## Essential Physics I

$$
\begin{gathered}
\text { 英語で物理学の } \\
\text { エッセンス }
\end{gathered}
$$

Lecture 7：30－05－I6

## Last lecture: review

Work:


$$
\begin{aligned}
W & =\bar{F} \cdot \Delta \bar{r} \\
& =F \Delta r \cos \theta \\
& =F_{x} \Delta r_{x}+F_{y} \Delta r_{y}+F_{z} \Delta r_{z} \\
& =\int_{x_{1}}^{x_{2}} F(x) d x
\end{aligned}
$$

Kinetic energy:

$$
K=\frac{1}{2} m v^{2}
$$

$$
\Delta K=W_{\text {net }} \quad \text { (Work-energy theorem) }
$$

Power:

$$
P=\frac{d W}{d t}=\bar{F} \cdot \bar{v}
$$

## Last lecture: review

## Quiz

3 forces:

$$
\begin{aligned}
& F_{1}=x^{1 / 2} \\
& F_{2}=x \\
& F_{3}=x^{2}
\end{aligned}
$$

$$
\text { ( } x \text { position in } \mathrm{m} \text { ) }
$$

act on an object from $x=0$ to $x=1 \mathrm{~m}$.
Each force has the same value at $\mathrm{x}=0$ and $\mathrm{x}=\mathrm{I}$, but which does the most work?
(a) $F_{1}=x^{1 / 2}$
(b) $F_{2}=x$
(c) $F_{3}=x^{2}$

## Last lecture: review

## Quiz

3 forces: $\quad F_{1}=x^{1 / 2}$

$$
F_{2}=x
$$

$$
F_{3}=x^{2} \quad(x \text { position in } \mathrm{m})
$$

act on an object from $\mathrm{x}=0$ to $\mathrm{x}=\mathrm{I} \mathrm{m}$.
Each force has the sar does the most work?

$$
\text { (a) } F_{1}=x^{1 / 2}
$$

(b) $F_{2}=x$
(c) $F_{3}=x^{2}$

## Last lecture: review

## Quiz

3 objects ( $\mathrm{A}, \mathrm{B}$ and C ) are moved $\Delta \bar{r}$
Object A is pushed in the direction of motion with a force, F
Object B is pushed at a $45^{\circ}$ angle in the direction of motion with a force, $2 F$

Object C is pushed at a $90^{\circ}$ angle in the direction of motion with a force, 2 F

Rank work done, smallest $\rightarrow$ largest
(a) $W_{C}>W_{B}>W_{A} \quad$ (c) $W_{B}>W_{A}>W_{C}$
(b) $W_{B}>W_{C}>W_{A}$
(d) $W_{A}>W_{B}>W_{C}$

## Lecture 7

## What happens when you drop a book?



## Last lecture

Work: $\quad W=\bar{F} \cdot \Delta \bar{r}$


Work-energy theorem:

$$
\Delta K=W_{\mathrm{net}}
$$

Work done is converted into kinetic energy ..... right?

## Energy

What about if you lift a book?
Book's weight: $\quad F_{g}=m g=10 \mathrm{~N}$
Displacement: $\quad \Delta r=\Delta y=1 \mathrm{~m}$
Work done by my force:

$$
W=(10 \mathrm{~N}) \times(1 \mathrm{~m})=10 J
$$

What about kinetic energy?

$$
K=\frac{1}{2} m v^{2}=?
$$

The book's velocity does not increase,
 so K does not increase: $\Delta K=0$

## Where did the energy go?

## Energy

Wait! There is a 2nd force.
Gravity exerts a downwards force on the book:
Work done by my force:

$$
W_{\mathrm{me}}=\bar{F} \cdot \Delta \bar{r}
$$

Work done by gravity's force:

$$
W_{\text {grav }}=-\bar{F} \cdot \Delta \bar{r}
$$

Net work:

$$
\begin{aligned}
W_{\mathrm{net}} & =W_{\mathrm{me}}+W_{\mathrm{grav}} \\
& =0 \mathrm{~J}
\end{aligned}
$$

Therefore:

$$
\Delta K=W_{\mathrm{net}}=0
$$



## Energy

We gave the book 10 J of energy


## Gravity took IO J of

 energy from the book
## 10. J

$W_{\mathrm{me}}=\bar{F} \cdot \Delta \bar{r}$
$\Delta K=W_{\text {net }}=0$
$W_{\text {grav }}=-\bar{F} \cdot \Delta \bar{r}$

!
Where did gravity's energy go?

