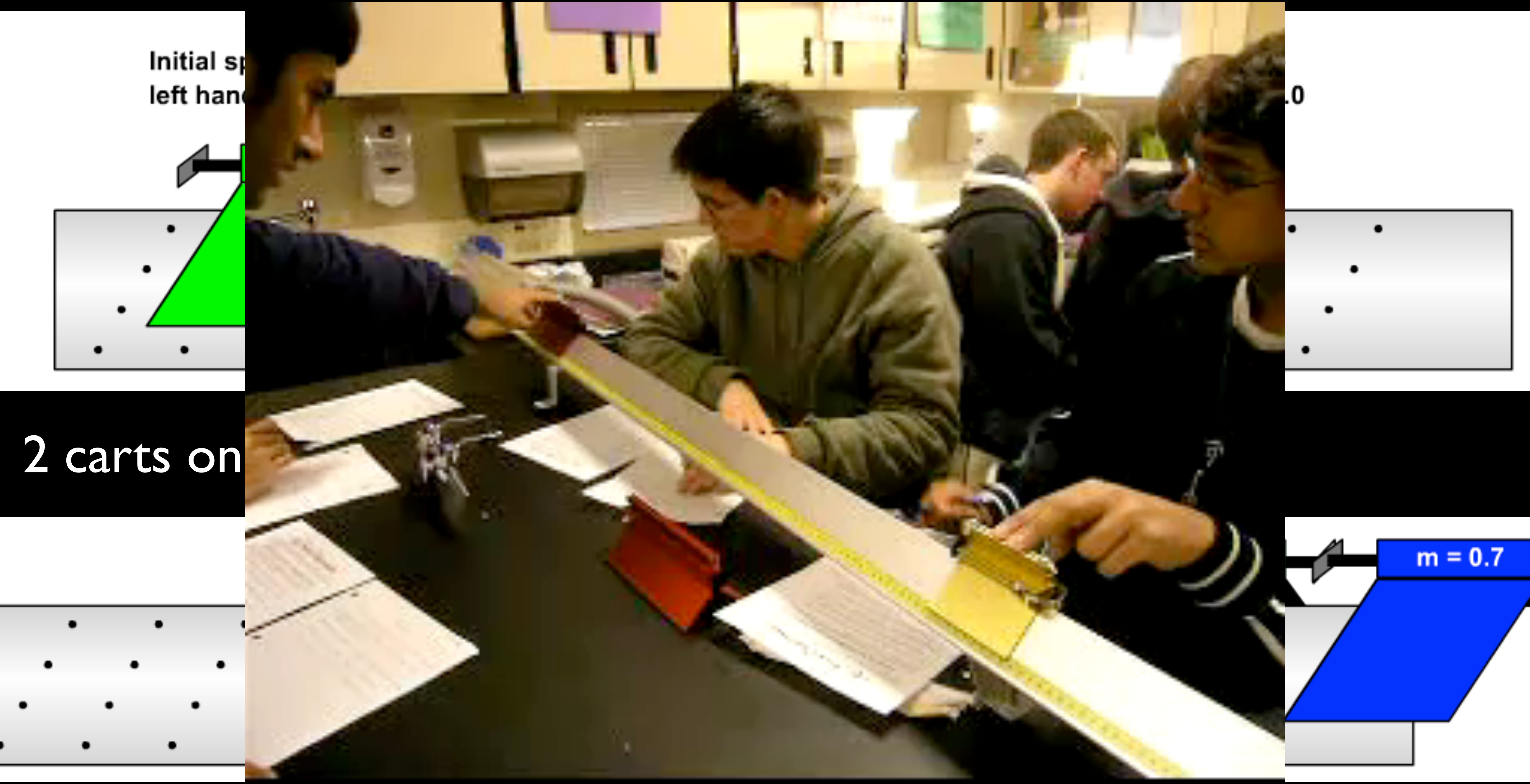


what is their combined velocity v_f ?

Collisions

Quiz

Inelastic collision 1D:



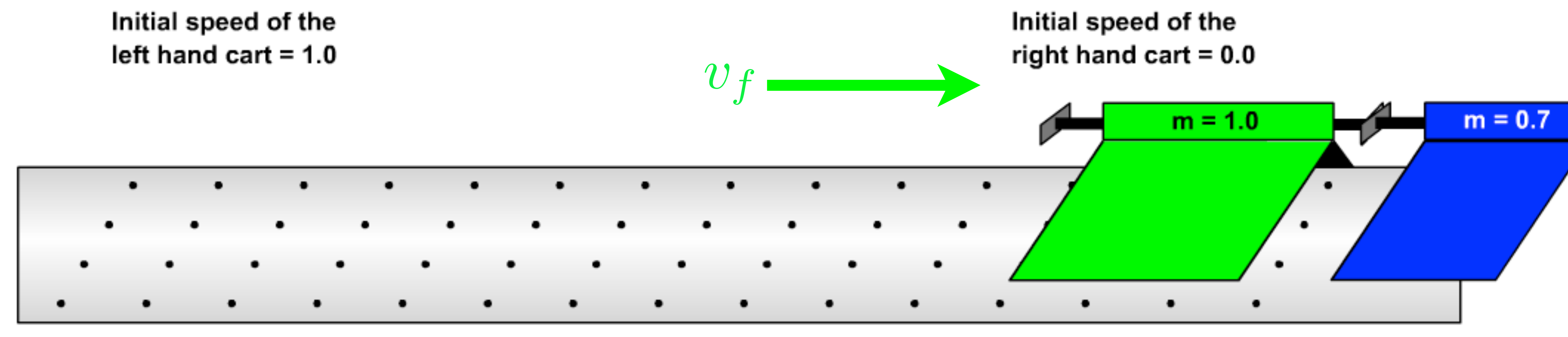
2 carts on

what is their combined velocity v_f ?

Collisions

Quiz

Inelastic collision 1D:



what is their combined velocity v_f ?

[✓ (a) 0.588]

(b) 1.0

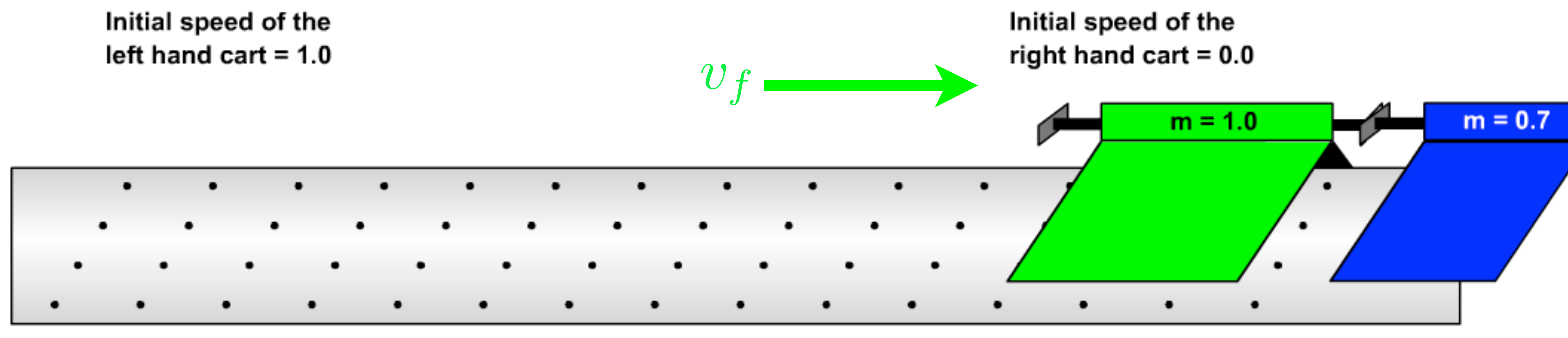
(c) 1.43

(d) 0.5

Collisions

Quiz

Inelastic collision 1D:



what is their combined velocity v_f ?

$$\bar{p}_{\text{before}} = m_L \bar{v}_L + m_R \bar{v}_R \quad = 1 \times 1 + 0.7 \times 0$$

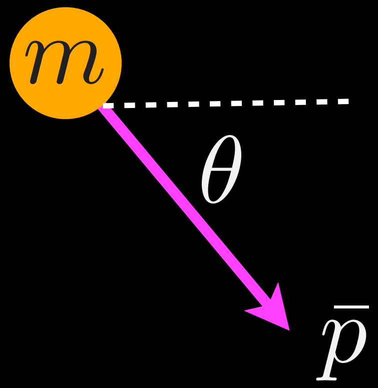
$$\bar{p}_{\text{after}} = m_{\text{tot}} \bar{v}_{\text{tot}} \quad = 1.7 \times \bar{v}_{\text{tot}}$$

$$\bar{p}_{\text{before}} = \bar{p}_{\text{after}}$$

$$\bar{v}_{\text{tot}} = \frac{1}{1.7} = 0.588$$

Collisions

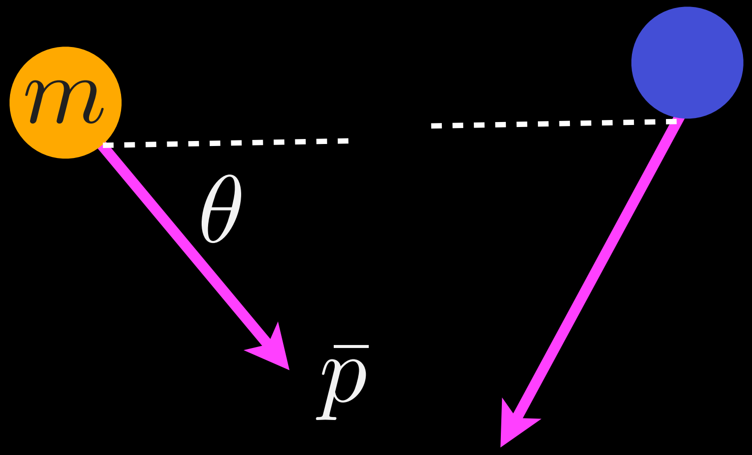
Inelastic: 2D



$$p_x = mv_x \quad = mv\cos\theta$$

$$p_y = mv_y \quad = mv\sin\theta$$

Collisions Inelastic: 2D

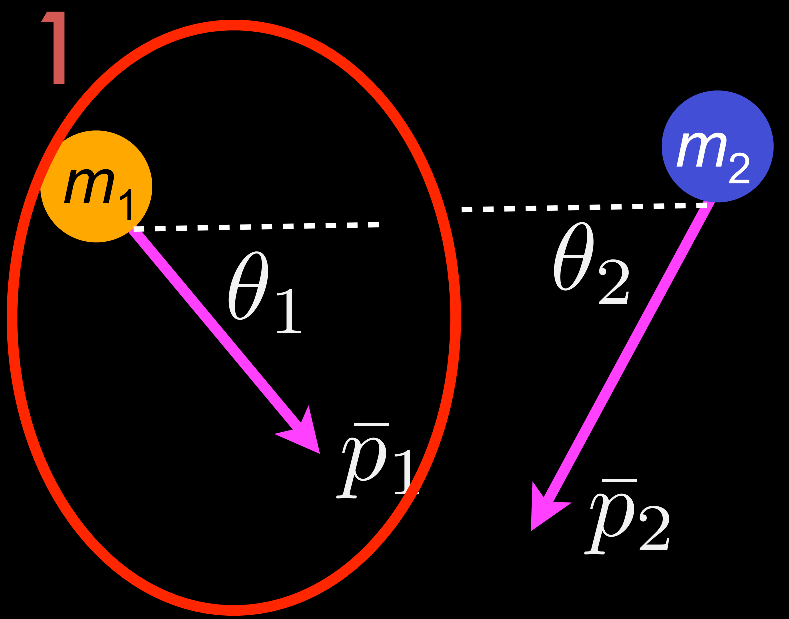


$$p_x = mv_x \quad = mv\cos\theta$$

$$p_y = mv_y \quad = mv\sin\theta$$

Collisions

Inelastic: 2D

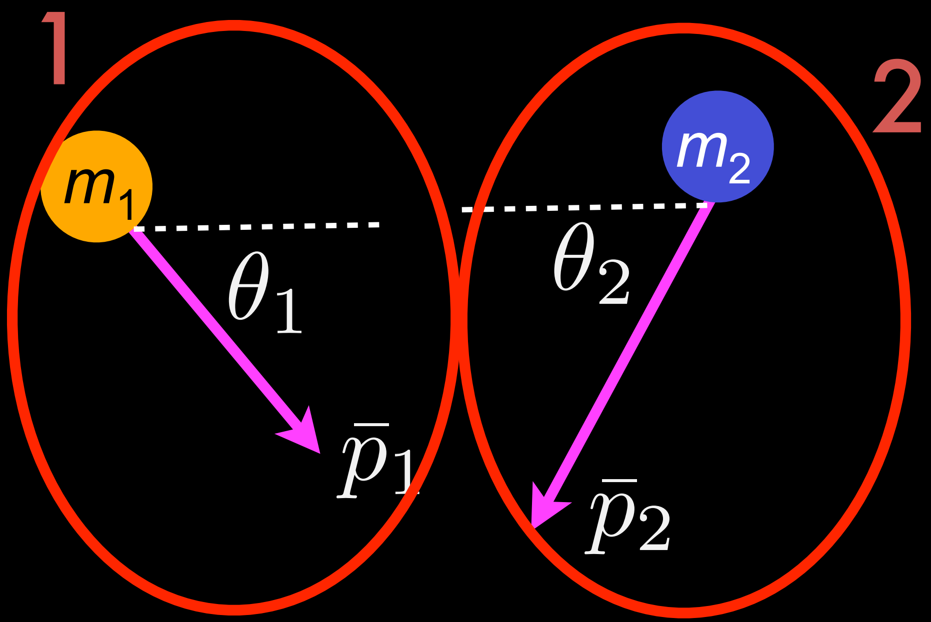


$$p_{1,x} = m_1 v_{1,x} = m_1 v_1 \cos\theta_1$$

$$p_{1,y} = m_1 v_{1,y} = m_1 v_1 \sin\theta_1$$

Collisions

Inelastic: 2D



$$p_{1,x} = m_1 v_{1,x} = m_1 v_1 \cos\theta_1$$

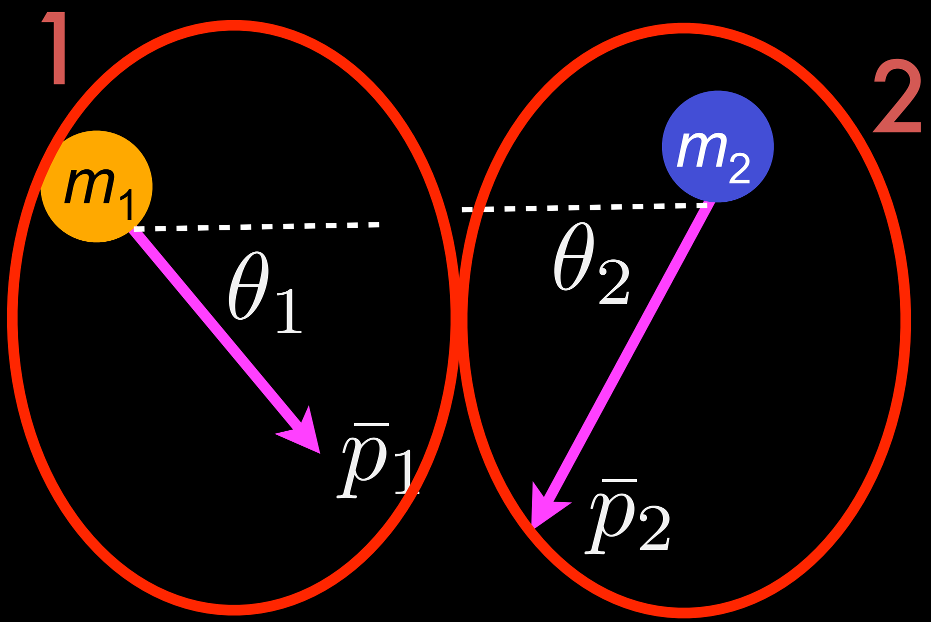
$$p_{1,y} = m_1 v_{1,y} = m_1 v_1 \sin\theta_1$$

