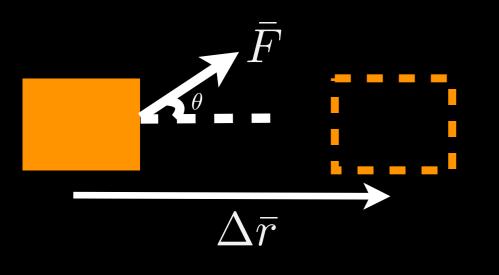
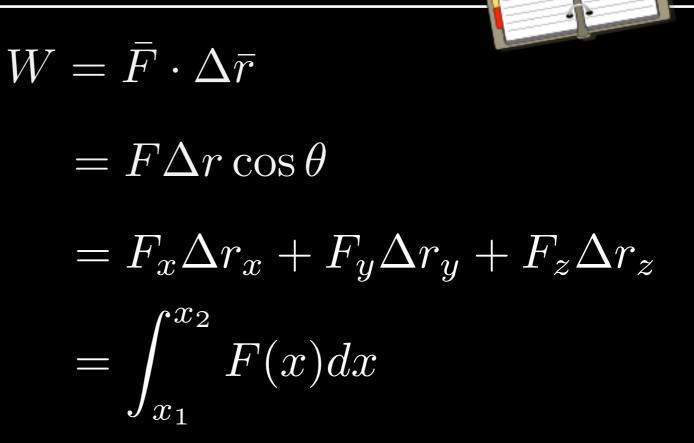
Essential Physics I

英語で物理学の エッセンス |

Lecture 7: 30-05-16







Kinetic energy:

$$K = \frac{1}{2}mv^2$$
 $\Delta K = W_{\rm net}$ (Work-energy theorem)

Power:

$$P = \frac{dW}{dt} = \bar{F} \cdot \bar{v}$$

3 forces: $F_1 = x^{1/2}$ $F_2 = x$

 $F_3 = x^2$ (x position in m)

Quiz

act on an object from x = 0 to x = 1 m.

Each force has the same value at x = 0 and x = 1, but which does the most work?

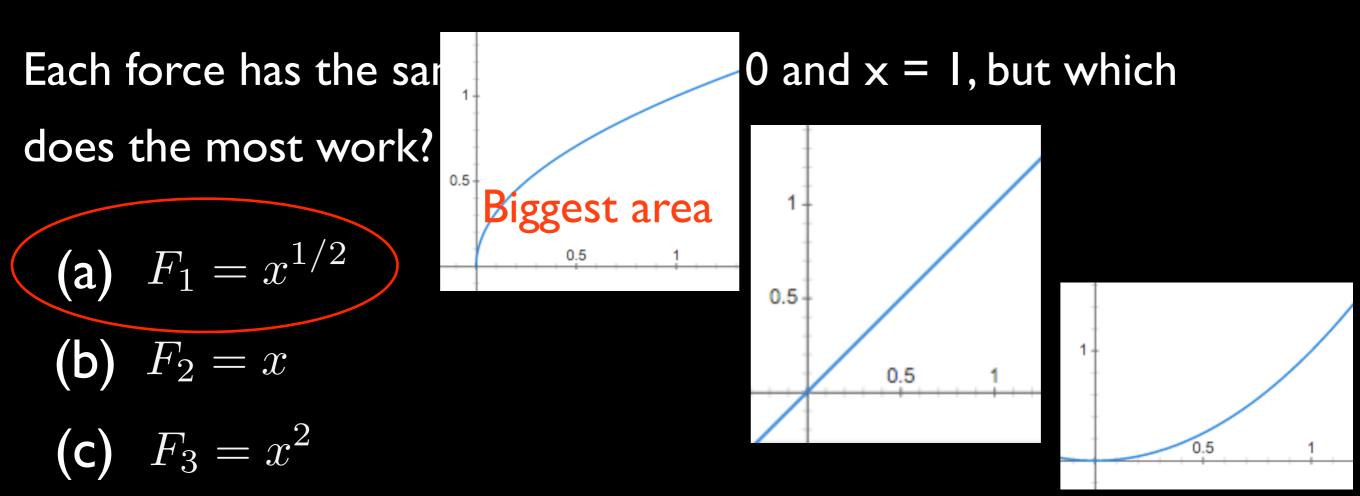
(a) $F_1 = x^{1/2}$ (b) $F_2 = x$ (c) $F_3 = x^2$

3 forces: $F_1 = x^{1/2}$ $F_2 = x$

 $F_3 = x^2$ (x position in m)

Quiz

act on an object from x = 0 to x = 1 m.



3 objects (A, B and C) are moved $\Delta \overline{r}$

Object A is pushed in the direction of motion with a force, F

Quiz

- Object B is pushed at a $\,45^\circ$ angle in the direction of motion with a force, 2F
- Object C is pushed at a $~90^\circ$ angle in the direction of motion with a force, 2F
- Rank work done, smallest \rightarrow largest

(a)
$$W_C > W_B > W_A$$
 (c) $W_B > W_A > W_C$

(b) $W_B > W_C > W_A$ (d) $W_A > W_B > W_C$

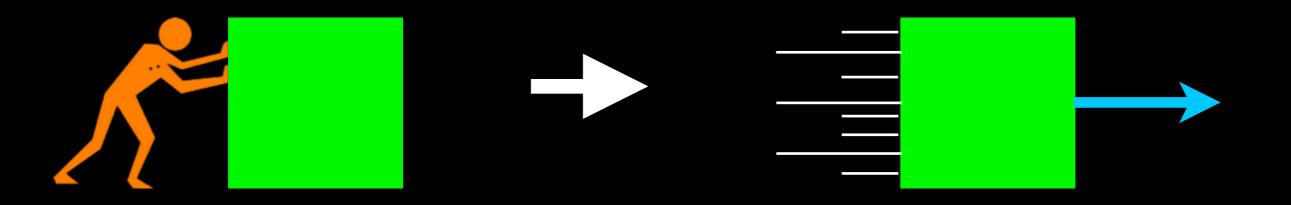
Lecture 7

What happens when you drop a book?



Last lecture

Work: $W = \overline{F} \cdot \Delta \overline{r}$



Work-energy theorem:

 $\Delta K = W_{\rm net}$

Work done is converted into kinetic energy right?



What about if you lift a book?

Book's weight: $F_g = mg = 10N$

Displacement: $\Delta r = \Delta y = 1 \text{m}$

Work done by my force:

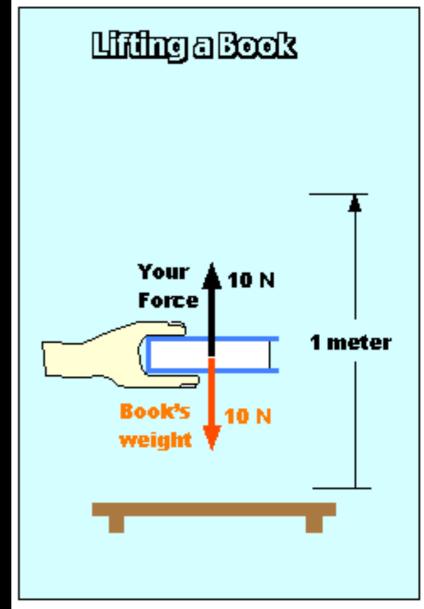
$$W = (10N) \times (1m) = 10J$$

What about kinetic energy?

$$K = \frac{1}{2}mv^2 = ?$$

The book's velocity does not increase, so K does not increase: $\Delta K = 0$

Where did the energy go?

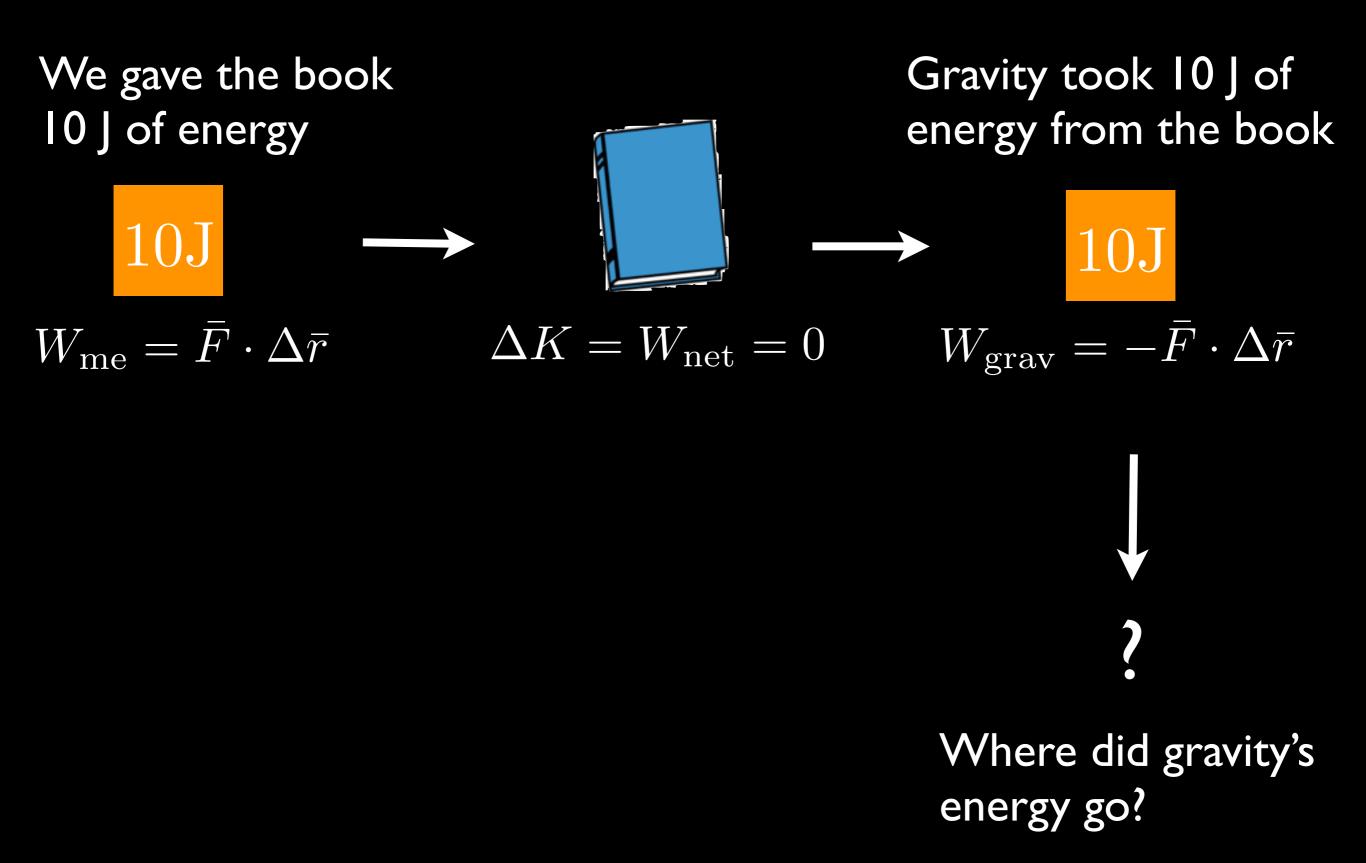




Wait! There is a 2nd force.

Gravity exerts a downwards force on the book:

Work done $W_{\rm me} = \bar{F} \cdot \Delta \bar{r}$ LiftingaBook by my force: Work done $W_{\rm grav} = -\bar{F} \cdot \Delta \bar{r}$ by gravity's force: Your 10 N Force 1 meter Book's Net work: 10 N $W_{\rm net} = W_{\rm me} + W_{\rm grav}$ weiaht = 0JTherefore: $\Delta K = W_{\text{net}} = 0$



What happens when you drop the book?

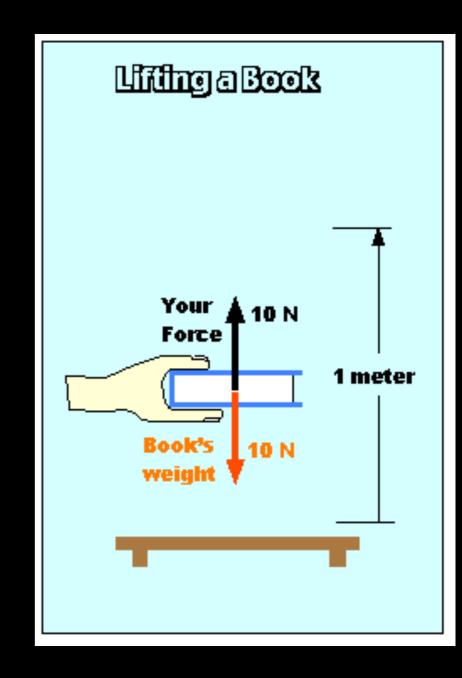
It accelerates down.

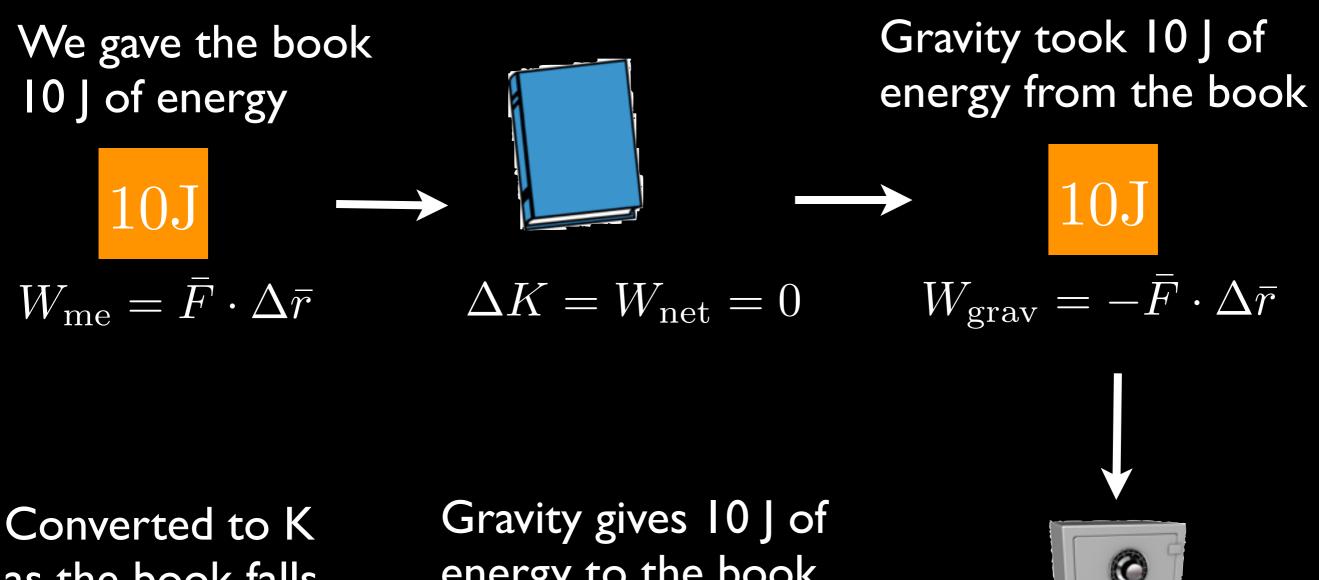
Gravity does positive work on the book, returning the I0 J:

$$W_{\rm grav} = \bar{F} \cdot \Delta \bar{r} = 10 {
m J}$$

Which is converted into kinetic energy:

$$K = \frac{1}{2}mv^2 = 10\mathbf{J}$$





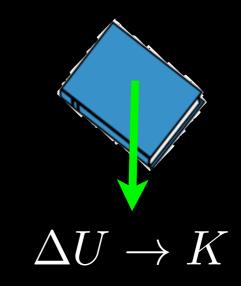
as the book falls energy to the book $10J \leftarrow 10J$ $K = \frac{1}{2}mv^2 = 10J$ $W_{\text{grav}} = \bar{F} \cdot \Delta \bar{r}$ Energy is stored until book is dropped

Potential Energy



"Stored energy" is called potential energy

It is 'stored', because we can get it back as kinetic energy

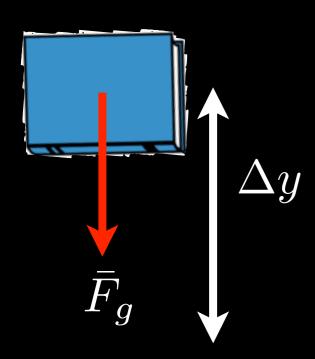


Forces that store the energy they use are called conservative forces e.g. gravity springs

negative work done by a conservative force:

$$W_{\rm force} = -\int_{A}^{B} \bar{F} \cdot d\bar{r} = \Delta U_{\rm AB}$$

e.g. when lifting a book:



Negative work done by gravity's force: $W_{\text{grav}} = -\bar{F} \cdot \Delta \bar{r}$ $= -mg \Delta y$ Potential energy stored

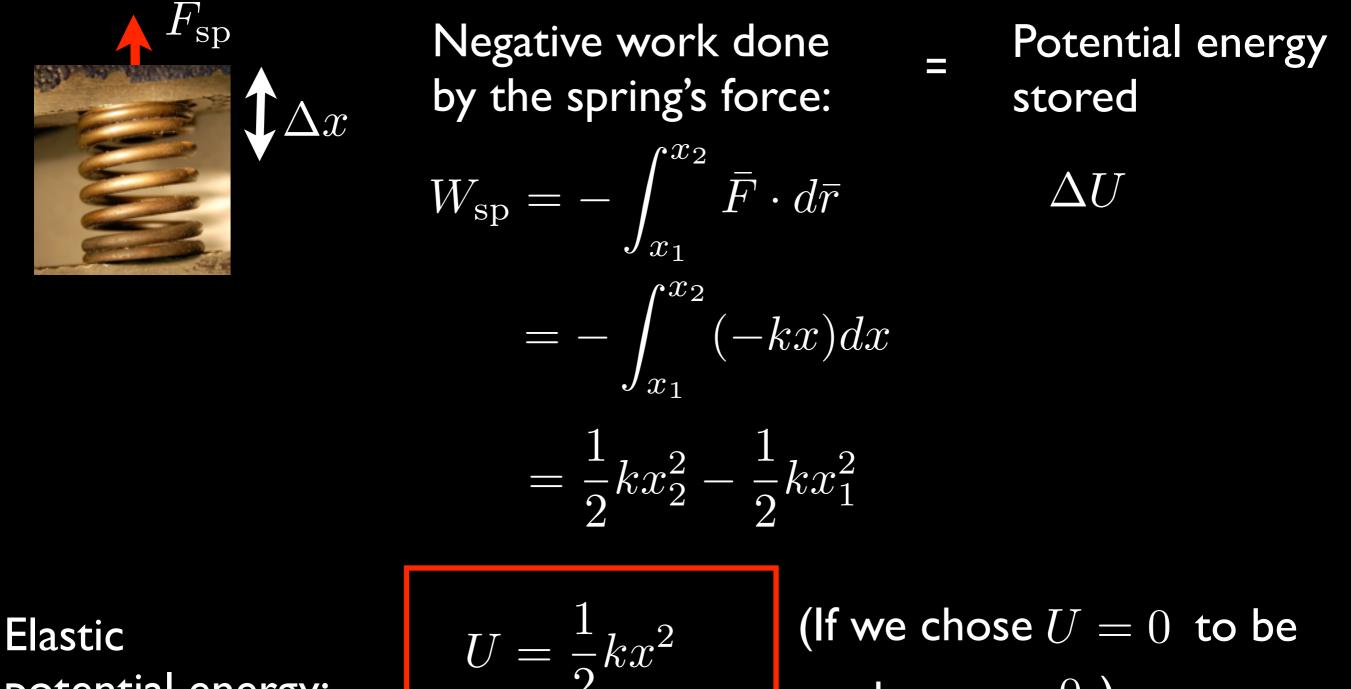
 ΔU

Gravitational potential energy:

$$\Delta U = -mg\Delta y$$

The change in potential energy, ΔU is the important quantity. Therefore, we can chose where U=0

e.g. when compressing a spring:



when x = 0)

potential energy:

You lift a 10 kg bag from the floor to a shelf 2m above the floor. Which of the following is true?

