Essential Physics I

英語で物理学の

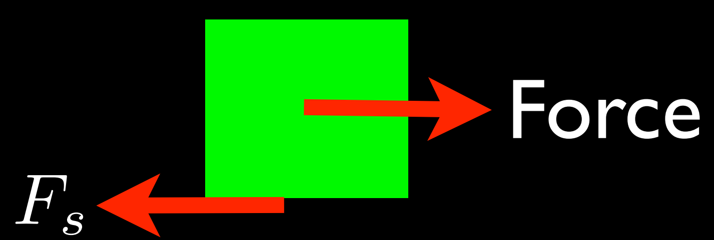
エッセンス I

Lecture 6:   23-05-16

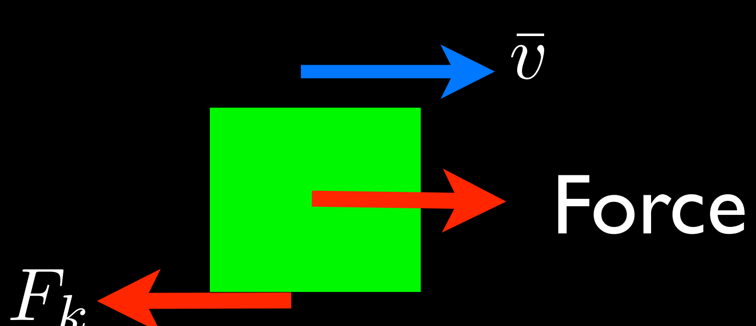
Last lecture

3 forces:

Friction

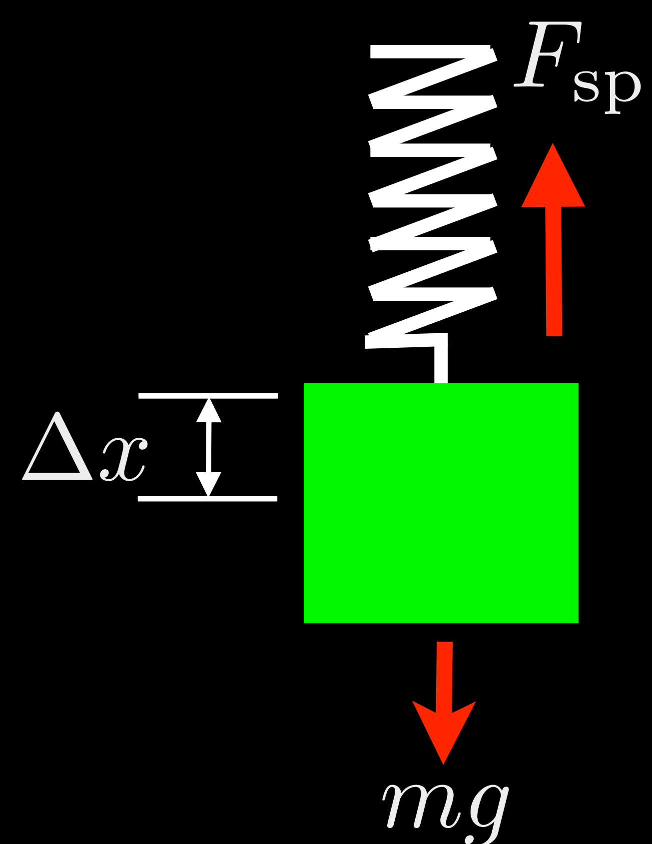


$$F_s \leq \mu_s N$$



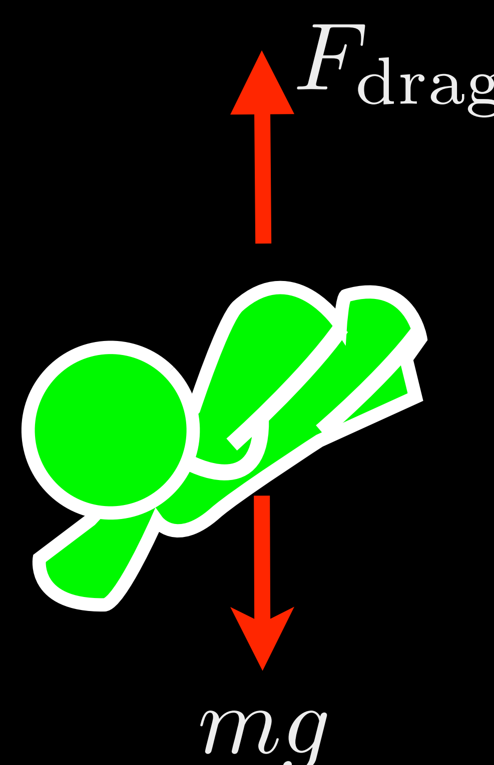
$$F_k = \mu_k N$$

Springs



$$\bar{F}_{\rm sp} = -k\Delta x$$

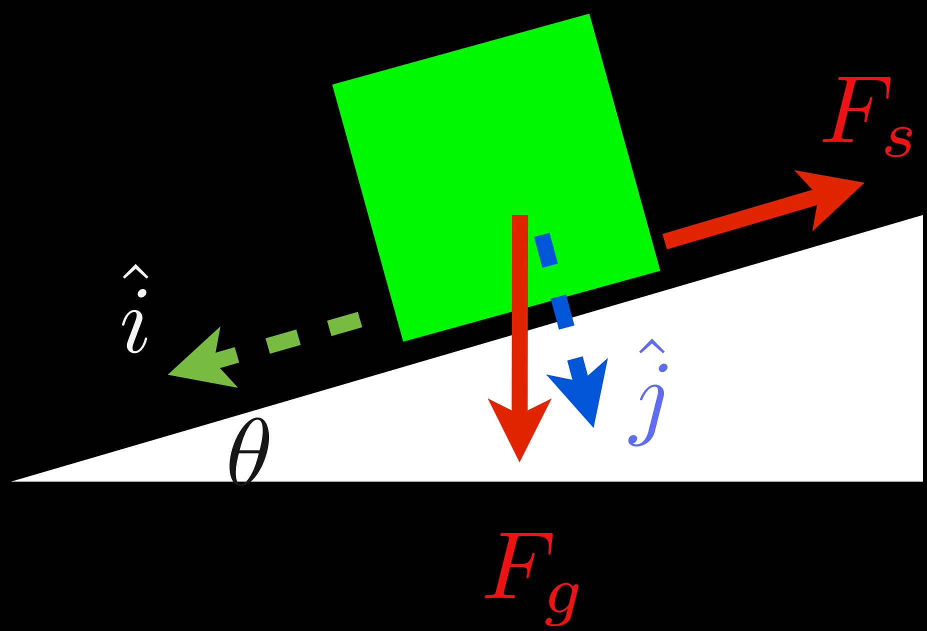
Drag



Last lecture: review

Quiz

A brick rests on an inclined plane.