$$p_{2,x} = -m_2 v_{2,x} = -m_2 v_2 \cos\theta_2$$

$$p_{2,y} = m_2 v_{2,y} = m_2 v_2 \sin\theta_2$$

Collisions

Inelastic: 2D



$$p_{1,x} = m_1 v_{1,x} = \boxed{m_1 v_1 \cos\theta_1}$$

$$p_{1,y} = m_1 v_{1,y} = m_1 v_1 \sin\theta_1$$

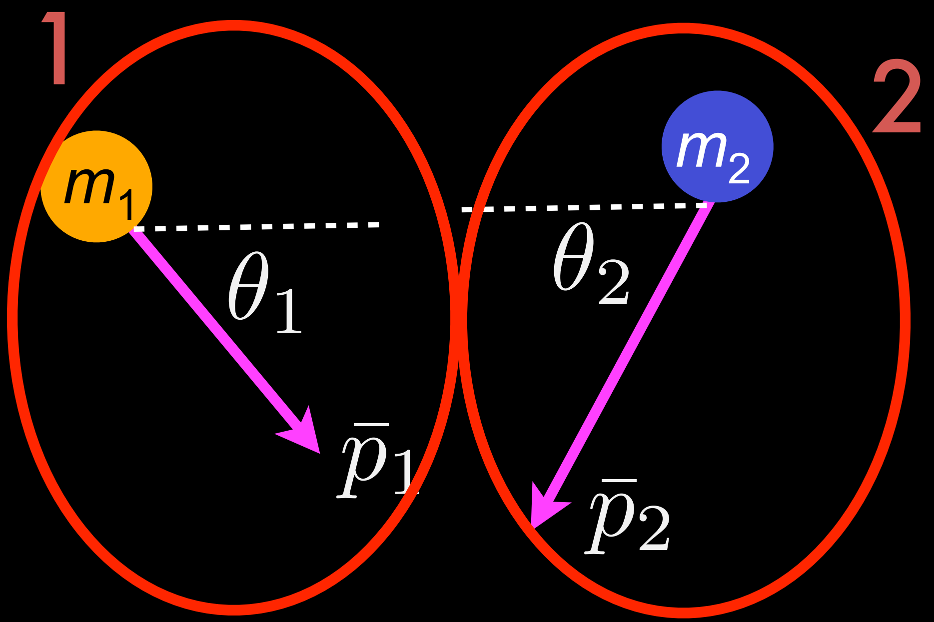
$$p_{2,x} = -m_2 v_{2,x} = -\boxed{m_2 v_2 \cos\theta_2}$$

$$p_{2,y} = m_2 v_{2,y} = m_2 v_2 \sin\theta_2$$

$$\begin{aligned} p_x &= p_{1,x} + p_{2,x} \\ &= m_1 v_1 \cos\theta_1 - m_2 v_2 \cos\theta_2 \end{aligned}$$

Collisions

Inelastic: 2D



$$p_{1,x} = m_1 v_{1,x} = m_1 v_1 \cos\theta_1$$

$$p_{1,y} = m_1 v_{1,y} = \boxed{m_1 v_1 \sin\theta_1}$$

$$p_{2,x} = -m_2 v_{2,x} = -m_2 v_2 \cos\theta_2$$

$$p_{2,y} = m_2 v_{2,y} = \boxed{m_2 v_2 \sin\theta_2}$$

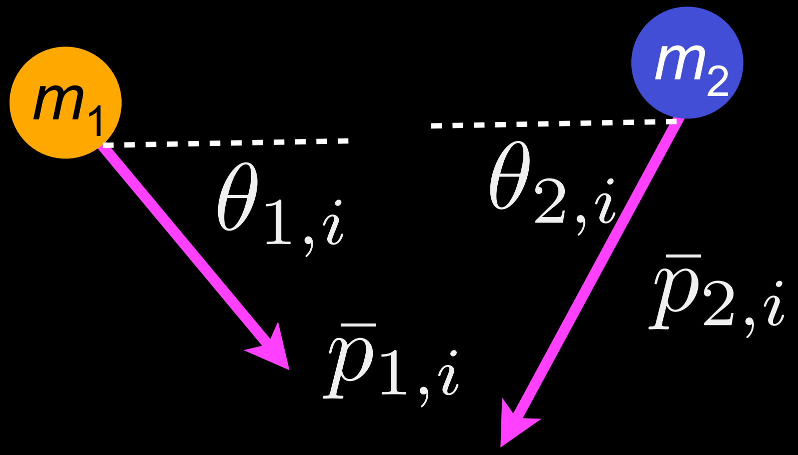
$$\begin{aligned} p_x &= p_{1,x} + p_{2,x} \\ &= m_1 v_1 \cos\theta_1 - m_2 v_2 \cos\theta_2 \end{aligned}$$

$$\begin{aligned} p_y &= p_{1,y} + p_{2,y} \\ &= m_1 v_1 \sin\theta_1 + m_2 v_2 \sin\theta_2 \end{aligned}$$

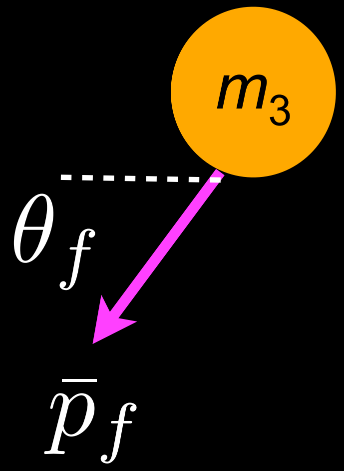
Collisions

Inelastic: 2D

before:



after:



$$\begin{aligned} p_x &= p_{1,x,i} + p_{2,x,i} \\ &= m_1 v_{1,i} \cos\theta_{1,i} - m_2 v_{2,i} \cos\theta_{2,i} \end{aligned}$$

$$p_x = -m_3 v_f \cos\theta_f$$

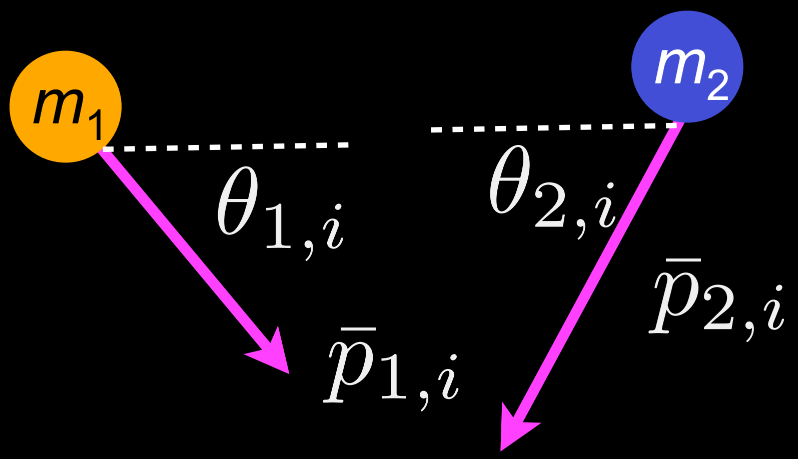
$$\begin{aligned} p_y &= p_{1,y,i} + p_{2,y,i} \\ &= m_1 v_{1,i} \sin\theta_{1,i} + m_2 v_{2,i} \sin\theta_{2,i} \end{aligned}$$

$$p_y = m_3 v_f \sin\theta_f$$

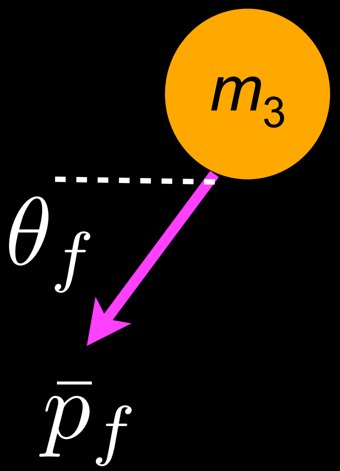
Collisions

Inelastic: 2D

before:



after:



x: $p_{x,\text{initial}} = p_{x,\text{final}}$

y: $p_{y,\text{initial}} = p_{y,\text{final}}$

Conservation of momentum:

$$p_x = m_1 v_{1,i} \cos\theta_{1,i} - m_2 v_{2,i} \cos\theta_{2,i}$$

$$p_y = m_1 v_{1,i} \sin\theta_{1,i} + m_2 v_{2,i} \sin\theta_{2,i}$$

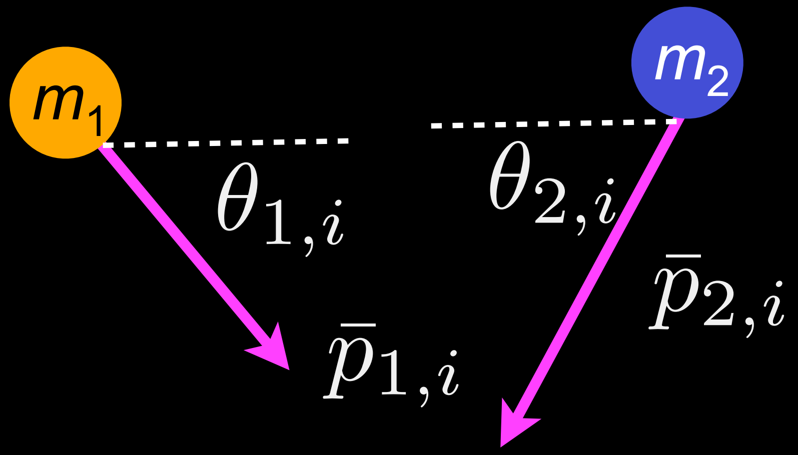
$$\begin{aligned} p_x &= -m_3 v_f \cos\theta_f \\ &= -(m_1 + m_2) v_f \cos\theta_f \end{aligned}$$

$$\begin{aligned} p_y &= m_3 v_f \sin\theta_f \\ &= (m_1 + m_2) v_f \sin\theta_f \end{aligned}$$

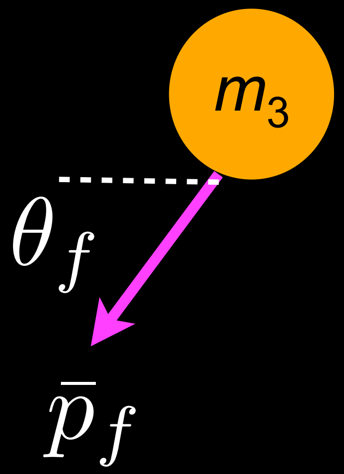
Collisions

Inelastic: 2D

before:



after:



Conservation of momentum:

$$p_x = m_1 v_{1,i} \cos\theta_{1,i} - m_2 v_{2,i} \cos\theta_{2,i}$$

$$p_y = m_1 v_{1,i} \sin\theta_{1,i} + m_2 v_{2,i} \sin\theta_{2,i}$$

$$p_x = -m_3 v_f \cos\theta_f$$

$$= -(m_1 + m_2) v_f \cos\theta_f$$

$$p_y = m_3 v_f \sin\theta_f$$

$$= (m_1 + m_2) v_f \sin\theta_f$$

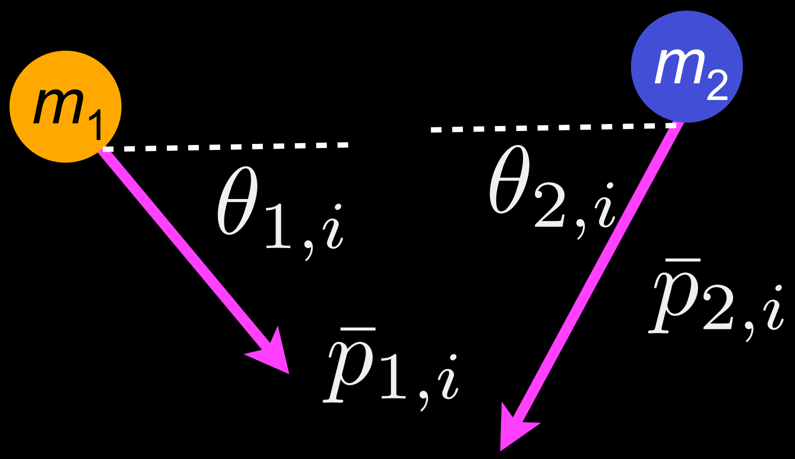
x: $m_1 v_{1,i} \cos\theta_{1,i} - m_2 v_{2,i} \cos\theta_{2,i} = -(m_1 + m_2) v_f \cos\theta_f$

y:

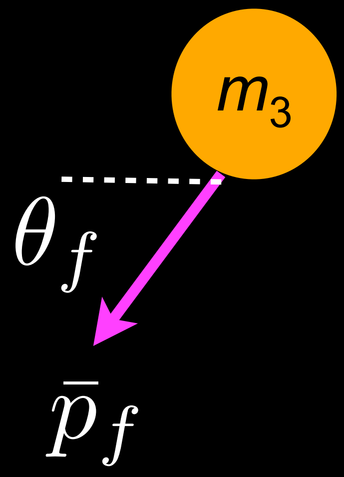
Collisions

Inelastic: 2D

before:



after:



Conservation of momentum:

$$p_x = m_1 v_{1,i} \cos\theta_{1,i} - m_2 v_{2,i} \cos\theta_{2,i}$$

$$p_y = m_1 v_{1,i} \sin\theta_{1,i} + m_2 v_{2,i} \sin\theta_{2,i}$$

$$p_x = -m_3 v_f \cos\theta_f$$
$$= -(m_1 + m_2) v_f \cos\theta_f$$

$$p_y = m_3 v_f \sin\theta_f$$
$$= (m_1 + m_2) v_f \sin\theta_f$$

x: $m_1 v_{1,i} \cos\theta_{1,i} - m_2 v_{2,i} \cos\theta_{2,i} = -(m_1 + m_2) v_f \cos\theta_f$

y: $m_1 v_{1,i} \sin\theta_{1,i} + m_2 v_{2,i} \sin\theta_{2,i} = (m_1 + m_2) v_f \sin\theta_f$

Collisions Quiz

A neutron (mass 1 u) strikes a deutron (mass 2 u) and they combine to form a tritium nucleus.