(A) You increased the gravitational potential energy and performed negative work on the bag.

(B) You increased the gravitational potential energy and the earth performed positive work on the bag.

(C) You increased the gravitational potential energy and the earth performed negative work on the bag.

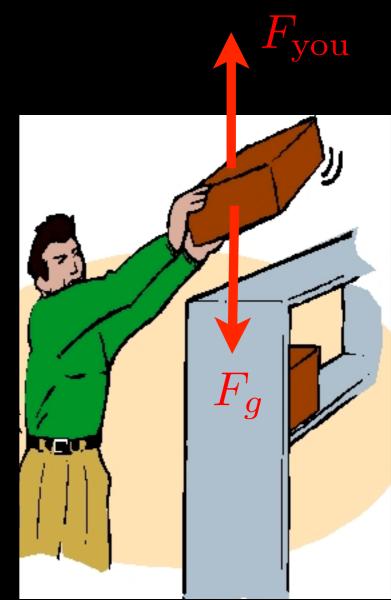
(D) You decreased the gravitational potential energy and the earth performed positive work on the bag.

(C) You increased the gravitational potential energy and the earth performed negative work on the bag.

You did positive work on the bag.

Gravity did negative work on the bag.

The gravitational potential energy (U) of the bag increased.



Quiz

A woman runs up a flight of stairs.

The gain in her gravitational potential energy is U.

If she runs up the same stairs with twice the speed, what is her gain in potential?

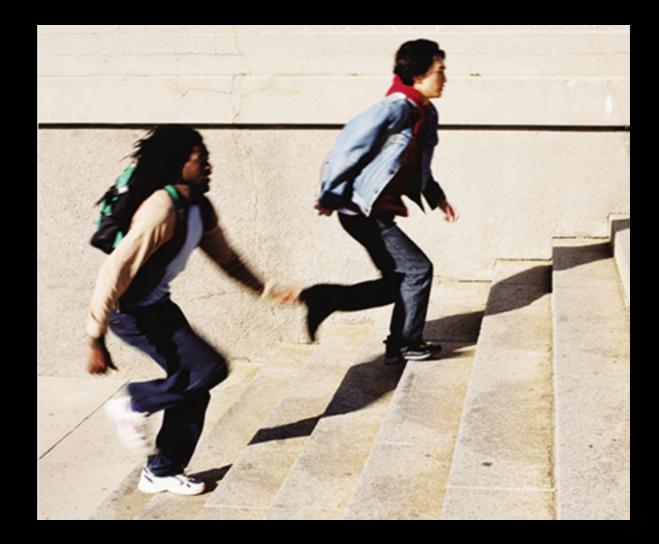
(A) U

(B) 2U

(C) U/2

(D) 4U

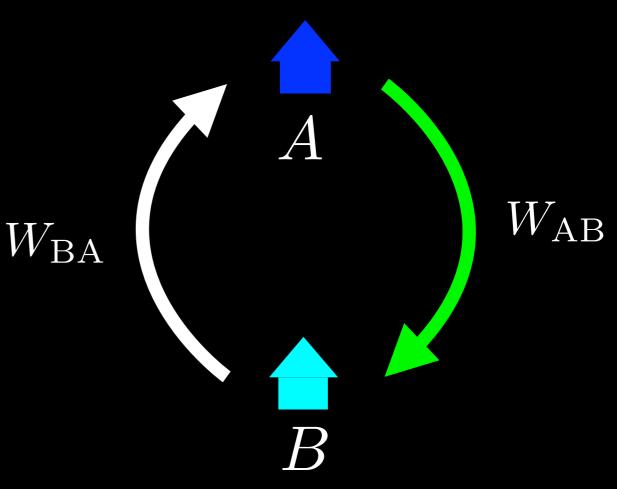
(E) U/4



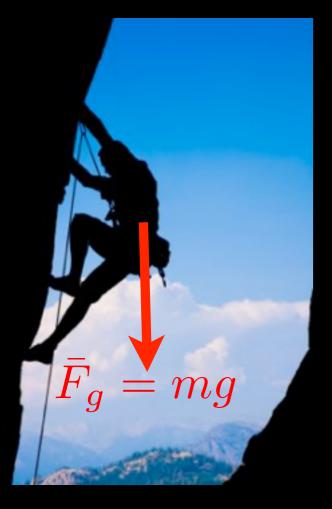
How do we know if a force is conservative?

If the total work done in moving from $A \to B \to A = 0$, then the force is conservative.

 $W_{\rm AB} + W_{\rm BA} = 0$



e.g. climbing a mountain



Climbing down Work done by gravity: $W_{
m grav, down} = mg \Delta y$

Climbing up

Work done by gravity:

$$W_{\rm grav,up} = -mg\Delta y$$



 $W_{\rm net} = -mg\Delta y + mg\Delta y = 0$

Gravity is conservative

e.g. pushing a box



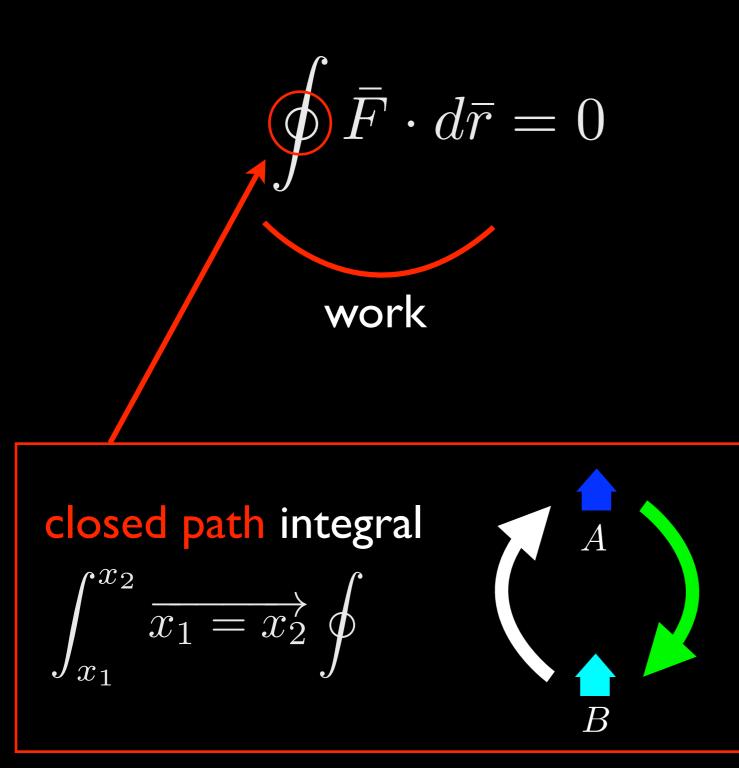
Pushed across room from $A \rightarrow B$ Work done by friction: $W_{\rm fric, A \rightarrow B} = -F_{\rm fric} \Delta x$

$$= -\mu_k N \Delta x = -\mu_k mg \Delta x$$

Pushed back from $B \to A$ Friction always opposes motion $W_{\rm fric, B \to A} = -\mu_k mg \Delta x$

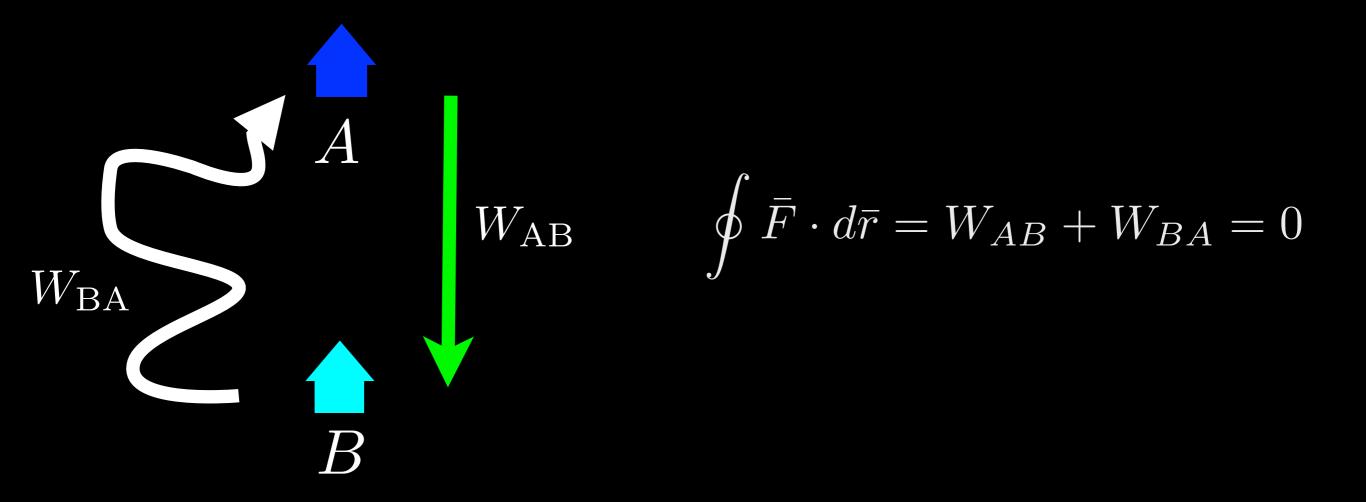
 $W_{\rm net} = -2\mu_k mg \Delta x \neq 0$ Friction is not conservative

Mathematically:



conservative force

For a conservative force, the path from $A \rightarrow B$ is not important.



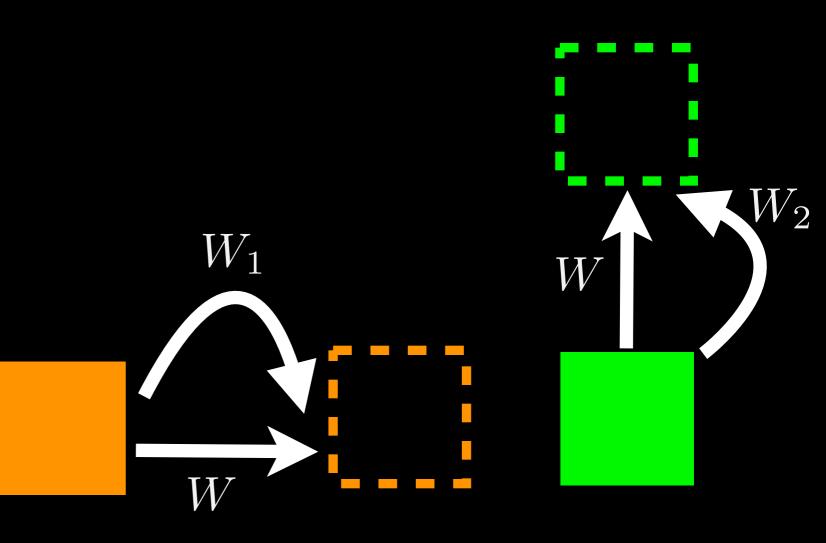
The work done by a conservative force depends only on the end points A and B, not on the path between them.

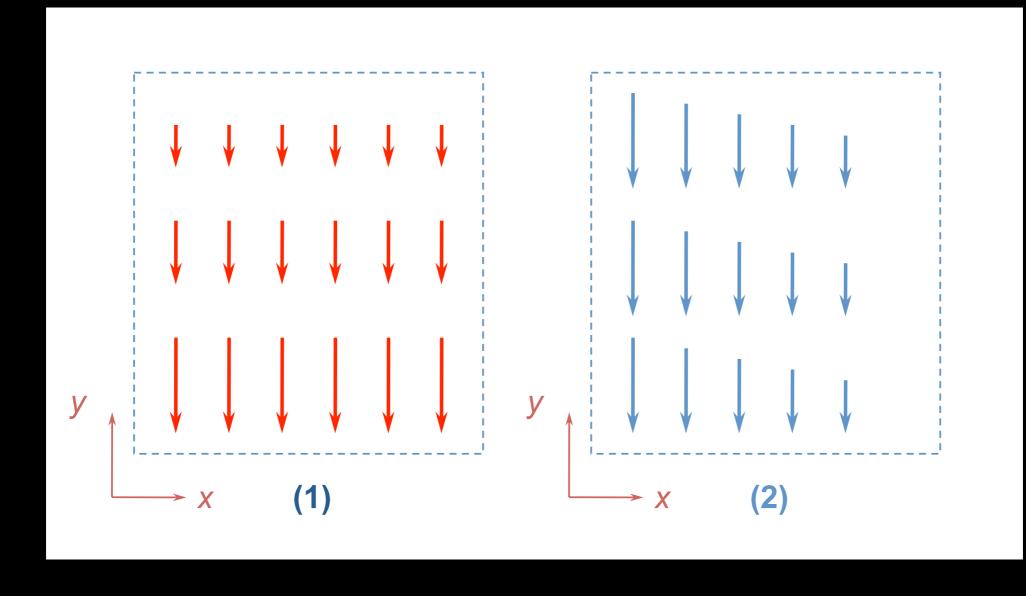
The same amount of work is done pushing a box across a rough floor and lifting a weight the same distance upwards.

How does the work compare if the box and weight are moved on curved paths between the start and end points?