## Energy

What happens when you drop the book? It accelerates down.


## Energy

We gave the book 10 J of energy


Gravity took IO J of energy from the book

$W_{\mathrm{me}}=\bar{F} \cdot \Delta \bar{r}$
$\Delta K=W_{\text {net }}=0$

$$
W_{\text {grav }}=-\bar{F} \cdot \Delta \bar{r}
$$

Converted to K as the book falls

Gravity gives 10 J of energy to the book

## Potential Energy

"Stored energy" is called potential energy

It is 'stored', because we can get it back as kinetic energy


$$
\Delta U \rightarrow K
$$

Forces that store the energy they use are called conservative forces e.g. gravity springs
negative work done by a conservative force:

$$
W_{\text {force }}=-\int_{A}^{B} \bar{F} \cdot d \bar{r} \quad=\Delta U_{\mathrm{AB}}
$$

## Conservative Forces

e.g. when lifting a book:


Negative work done
$=$
Potential energy by gravity's force:

$$
\begin{aligned}
W_{\text {grav }} & =-\bar{F} \cdot \Delta \bar{r} \\
& =-m g \Delta y
\end{aligned}
$$

stored
$\Delta U$

Gravitational potential energy: $\quad \Delta U=-m g \Delta y$

The change in potential energy, $\Delta U$ is the important quantity. Therefore, we can chose where $U=0$

## Conservative Forces

e.g. when compressing a spring:


Negative work done by the spring's force:

$$
\begin{aligned}
W_{\mathrm{sp}} & =-\int_{x_{1}}^{x_{2}} \bar{F} \cdot d \bar{r} \\
& =-\int_{x_{1}}^{x_{2}}(-k x) d x \\
& =\frac{1}{2} k x_{2}^{2}-\frac{1}{2} k x_{1}^{2}
\end{aligned}
$$

$=$ Potential energy stored

Elastic
potential energy:

$$
U=\frac{1}{2} k x^{2}
$$

(If we chose $U=0$ to be when $x=0$ )

## Conservative Forces

You lift a 10 kg bag from the floor to a shelf 2 m above the floor. Which of the following is true?
(A) You increased the gravitational potential energy and performed negative work on the bag.
(B) You increased the gravitational potential energy and the earth performed positive work on the bag.
(C) You increased the gravitational potential energy and the earth performed negative work on the bag.
(D) You decreased the gravitational potential energy and the earth performed positive work on the bag.

## Conservative Forces

## Quiz

(C) You increased the gravitational potential energy and the earth performed negative work on the bag.

You did positive work on the bag.

Gravity did negative work on the bag.

The gravitational potential energy (U) of the bag increased.


## Conservative Forces

## Quiz

A woman runs up a flight of stairs.
The gain in her gravitational potential energy is U .
If she runs up the same stairs with twice the speed, what is her gain in potential?

## (A) U

(B) 2 U
(C) U/2
(D) 4 U
(E) U/4


## Conservative Forces

How do we know if a force is conservative?

If the total work done in moving from $A \rightarrow B \rightarrow A=0$, then the force is conservative.
$W_{\mathrm{AB}}+W_{\mathrm{BA}}=0$


## Conservative Forces

e.g. climbing a mountain


## Climbing up

Work done by gravity:

$$
W_{\text {grav, up }}=-m g \Delta y
$$

Climbing down
Work done by gravity:

$$
W_{\text {grav, down }}=m g \Delta y
$$



$$
W_{\mathrm{net}}=-m g \Delta y+m g \Delta y=0
$$

## Conservative Forces

e.g. pushing a box


Pushed across room from $A \rightarrow B$
Work done by friction:

$$
\begin{aligned}
W_{\text {fric }, \mathrm{A} \rightarrow \mathrm{~B}} & =-F_{\text {fric }} \Delta x \\
& =-\mu_{k} N \Delta x=-\mu_{k} m g \Delta x
\end{aligned}
$$

Pushed back from $B \rightarrow A$
Friction always opposes motion

$$
W_{\text {fric }, \mathrm{B} \rightarrow \mathrm{~A}}=-\mu_{k} m g \Delta x
$$

$W_{\text {net }}=-2 \mu_{k} m g \Delta x \neq 0$
Friction is not conservative

## Conservative Forces

Mathematically:


## Conservative Forces

For a conservative force, the path from $A \rightarrow B$ is not important.