If the neutron's initial velocity was $28\bar{i} + 17\bar{j}\mathrm{Mm/s}$

and the tritium's final velocity is $12\bar{i} + 20\bar{j}\mathrm{Mm/s}$

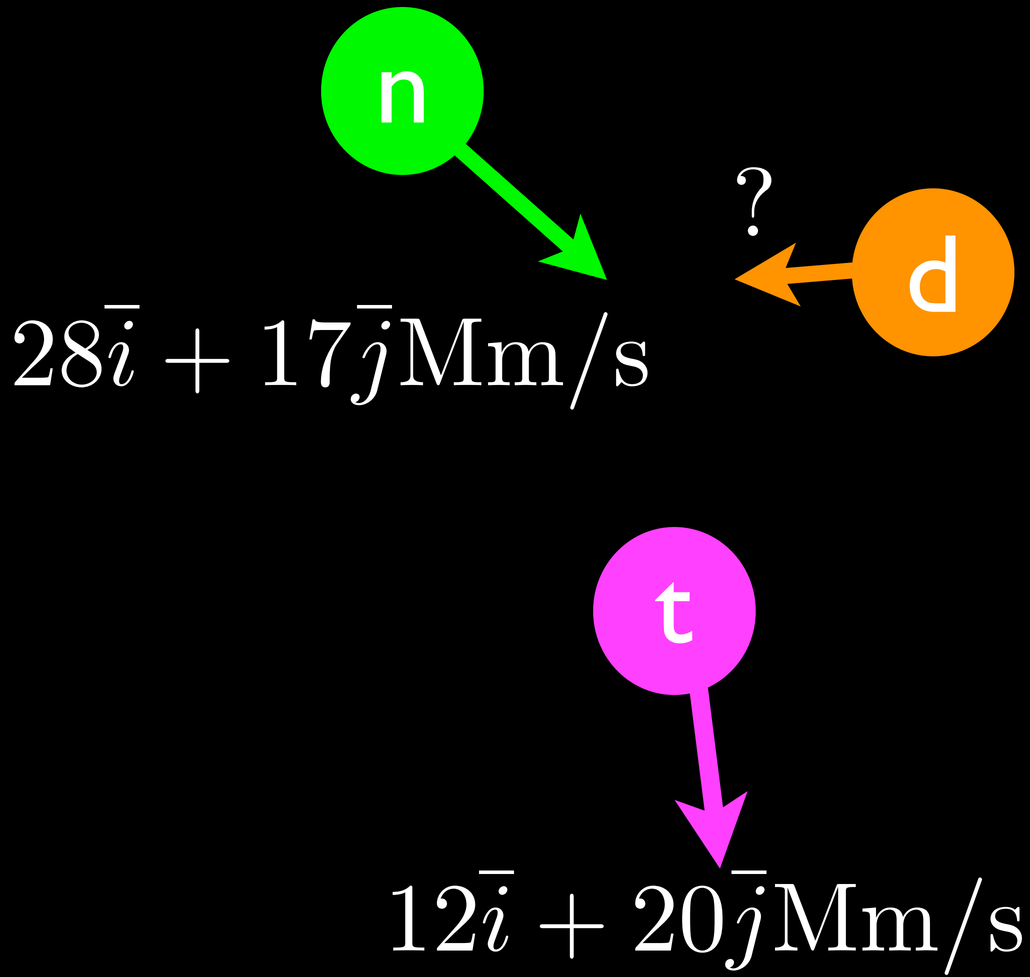
what is the deutron's velocity?

(a) $40\bar{i} - 37\bar{j}\mathrm{Mm/s}$

(b) $32\bar{i} + 38.5\bar{j}\mathrm{Mm/s}$

(c) $16\bar{i} - 3\bar{j}\mathrm{Mm/s}$

[✓ (d) $4\bar{i} + 21.5\bar{j}\mathrm{Mm/s}$]



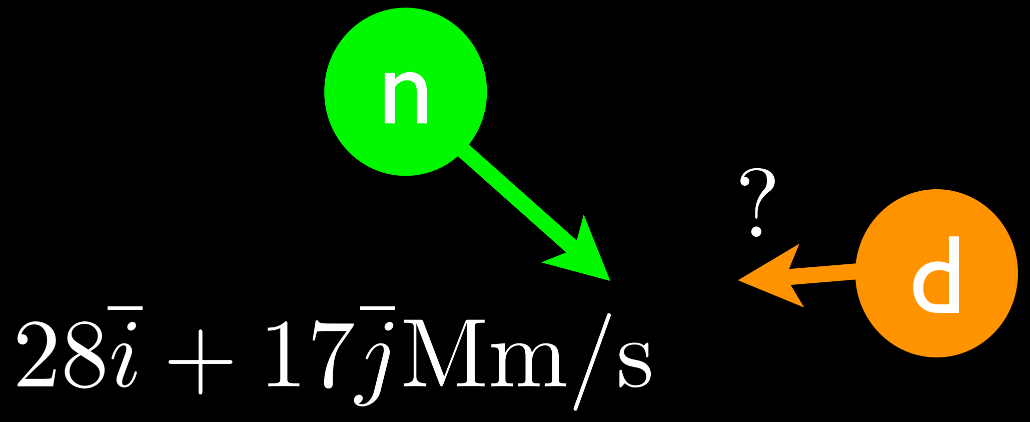
Collisions

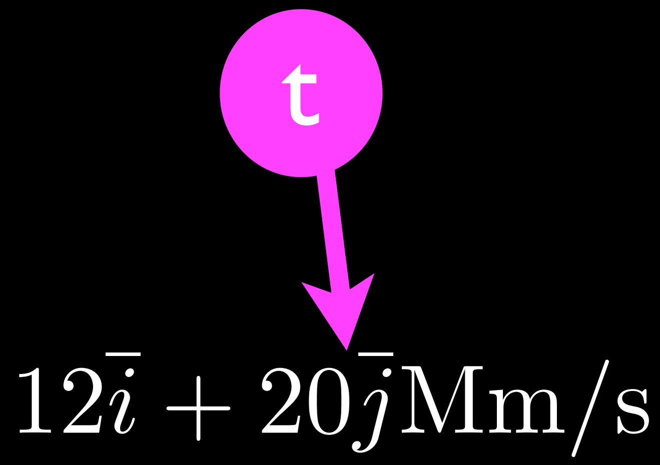
Quiz

Just for interest:

Atomic mass unit: $1u = 1.67 \times 10^{-27}\,\mathrm{kg}$

$d = 1\mathrm{proton} + 1\mathrm{neutron}$





$$\bar{p}_{\mathrm{before}} = m_n\bar{v}_n + m_d\bar{v}_d$$

$$\bar{p}_{\mathrm{after}} = m_t\bar{v}_t$$

$$\bar{p}_{\mathrm{before}} = \bar{p}_{\mathrm{after}}$$

$$\bar{v}_d = \frac{m_t\bar{v}_t - m_n\bar{v}_n}{m_d}$$

x: $$\bar{v}_{d,x} = \frac{(3u \times 12) - (u \times 28)}{2u} = 4\,\mathrm{Mm/s}$$

y: $$\bar{v}_{d,y} = \frac{(3u \times 20) - (u \times 17)}{2u} = 21.5\,\mathrm{Mm/s}$$

$$(4\hat{i} + 21.5\hat{j})\,\mathrm{Mm/s}$$

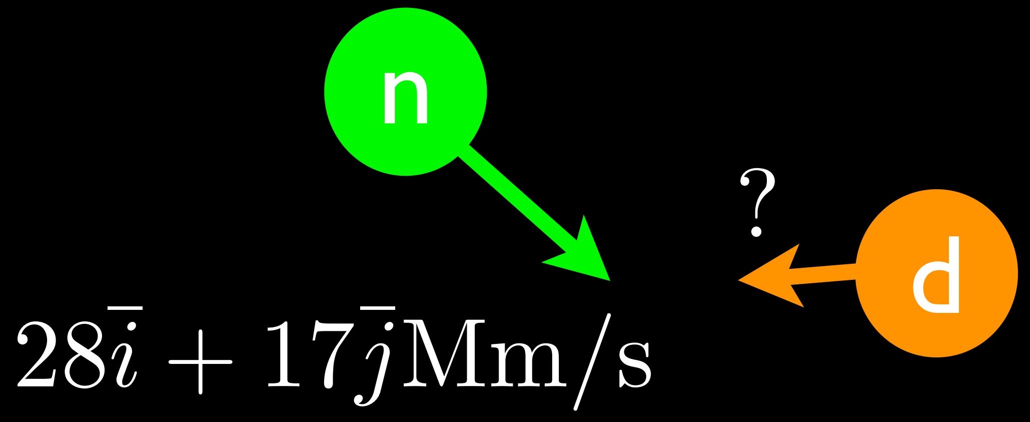
Collisions

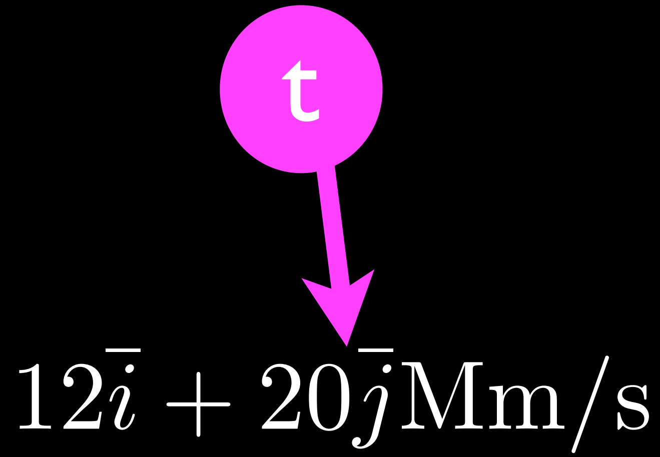
Quiz

Just for interest:

Atomic mass unit: $1u = 1.67 \times 10^{-27}\,\mathrm{kg}$

$d = 1\mathrm{proton} + 1\mathrm{neutron}$





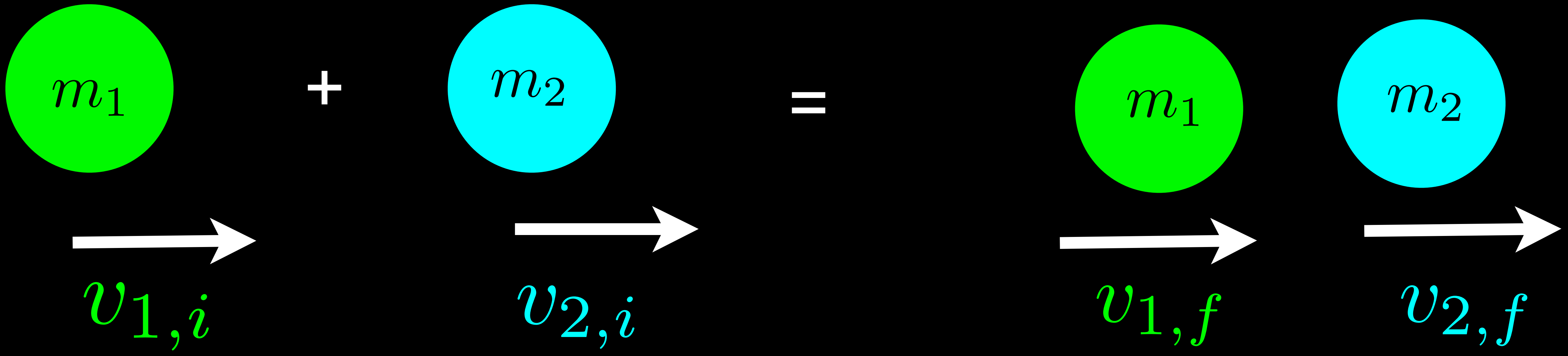
$$\left.\begin{array}{l} \bar{p}_{\mathrm{before}} = m_n\bar{v}_n + m_d\bar{v}_d \\ \bar{p}_{\mathrm{after}} = m_t\bar{v}_t \end{array}\right\} \quad \bar{p}_{\mathrm{before}} = \bar{p}_{\mathrm{after}}$$

$$\bar{v}_d = \frac{m_t\bar{v}_t - m_n\bar{v}_n}{m_d}$$

$$= \frac{3u(12\hat{i} + 20\hat{j}) - u(28\hat{i} + 17\hat{j})}{2u} \quad = 4\bar{i} + 21.5\bar{j}\mathrm{Mm/s}$$

Collisions

Elastic: 1D



Conservation of momentum, p:

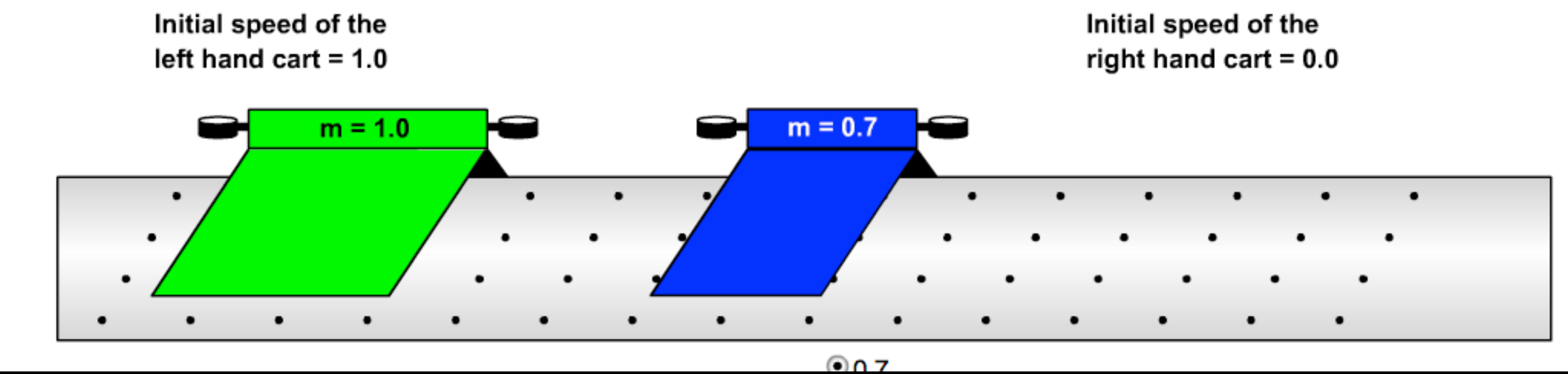
$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

Conservation of kinetic energy, K:

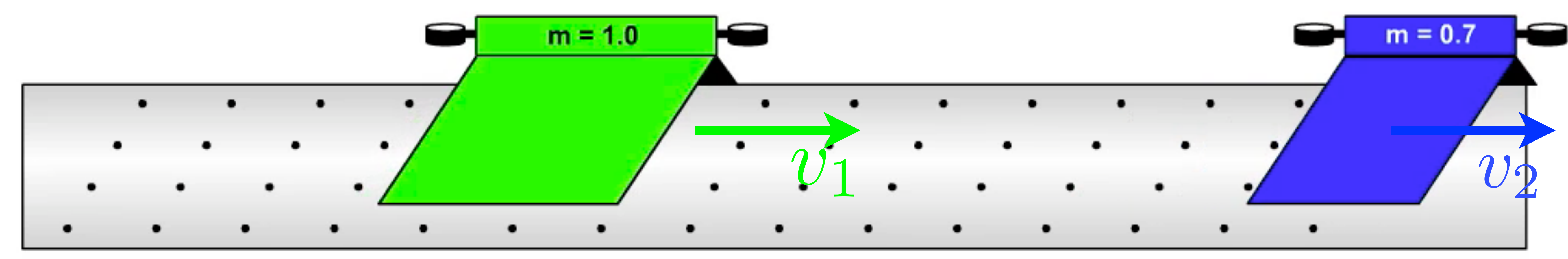
$$\frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

Collisions

Elastic collision 1D:



2 carts on frictionless track collide in an elastic collision



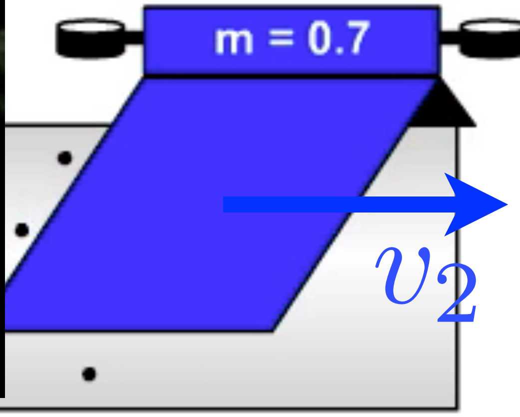
find: v_1 and v_2

Collisions

Elastic collision 1D:

Initial speed f the
left hand ca t = 0.0

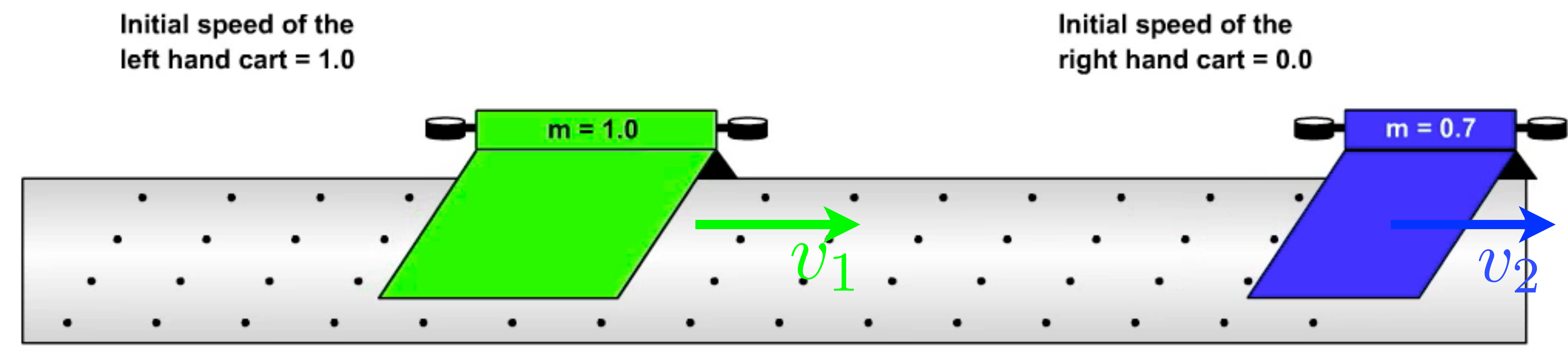
2 carts on fr



find: v_1 and v_2

Collisions

Elastic collision 1D:



find: v_1 and v_2

[✓ (a) 0.18 & 1.18]

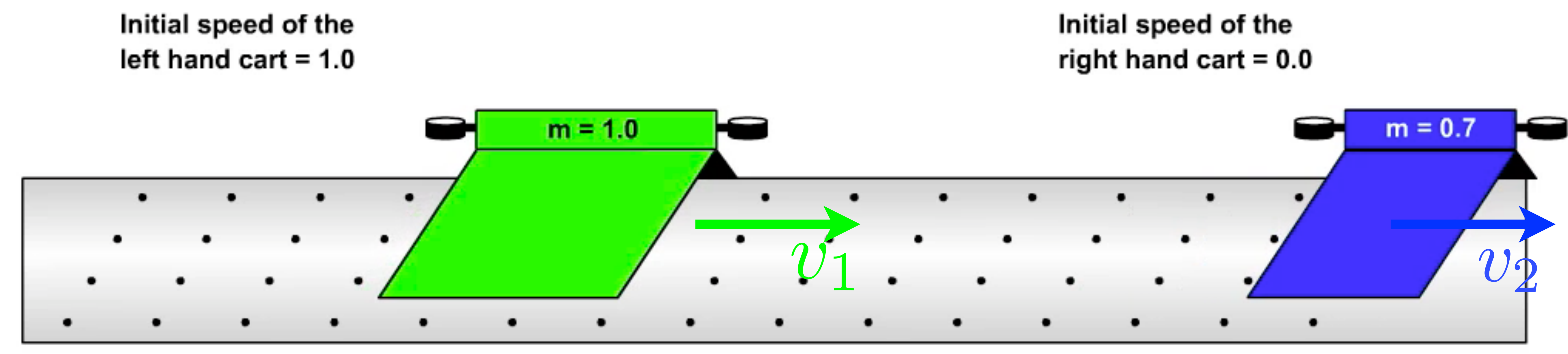
(b) 0.0 & 1.0

(c) 0.51 & 0.11

(d) 0.5 & -0.5

Collisions

Elastic collision 1D: find: v_1 and v_2

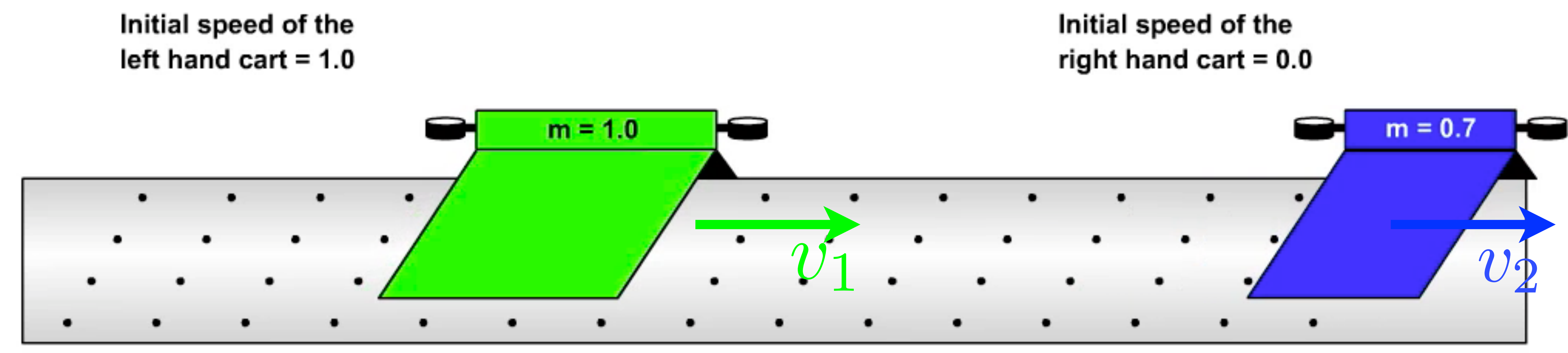


$$\left.\begin{array}{l} \bar{p}_{\text{before}} = m_L\bar{v}_{L,1} + m_R\bar{v}_{R,1} = 1\times 1 + 0.7\times 0 \\ \bar{p}_{\text{after}} = m_L\bar{v}_{L,2} + m_R\bar{v}_{R,2} = 1\times \bar{v}_{L,2} + 0.7\times \bar{v}_{\mathrm{R},2} \end{array}\right\} =$$

$$\left.\begin{array}{l} K_{\text{before}} = \frac{1}{2}m_L\bar{v}_{L,1}^2 + \frac{1}{2}m_R\bar{v}_{R,1}^2 = \frac{1}{2}\times 1\times 1^2 + 0 \\ K_{\text{after}} = \frac{1}{2}m_L\bar{v}_{L,2}^2 + \frac{1}{2}m_R\bar{v}_{R,2}^2 = \frac{1}{2}1\bar{v}_{L,2}^2 + \frac{1}{2}0.7\bar{v}_{R,2}^2 \end{array}\right\} =$$

Collisions

Elastic collision 1D: find: v_1 and v_2



$$\bar{p}_{\text{before}} = \bar{p}_{\text{after}}$$

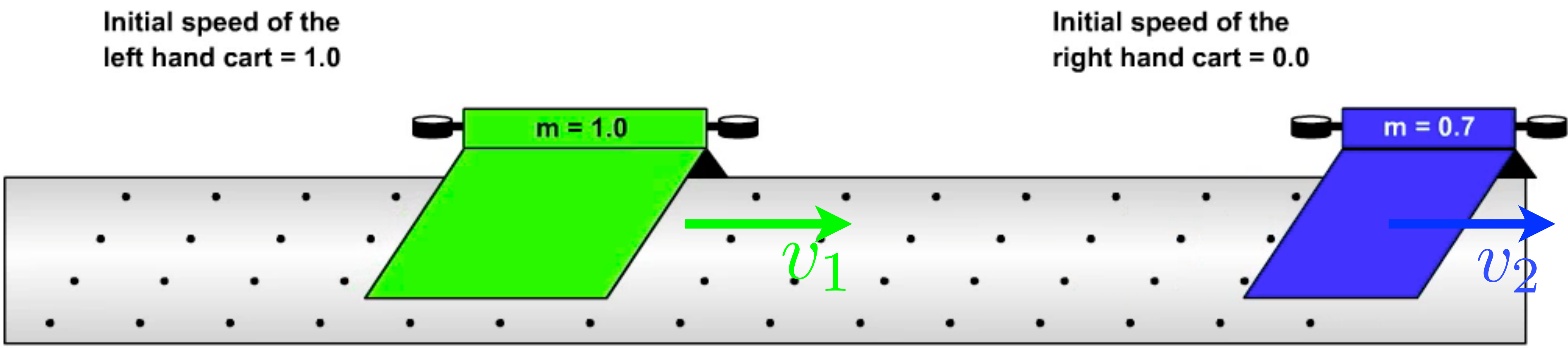
$$1 = \bar{v}_{L,2} + 0.7\bar{v}_{R,2} \longrightarrow \boxed{\bar{v}_{L,2} = 1 - 0.7\bar{v}_{R,2}}$$

$$K_{\text{before}} = K_{\text{after}}$$

$$\frac{1}{2} = \frac{1}{2}\bar{v}^2_{\mathrm{L},2} + \frac{1}{2}0.7\bar{v}^2_{R,2} \longrightarrow 1 = \bar{v}^2_{L,2} + 0.7\bar{v}^2_{R,2}$$

Collisions

Elastic collision 1D: find: v_1 and v_2



Combining:

$$1 = (1 - 0.7\bar{v}_{R,2})^2 + 0.7\bar{v}_{R,2}^2$$

$$= 1 - 1.4\bar{v}_{R,2} + 0.49\bar{v}_{\mathrm{R},2}^2 + 0.7\bar{v}_{\mathrm{R},2}^2$$

$$\bar{v}_{L,2} = 1 - 0.7\bar{v}_{R,2}$$

$$0 = \bar{v}_{R,2}(1.19\bar{v}_{R,2} - 1.4) \longrightarrow \bar{v}_{R,2} = 0 \text{ or } 1.18$$

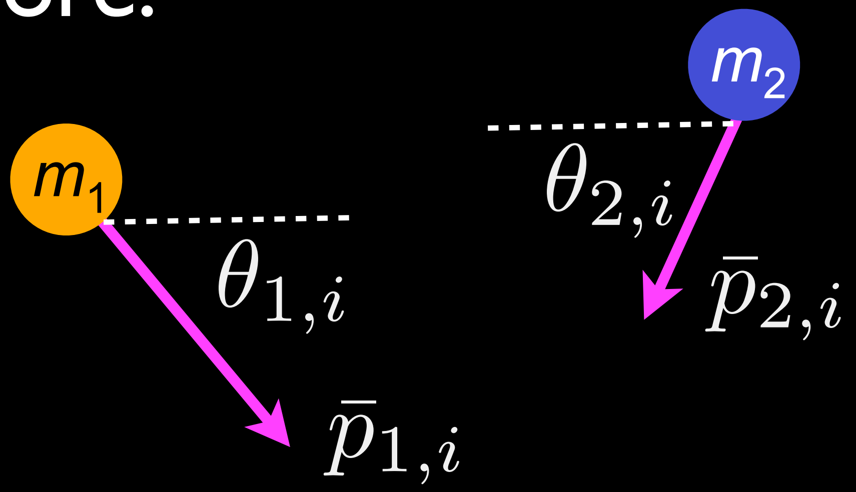
$$\longrightarrow \bar{v}_{L,2} = 1 \text{ or } 0.18$$

(0,1 = initial conditions)

Collisions

Elastic: 2D

before:

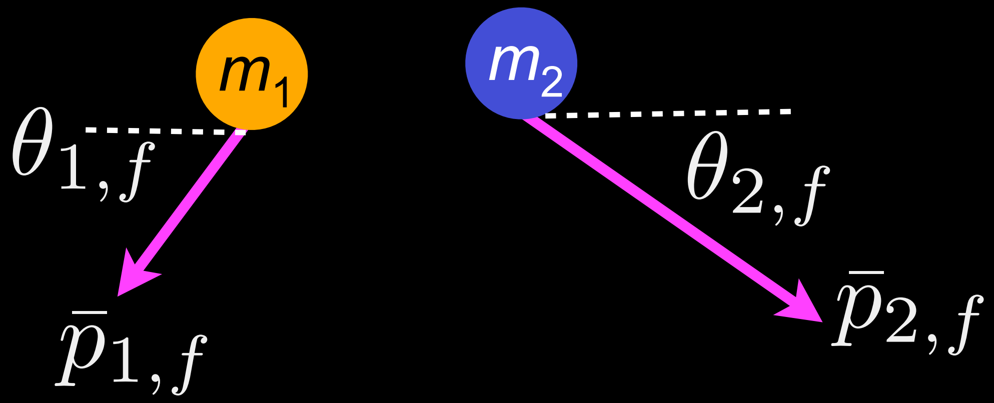


Conservation of momentum:

$$p_x = m_1 v_{1,i} \cos\theta_{1,i} - m_2 v_{2,i} \cos\theta_{2,i}$$

$$p_y = m_1 v_{1,i} \sin\theta_{1,i} + m_2 v_{2,i} \sin\theta_{2,i}$$

after:



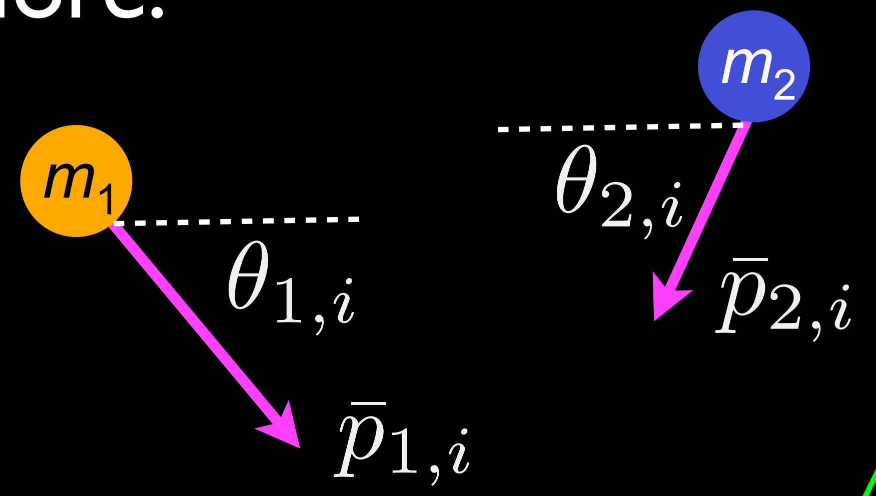
$$p_x = -m_1 v_{1,f} \cos\theta_{1,f} + m_2 v_{2,f} \cos\theta_{2,f}$$

$$p_y = m_1 v_{1,f} \sin\theta_{1,f} + m_2 v_{2,f} \sin\theta_{2,f}$$

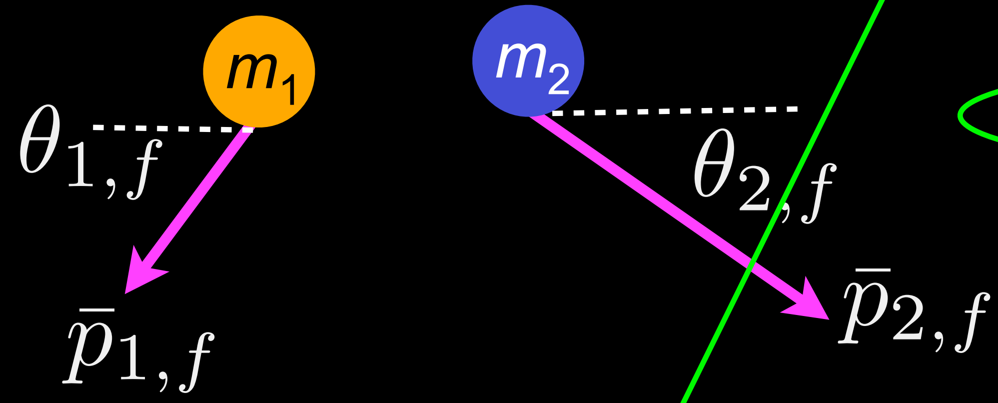
Collisions

Elastic: 2D

before:



after:



Conservation of momentum:

$$p_x = m_1 v_{1,i} \cos\theta_{1,i} - m_2 v_{2,i} \cos\theta_{2,i}$$

$$p_y = m_1 v_{1,i} \sin\theta_{1,i} + m_2 v_{2,i} \sin\theta_{2,i}$$

$$p_x = -m_1 v_{1,f} \cos\theta_{1,f} + m_2 v_{2,f} \cos\theta_{2,f}$$

$$p_y = m_1 v_{1,f} \sin\theta_{1,f} + m_2 v_{2,f} \sin\theta_{2,f}$$

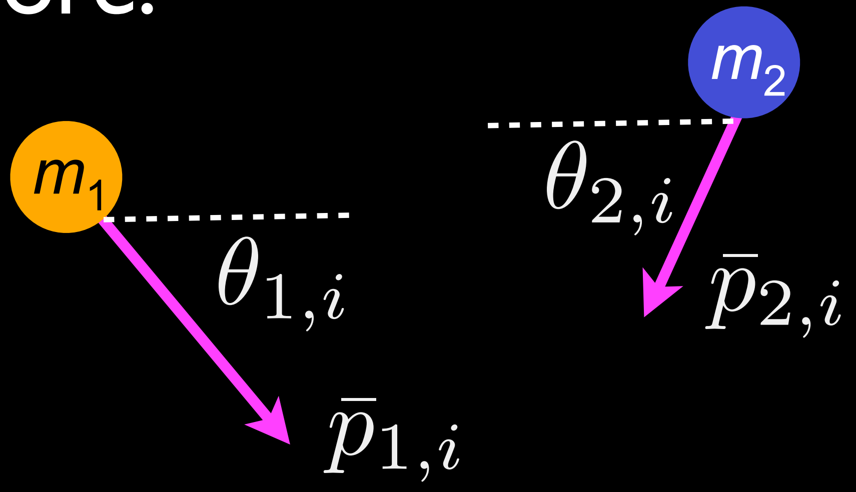
$$m_1 v_{1,i} \cos\theta_{1,i} - m_2 v_{2,i} \cos\theta_{2,i} =$$

$$-m_1 v_{1,f} \cos\theta_{1,f} + m_2 v_{2,f} \cos\theta_{2,f}$$

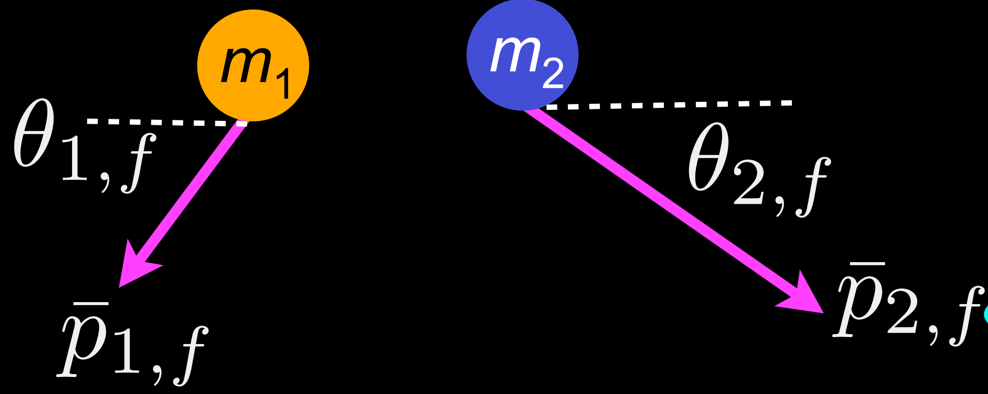
Collisions

Elastic: 2D

before:



after:



Conservation of momentum:

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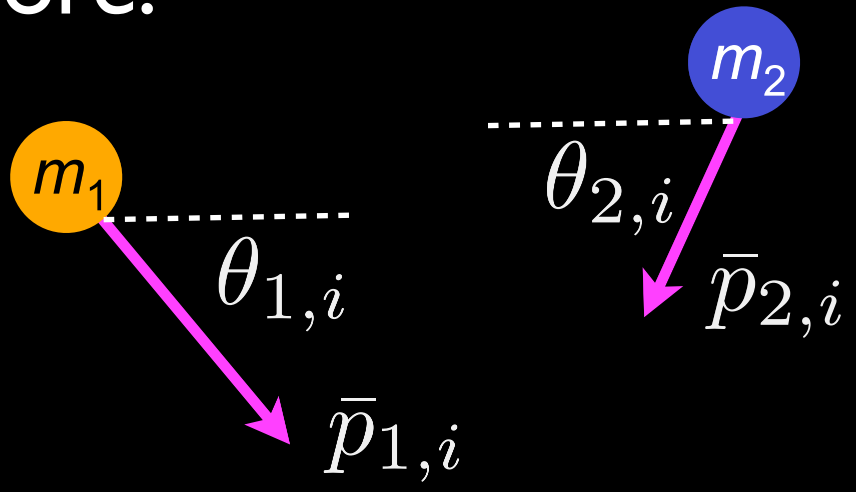
$$m_1 v_{1,i} \cos\theta_{1,i} - m_2 v_{2,i} \cos\theta_{2,i} = -m_1 v_{1,f} \cos\theta_{1,f} + m_2 v_{2,f} \cos\theta_{2,f}$$

$$m_1 v_{1,i} \sin\theta_{1,i} + m_2 v_{2,i} \sin\theta_{2,i} = m_1 v_{1,f} \sin\theta_{1,f} + m_2 v_{2,f} \sin\theta_{2,f}$$

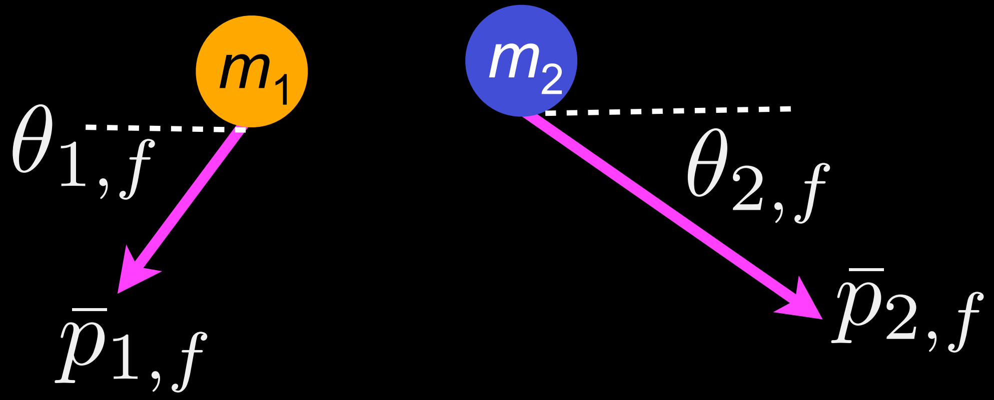
Collisions

Elastic: 2D

before:



after:



Conservation of kinetic energy:

$$\frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2$$

$$\frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2$$

$$\frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2 = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2$$

Collisions

Example

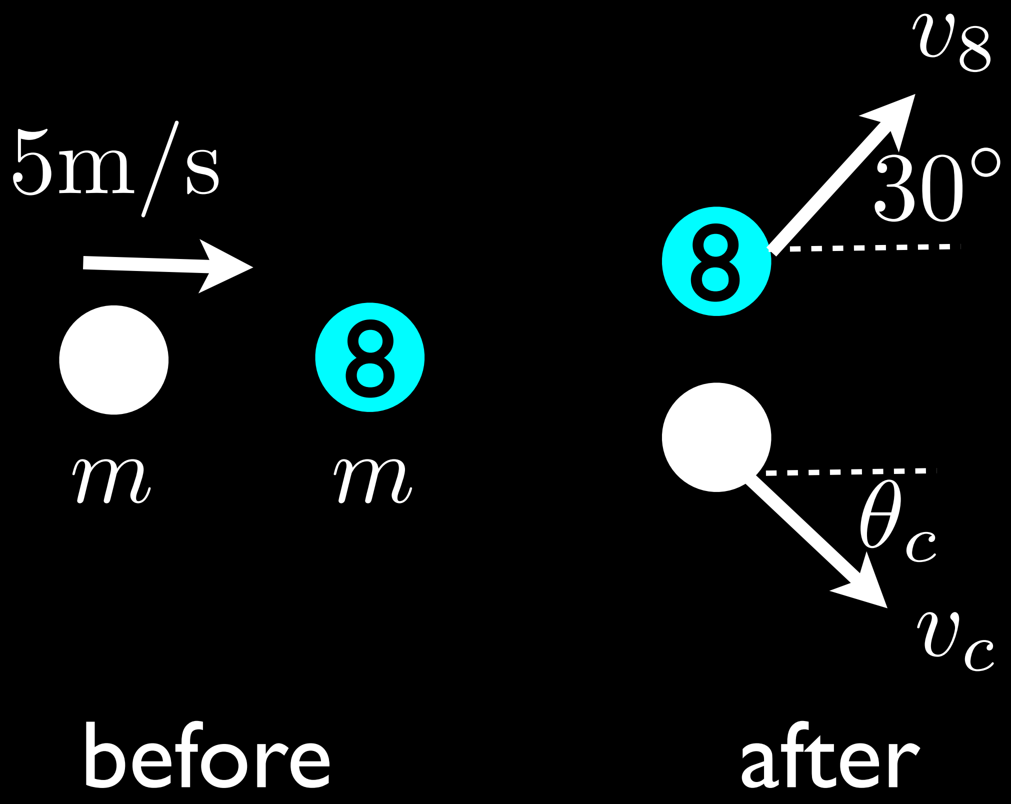
In a pool game, the cue ball, with an initial speed of 5 m/s, makes and elastic collision with the 8-ball, which is initially at rest.

After the collision, the 8-ball moves at an angle of 30° with the original direction of the cue ball.

(a) find the direction of motion of the cue ball

(b) speed of each ball

Assume balls are of equal mass.



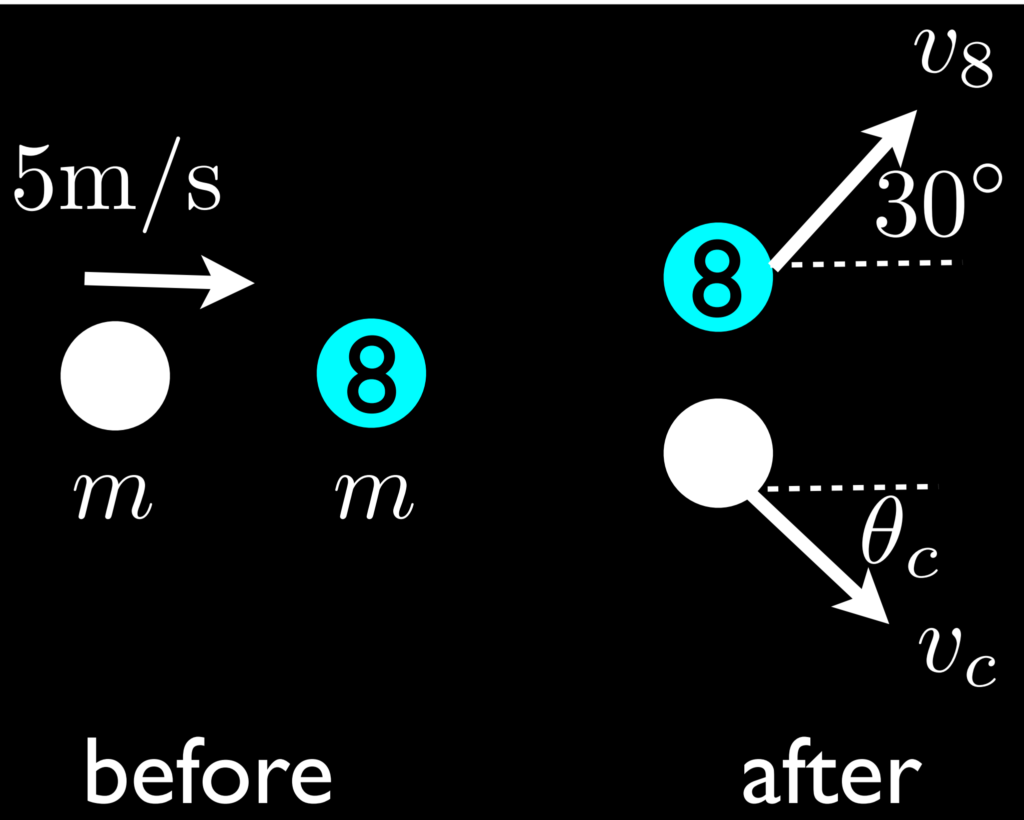
Collisions

Example

Conservation of momentum:

X: $5m = mv_8 \cos 30 + mv_c \cos \theta_c$

Y: $0 = mv_8 \sin 30 + mv_c \sin \theta_c$



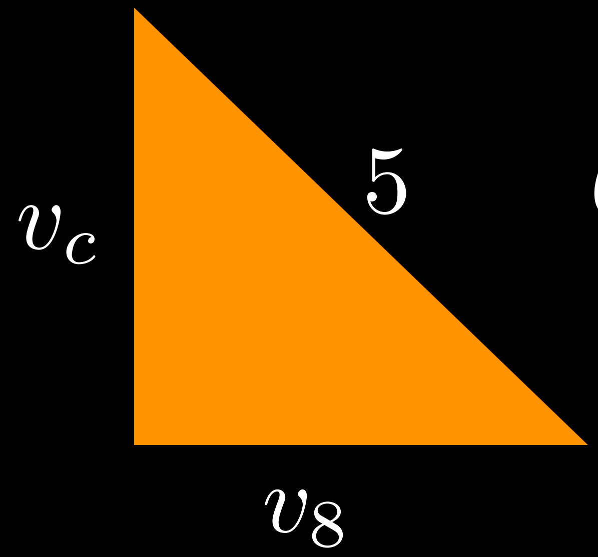
Conservation of kinetic energy:

$$\frac{1}{2}m5^2 = \frac{1}{2}mv_8^2 + \frac{1}{2}mv_c^2$$

when mass is equal:

$$5^2 = v_8^2 + v_c^2 \longrightarrow$$

Pythagorus: right-angled triangle equation

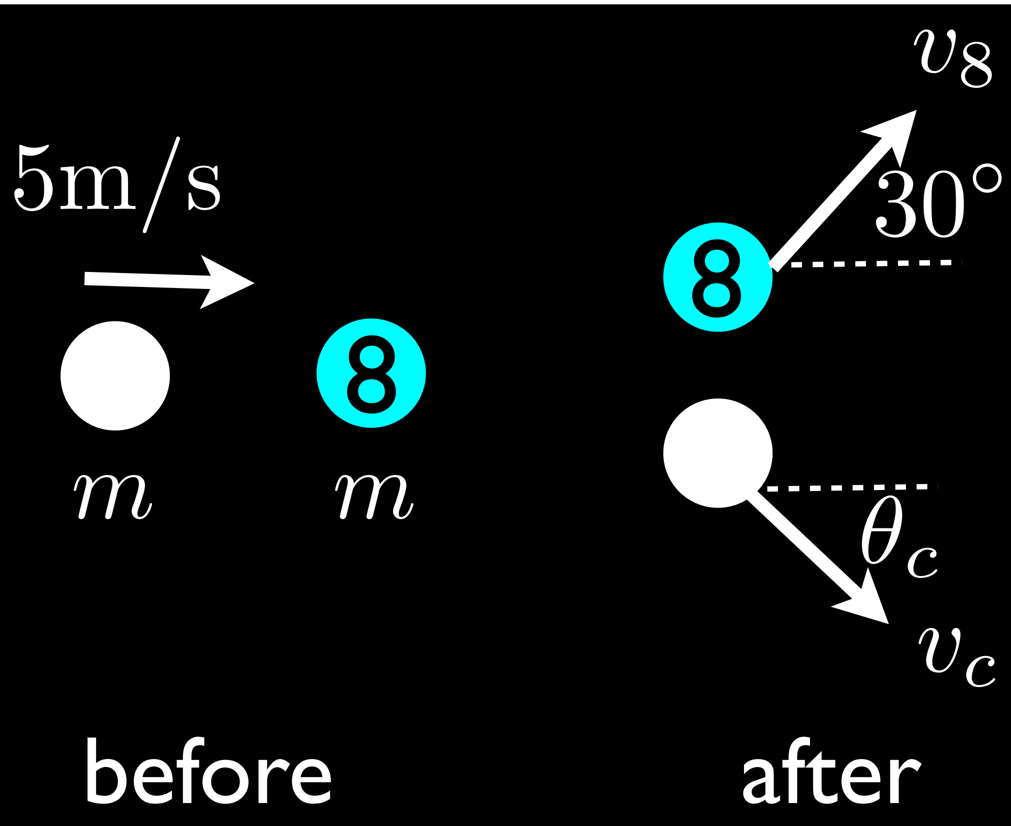


$$\theta_c + \theta_8 = 90$$

$$\boxed{\theta_c = 60°}$$

Collisions

Example



From:

$$0 = mv_8 \sin 30 + mv_c \sin\theta_c$$

$$v_8 = v_c \frac{\sin 60}{\sin 30}$$

From:

$$5m = mv_8 \cos 30 + mv_c \cos\theta_c$$

$$v_c = \frac{5.0}{(\sin 60 \cot 30 + \cos 60)} \quad = 2.5\text{m/s}$$

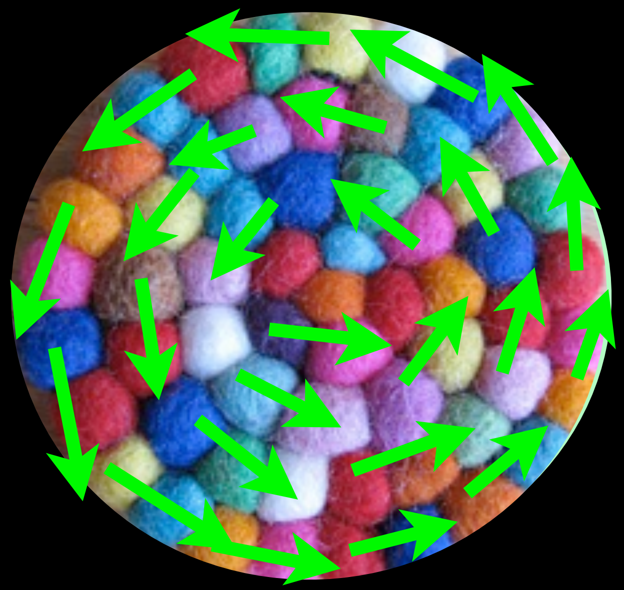
and:

$$v_8 = (2.5\text{m/s})\frac{\sin 60}{\sin 30} \quad = 4.33\text{m/s}$$

Rotation

Rotation

How do we calculate the motion of a rotating disc?

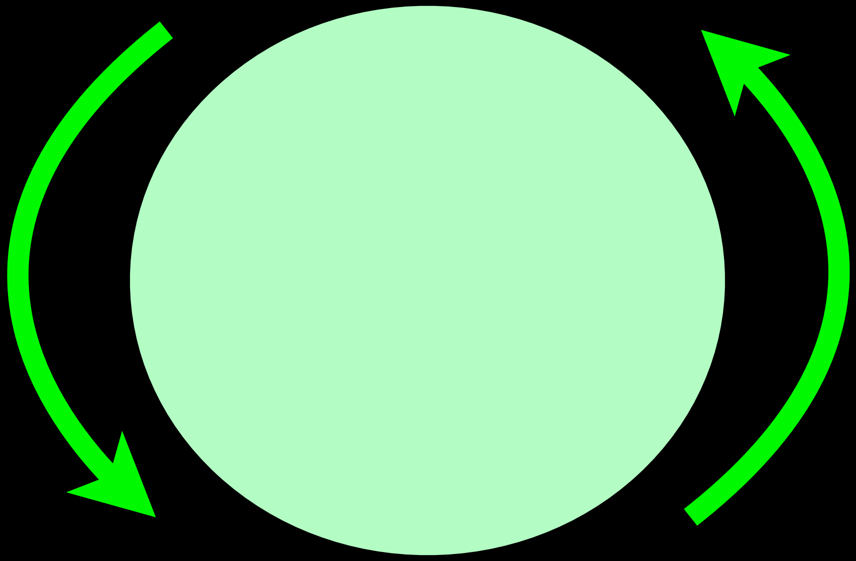


We could use circular motion to give each point a speed and direction .....

.... but that would be slow!

Rotation

How do we calculate the motion of a rotating disc?



rigid body: all points remain fixed relative to one another

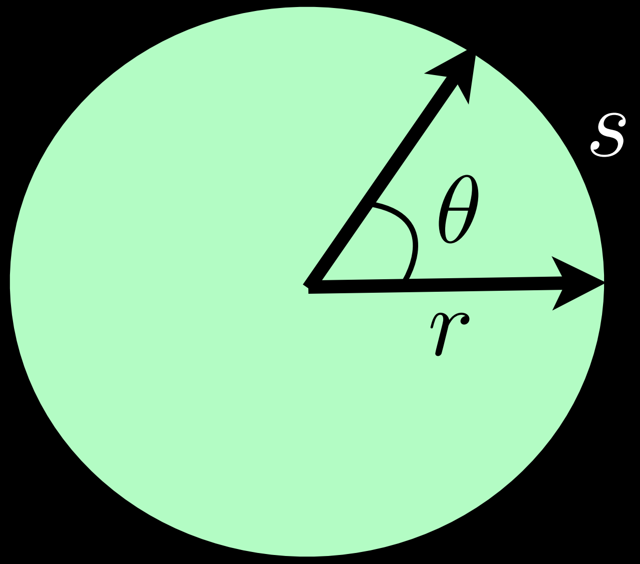
We could use circular motion to give each point a speed and direction .....

.... but that would be slow!

Easier to say that the disc rotates with 800 revolutions per minute (rpm)

Rotation

Angular velocity



Rate of rotation: change in angle with time

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \ \text{rad/s}$$ average angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$ instantaneous angular velocity

For small θ : $\theta = \frac{s}{r}$ [rad]

$$\underset{\omega}{\frac{d\theta}{dt}} = \frac{1}{r} \underset{v}{\frac{ds}{dt}} \rightarrow \boxed{v = rw}$$

Rotation

Quiz

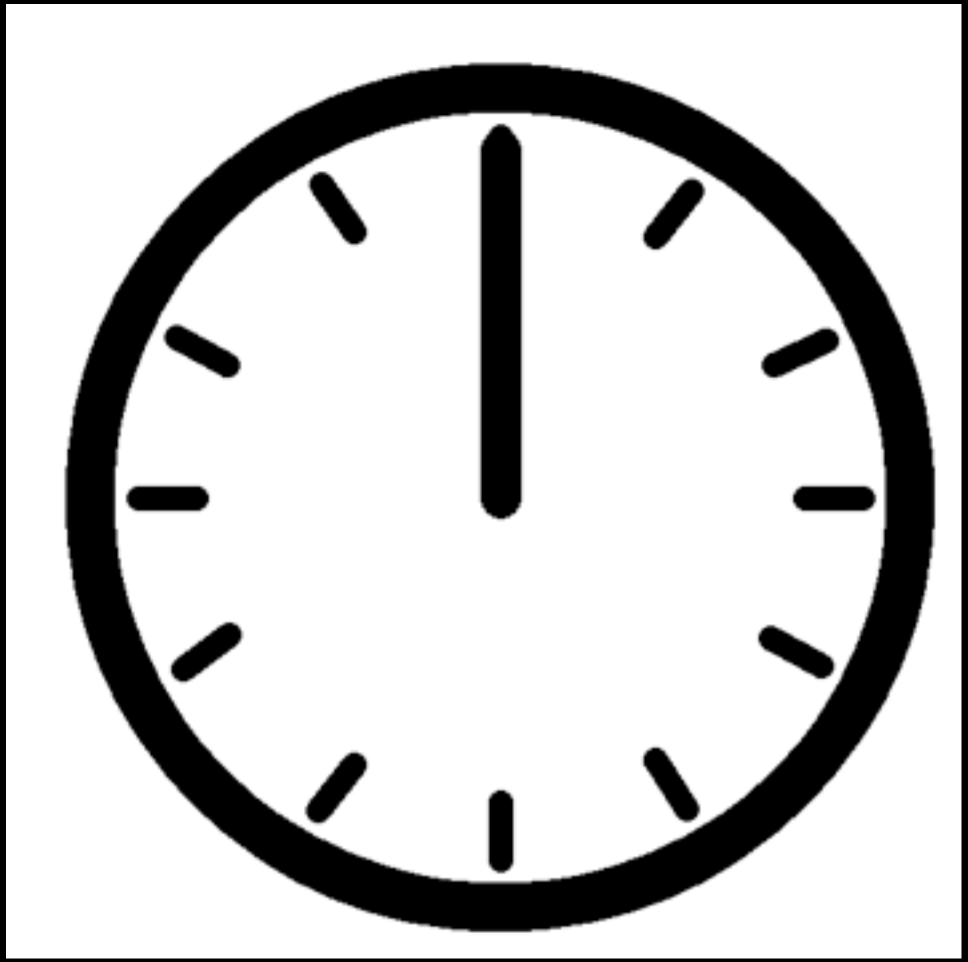
What is the angular speed of the hour hand of a clock?

[✓ (a) $1.45 \times 10^{-4} \quad \mathrm{rads}^{-1}$]

(b) $1.75 \times 10^{-3} \quad \mathrm{rads}^{-1}$

(c) $0.0083 \quad \mathrm{rads}^{-1}$

(d) $0.1 \quad \mathrm{rads}^{-1}$



Rotation

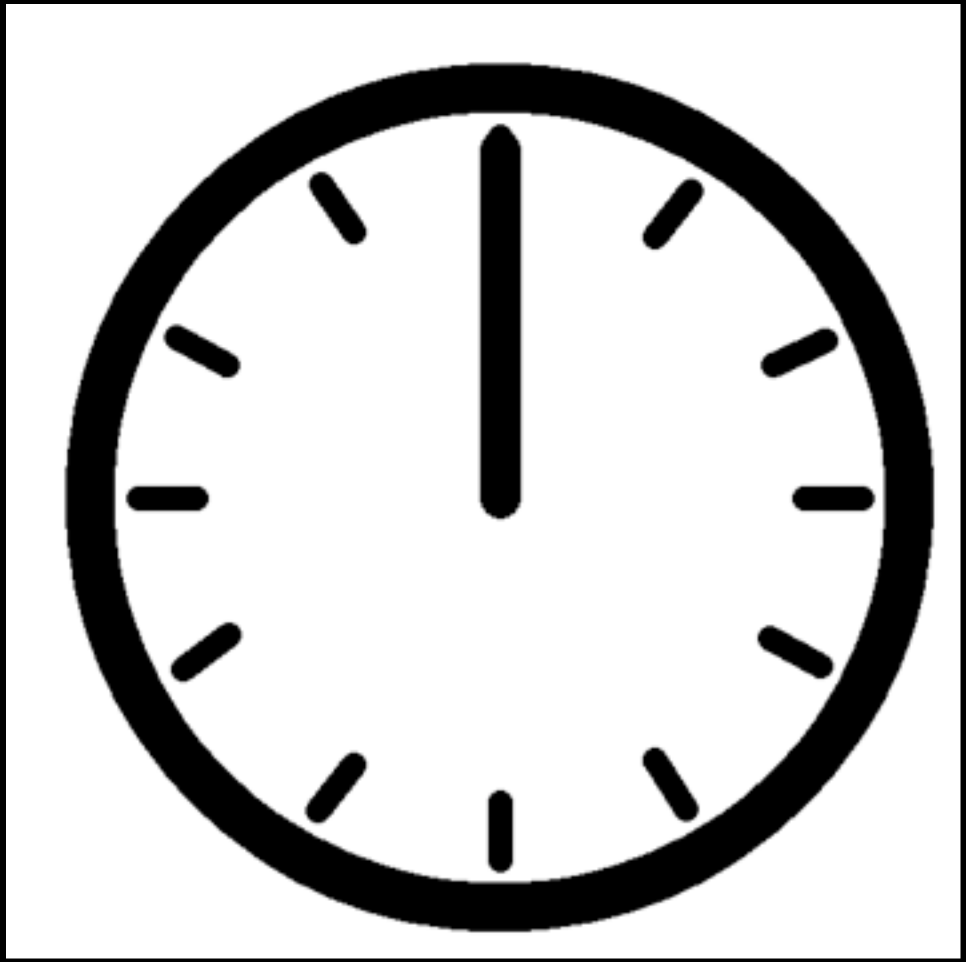
Quiz

What is the angular speed of the hour hand of a clock?

$$\omega_{\mathrm{hr}} = \frac{1\,\mathrm{rev}}{12\,\mathrm{hr}}$$

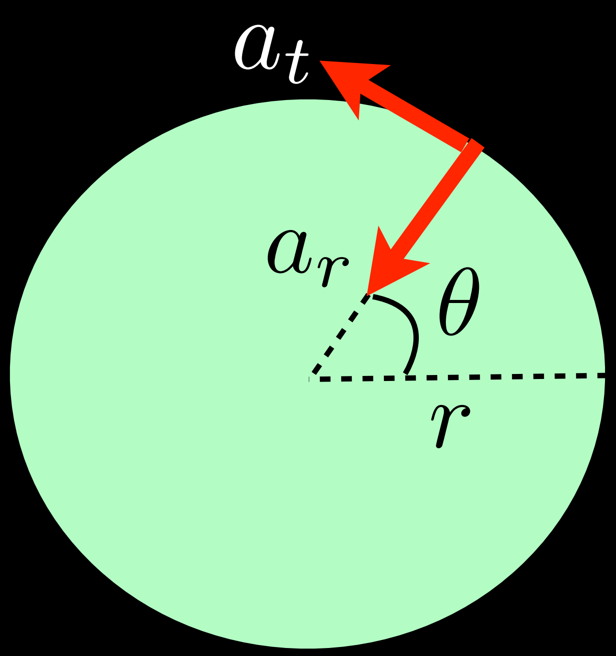
$$= \frac{2\pi}{12 \times 3600}$$

$$= 1.45 \times 10^{-4}\,\mathrm{rads}^{-1}$$



Rotation

Angular acceleration



$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} \quad \mathrm{rad/s^2}$$

Tangential acceleration speeds up or slows down rotation:

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$$

There is still radial acceleration:

$$a_r = \frac{v^2}{r} \quad \overrightarrow{v = r\omega} \quad a_r = \omega^2 r$$

Rotation

Linear and angular quantities:

Linear Quantity	Angular Quantity
Position x	Angular Position θ
Velocity $v = \frac{dx}{dt}$	Angular velocity $w = \frac{d\theta}{dt}$
Acceleration $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$	Angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Rotation

Eq. for constant linear acceleration	Eq. for angular acceleration
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Rotation

Quiz

A merry-go-round starts from rest and accelerates with angular acceleration of $0.010 \quad \mathrm{rad/s^2}$ for 14 s.