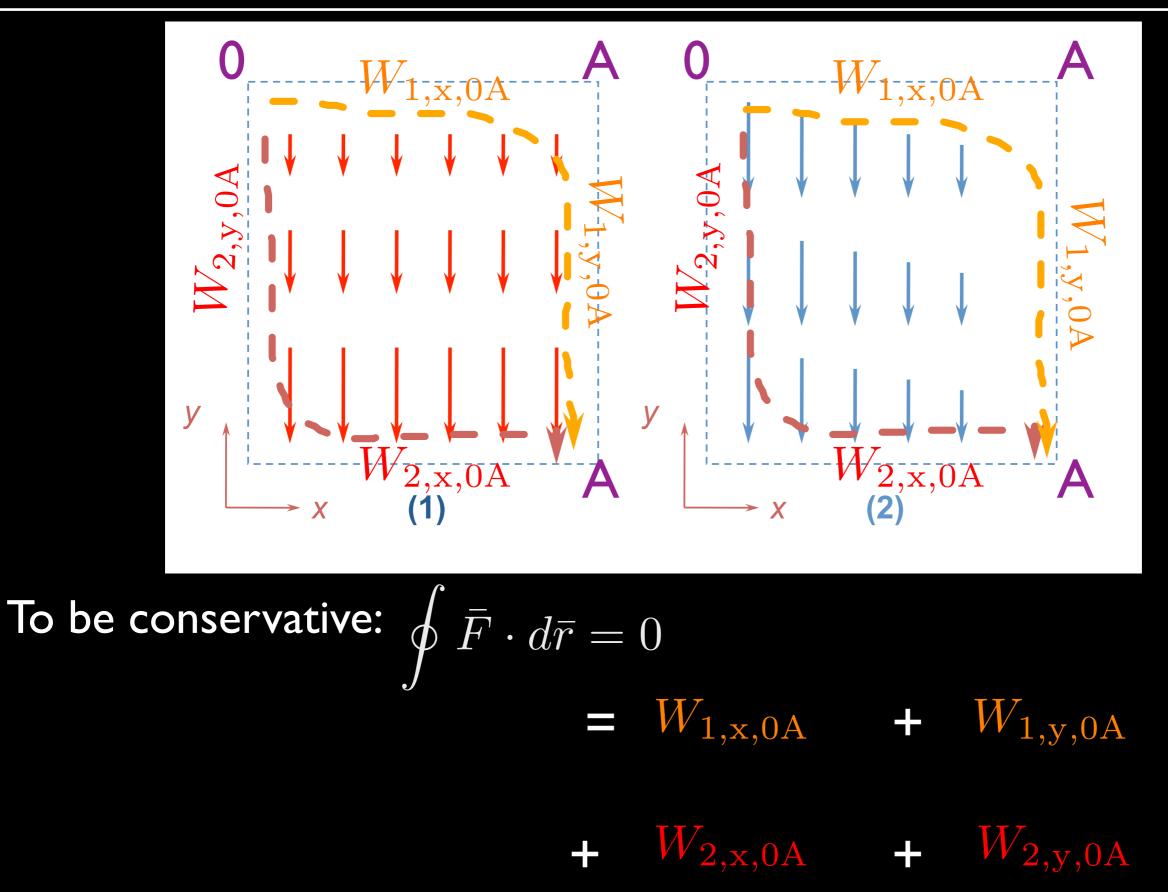
(a)
$$W = W_1 = W_2$$

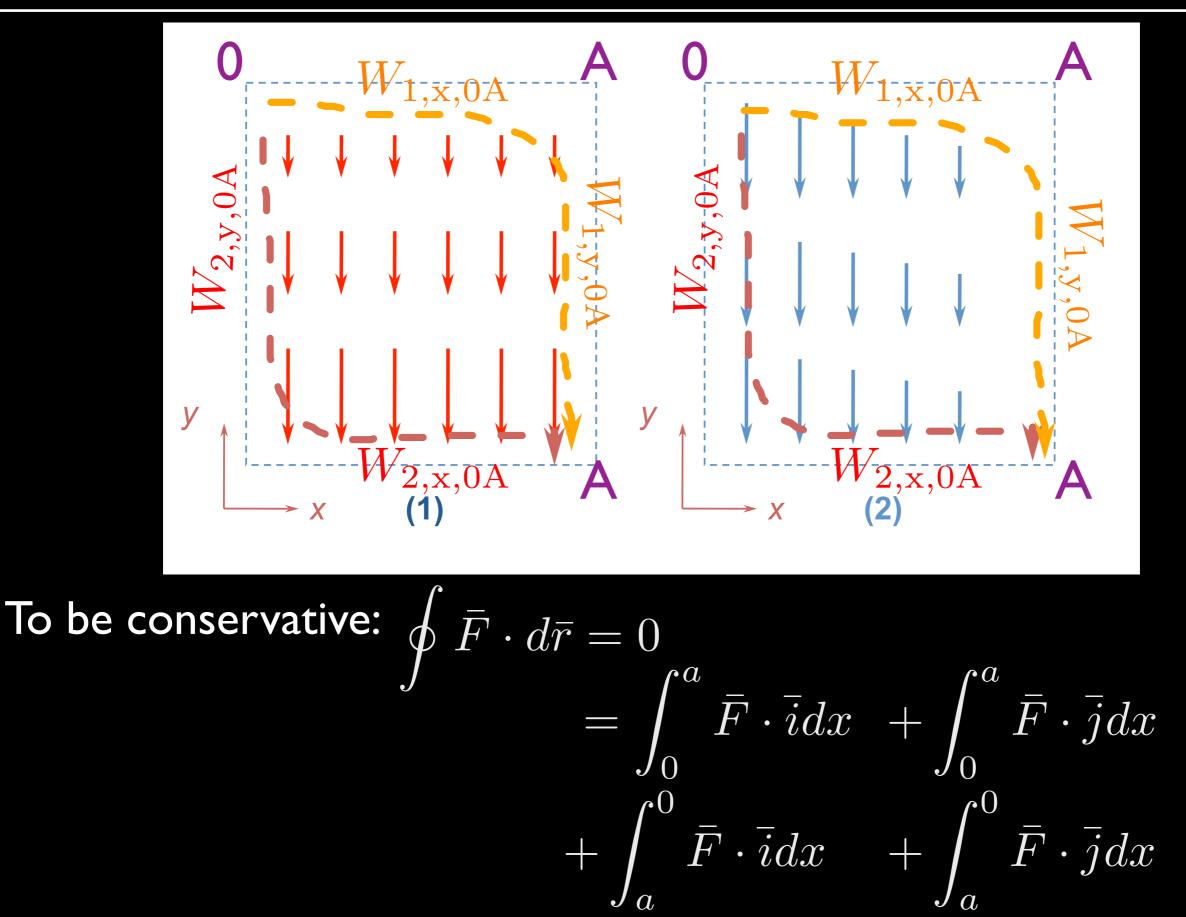
(b) $W_1 > W$
(c) $W_2 > W$

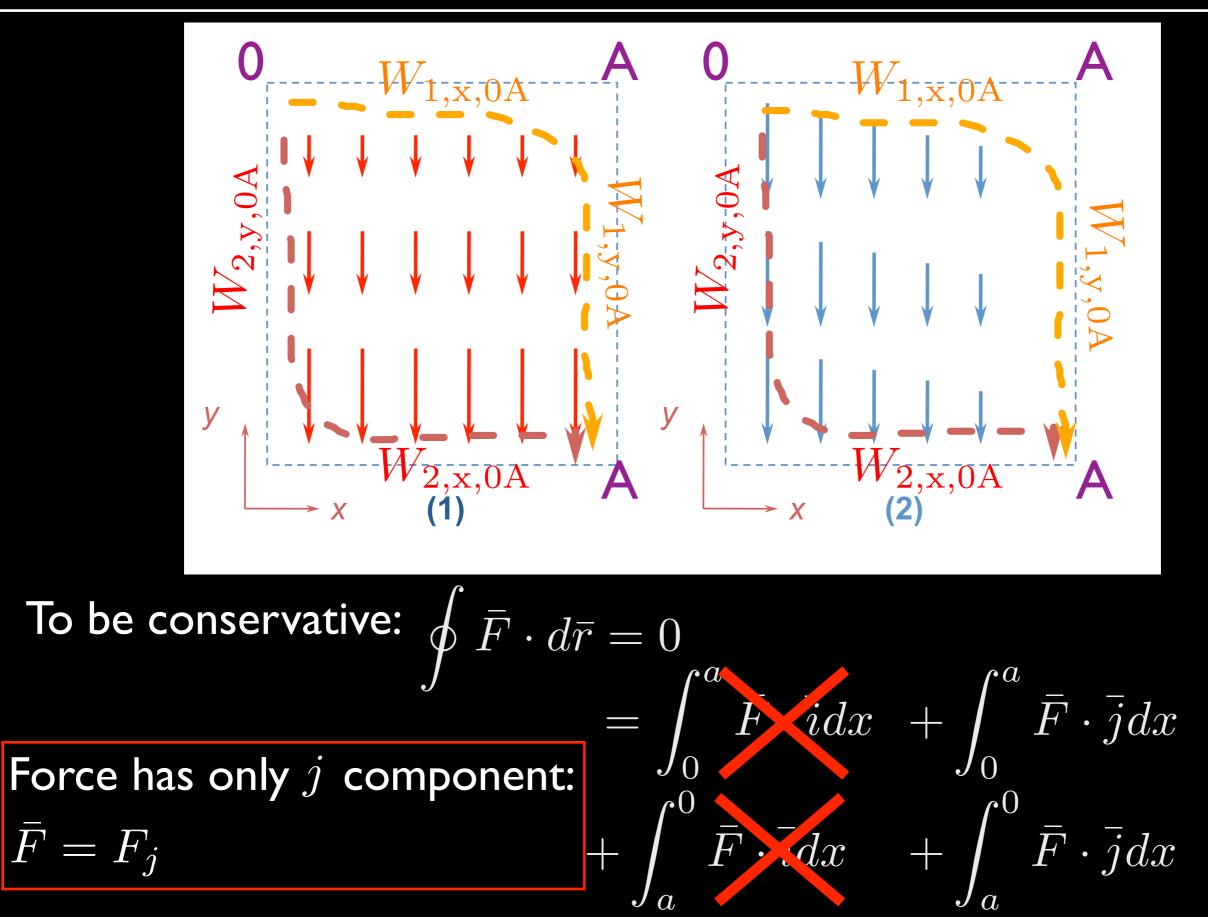


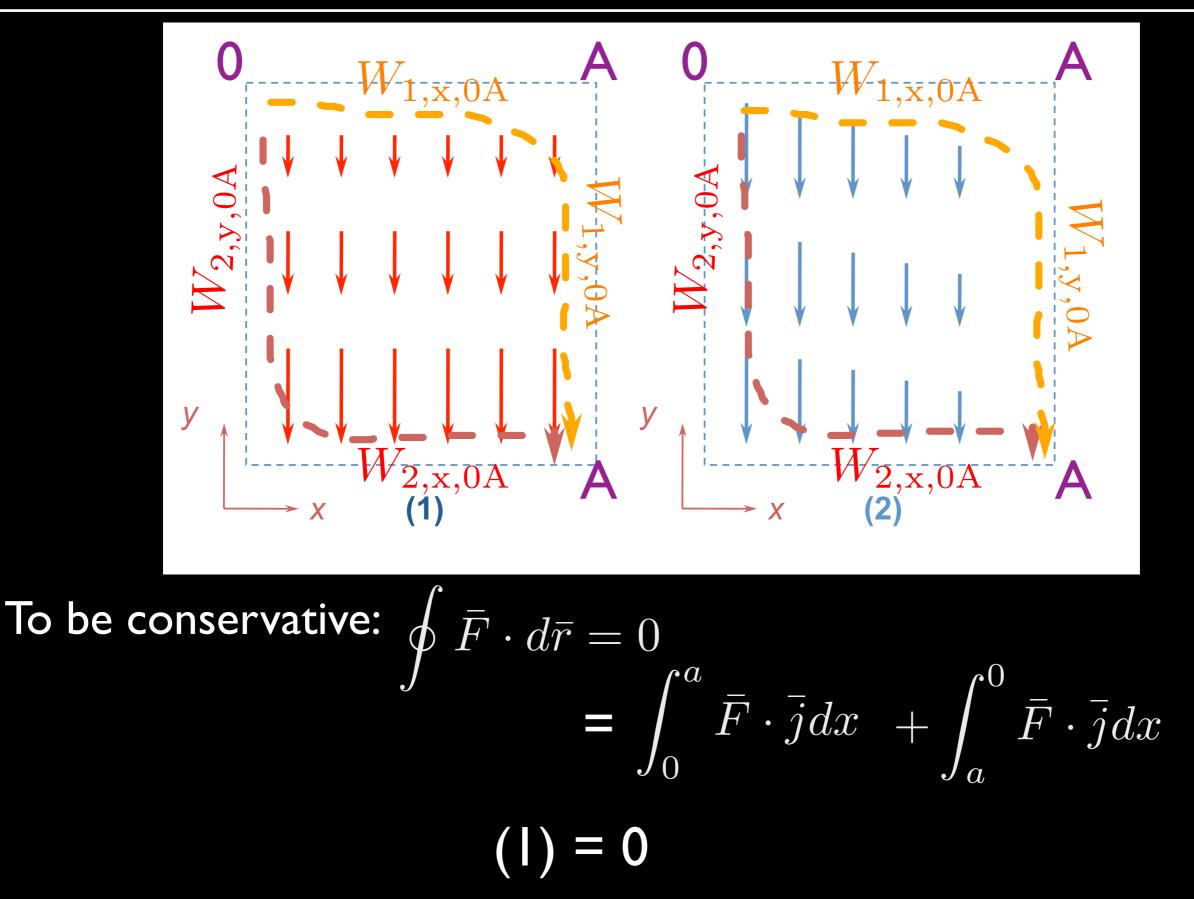


Which of these force fields is conservative?



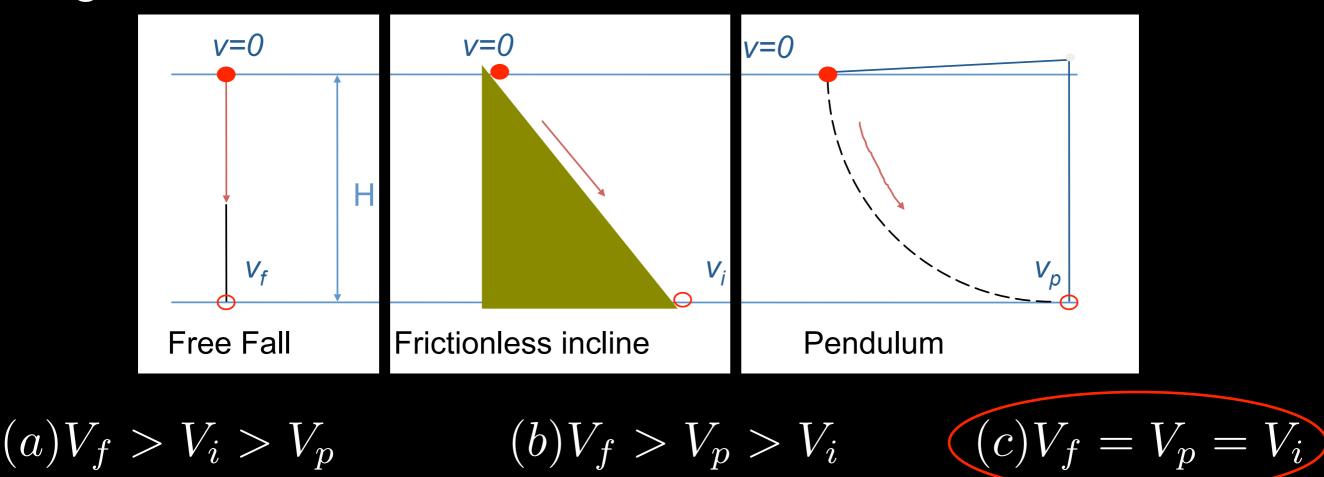




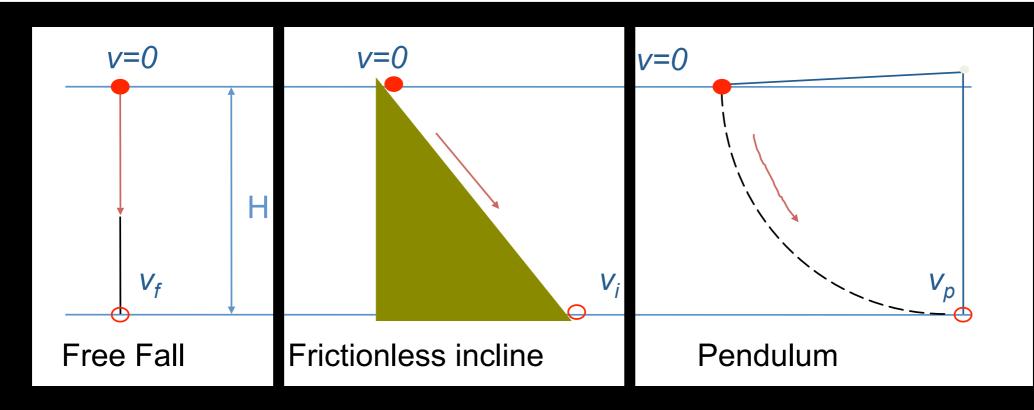




- 3 objects of mass m begin at height h with velocity 0.0
- 1 falls straight down
- 1 slides down a frictionless inclined plane
- 1 swings at the end of a pendulum
- What is the relationship between their speeds when they reach height 0?



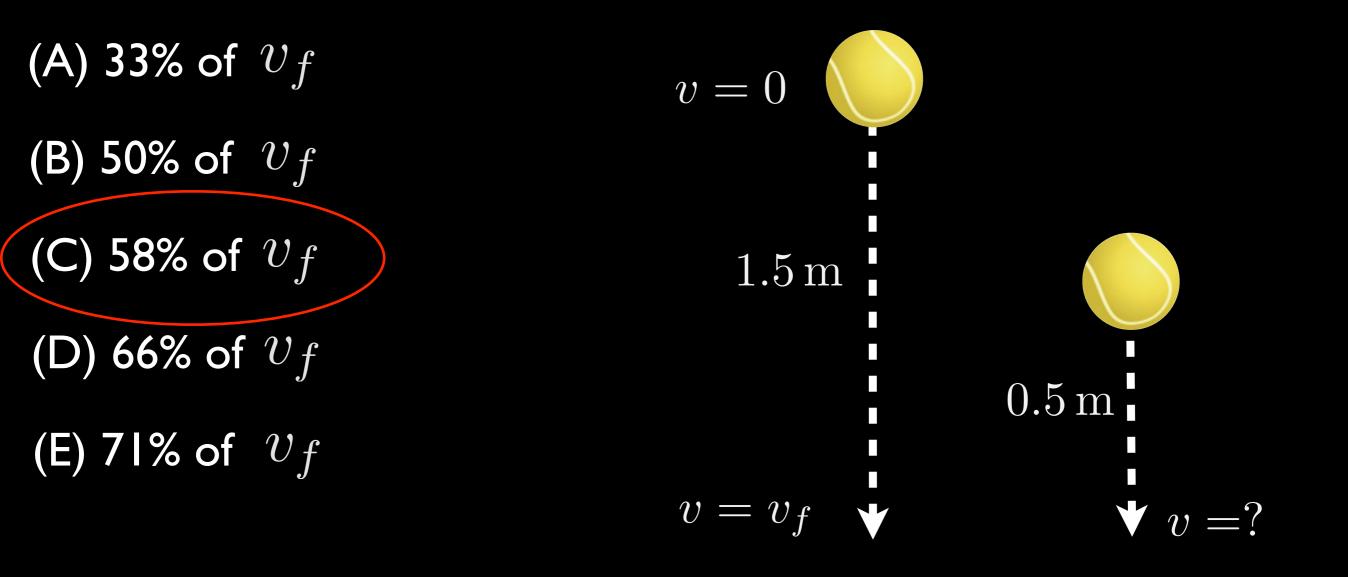




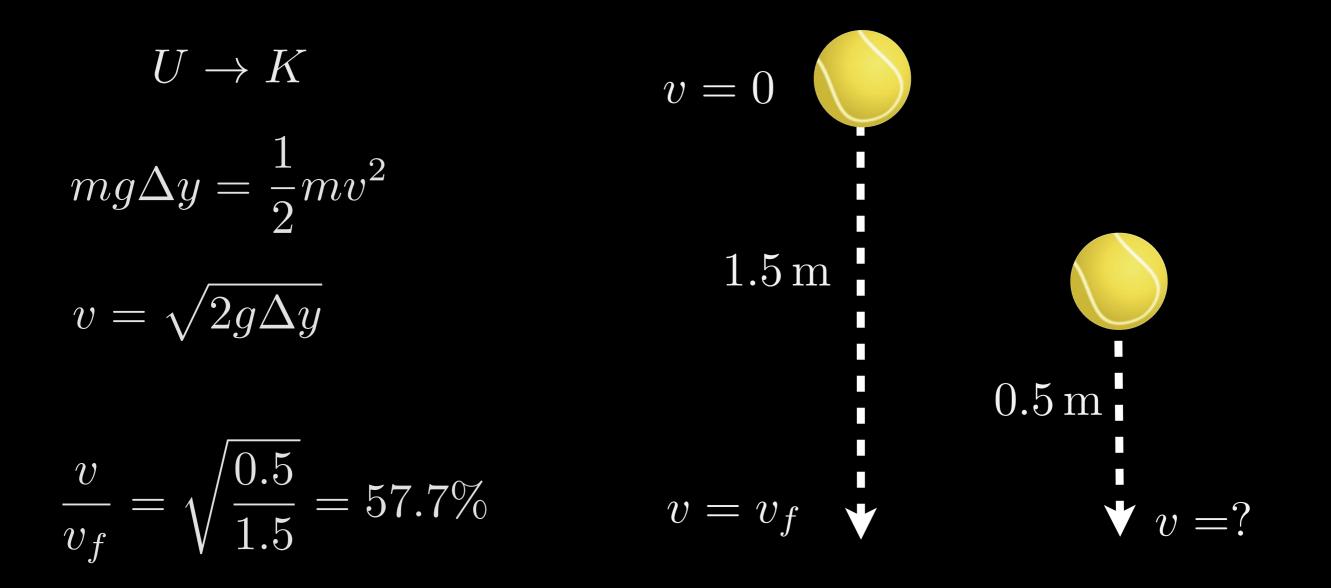
The only work is done by gravity, a conservative force Therefore, the path doesn't matter

$$U_f = U_p = U_i = mgh = K = \frac{1}{2}mv^2$$
$$V_f = V_p = V_i = \sqrt{2gH}$$

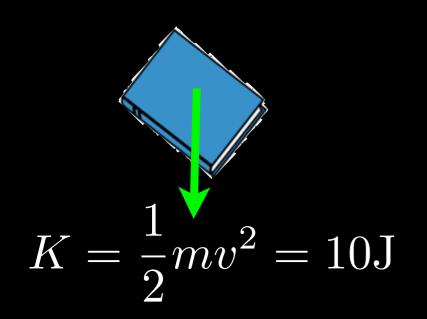
- A ball, initially at rest ($v_i=0$), is dropped from 1.5m.
- It is observed to have a velocity v_f just before it hits the floor.
- If instead the ball is dropped from 0.5 m what is its velocity?



A ball, initially at rest ($v_i = 0$), is dropped from 1.5m. It is observed to have a velocity v_f just before it hits the floor. If instead the ball is dropped from 0.5 m what is its velocity?



Energy conservation



The book's potential energy (U) becomes kinetic (K)

Then what?

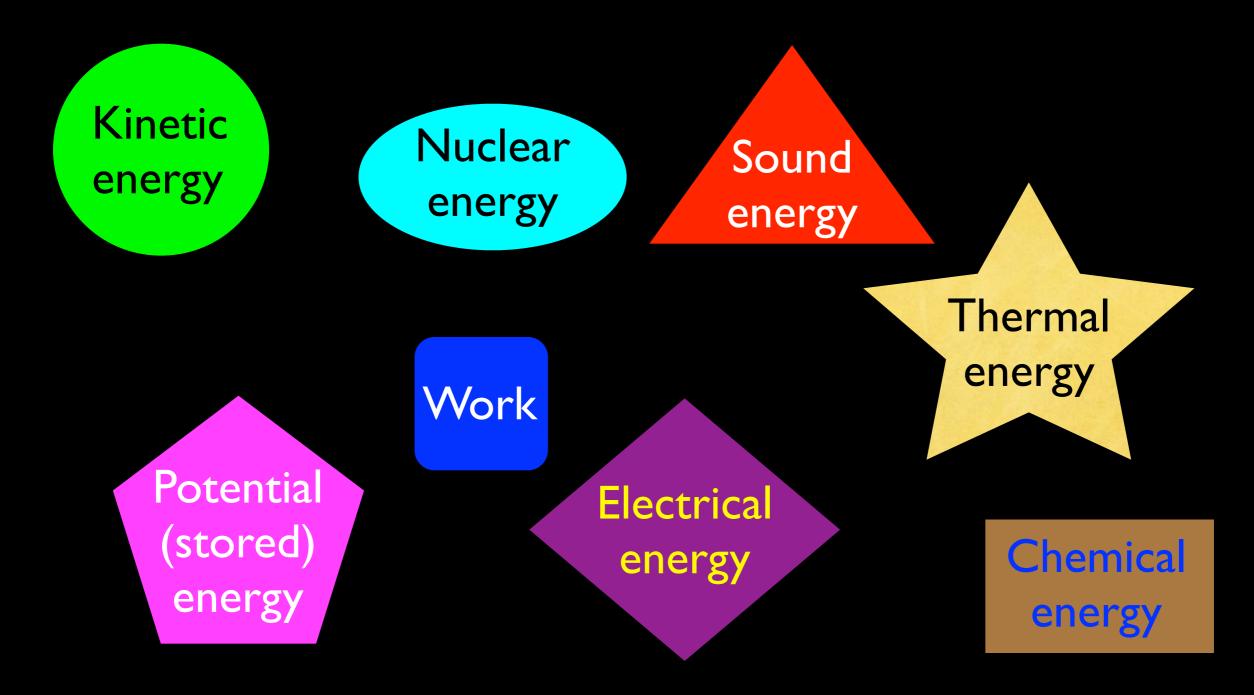
Where did the kinetic energy go?