The work done by a conservative force depends only on the end points $A$ and $B$, not on the path between them.

## Conservative Forces

The same amount of work is done pushing a box across a rough floor and lifting a weight the same distance upwards.

How does the work compare if the box and weight are moved on curved paths between the start and end points?
(a) $W=W_{1}=W_{2}$
(b) $W_{1}>W$
(c) $\quad W_{2}>W$


## Conservative Forces



Which of these force fields is conservative?

## Conservative Forces



To be conservative:

$$
\begin{aligned}
\oint \bar{F} \cdot d \bar{r} & =0 \\
& =W_{1, \mathrm{x}, 0 \mathrm{~A}}+W_{1, \mathrm{y}, 0 \mathrm{~A}} \\
& +W_{2, \mathrm{x}, 0 \mathrm{~A}}+W_{2, \mathrm{y}, 0 \mathrm{~A}}
\end{aligned}
$$

## Conservative Forces



To be conservative:

$$
\begin{aligned}
\oint \bar{F} \cdot d \bar{r} & =0 \\
& =\int_{0}^{a} \bar{F} \cdot \bar{i} d x+\int_{0}^{a} \bar{F} \cdot \bar{j} d x \\
& +\int_{a}^{0} \bar{F} \cdot \bar{i} d x+\int_{a}^{0} \bar{F} \cdot \bar{j} d x
\end{aligned}
$$

## Conservative Forces



To be conservative: $\oint \bar{F} \cdot d \bar{r}=0$
Force has only $j$ component: $\bar{F}=F_{j}$

$$
\begin{aligned}
& =\int_{0}^{a} \dot{y} d d x+\int_{0}^{a} \bar{F} \cdot \bar{j} d x \\
& +\int_{a}^{0} \vec{F}=d x+\int_{a}^{0} \bar{F} \cdot \bar{j} d x
\end{aligned}
$$

## Conservative Forces



To be conservative:

$$
\begin{aligned}
\oint \bar{F} \cdot d \bar{r} & =0 \\
& =\int_{0}^{a} \bar{F} \cdot \bar{j} d x+\int_{a}^{0} \bar{F} \cdot \bar{j} d x \\
(\mathrm{I}) & =0
\end{aligned}
$$

## Conservative Forces

3 objects of mass $m$ begin at height $h$ with velocity 0.0
1 falls straight down
1 slides down a frictionless inclined plane
1 swings at the end of a pendulum
What is the relationship between their speeds when they reach height 0 ?


Free Fall


Frictionless incline


Pendulum
$(a) V_{f}>V_{i}>V_{p}$
(b) $V_{f}>V_{p}>V_{i}$
(c) $V_{f}=V_{p}=V_{i}$

## Conservative Forces



Free Fall


Pendulum

The only work is done by gravity, a conservative force
Therefore, the path doesn't matter

$$
\begin{aligned}
& U_{f}=U_{p}=U_{i}=m g h=K=\frac{1}{2} m v^{2} \\
& V_{f}=V_{p}=V_{i}=\sqrt{2 g H}
\end{aligned}
$$

## Conservative Forces

A ball, initially at rest ( $v_{i}=0$ ), is dropped from I.5m.
It is observed to have a velocity $v_{f}$ just before it hits the floor.
If instead the ball is dropped from 0.5 m what is its velocity?
(A) $33 \%$ of $v_{f}$
(B) $50 \%$ of $v_{f}$
(C) $58 \%$ of $v_{f}$
(D) $66 \%$ of $v_{f}$
(E) $71 \%$ of $v_{f}$

$$
\begin{aligned}
& v=0 \\
& v=v_{f}
\end{aligned}
$$

## Conservative Forces

A ball, initially at rest ( $v_{i}=0$ ), is dropped from 1.5 m . It is observed to have a velocity $v_{f}$ just before it hits the floor. If instead the ball is dropped from 0.5 m what is its velocity?

$$
\begin{gathered}
U \rightarrow K \\
m g \Delta y=\frac{1}{2} m v^{2} \\
v=\sqrt{2 g \Delta y} \\
\frac{v}{v_{f}}=\sqrt{\frac{0.5}{1.5}}=57.7 \%
\end{gathered}
$$

$$
v=0
$$



## Energy conservation



The book's potential energy (U) becomes kinetic (K)

Then what?
Where did the kinetic energy go?