The friction force is:

(a) zero

(b) = weight of brick

(c) > weight of brick

(d) < weight of brick

Forces along plane: $\hat{i}$

$$F_{g,i} - F_s = m \times 0$$

$$= F_g \sin\theta$$

$$F_g \sin\theta = F_s \qquad F_g > F_g \sin\theta$$

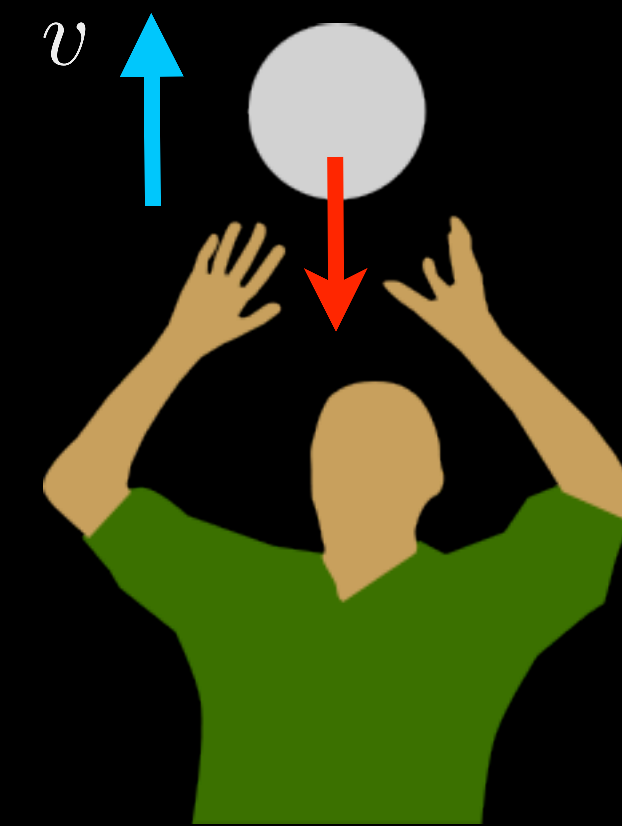
$$F_s < F_g$$

Last lecture: review

Quiz

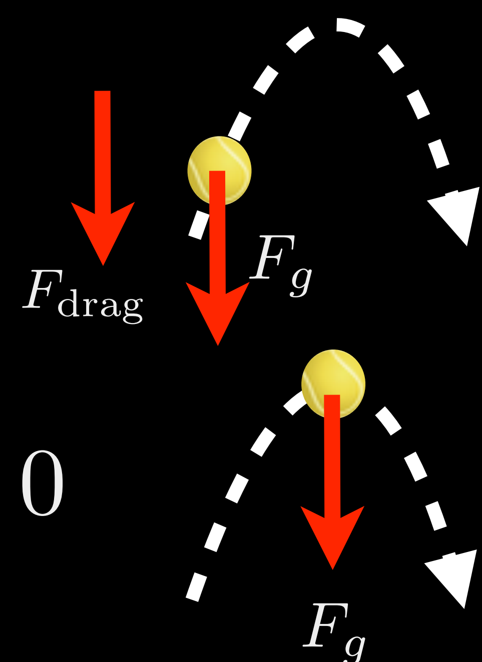
If drag is proportional to a ball's speed:

$$F_{\text{drag}} \propto |\bar{v}|$$



and you throw the ball upward, the magnitude of the acceleration is largest...

(a) Just after ball is released



$$F_{\text{drag}} + F_g = ma_a$$

(b) At top of trajectory $v = 0$

$$F_g = ma_b$$

(c) Acceleration is always the same

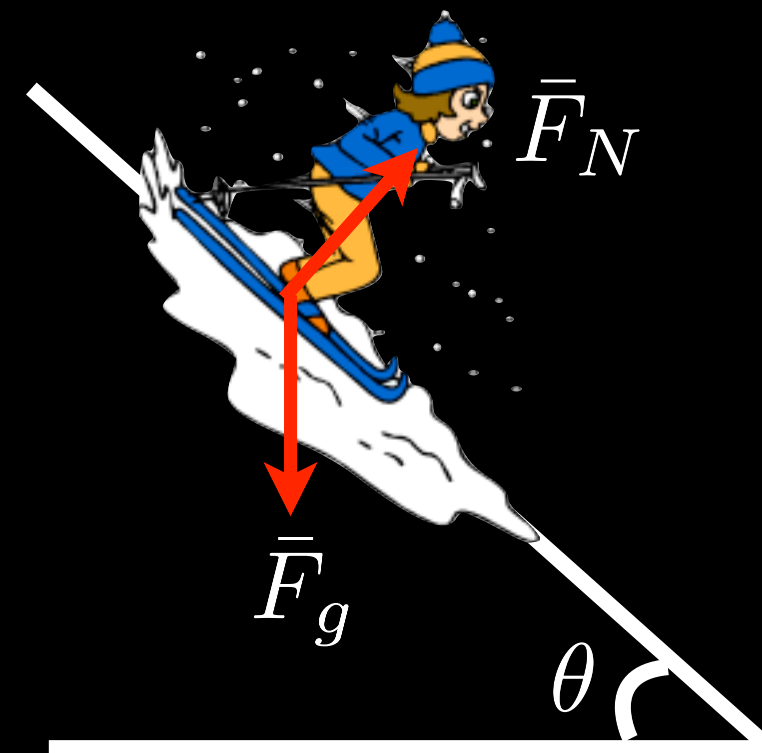
This lecture: work & energy

- The concept of work
- How to calculate work
  - with a constant force
  - with a varying (changing) force
- The concept of kinetic energy
- The relation between work and kinetic energy
- The concept of power and its relation to energy

Work

When the force is constant, we can easily (!) find the speed

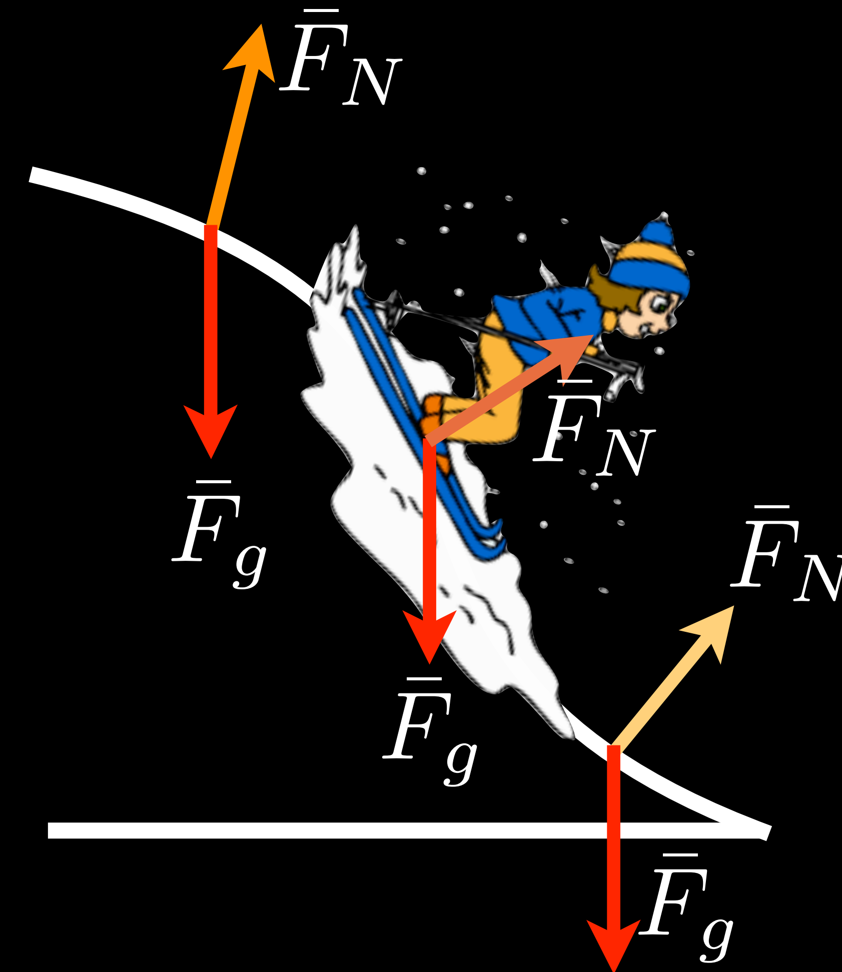
$$\bar{F}_{\text{net}} = \bar{F}_g + \bar{F}_N = \text{constant}$$



But if the slope changes

$\bar{F}_N$ changes direction,

so $\bar{F}_{\text{net}}$ is not constant.