(a) How many revolutions does it make during this time?

(b) What is the average angular speed?

(1) 0.16 rev, 0.01 rad/s

(2) 0.98 rev, 0.07 rad/s

[✓ (3) 0.16 rev, 0.07 rad/s]

(4) 0.98 rev, 0.01 rad/s

Rotation

Quiz

A merry-go-round starts from rest and accelerates with angular acceleration of $0.010 \quad \mathrm{rad/s^2}$ for 14 s.

$$\theta = \theta_0 + \underset{0}{\cancel{\omega_o}} t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = \theta - \theta_0 = \frac{1}{2}(0.010\,\mathrm{rads}^{-1})(14\,\mathrm{s})^2$$

$$= 0.98\,\mathrm{rad} \;= 0.98\,\mathrm{rad}\left(\frac{1\,\mathrm{rev}}{2\pi\,\mathrm{rad}}\right) = 0.16\,\mathrm{rev}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{0.98\,\mathrm{rad}}{14\,\mathrm{s}} = 0.07\,\mathrm{rad/s}$$

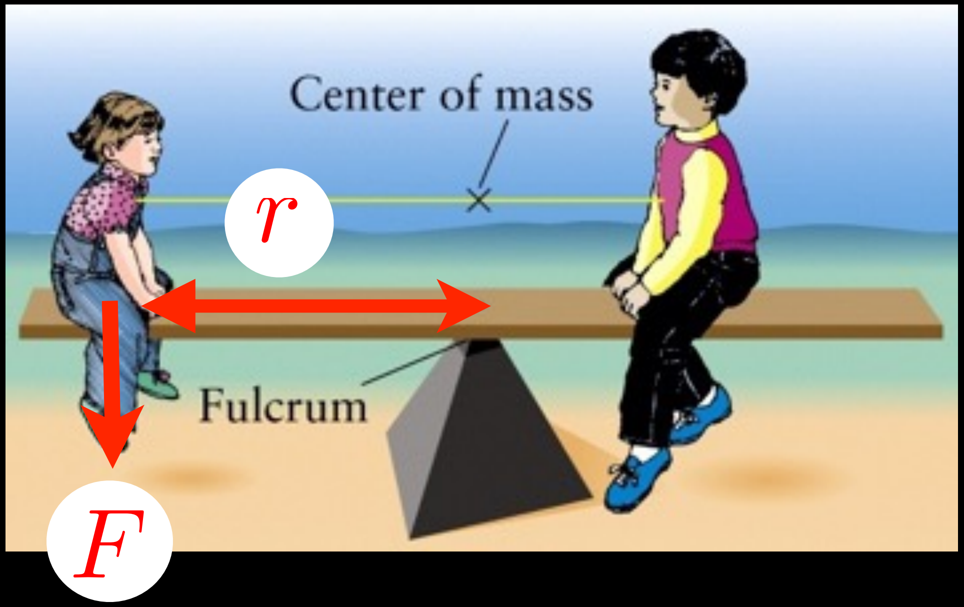
Rotation

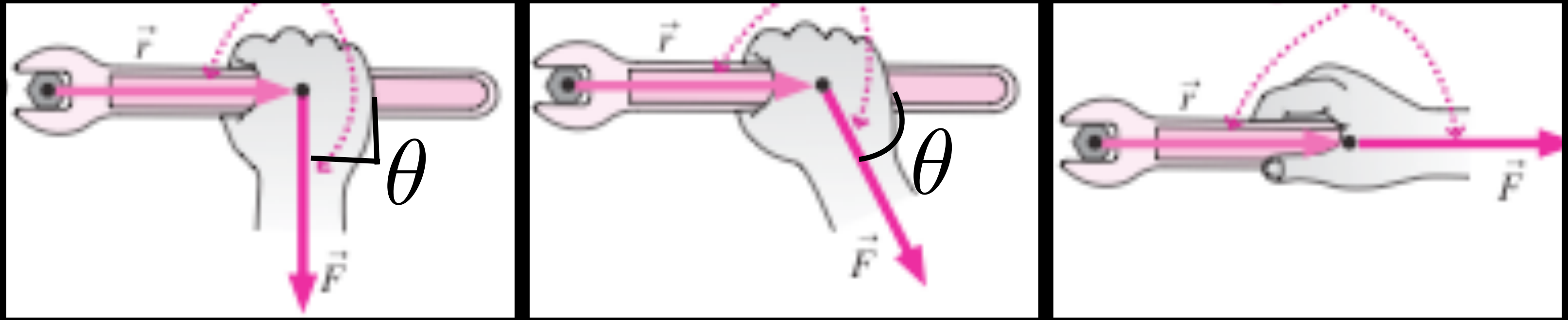
We have angular equivalents of x, v and a.

What about for $\bar{F} = m\bar{a}$?

Need angular equivalents for *force* and *mass*.

Forces that change rotational motion depend on.....





magnitude of the force and distance from the axis

direction of applied force

Angular force, Torque: $$\boxed{\tau = rF\sin\theta}$$ [Nm] “twisting force”

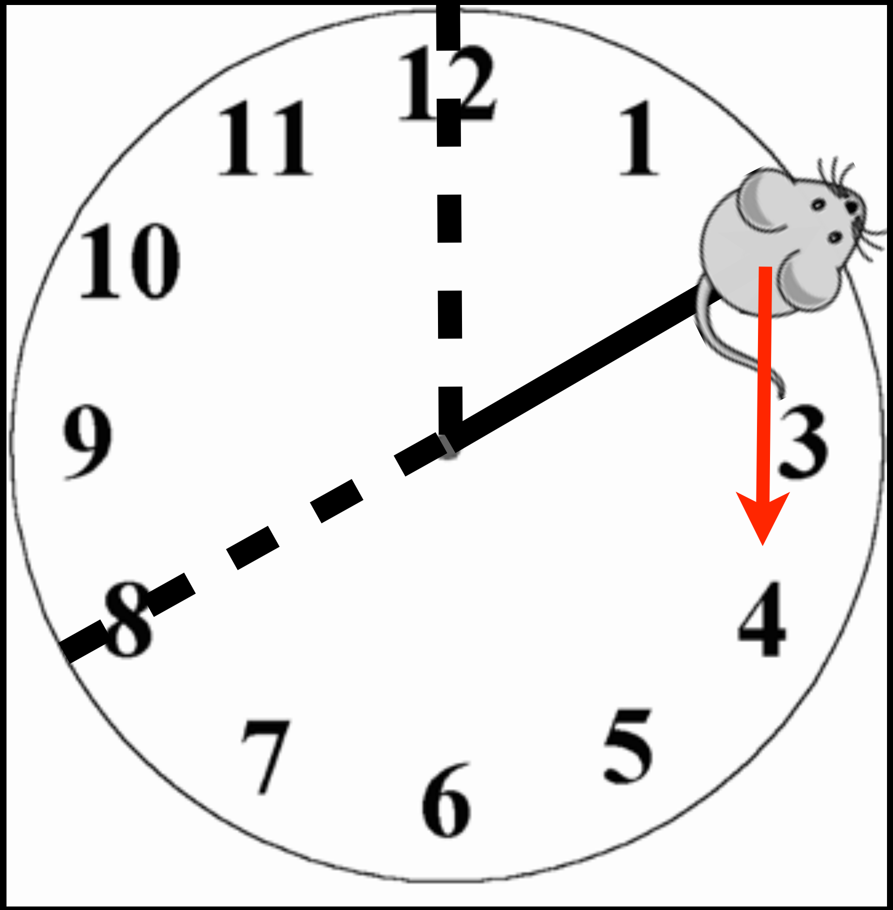
Rotation

Quiz

A 55-g mouse runs out to the end of the 17-cm long minute hand of a clock when the clock is at 10 past the hour.

What torque does the mouse's weight exert about the rotation axis of the clock hand?

- (1) 0.1Nm
- (2) 0.016Nm
- (3) 0.09Nm
- [✓ (4) 7.9×10^{-2}Nm]



Rotation

Quiz

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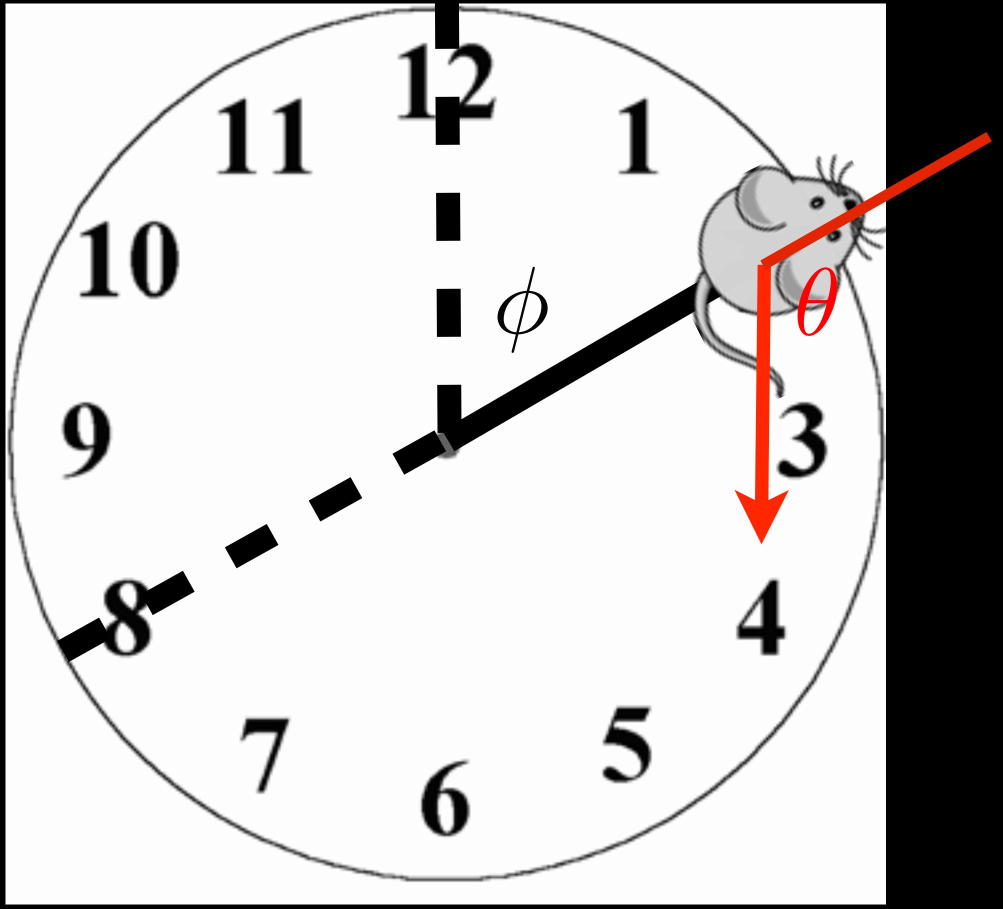


From clock face: $\phi = \frac{180^\circ}{3} = 60^\circ$

$$\theta = 180^\circ - 60^\circ = 120^\circ$$

$$\tau = rF\sin\theta$$

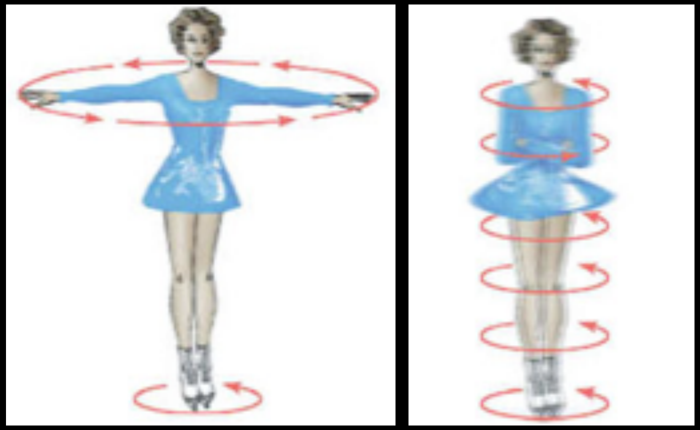
$$= (0.17\,\mathrm{m})(0.055\,\mathrm{kg})(9.81\,\mathrm{m/s^2})\sin(120^\circ)$$

$$= 7.9 \times 10^{-2}\,\mathrm{Nm}$$

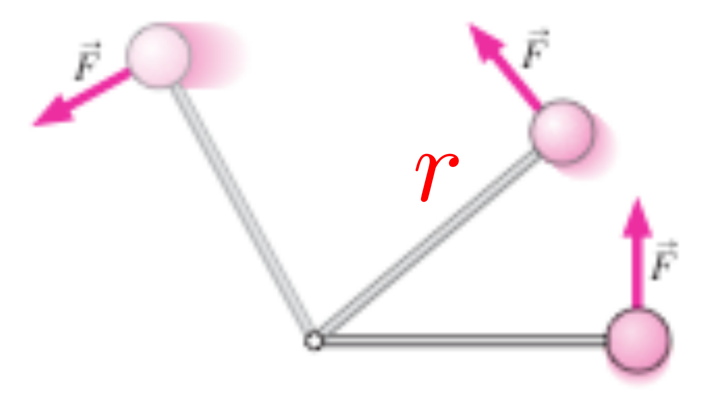
Rotation

What about mass?