Thermal (heat) energy and sound energy

## Energy conservation

Energy can be converted between forms:

but it can NEVER be destroyed

## Energy conservation



Frictionless track


## $\min U$ <br> $\max \mathrm{K}$



## Highest points

$\max U$
$\min K$


## Potential energy



## $\downarrow$

Kinetic energy

Potential energy

Kinetic energy



What about this track?


Initially all energy is potential energy, U , and starts to change to K
The track impacts create thermal energy (heat)


Initially all energy is potential energy, U , and starts to change to K
The track impacts create thermal energy (heat)
Less energy is shared between U and K


Initially all energy is potential energy, U , and starts to change to K
The track impacts create thermal energy (heat)
Less energy is shared bet, veen U and K
Total energy is constant
(can never destroy energy)

## What if we add friction

 to the track?


Friction creates thermal energy
The more the skater moves, the more thermal energy is generated
Less and less energy is available for K and U
Total energy is constant

## Energy conservation

When an object is acted on by only conservative forces, the sum of its kinetic and potential energies does not change.
(a) True
(b) False

## Energy conservation

A bowling ball is tied to a rope and hangs from the ceiling.
A girl holds the ball to her nose and releases it.
Should she duck as it swings back?
(a) Yes!
(b) No!


## Energy conservation

If the ball was in a vacuum...
Should she duck?
(a) Yes!
(b) No!


Cannot gain MORE energy than you begin with

## Energy conservation

## example

What height must the cart start to complete the loop-the-loop?


To stay on the track: $\quad F_{N}>0$
$\bar{F}_{N}+\bar{F}_{g}=m \bar{a}$
$F_{N}+m g \cos \theta=m \frac{v^{2}}{r}$


## Energy conservation

## example

What height must the cart start to complete the loop-the-loop?


To stay on the track: $\quad F_{N}>0$
$\bar{F}_{N}+\bar{F}_{g}=m \bar{a}$
$F_{N}+m g \cos \theta=m \frac{v^{2}}{r}$
At loop top: $F_{N}+m g=m \frac{v^{2}}{r}$

$$
\xrightarrow{F_{N}=0} \quad v_{\min }=\sqrt{r g}
$$

## Energy conservation

## example

What height must the cart start to complete the loop-the-loop?


Energy:

$$
\begin{aligned}
& K_{\text {top }}=\frac{1}{2} m v_{\text {top }}^{2}=\frac{1}{2} m(r g) \\
& U_{\text {top }}=m g 2 r \\
& E_{\text {top,total }}=\frac{1}{2} m g r+2 m g r
\end{aligned}
$$



## Energy conservation

## example

What height must the cart start to complete the loop-the-loop?


Energy conservation:

$$
\begin{aligned}
E_{\text {top }, \text { total }} & =E_{\text {start }} \\
& =U_{\text {start }}+K_{\text {start }}(=0) \\
& =m g \Delta h \\
\frac{5}{2} m g r & =m g \Delta h \\
\Delta h & =\frac{5}{2} r
\end{aligned}
$$



## Energy conservation



A block is launched up a frictionless slope at 40 degrees, with speed $v$. It reaches a vertical height $h$.

Speed is changed to 2 v . What is the new height?
(a) h
(c) 2 h
(e) 8 h
(b) h/2
(d) 4 h

## Energy conservation <br> Quiz



A block is launched up a frictionless slope at 40 degrees, with speed $v$. It reaches a vertical height $h$.

Speed is changed to 2 v . What is the new height?

$$
\begin{aligned}
K E \rightarrow U \quad & \frac{1}{2} m v^{2}=m g h \longrightarrow h=\frac{v^{2}}{2 g} \\
& \frac{1}{2} m(2 v)^{2}=m g h_{2} \longrightarrow h_{2}=\frac{(2 v)^{2}}{2 g}=4 h
\end{aligned}
$$

## Energy conservation

A I kg block slides 4 m down a frictionless plane inclined at $30^{\circ}$ to the horizontal.