How do we find the speed of the skier?

Work

New concept: WORK

$$W = F_x \Delta x$$

Work increases...

The bigger the force you apply

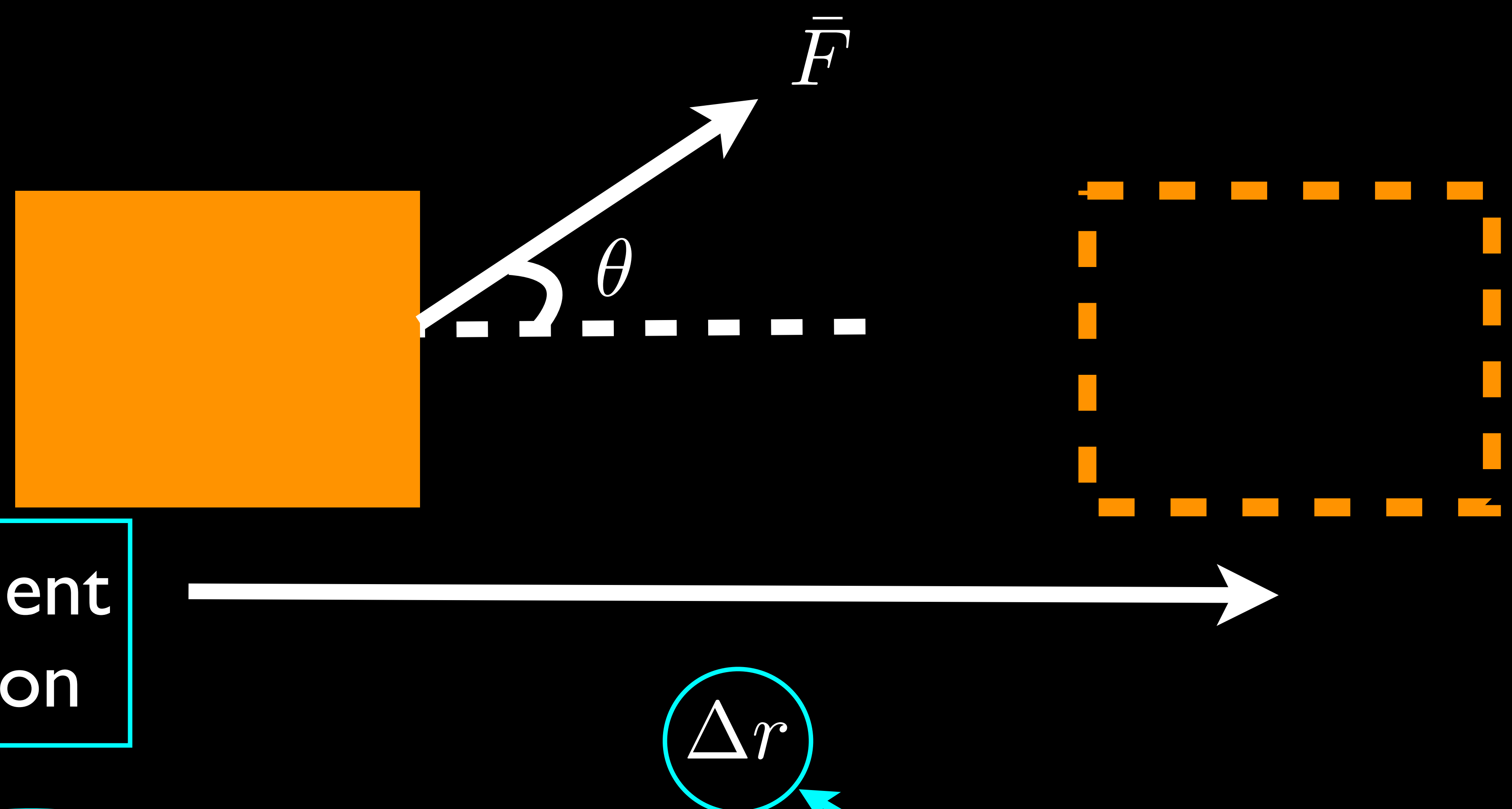
The further you move an object...

.... in the direction of the force.

Units of work: $Nm = J$ (Joules)

Work

New concept: WORK



Force component in the r-direction

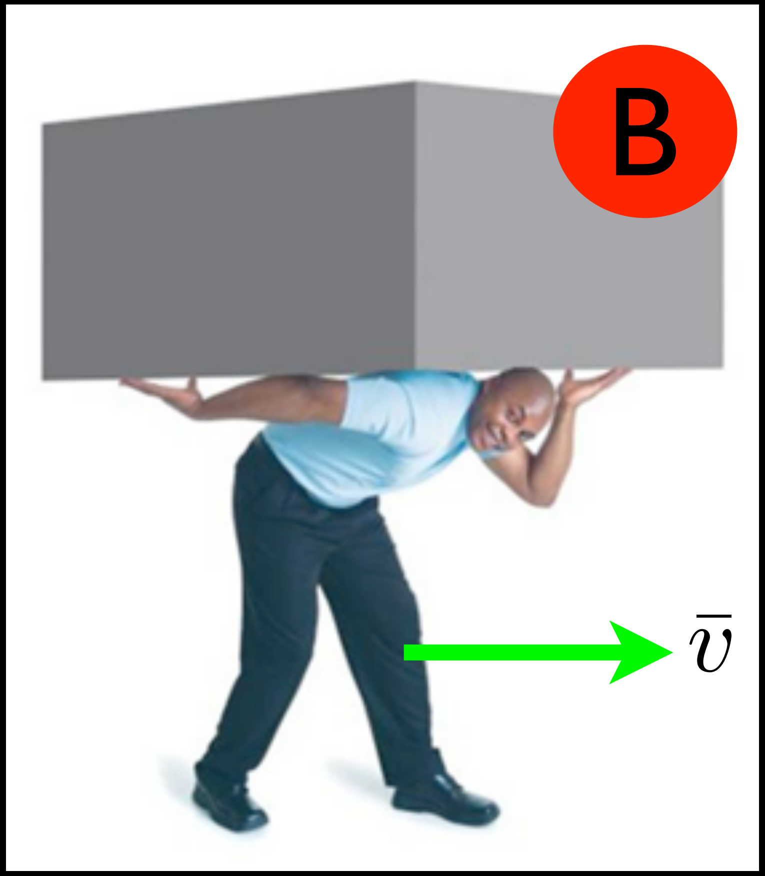
$$W = (F\cos\theta)(\Delta r)$$
$$= F\Delta r\cos\theta$$

'r' because displacement can be *any* vector, e.g. $r = 3\bar{i} + 4\bar{j}$

Work

Quiz

Which of these does work?