Thermal (heat) energy and sound energy

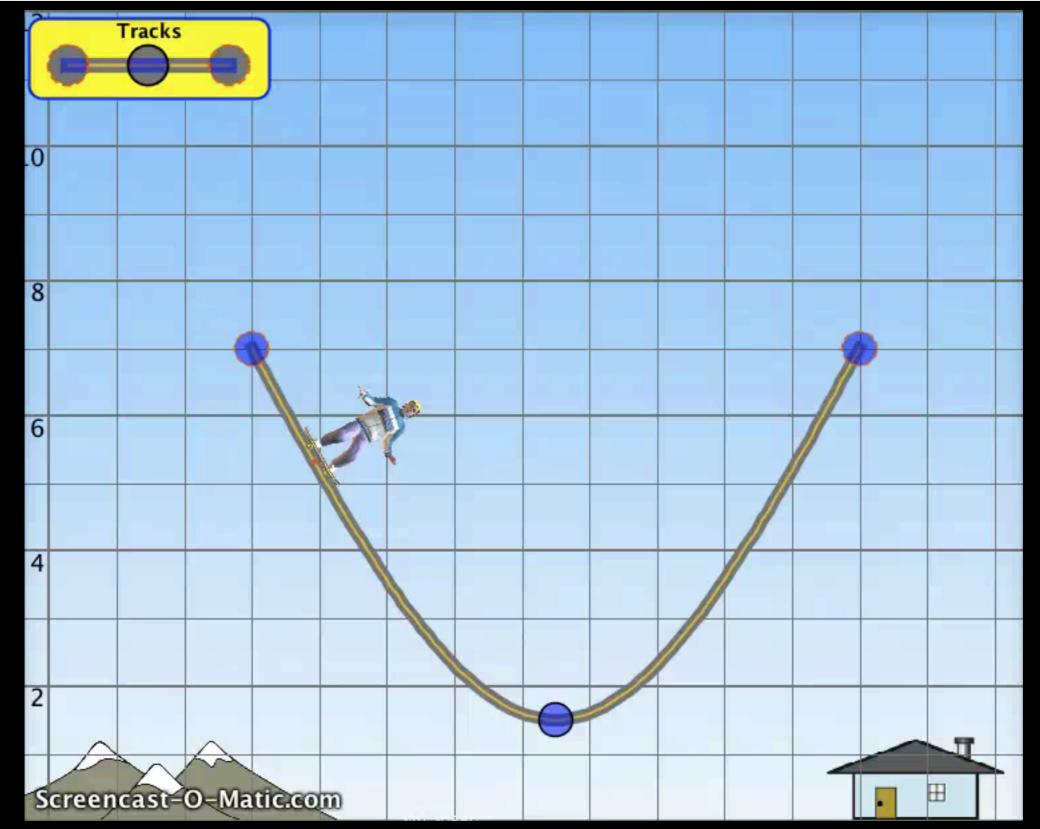
Energy conservation

Energy can be converted between forms:

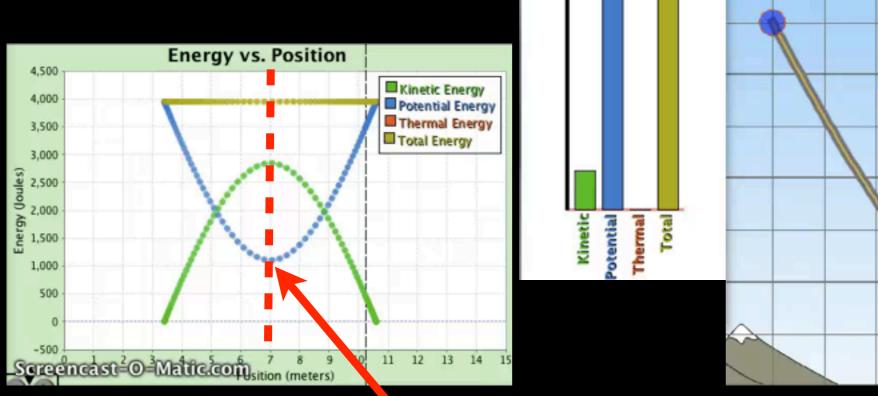


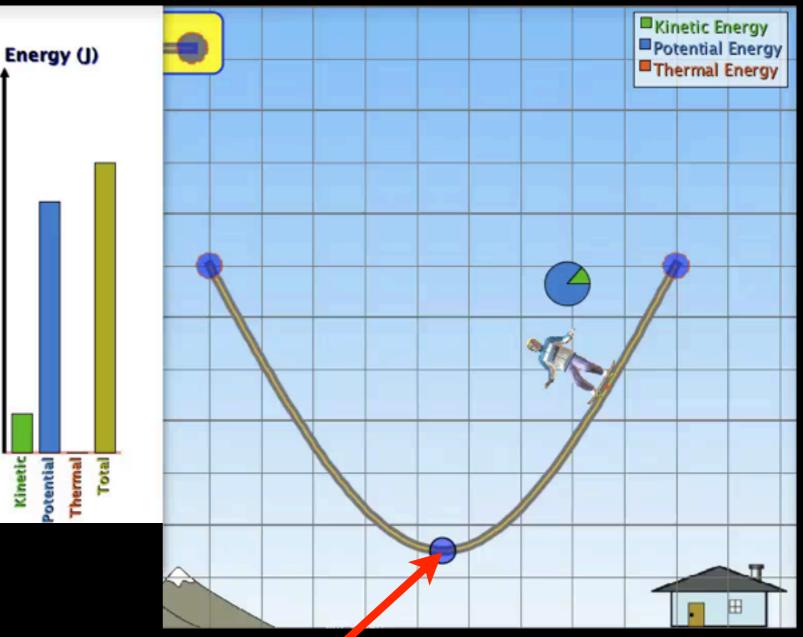
but it can NEVER be destroyed

Energy conservation



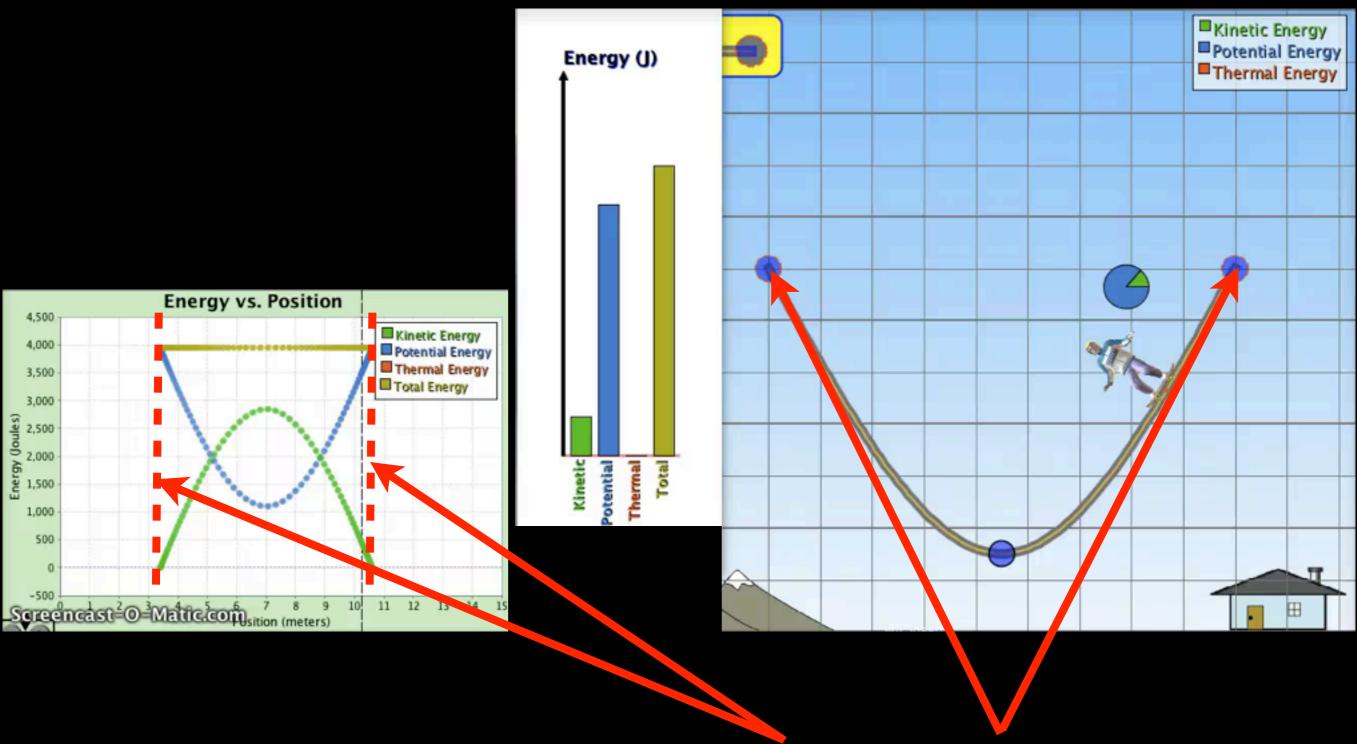
Frictionless track



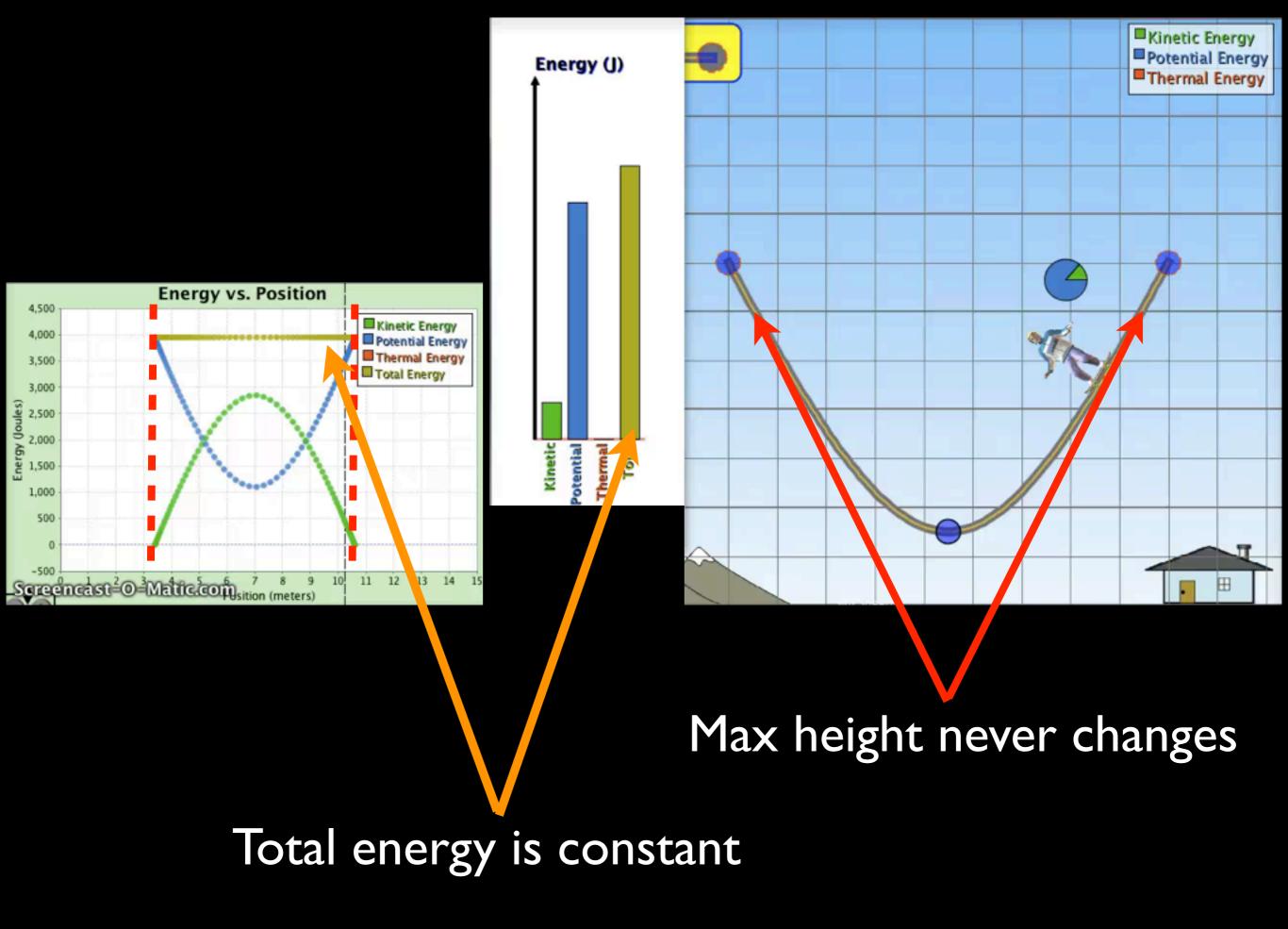


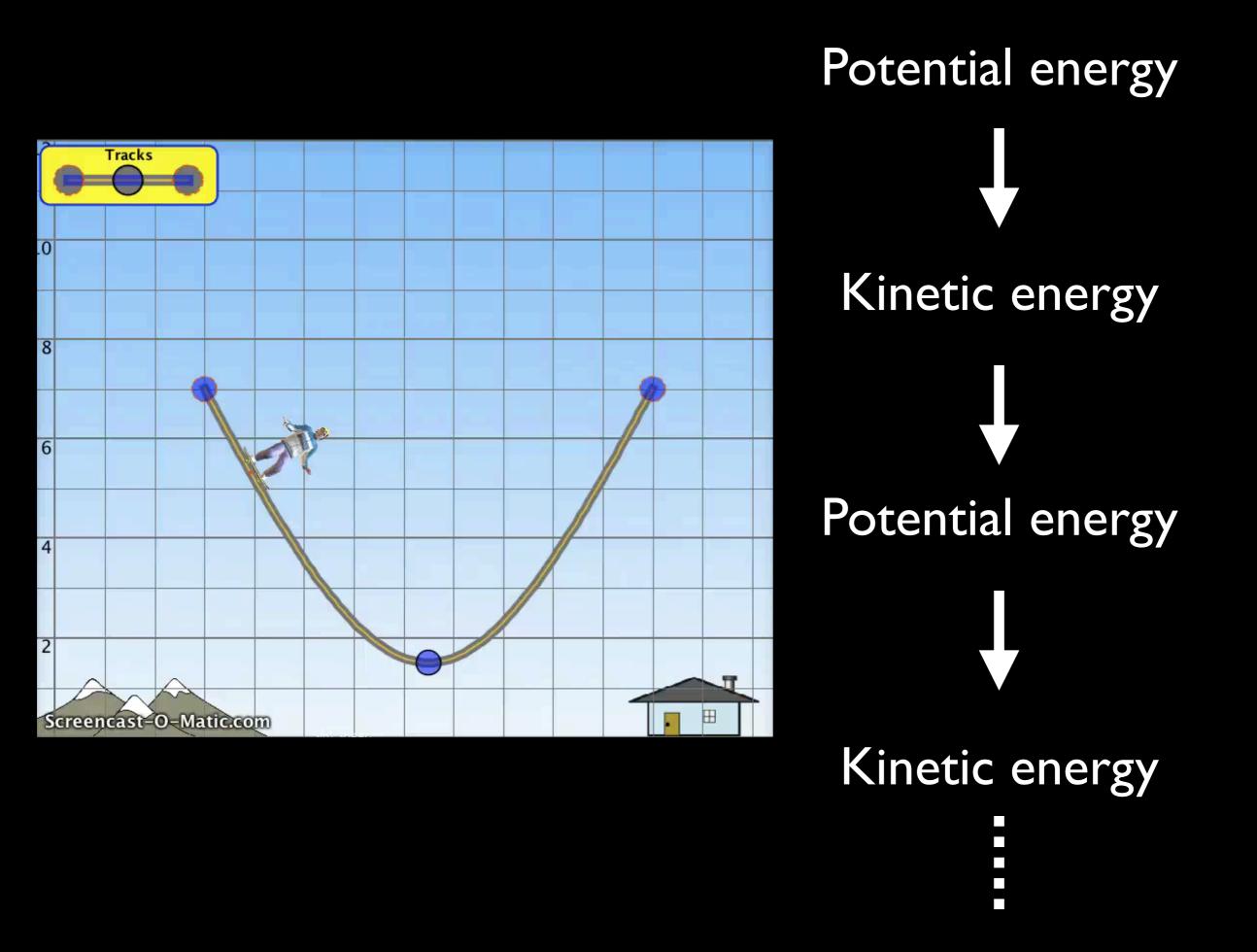
Lowest point min U

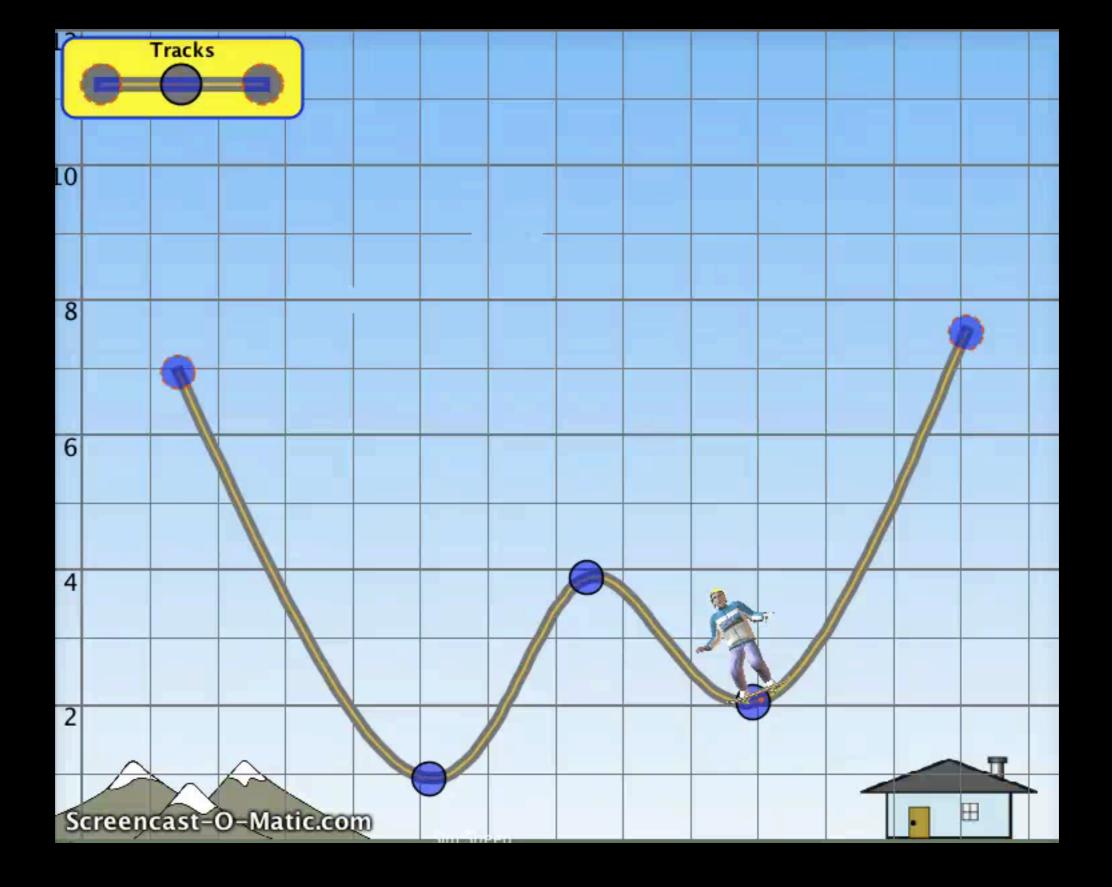
max K



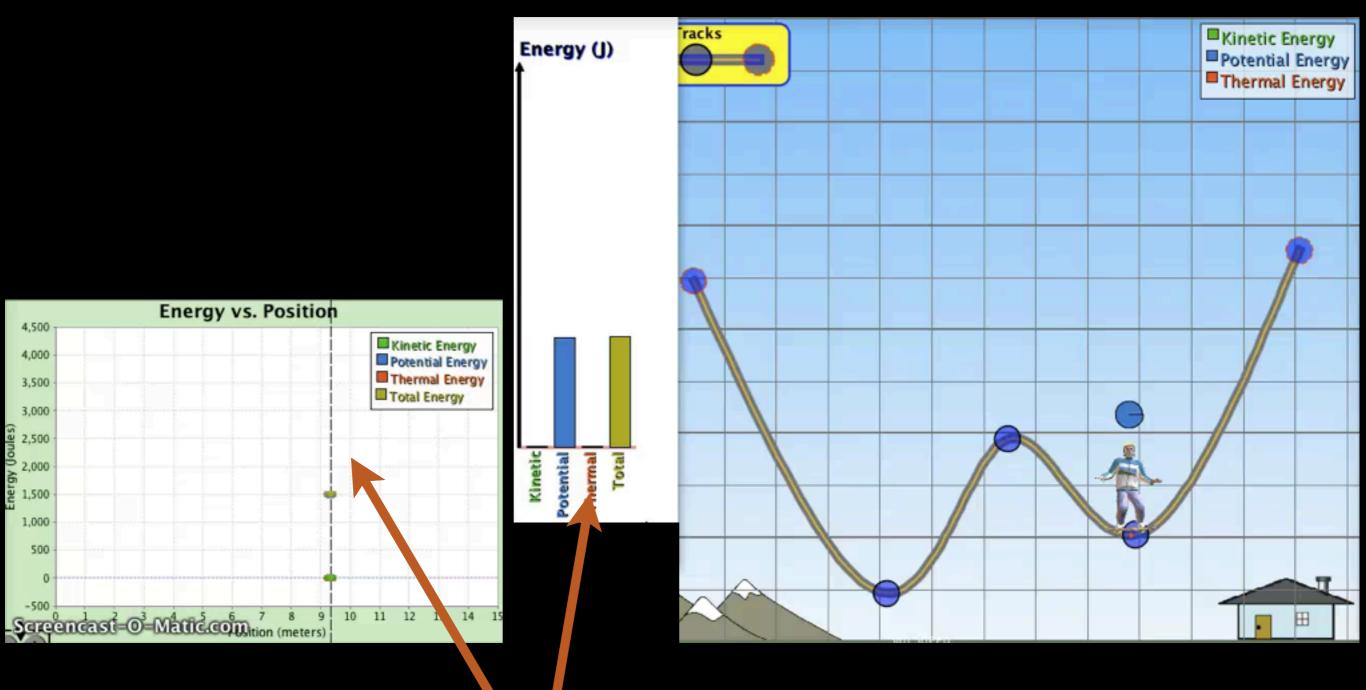
Highest points max U min K



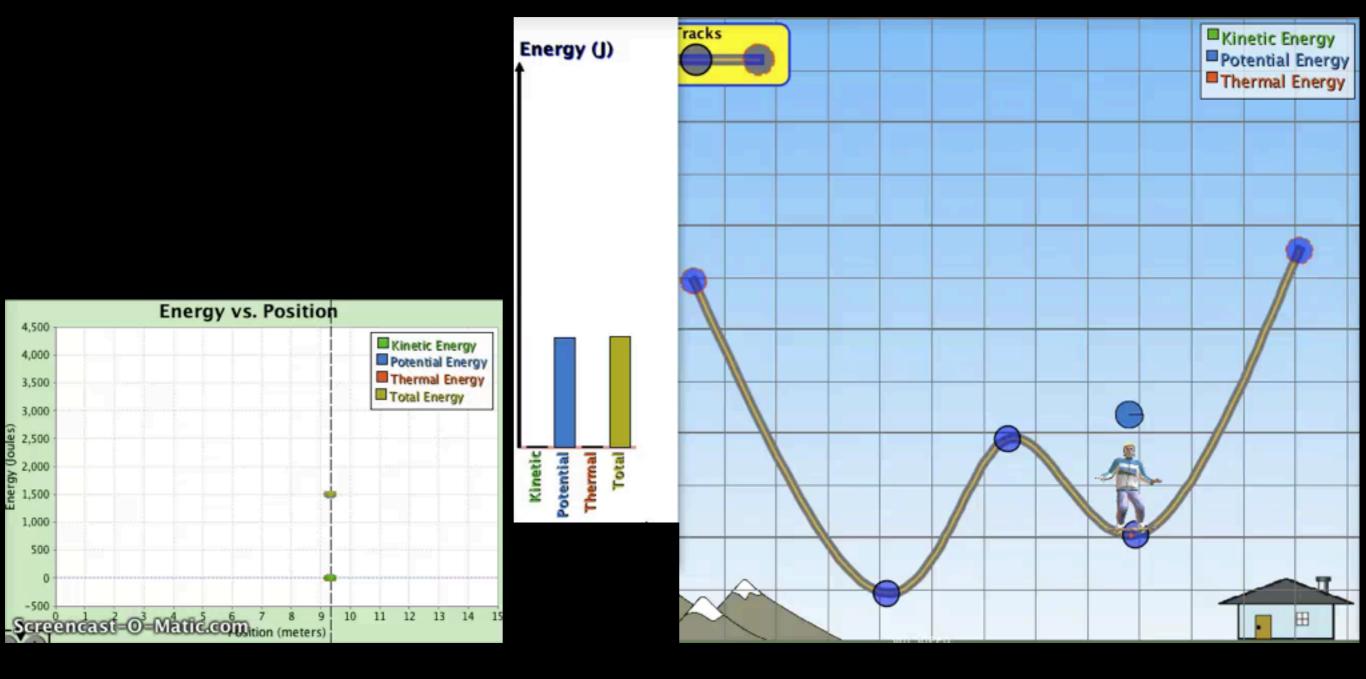




What about this track?



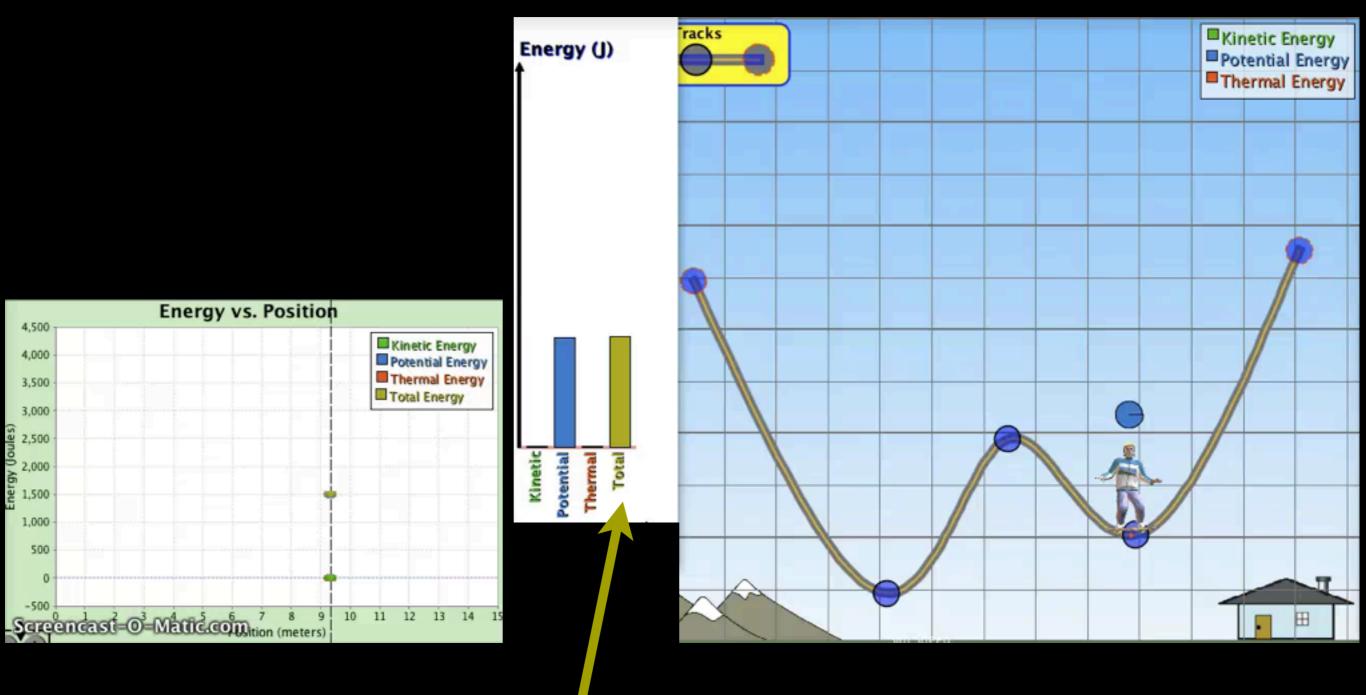
Initially all energy is potential energy, U, and starts to change to K The track impacts create thermal energy (heat)



Initially all energy is potential energy, U, and starts to change to K

The track impacts create thermal energy (heat)

Less energy is shared between U and K

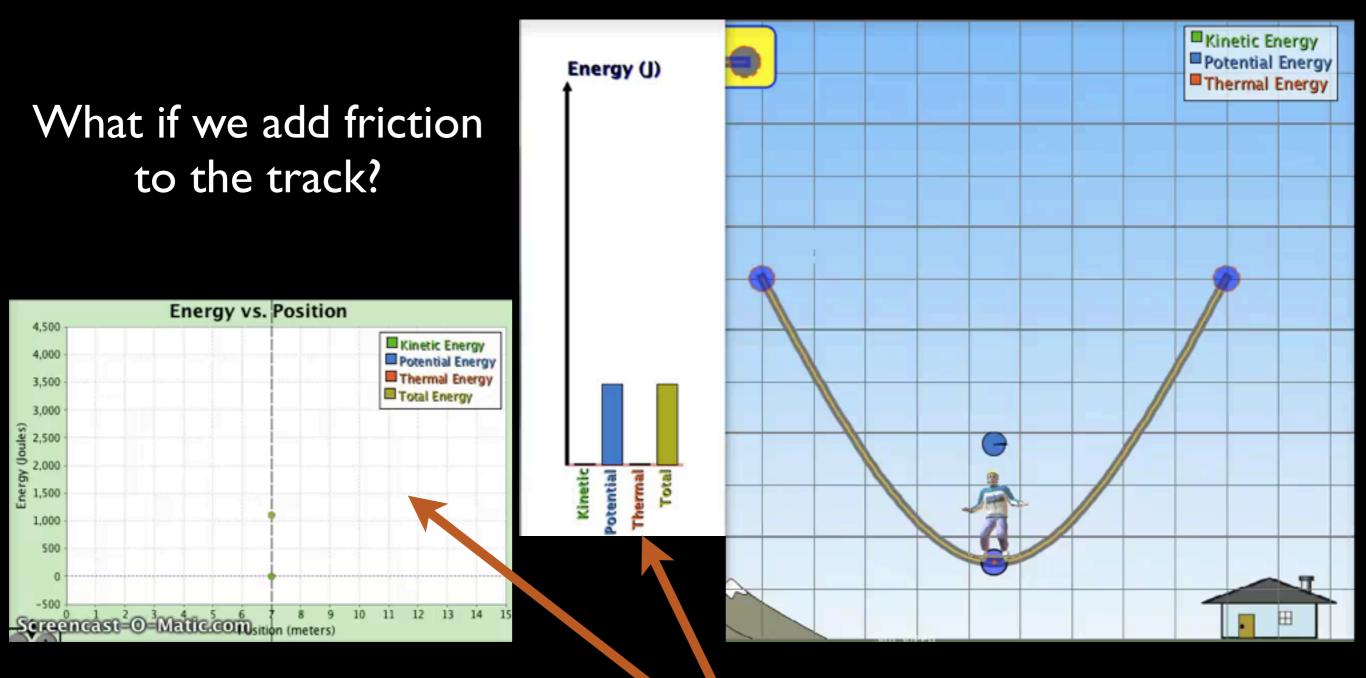


Initially all energy is potential energy, U, and starts to change to K

The track impacts create thermal energy (heat)

Less energy is shared betyveen U and K

Total energy is constant (can never destroy energy)



Friction creates thermal energy

The more the skater moves, the more thermal energy is generated

Less and less energy is available for K and U

Total energy is constant

When an object is acted on by only conservative forces, the sum of its kinetic and potential energies does not change.

UÌZ

(a) True

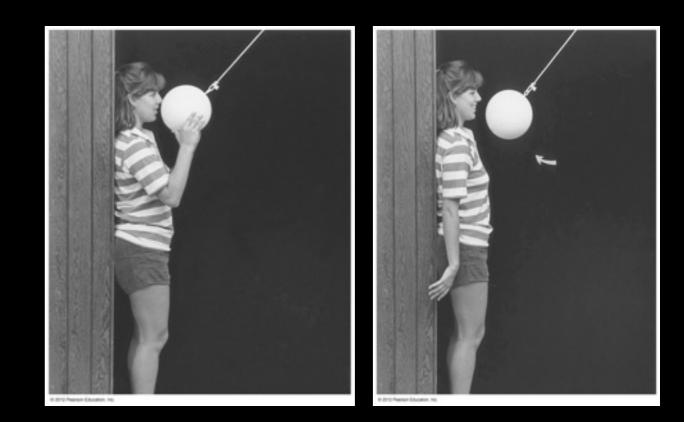
(b) False



- A bowling ball is tied to a rope and hangs from the ceiling.
- A girl holds the ball to her nose and releases it.
- Should she duck as it swings back?

Yes! (a)

(b) No!

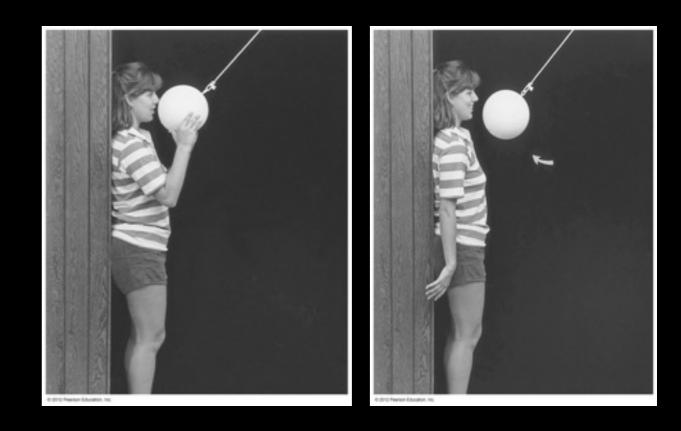


If the ball was in a vacuum...

Should she duck?

(a) Yes!

(b) No!



UIZ

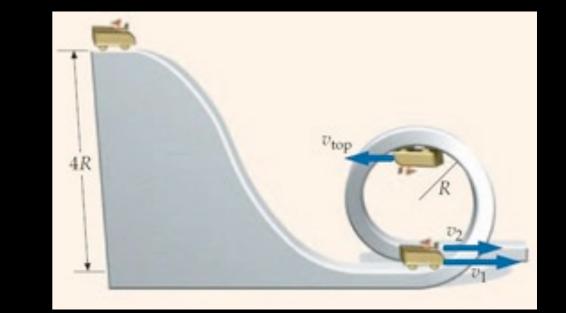
Cannot gain MORE energy than you begin with

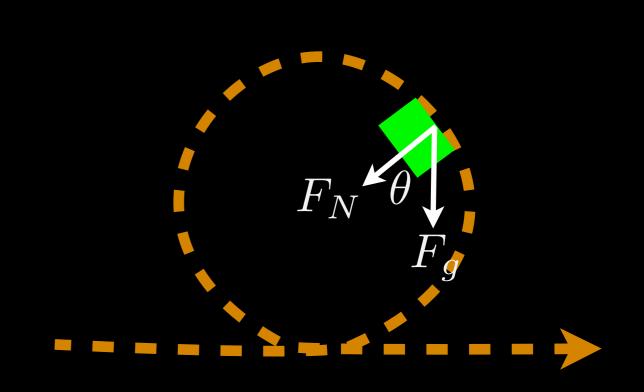
What height must the cart start to complete the loop-the-loop?

To stay on the track: $F_N > 0$

$$\bar{F}_N + \bar{F}_g = m\bar{a}$$

$$F_N + mg\cos\theta = m\frac{v^2}{r}$$







What height must the cart start to complete the loop-the-loop?

To stay on the track: $F_N > 0$

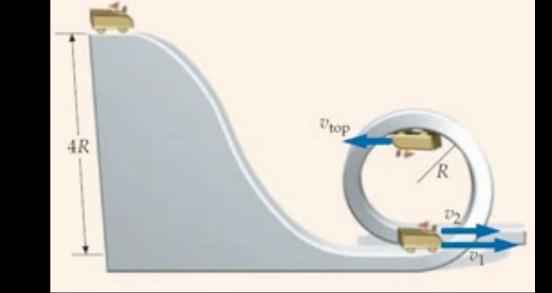
$$\bar{F}_N + \bar{F}_g = m\bar{a}$$

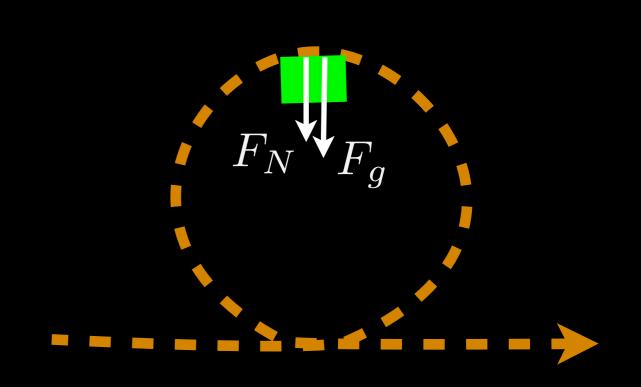
$$F_N + mg\cos\theta = m\frac{v^2}{r}$$

At loop top: $F_N + mg = m \frac{1}{r}$

 v^2

 $\xrightarrow{F_N=0} v_{\min} = \sqrt{rg}$





example

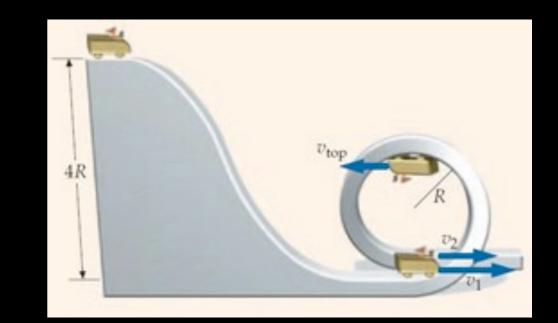
What height must the cart start to complete the loop-the-loop?

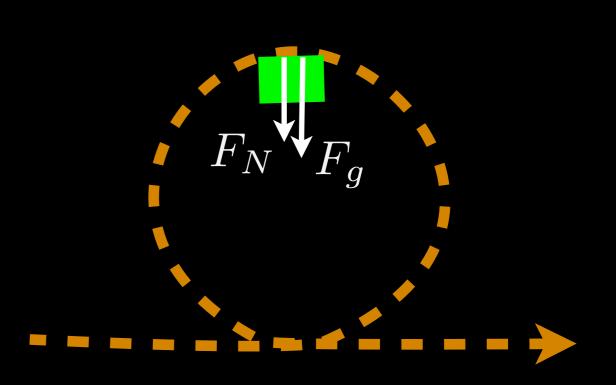
Energy:

$$K_{\text{top}} = \frac{1}{2}mv_{\text{top}}^2 = \frac{1}{2}m(rg)$$
$$U_{\text{top}} = mg2r$$

$$E_{\rm top,total} = \frac{1}{2}mgr + 2mgr$$





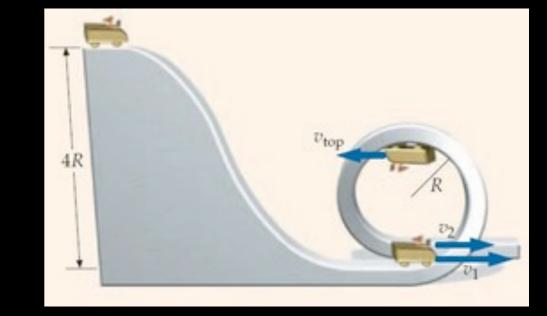


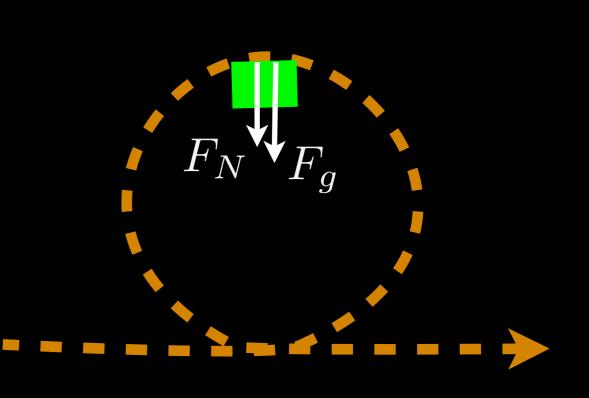
example

What height must the cart start to complete the loop-the-loop?