After reaching the bottom, it slides along a frictionless horizontal plane and strikes a spring with constant $\mathrm{k}=314 \mathrm{~N} / \mathrm{m}$.

How far is the spring compressed when it stops the block?


Energy conservation:

$$
U_{\text {slope }} \rightarrow K_{\text {block }} \rightarrow U_{\text {spring }}
$$

## Energy conservation

 Quiz1 kg

(a) 35 cm
(b) 50 cm
(c) 0.4 cm
(d) 29 cm

## Energy conservation



Energy conservation:


$$
U_{\text {slope }} \rightarrow K_{\text {block }} \rightarrow U_{\text {spring }}
$$

$$
U_{\text {slope }}=m g L \sin \theta=K_{\mathrm{block}}
$$

$$
K_{\text {block }}=U_{\text {spring }}=\frac{1}{2} k x^{2}
$$

$$
\frac{1}{2} k x^{2}=m g L \sin \theta
$$

$$
x=\sqrt{\frac{2 m g L \sin \theta}{k}}=35 \mathrm{~cm}
$$

## Potential energy curve



Potential energy vs position is called a potential energy curve

## Potential energy curve



We can use it to understand the skater's motion

## Potential energy curve



Initially, the skater's potential + kinetic energy > track's potential: $U_{\mathrm{s}}+K_{\mathrm{s}}>U_{\mathrm{T}}$

## Potential energy curve



Initially, the skater's potential + kinetic energy > track's potential: $U_{\mathrm{s}}+K_{\mathrm{s}}>U_{\mathrm{T}}$
Track impact creates thermal energy, decreasing $U_{s}+K_{s}$ It is no longer larger than all of the track's potential

## Potential energy curve



Initially, the skater's potential + kinetic energy > track's potential: $U_{\mathrm{s}}+K_{\mathrm{s}}>U_{\mathrm{T}}$
Track impact creates thermal energy, decreasing $U_{s}+K_{s}$ It is no longer larger than all of the track's potential

The skater gets trapped in a potential barrier

## Potential energy curve

Potential barriers are not just gravity


## Force and potential energy

The slope of the potential is the force In ID:

$$
\Delta U_{A B}=-\int_{A}^{B} F_{x} d x
$$

Therefore:


The force acts in the direction of the potential minimum

## Force and potential energy

Where the slope is zero, there is no force:

$$
-\frac{d U}{d x}=0 \rightarrow F_{x}=0
$$

This is called an equilibrium position


If a small displacement creates a force towards the equilibrium position, it is a stable equilibrium.


If a small displacement creates a force away the equilibrium position, it is a unstable equilibrium.


## Force and potential energy

## Quiz

Potential energy for an electron in a microelectronic device


Where is the force greatest?
(a) A
(c) C
(e) E
(b) B
(d) D

## Force and potential energy

## Quiz

Potential energy for an electron in a microelectronic device


Find the right-most point (one most right) where the force points to the left.
(a) A
(c) C
(b) B
(d) D
(e) E

## Force and potential energy <br> Quiz



Force in segment (a)?
(a) -2 N
(b) 2 N
(c) 0.5 N
(d) -0.5 N
(d) 0 N

## Force and potential energy <br> Quiz



Force in segment (a)?

$$
F(x)=-\frac{\Delta U}{\Delta x}=-\frac{3 J}{1.5 m}=-2 N
$$

## Force and potential energy <br> Quiz



Force in segment (b)?
(a) -3 N
(b) 3 N
(c) 1.2 N
(d) -1.2 N
(d) 0 N

## Force and potential energy <br> Quiz



Force in segment (c)?
(a) -8 N
(b) 8 N
(c) -4 N
(d) 4 N
(d) 0 N

## Force and potential energy <br> Quiz



Force in segment (c)?