(1) A

(2) B

(3) C

(4) A & B & C

(5) A & B

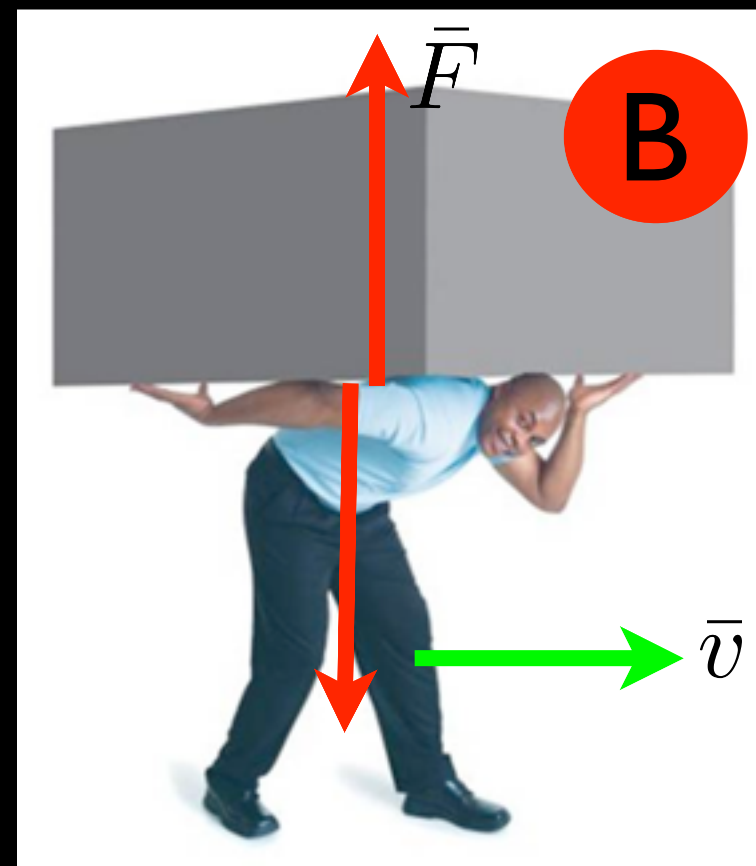
(6) A & C

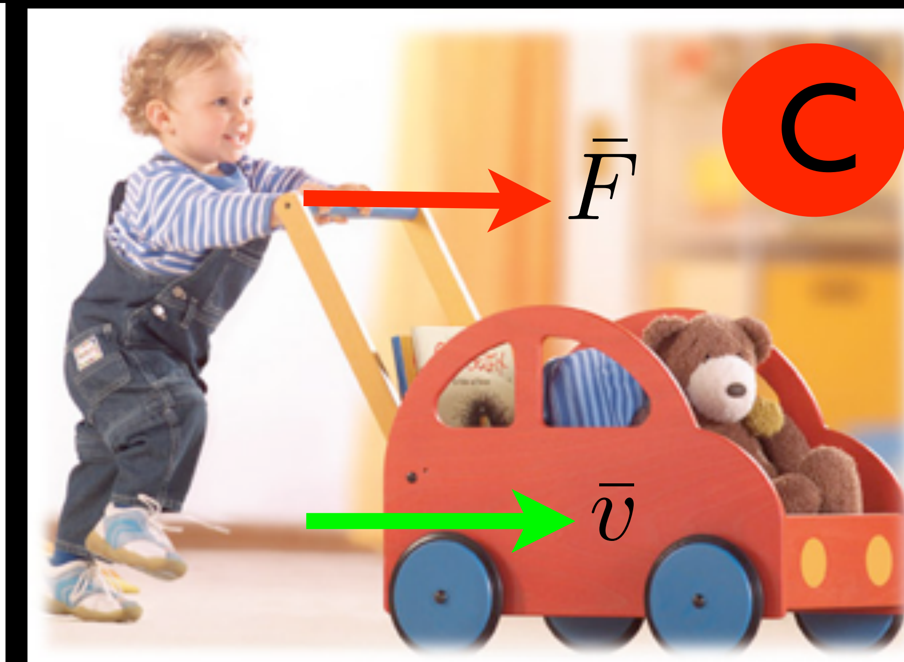
(7) B & C

Work

Quiz

Which of these does work?





No mechanical forces

$$F_x = 0$$

No forces in direction of motion

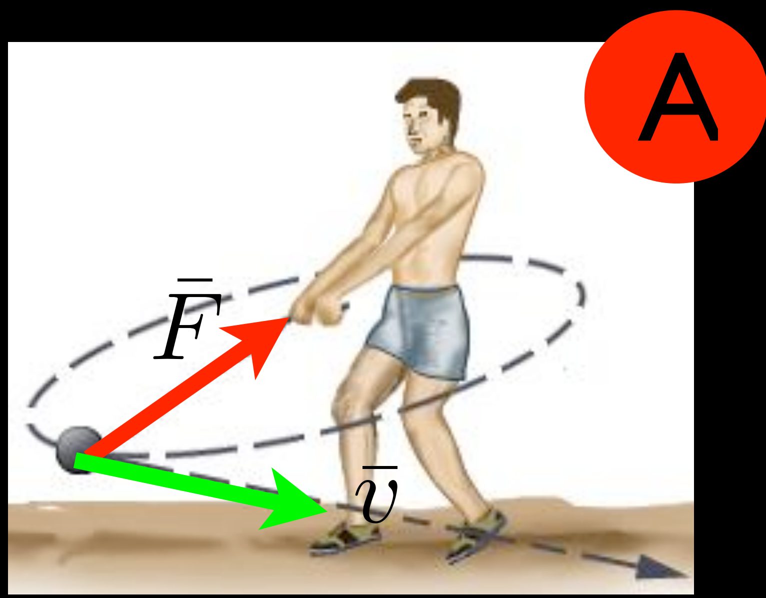
$$F_x = 0$$

$$W = F_x \Delta x$$

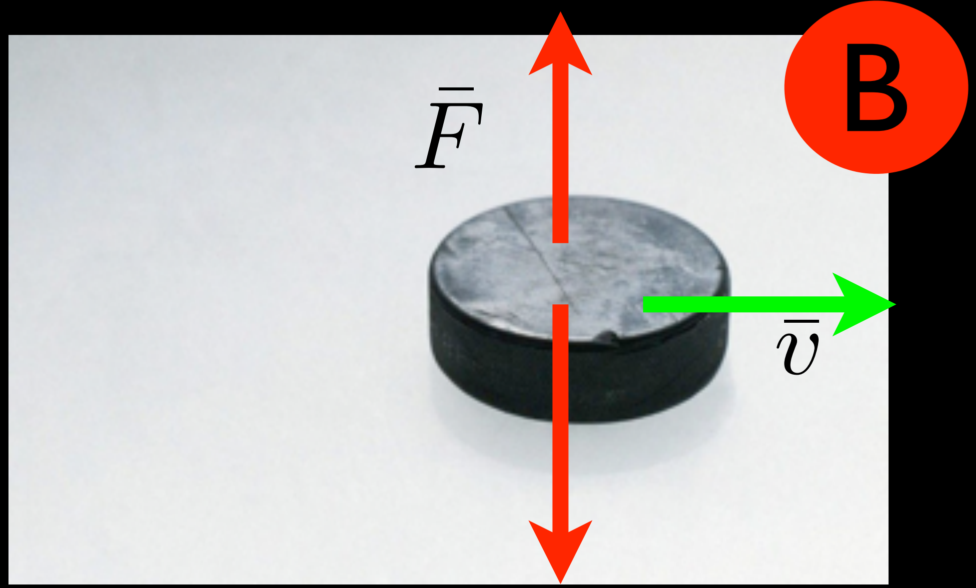
Work

Quiz

Which of these does work?



circular motion



puck sliding on
(frictionless) ice

(1) A

(3) A + B

(2) B

(4) Neither A or B

Work

Quiz

A crane lifts a 650 kg beam vertically upward 23 m and then swings it eastward 18 m.