Energy conservation:

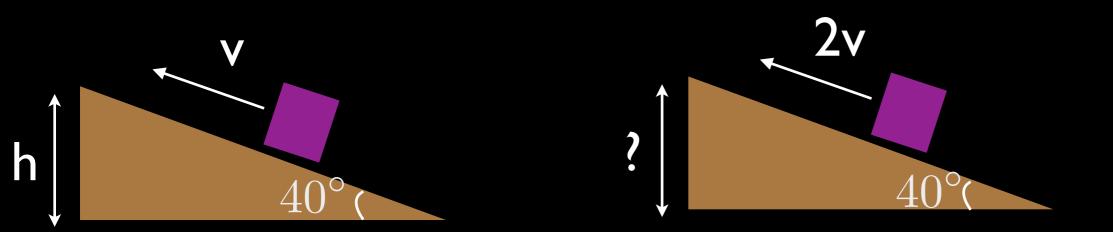
 $E_{\text{top,total}} = E_{\text{start}}$ $= U_{\text{start}} + K_{\text{start}} (= 0)$ $= mg\Delta h$ $\frac{5}{2}mgr = mg\Delta h$ $\Delta h = \frac{5}{2}r$





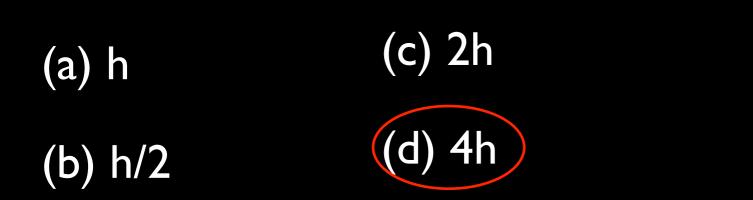


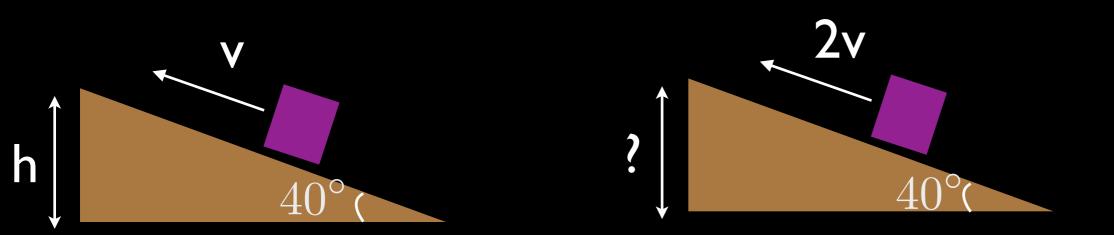
(e) 8h



A block is launched up a frictionless slope at 40 degrees, with speed v. It reaches a vertical height h.

Speed is changed to 2v. What is the new height?





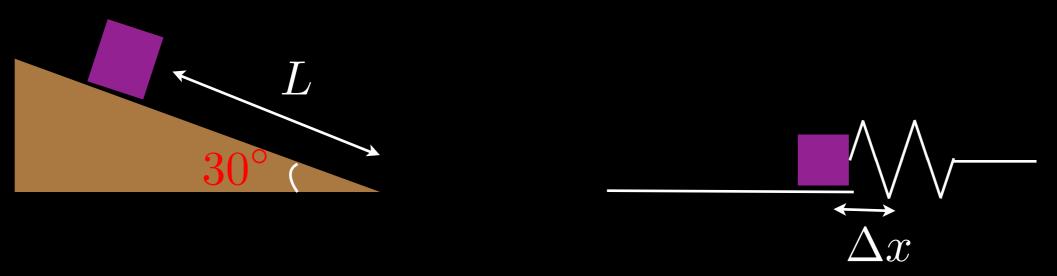
A block is launched up a frictionless slope at 40 degrees, with speed v. It reaches a vertical height h.

Speed is changed to 2v. What is the new height?

$$KE \to U$$
 $\frac{1}{2}mv^2 = mgh \longrightarrow h = \frac{v^2}{2g}$

$$\frac{1}{2}m(2v)^2 = mgh_2 \quad \longrightarrow \quad h_2 = \frac{(2v)^2}{2g} = 4h$$

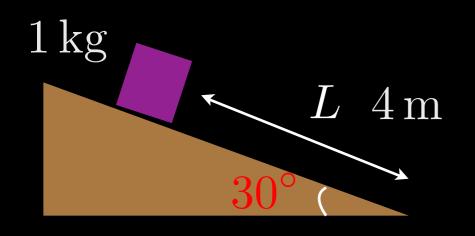
- A I kg block slides 4m down a frictionless plane inclined at 30° to the horizontal.
- After reaching the bottom, it slides along a frictionless horizontal plane and strikes a spring with constant k = 314 N/m.
- How far is the spring compressed when it stops the block?

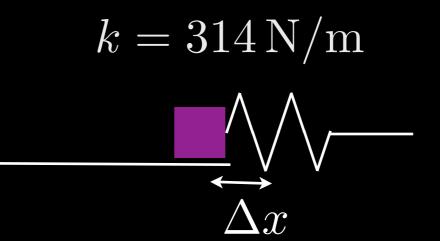


Energy conservation:

 $U_{\text{slope}} \to K_{\text{block}} \to U_{\text{spring}}$

Quiz





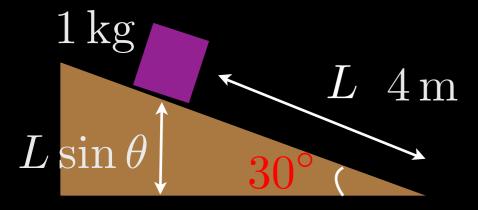
(a) 35 cm

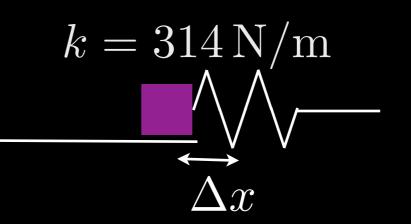
(b) 50 cm

(c) 0.4 cm

(d) 29 cm







Energy conservation:

$$U_{\text{slope}} \to K_{\text{block}} \to U_{\text{spring}}$$

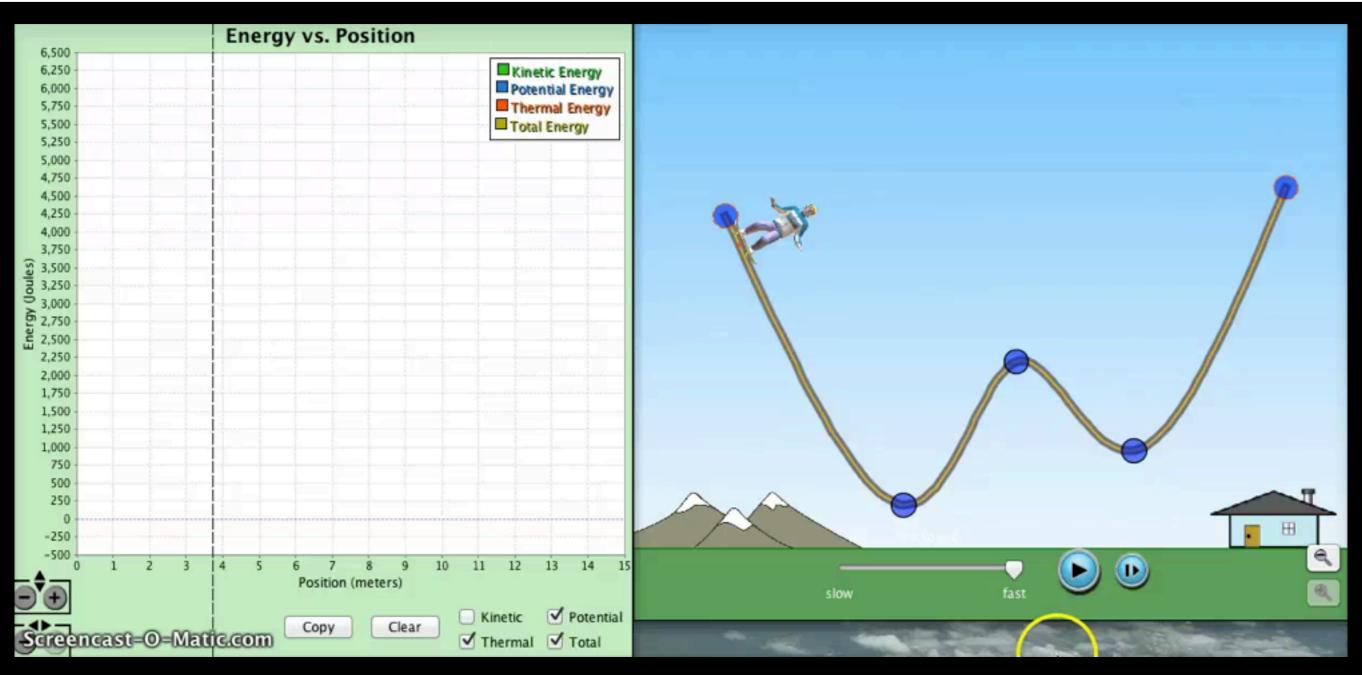
$$U_{\text{slope}} = mgL\sin\theta = K_{\text{bloc}}$$
$$K_{\text{block}} = U_{\text{spring}} = \frac{1}{2}kx^2$$

$$\frac{1}{2}kx^2 = mgL\sin\theta$$

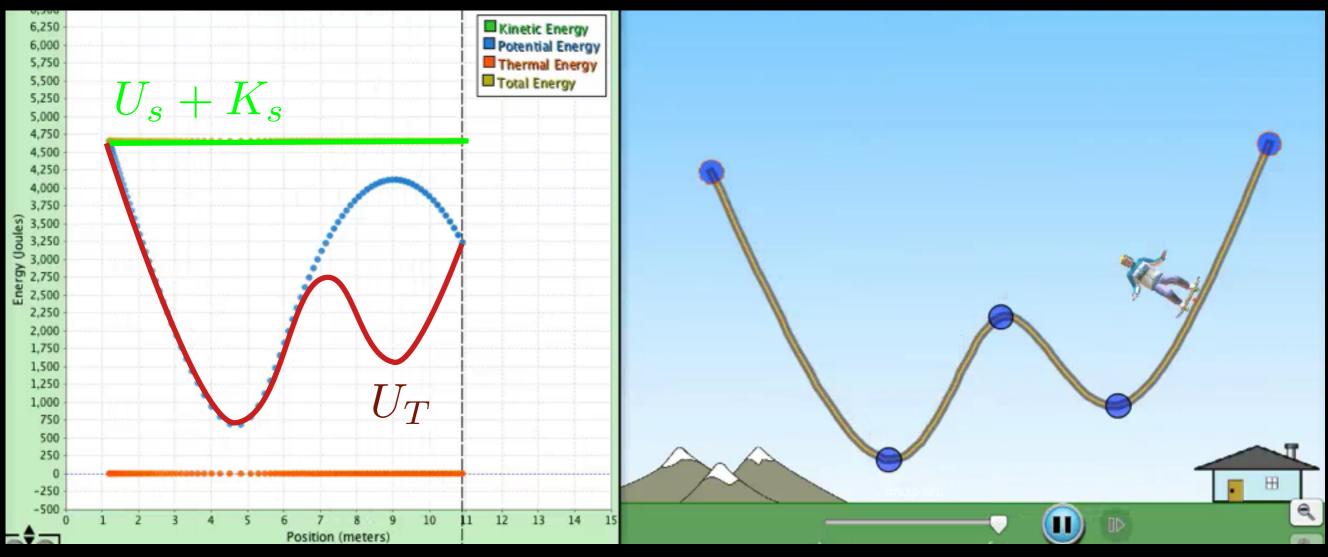
$$x = \sqrt{\frac{2mgL\sin\theta}{k}} = 35\text{cm}$$



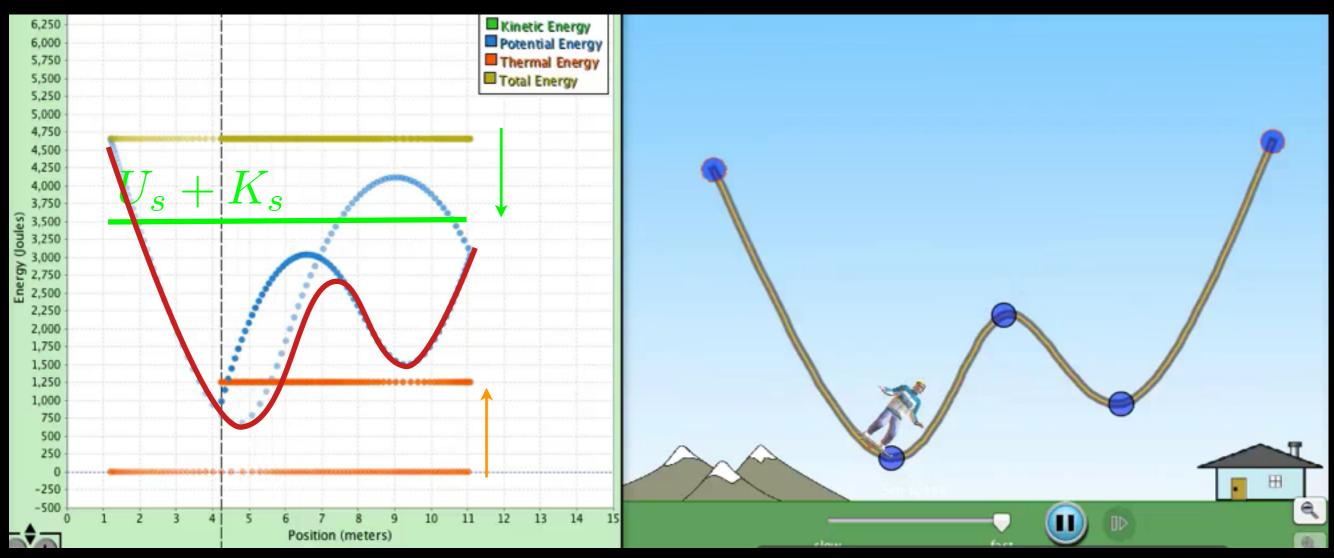
Potential energy vs position is called a potential energy curve



We can use it to understand the skater's motion

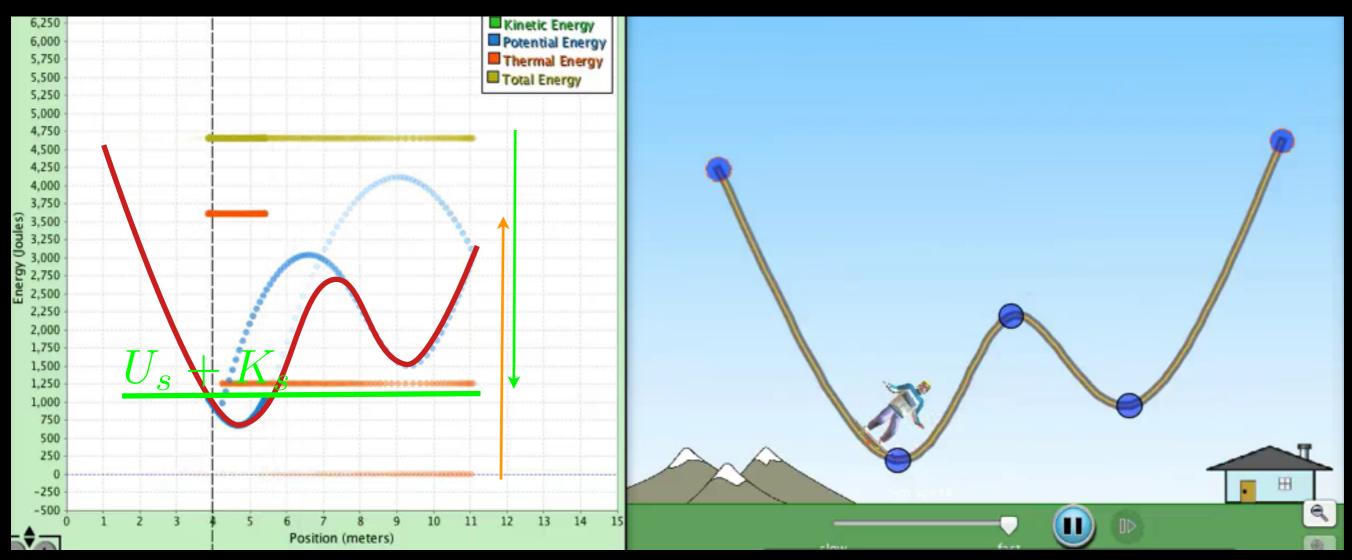


Initially, the skater's potential + kinetic energy > track's potential: $U_{\rm s}+K_{\rm s}>U_{\rm T}$



Initially, the skater's potential + kinetic energy > track's potential: $U_{\rm s}+K_{\rm s}>U_{\rm T}$

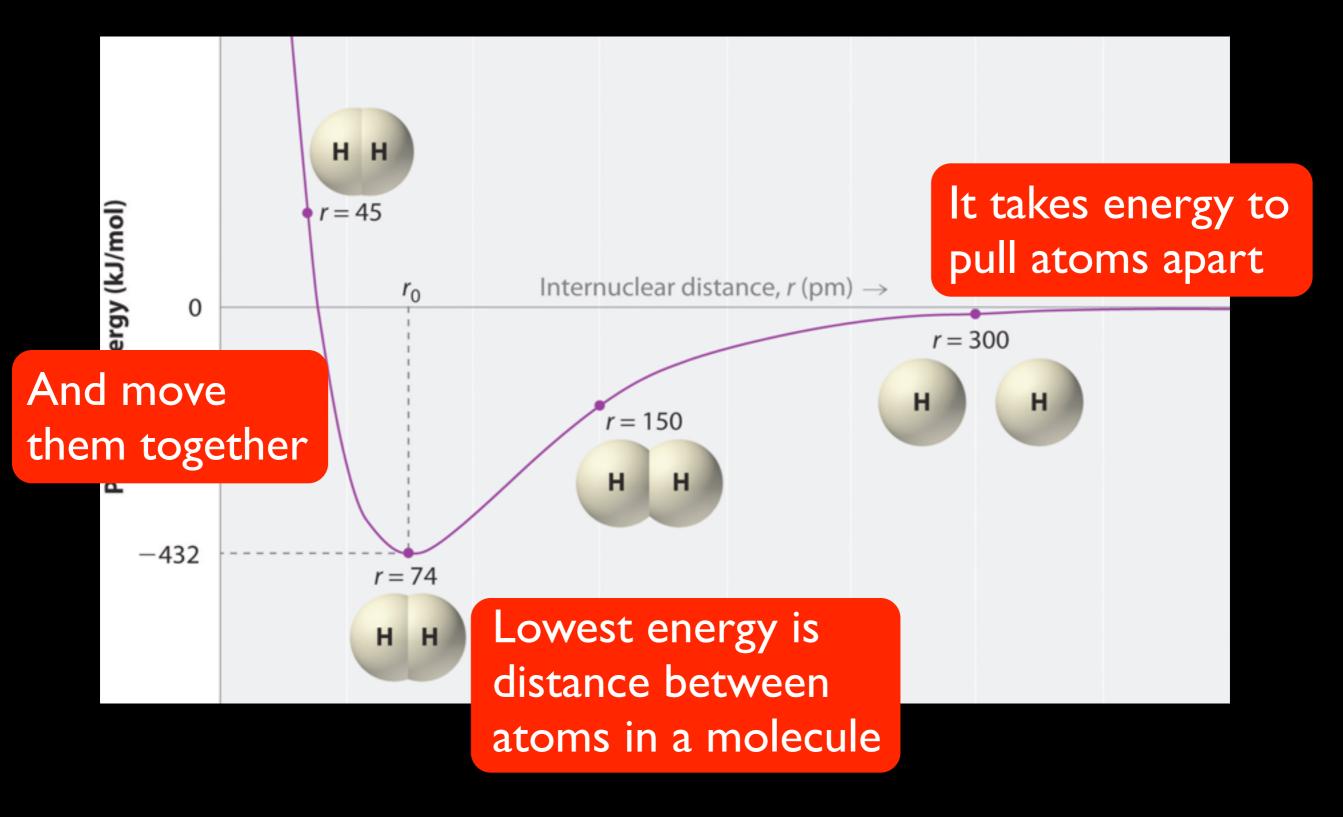
Track impact creates thermal energy, decreasing $U_s + K_s$ It is no longer larger than all of the track's potential