It is easier to move an object if its mass is low



It is easier to rotate an object if its mass is centred near the rotation axis



$$F = ma_t \; = m\alpha r$$

$$\tau = rF\sin 90 \; = rF$$

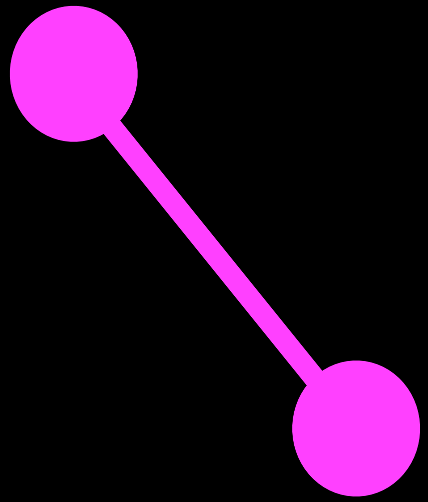
$$\tau = \boxed{(mr^2)}\alpha$$

rotational inertia, I

Newton's 2nd law for rotation: $\boxed{\tau = I\alpha}$

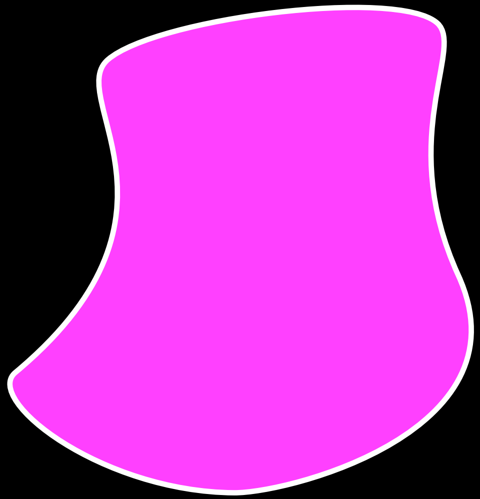
Rotation

Calculating rotational inertia



Discrete mass points:

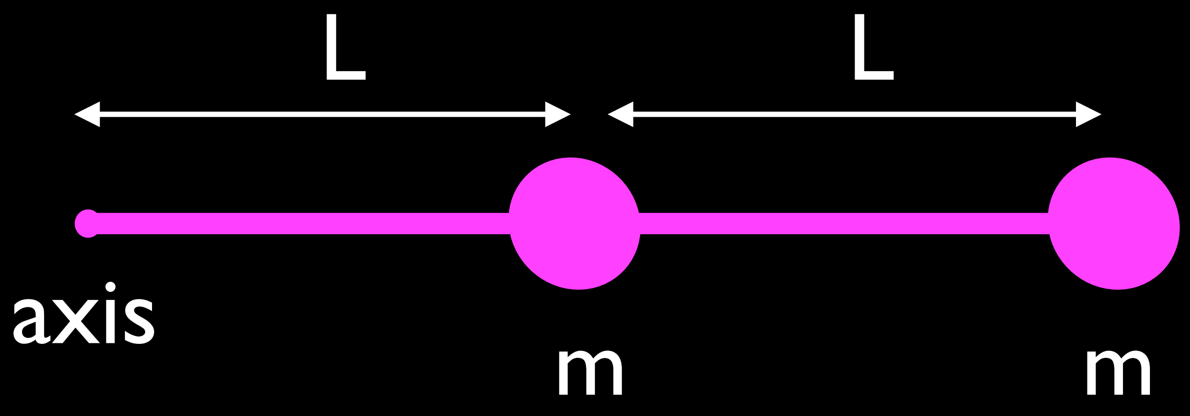
$$I = \Sigma m_i r_i^2$$



Continuous matter:

$$I = \int r^2 dm$$

Rotation



A light (no mass) rod of length = 2L.
2 heavy masses (each mass = m) attached at the end and middle.
What is the rotational inertia about the axis?

(1) mL^2

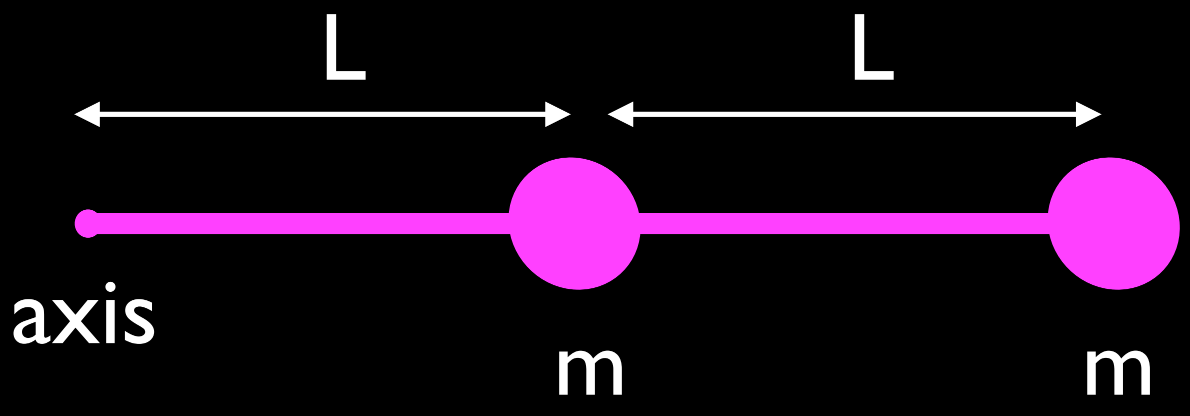
(2) $2mL^2$

(3) $4mL^2$

[✓ (4) $5mL^2$]

(5) $9mL^2$

Rotation



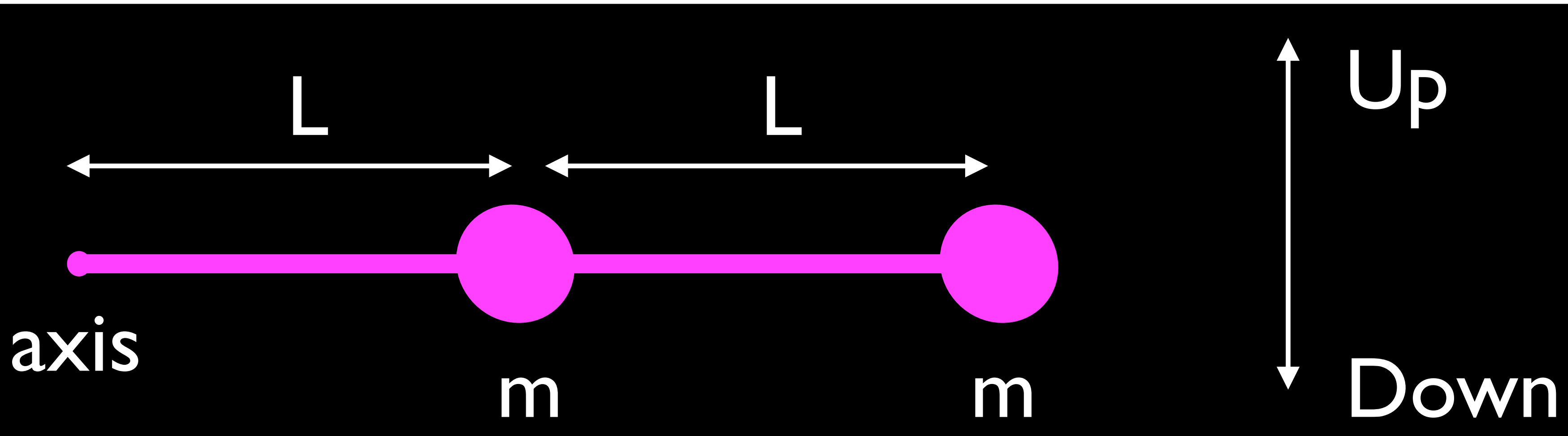
A light (no mass) rod of length = 2L.
2 heavy masses (each mass = m) attached at the end and middle.
What is the rotational inertia about the axis?

Centre mass: $I = mL^2$

End mass: $I = m(2L)^2$

Total: $I = \Sigma m_i r_i^2$ $= 5mL^2$

Rotation



$I = 5mL^2$

What is the net torque when it's released?

(1) 2mgL

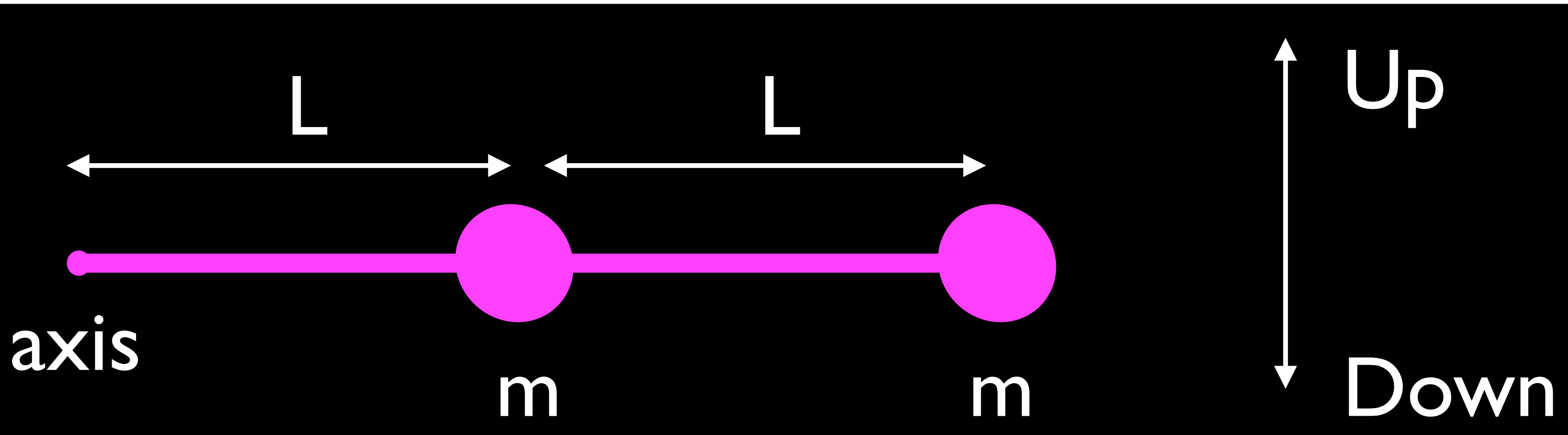
(2) -2mgL

(3) 3mgL

[✓ (4) -3mgL]

(5) 4mgL

Rotation



$I = 5mL^2$

What is the net torque when it's released?

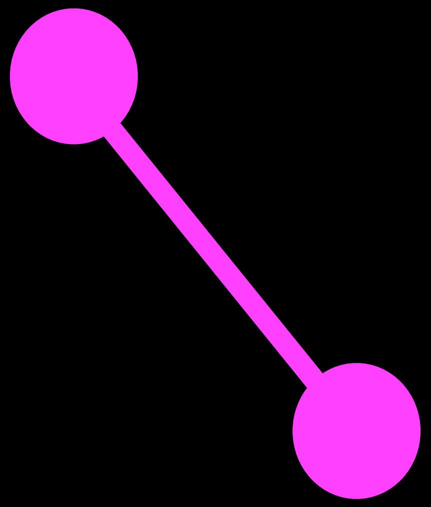
$$\tau = rF\sin 90$$

$$= L(-mg)\sin(0) + (2L)(-mg)\sin(0)$$

$$= -3mgL$$

Rotation

Calculating rotational inertia



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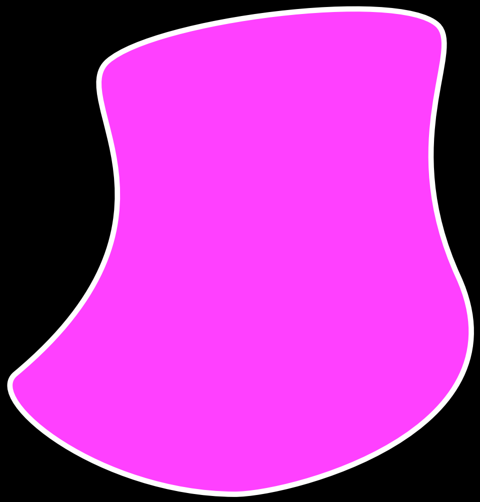
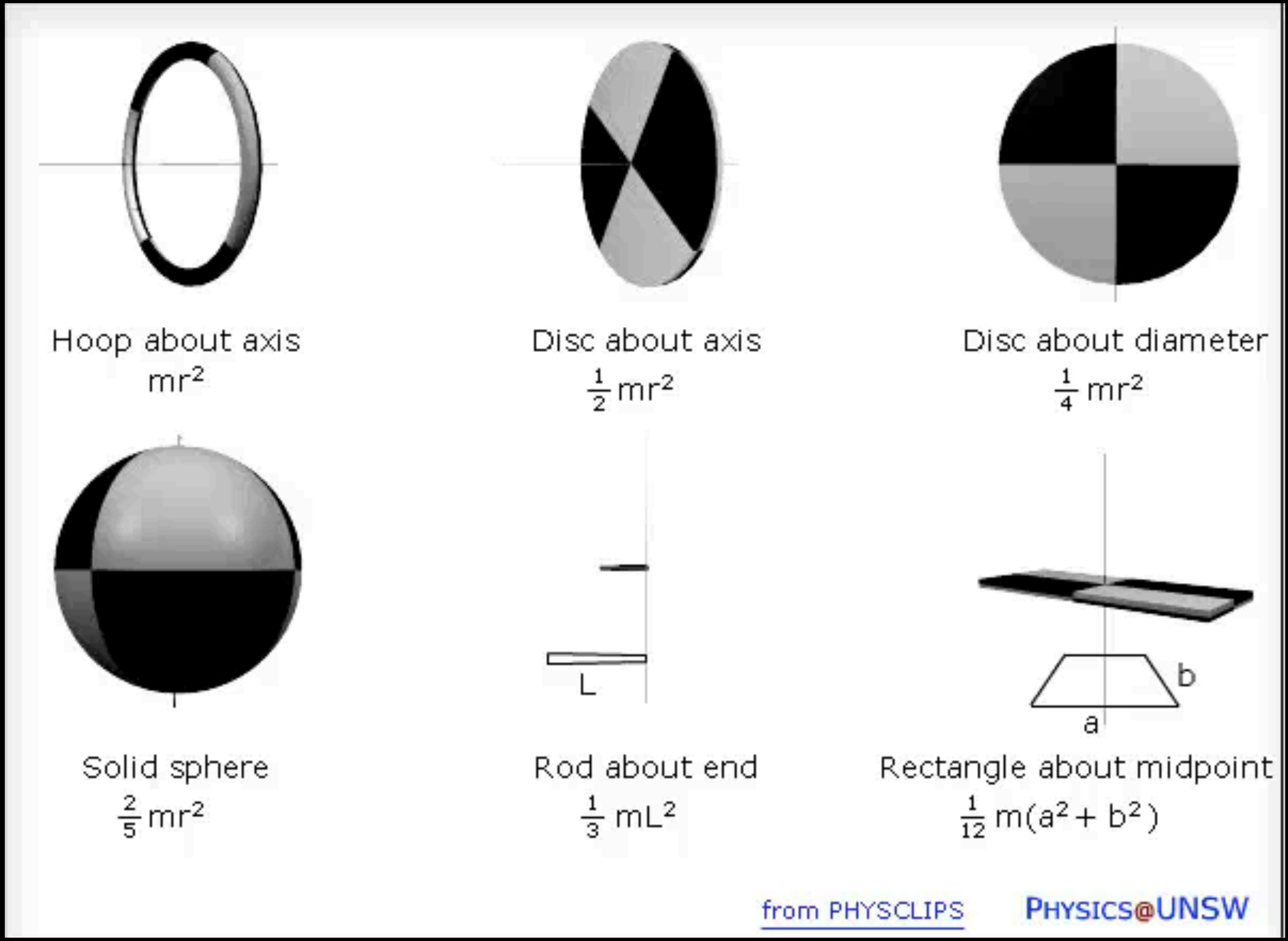


Continuous matter:

$$I = \int r^2 dm$$

Rotation

Rotational inertia



Also p 163 in textbook

Rotation

If we know I through the centre of mass of the object:

$$I = \frac{2}{5}Mr^2$$

$$I = \frac{2}{5}Mr^2 + Md^2$$
$$= \frac{7}{5}Mr^2$$

we can calculate I through any parallel axis. $\boxed{I = I_{\mathrm{cm}} + Md^2}$