How much work does the crane do?

(Neglect friction, assume beam velocity is constant)

(a) 0 kJ

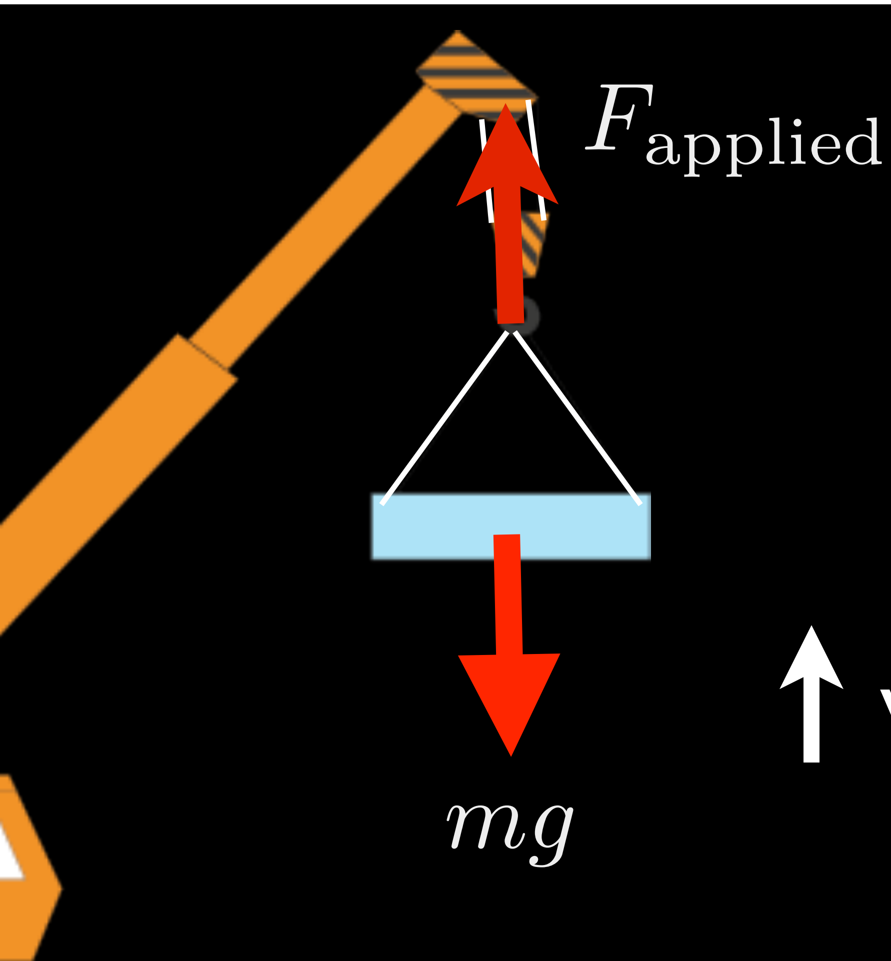
(b) 261 kJ

(c) 115 kJ

(d) 147 kJ

Work

Quiz



How much work does the crane do?

$$F_{\text{applied}} - mg = m \times 0$$
$$F_{\text{applied}} = mg$$

↑ **Vertical motion:** $W = F_{\text{applied,y}} \Delta y = mg\Delta y$

$$= (650\,\text{kg})(9.81\,\text{m/s}^2)(23\,\text{m})$$
$$= 147\,kJ$$

← **East motion:** $W = F_{\text{applied,x}} \Delta x$

$$= 0\Delta x$$
$$= 0$$

Work

This combination:

$$W = F \Delta r \cos\theta$$

vector magnitude x vector magnitude x cosine angle between vectors

is called a scalar product, $\bar{F} \cdot \Delta \bar{r}$

The result (e.g. work) is not a vector, although it can be negative.

In component form:

$$\bar{F} \cdot \Delta \bar{r} = F_x \Delta r_x + F_y \Delta r_y + F_z \Delta r_z \ \ (= F \Delta r \cos\theta)$$

Work

e.g. We could have solved the crane problem using components of the force:

$$\Delta\bar{r} = 18\hat{i} + 23\hat{j}\,\mathrm{m}$$

$$\bar{F} = (650\,\mathrm{kg} \times 9.81\,\mathrm{m/s^2})\hat{j}$$

$$W = \bar{F} \cdot \Delta r = F_x \Delta r_x + F_y \Delta r_y$$

$$= (18\mathrm{m} \times 0) + (23\mathrm{m} \times (650\mathrm{kg} \times 9.81\mathrm{m/s^2})) = 146.7\mathrm{kJ}$$

Work

Quiz

Find the work done by a force $\bar{F} = (1.8\hat{i} + 2.2\hat{j})\,\mathrm{N}$ as it acts on an object moving from the origin to point $(56\hat{i} + 31\hat{j})\,\mathrm{m}$

(a) 100 J

(b) 169 J

(c) 68 J

(d) 179 J

$$W = (1.8\hat{i} + 2.2\hat{j})\,\mathrm{N} \cdot (56\hat{i} + 31\hat{j})\,\mathrm{m}$$

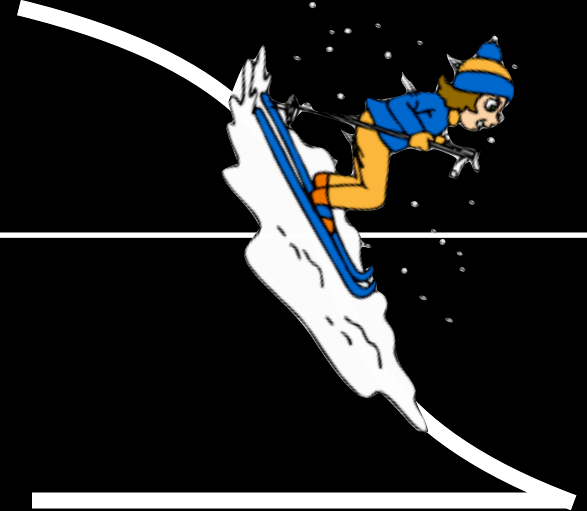
$$= (1.8\,\mathrm{N})(56\,\mathrm{m}) + (2.2\,\mathrm{N})(31\,\mathrm{m})$$