Initially, the skater's potential + kinetic energy > track's potential: $U_{\rm s}+K_{\rm s}>U_{\rm T}$

Track impact creates thermal energy, decreasing $U_s + K_s$ It is no longer larger than all of the track's potential The skater gets trapped in a potential barrier

Potential barriers are not just gravity



The slope of the potential is the force

In ID:

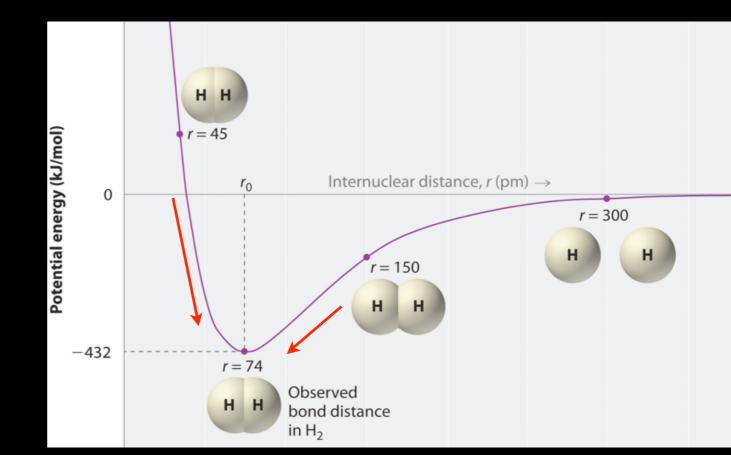
$$\Delta U_{AB} = -\int_{A}^{B} F_{x} dx$$

dU

dx

Therefore:

 F_x

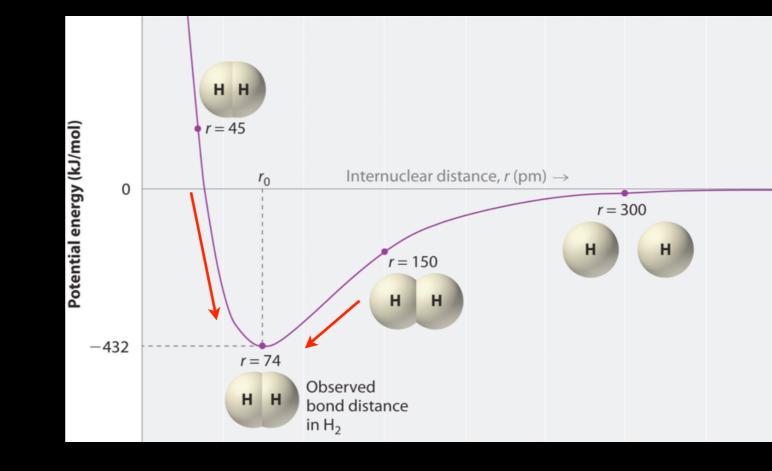


The force acts in the direction of the potential minimum

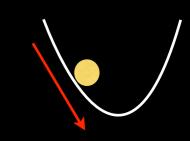
Where the slope is zero, there is no force:

$$-\frac{dU}{dx} = 0 \to F_x = 0$$

This is called an equilibrium position

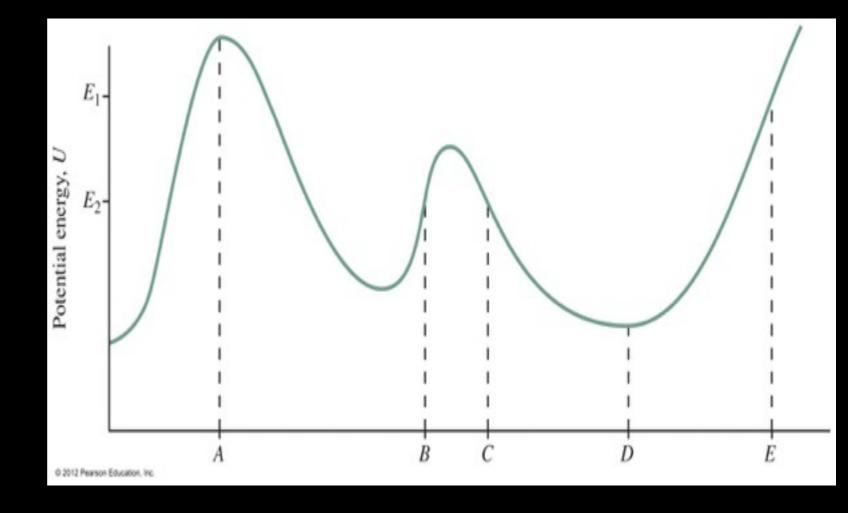


- If a small displacement creates a force towards the equilibrium position, it is a stable equilibrium.
- If a small displacement creates a force away the equilibrium position, it is a unstable equilibrium.





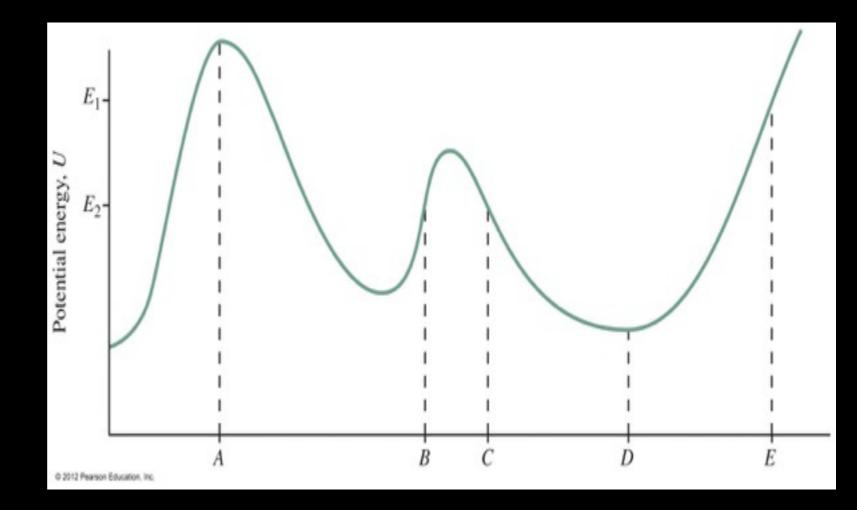
Potential energy for an electron in a microelectronic device



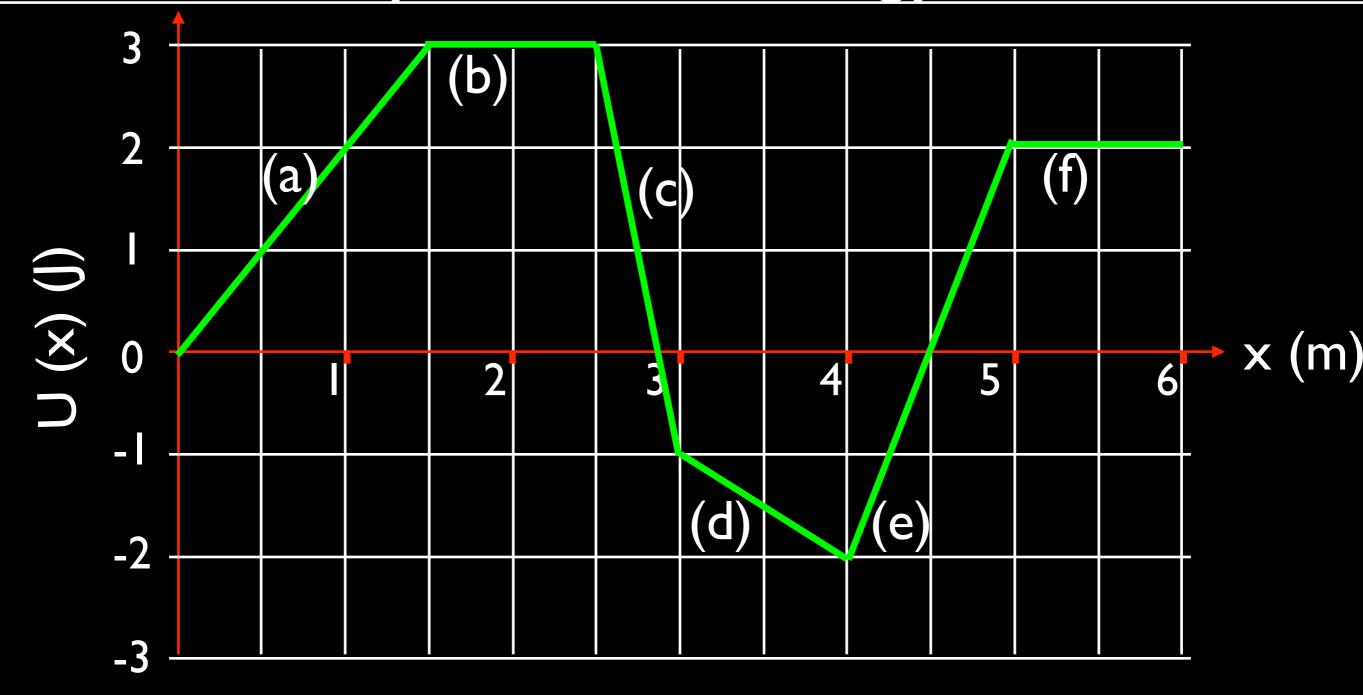
Where is the force greatest?

(a) A (c) C (e) E (b) B (d) D

Potential energy for an electron in a microelectronic device



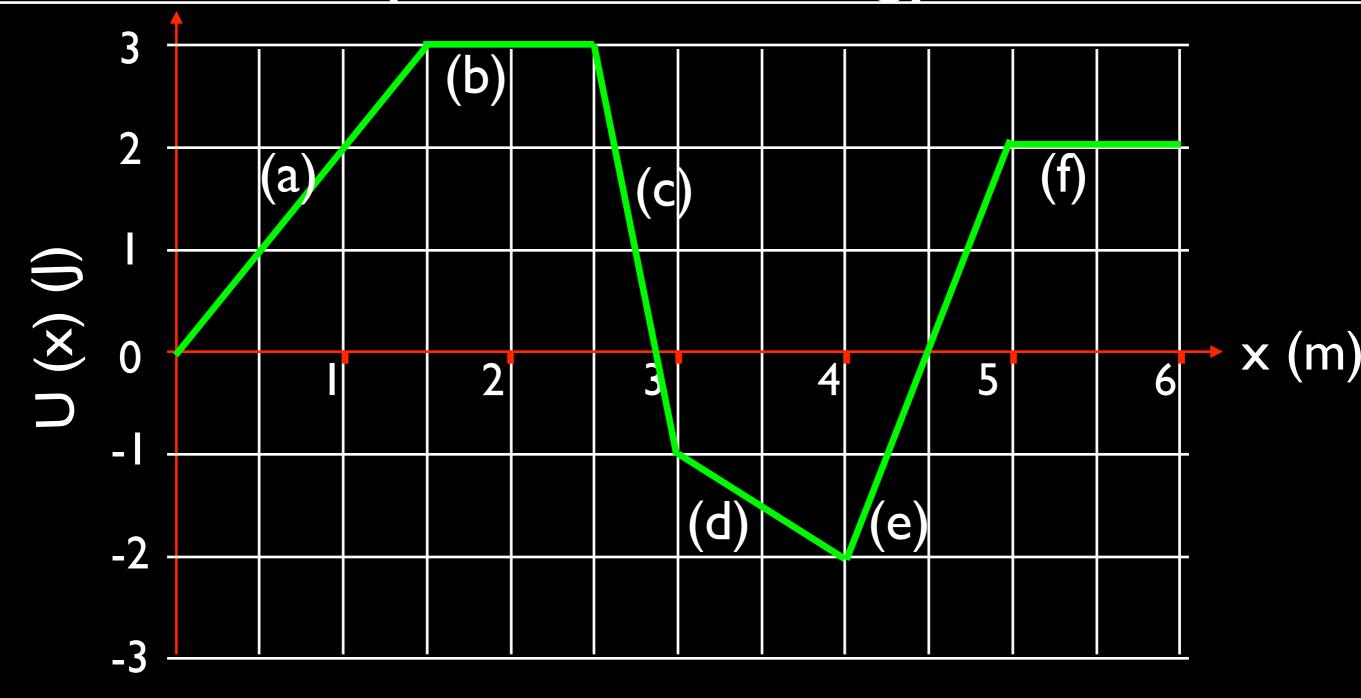
Find the right-most point (one most right) where the force points to the left.



Juiz

Force in segment (a)?

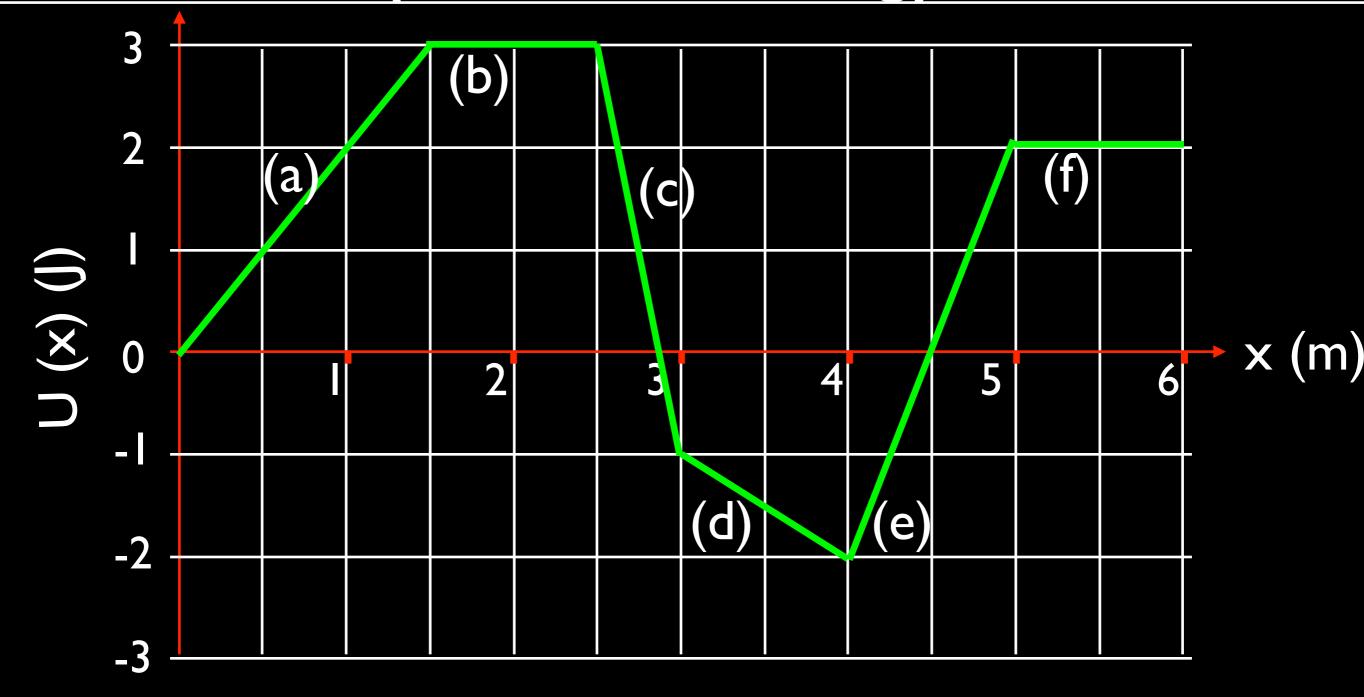
(a) -2N (b) 2N (c) 0.5N (d) -0.5N (d) 0 N



Force in segment (a)?

$$F(x) = -\frac{\Delta U}{\Delta x} = -\frac{3J}{1.5m} = -2N$$

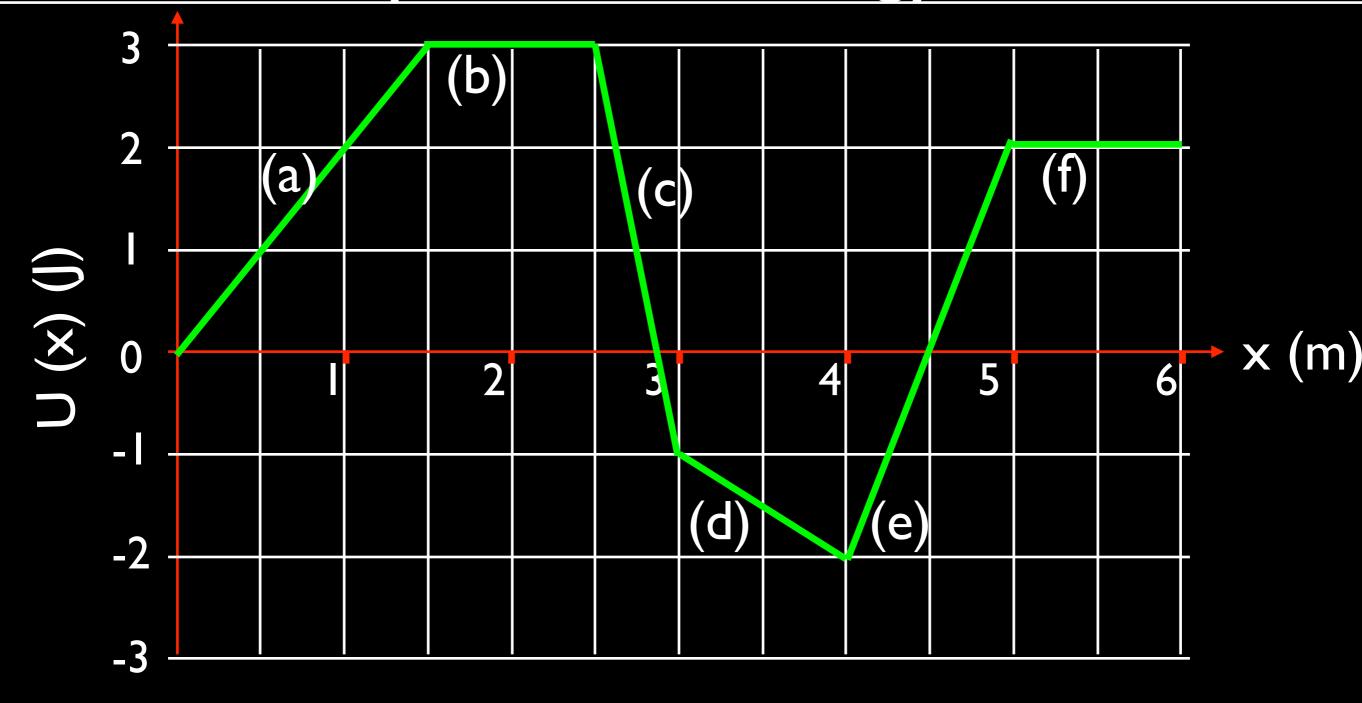
Juiz



Juiz

Force in segment (b)?