$$
F(x)=-\frac{\Delta U}{\Delta x}=-\frac{-4 J}{0.5 m}=8 N
$$

## Force and potential energy <br> Quiz



Force in segment (d)?
(a) - IN
(b) $I \mathrm{~N}$
(c) -0.5 N
(d) 0.5 N
(d) 0 N

## Force and potential energy <br> Quiz



Force in segment (e)?
(a) -4 N
(b) 4 N
(c) -8 N
(d) 8 N
(d) 0 N

## Force and potential energy <br> Quiz



Force in segment (f)?
(a) -2 N
(b) 2 N
(c) -IN
(d) $I \mathrm{~N}$
(d) 0 N

## Quiz

'Box I' and 'Box 2' have the same mass, $m$.
They start at the top of 2 inclined planes at the same height.
The planes are inclined at $30^{\circ}$ and $60^{\circ}$.
If the coefficient of friction, $\mu_{K}$, is the same, which of the boxes is going faster when it reaches the bottom?
(a) Both blocks have the same speed
(b) Block I is faster
(c) Block 2 is faster

(d) We need to know more information

## Quiz

'Box I' and 'Box 2 ' have the same mass, $m$.
They start at the top of 2 inclined planes at the same height.
The planes are inclined at $30^{\circ}$ and $60^{\circ}$.
If the coefficient of friction, $\mu_{K}$, is the same, which of the boxes is going faster when it reaches the bottom?
(a) Both blocks have the same speed
(b) Block I is faster
(c) Block 2 is faster

$$
d_{1}>d_{2}
$$


(d) We need to know more information

## Quiz

A 50 kg skier starts from rest and travels 200 m down a hill inclined at $20^{\circ}$.

When she reaches the bottom of the hill, her speed is $30 \mathrm{~m} / \mathrm{s}$.
How much work is done by friction as the skier comes down the hill?
(a) $\quad-11,050 \mathrm{~J}$
(b) $-33,550 \mathrm{~J}$
(c) $-22,550 \mathrm{~J}$
(d) $-56,050 \mathrm{~J}$

## Quiz

A 50 kg skier starts from rest and travels 200 m down a hill inclined at $20^{\circ}$.

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(a) $-11,050 \mathrm{~J}$
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(c) $\quad-22,550 \mathrm{~J}$
(d) $\quad-56,050 \mathrm{~J}$

## Quiz

A 50 kg skier starts from rest and travels 200 m down a hill inclined at $20^{\circ}$.

When she reaches the bottom of the hill, her speed is $30 \mathrm{~m} / \mathrm{s}$.
How much work is done by friction as the skier comes down the hill?

Energy: $\quad E_{\text {tot }}=K-U-W_{\text {friction }}=0$


$$
\text { Height: } \quad \begin{aligned}
h & =(200 \mathrm{~m}) \sin 20^{\circ} \\
& =68.4 \mathrm{~m}
\end{aligned}
$$

Potential $\quad U=m g h$ energy:
$=(50 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(68.4 \mathrm{~m})$
$=33,550 \mathrm{~J}$

## Quiz

A 50 kg skier starts from rest and travels 200 m down a hill inclined at $20^{\circ}$.

When she reaches the bottom of the hill, her speed is $30 \mathrm{~m} / \mathrm{s}$.
How much work is done by friction as the skier comes down the hill?

Energy: $\quad E_{\text {tot }}=K-U-W_{\text {friction }}=0$
$\begin{aligned} & \text { Kinetic } \\ & \text { energy: }\end{aligned} \quad K=\frac{1}{2} m v^{2}$
$=\frac{1}{2}(50 \mathrm{~kg})(30 \mathrm{~m} / \mathrm{s})^{2}$
$=22,500 \mathrm{~J}$

## Quiz

A 50 kg skier starts from rest and travels 200 m down a hill inclined at $20^{\circ}$.

When she reaches the bottom of the hill, her speed is $30 \mathrm{~m} / \mathrm{s}$.
How much work is done by friction as the skier comes down the hill?

Energy: $\quad E_{\text {tot }}=K-U-W_{\text {friction }}=0$

$$
\begin{aligned}
W_{\text {friction }} & =K-U \\
& =22,500 \mathrm{~J}-33,550 \mathrm{~J} \\
& =-11,050 \mathrm{~J}
\end{aligned}
$$

## Lecture 7 : Summary

The difference between conservative and non-conservative forces

Potential energy: what it is \& how to calculate it

Conservation of energy

Potential energy curves