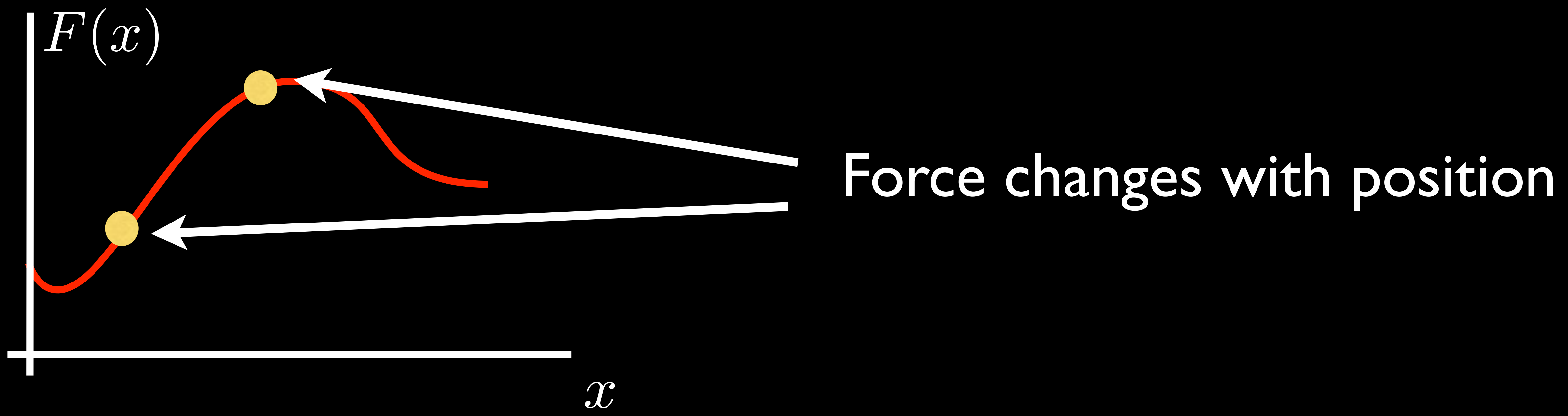
$$F_x \Delta r_x \qquad F_y \Delta r_y$$

$$= 169\,\mathrm{J}$$

Work

How does this help with forces that vary?





Work

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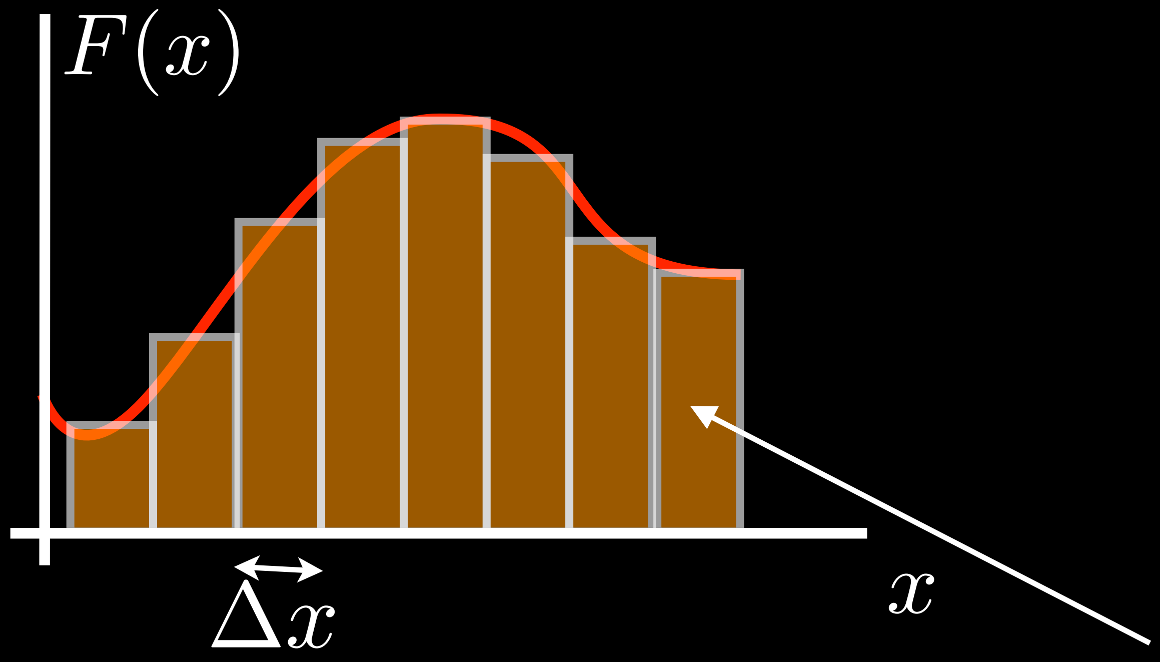
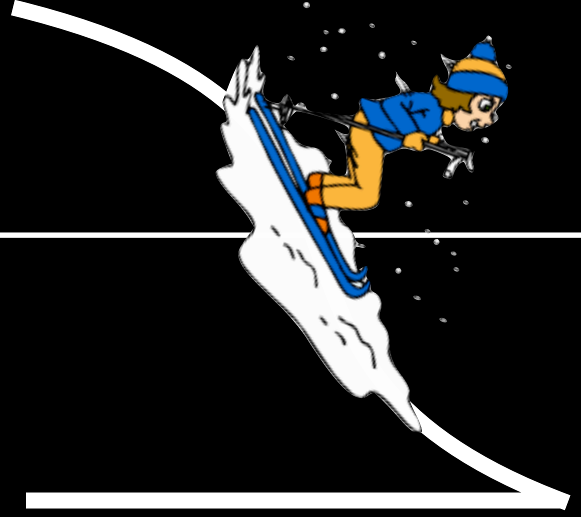


Divide region into small rectangles with width Δx

$F \approx$ constant over Δx

$$\Delta W = F(x)\Delta x$$

$$W \simeq \sum_{i=0}^{N} \Delta W_i = \sum_{i=0}^{N} F(x_i)\Delta x$$



Work

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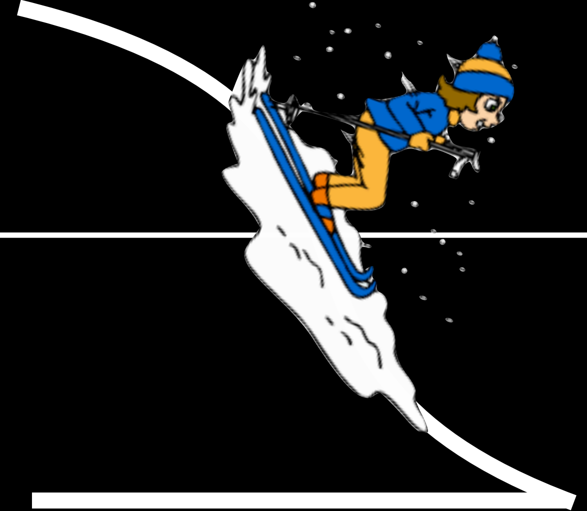
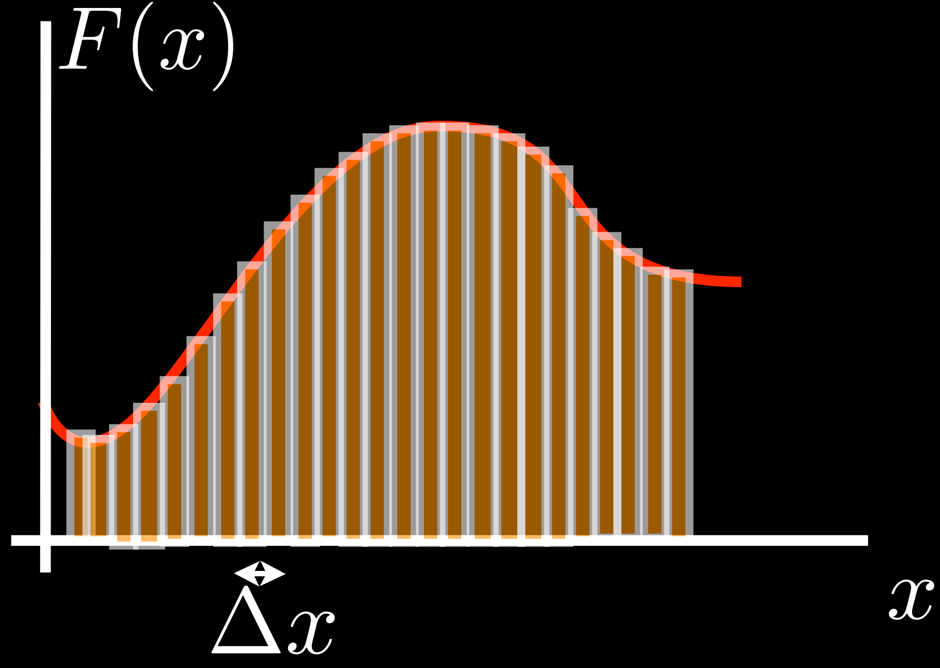