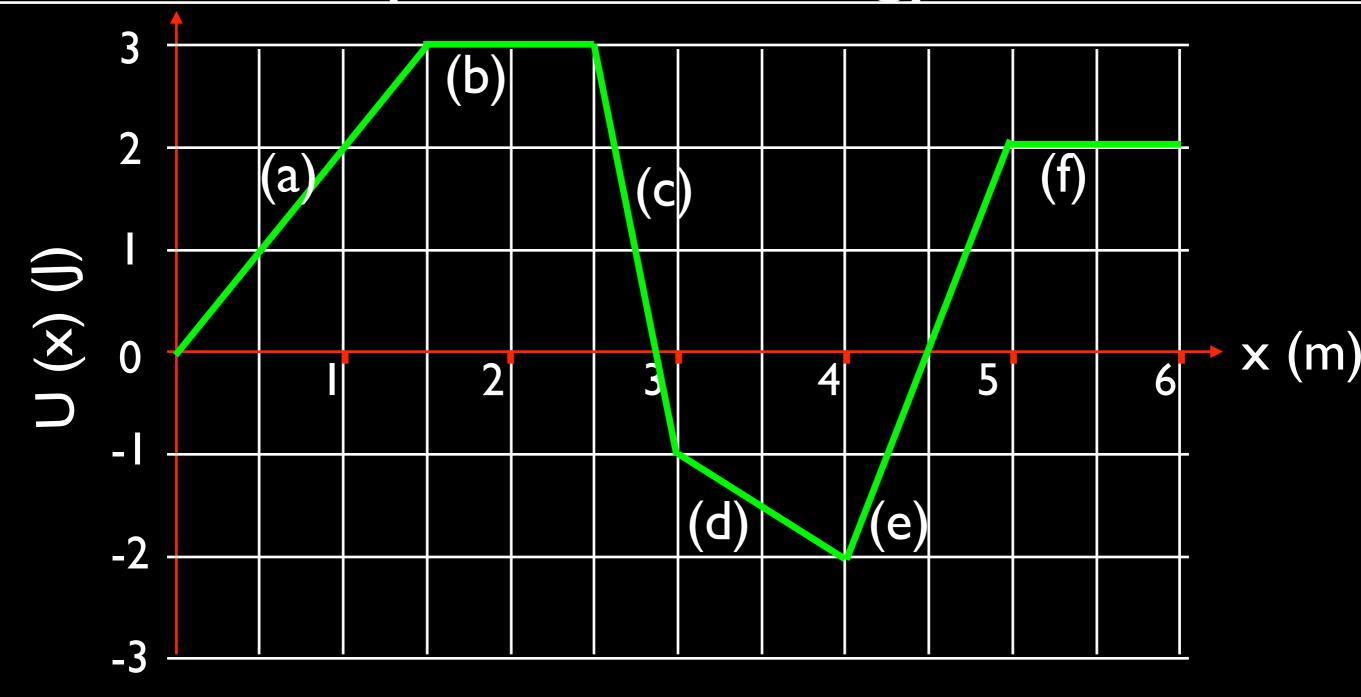
(a) -3N (b) 3N (c) 1.2N (d) -1.2N (d) 0 N



Juiz

Force in segment (c)?

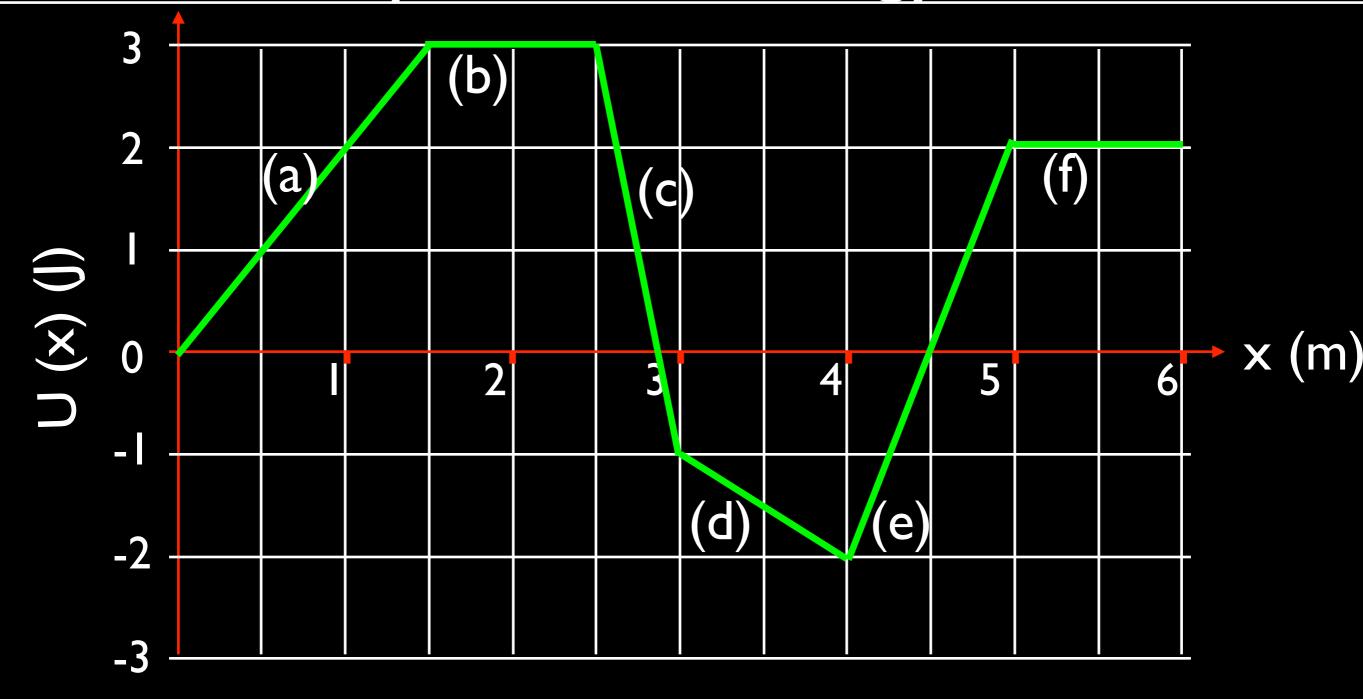
(a) -8N (b) 8N (c) -4N (d) 4N (d) 0 N



Force in segment (c)?

$$F(x) = -\frac{\Delta U}{\Delta x} = -\frac{-4J}{0.5m} = 8N$$

Juiz

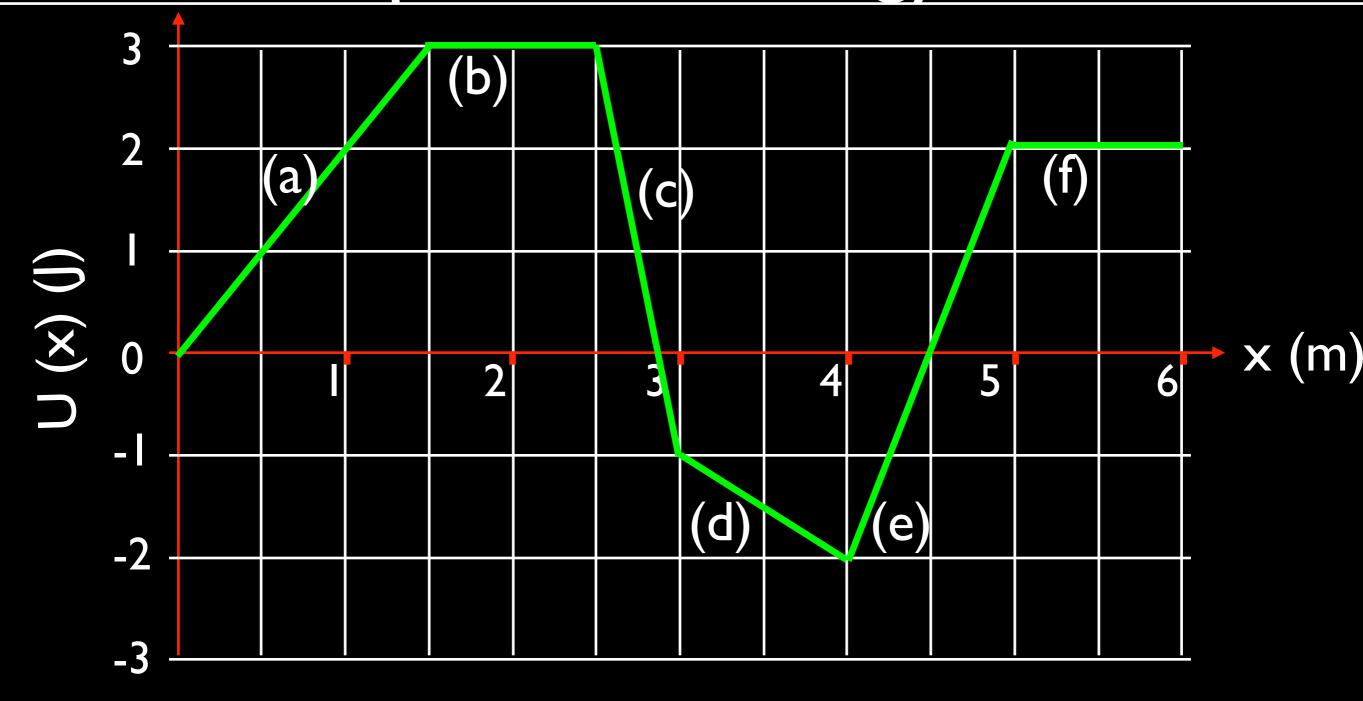


Juiz

Force in segment (d)?

(a) - IN (b) IN (c) -0.5N (d) 0.5N (d) 0 N

(b) 4N (c) -8N (d) 8N

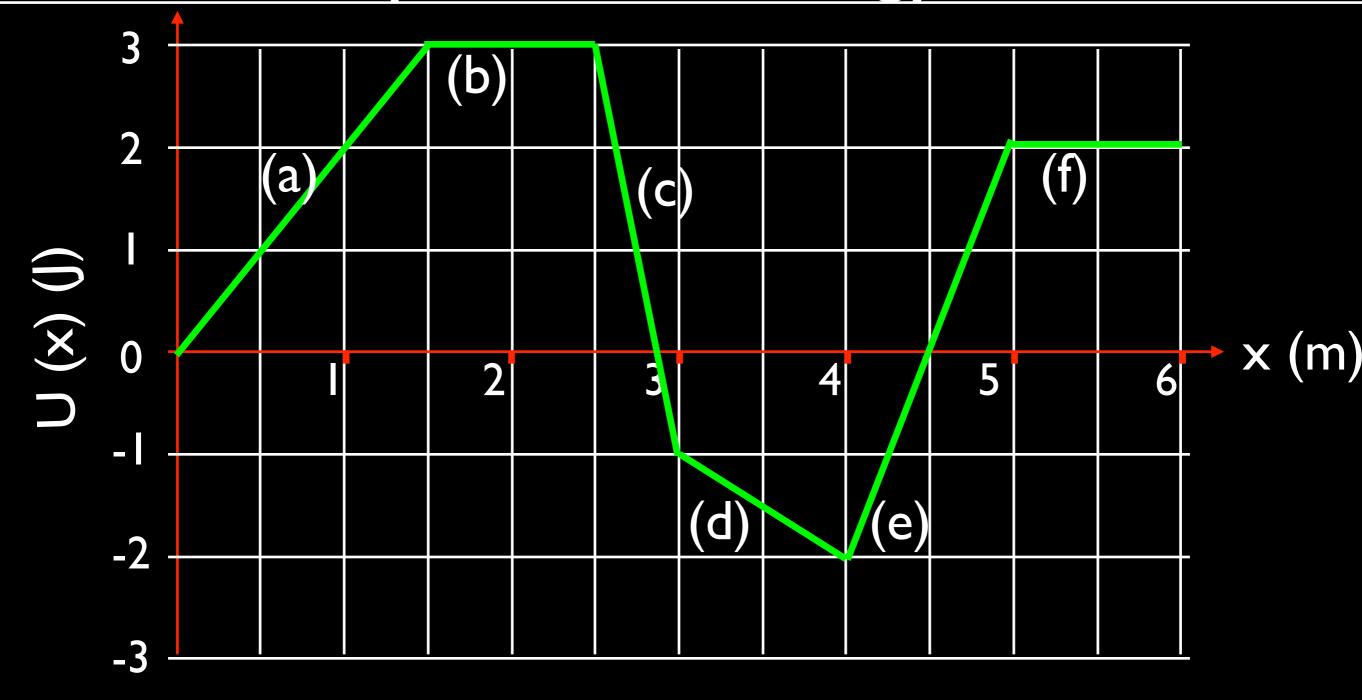


(d) 0 N

Juiz

Force in segment (e)?

(a) -4N



(d) 0 N

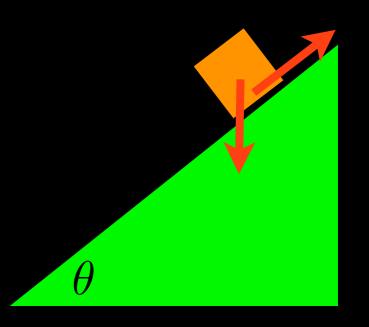
Juiz

Force in segment (f)?

(a) -2N (b) 2N (c) -1N (d) 1N

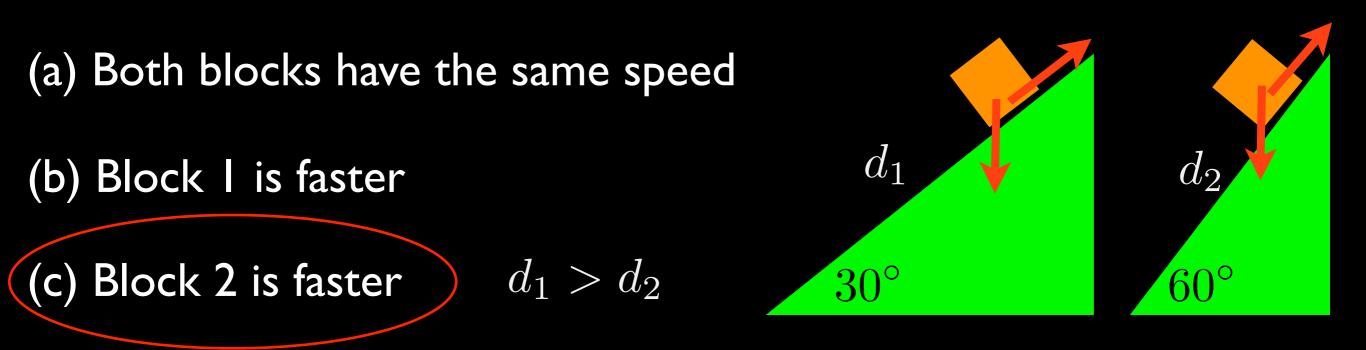
Quiz

- 'Box I' and 'Box 2' have the same mass, m.
- They start at the top of 2 inclined planes at the same height.
- The planes are inclined at 30° and 60° .
- If the coefficient of friction, μ_K , is the same, which of the boxes is going faster when it reaches the bottom?
- (a) Both blocks have the same speed
- (b) Block I is faster
- (c) Block 2 is faster
- (d) We need to know more information



Quiz

- 'Box I' and 'Box 2' have the same mass, m.
- They start at the top of 2 inclined planes at the same height.
- The planes are inclined at 30° and 60° .
- If the coefficient of friction, μ_K , is the same, which of the boxes is going faster when it reaches the bottom?

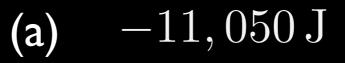


(d) We need to know more information

- A 50 kg skier starts from rest and travels 200m down a hill inclined at $20^{\circ}\!.$
- When she reaches the bottom of the hill, her speed is 30 m/s.

200m

How much work is done by friction as the skier comes down the hill?



- (b) $-33,550 \,\mathrm{J}$
- (c) $-22,550 \,\mathrm{J}$

(d) $-56,050 \,\mathrm{J}$

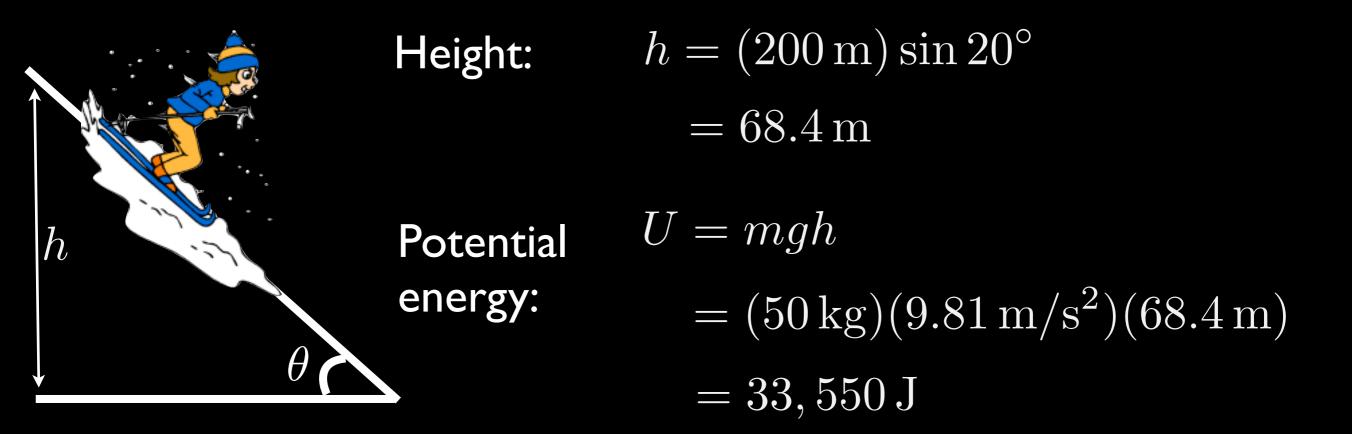
- A 50 kg skier starts from rest and travels 200m down a hill inclined at 20° .
- When she reaches the bottom of the hill, her speed is 30 m/s.
- How much work is done by friction as the skier comes down the hill?

(a)
$$-11,050 \text{ J}$$

(b) $-33,550 \text{ J}$
(c) $-22,550 \text{ J}$
(d) $-56,050 \text{ J}$

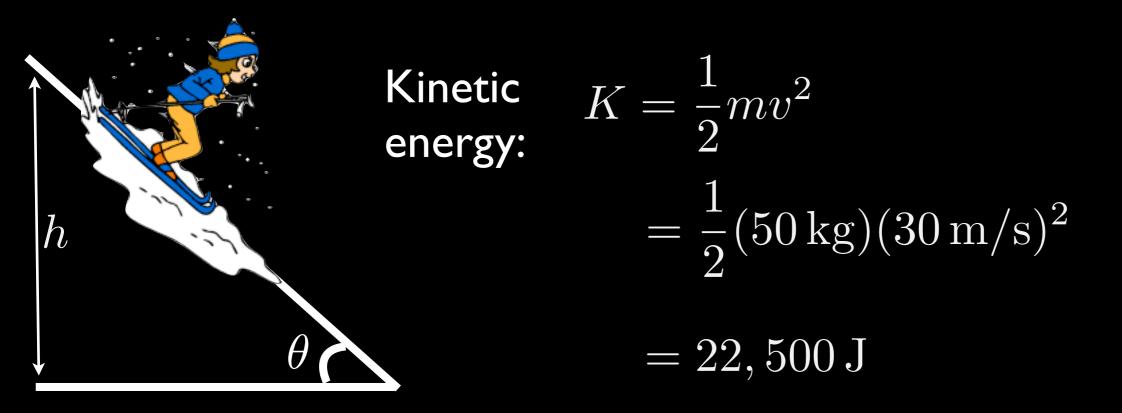
- A 50 kg skier starts from rest and travels 200m down a hill inclined at $20^{\circ}\!.$
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Energy:
$$E_{\text{tot}} = K - U - W_{\text{friction}} = 0$$



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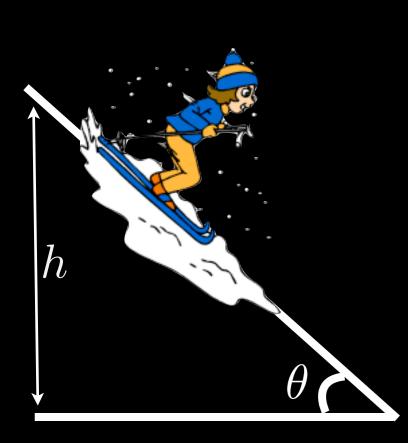
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nergy:
$$E_{tot} = K - U - W_{friction} = 0$$

 $W_{friction} = K - U$
 $= 22,500 \text{ J} - 33,550 \text{ J}$
 $= -11,050 \text{ J}$



E

Lecture 7 : Summary

The difference between conservative and non-conservative forces



Potential energy: what it is & how to calculate it