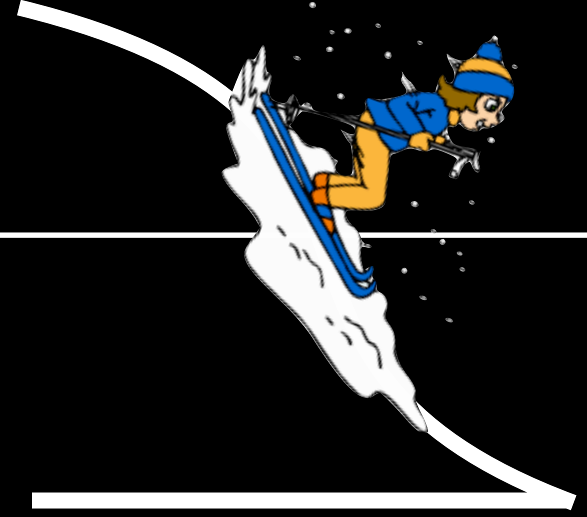


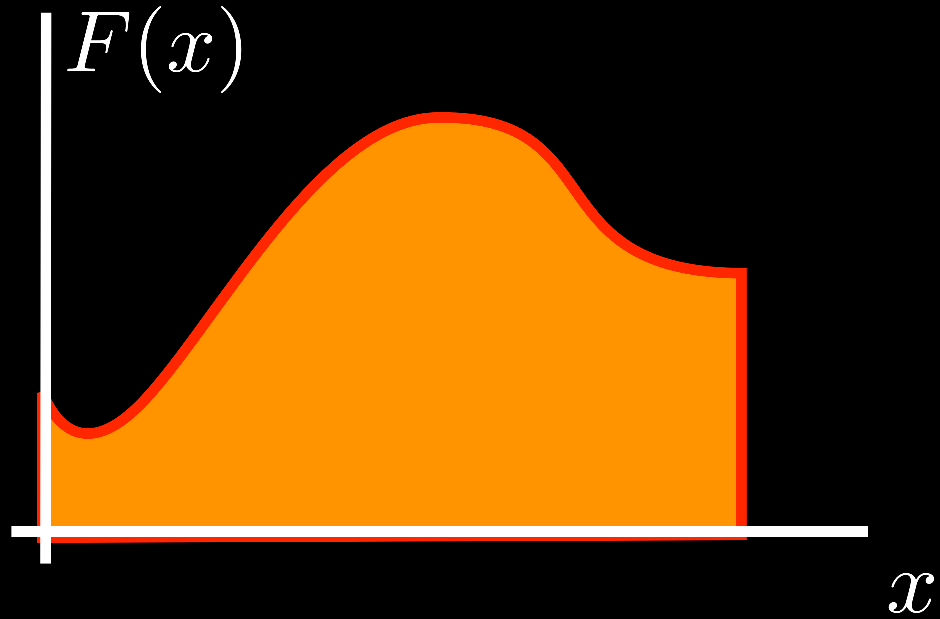
$\Delta x \rightarrow$ smaller, the approximation improves.

$$W \simeq \sum_{i=0}^{N} \Delta W_i = \sum_{i=0}^{N} F(x_i) \Delta x$$

Work

How does this help with forces that vary?





As $\Delta x \to 0$

$$W = \lim_{\Delta x \to 0} \sum_{i=0}^{N} F(x_i)\Delta x \quad = \boxed{\int_{x_1}^{x_2} F(x)dx}$$

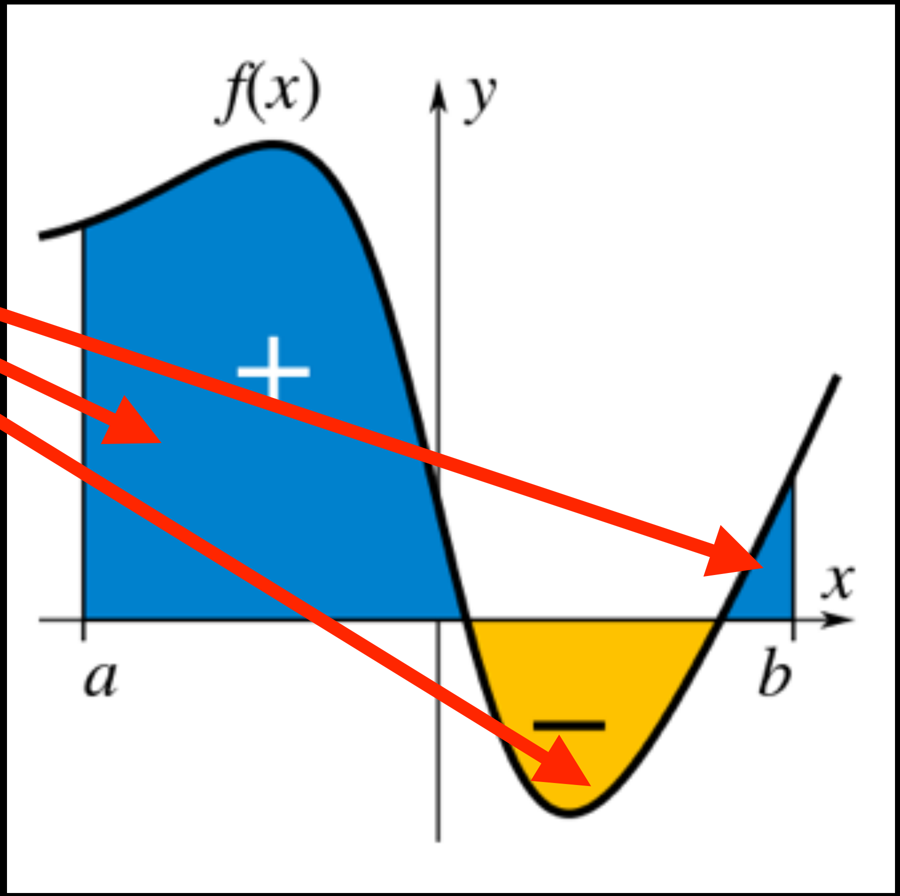
Work done by a force varying in 1D

Integration

Integral is total area under graph

+

$$\int_a^b x^r\,dx = \left[\frac{x^{r+1}}{r+1}\right]_a^b$$



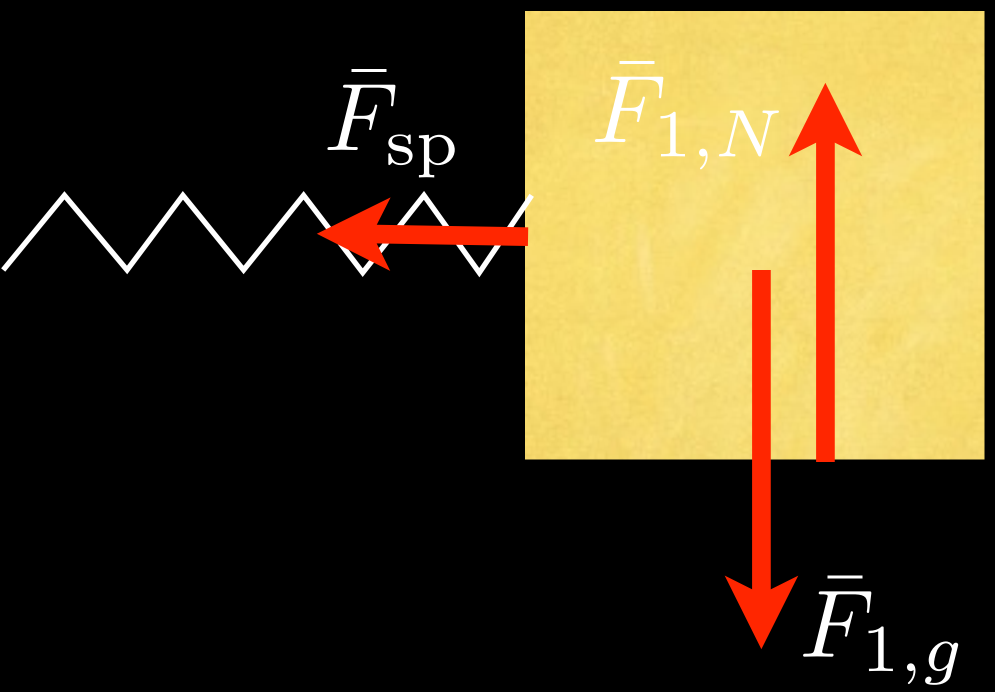
$$= \frac{b^{r+1}}{r+1} - \frac{a^{r+1}}{r+1}$$

Work

A 4 kg block on a frictionless table is attached to a horizontal spring.

The spring constant is k = 400 N/m. It is compressed to $x_1 = -5\text{cm}$

Find the work done by the spring.



Hooke's law: $F_x = -kx$

Work

Example

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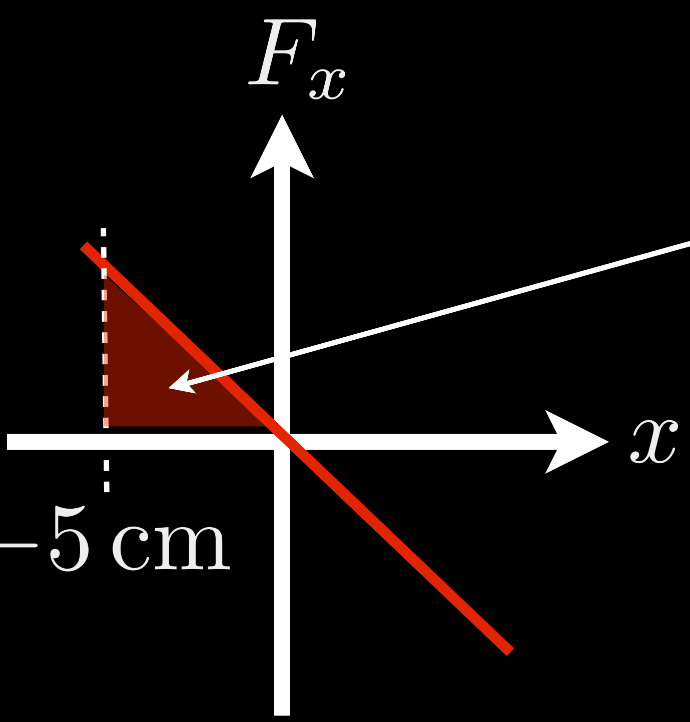

$W =$ area under graph

$$W = \int_{-0.05}^{0} F_x dx = \int_{-0.05}^{0} -kx dx$$

$$= \left[-\frac{1}{2}kx^2\right]_{-0.05}^{0}$$

Hooke's law: $F_x = -kx$

$$= \left(-\frac{1}{2}(400\,\text{N/M})(0)\right) - \left(-\frac{1}{2}(400\,\text{N/M})(-0.05\,\text{m})^2\right)$$

$$= 0.5\,\text{J}$$

Work

Example

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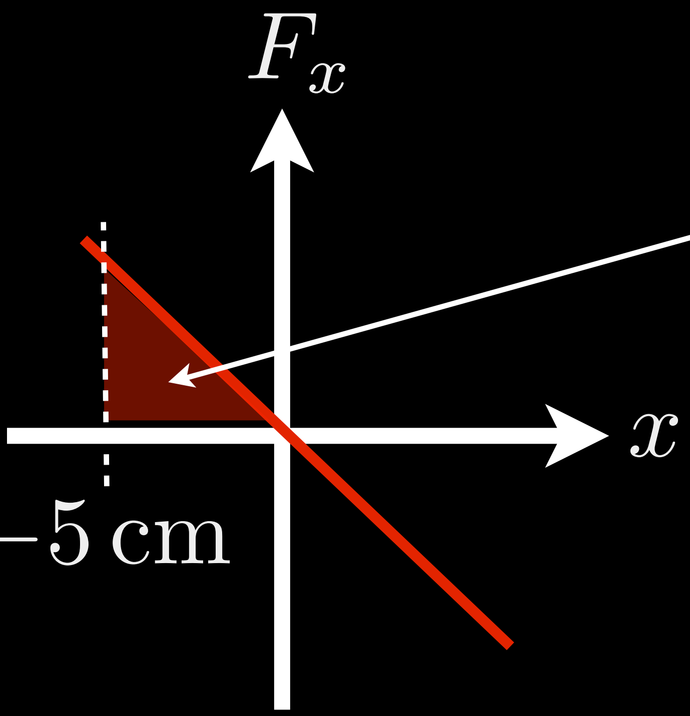

$W =$ area under graph

Hooke's law: $F_x = -kx$

Alternatively:

Area of triangle: $\frac{1}{2}\text{base} \times \text{height}$

$$= \frac{1}{2}(0.05) \times F_x$$

$$= \frac{1}{2}(0.05)(kx) = \frac{1}{2}(0.05)(400 \times 0.05)$$

$$= 0.5\,\text{J}$$

Work

Quiz

Spider silk is remarkably elastic. A silk strand has a spring constant k = 70 mN/m and stretches 9.6 cm when a fly hits it.

How much work did the fly's impact do on the silk strand?

(a) 3.36 mJ

(b) 6.72 mJ

(c) 0.32 mJ

(d) 0.15 mJ

Work

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$$W = \int_0^{0.096\,\mathrm{m}} F_x dx$$

$$= \int_0^{0.096\,\mathrm{m}} -kx dx = \left[-\frac{1}{2}kx^2\right]_0^{0.096}$$

$$= -\frac{1}{2}(70\,\mathrm{mN/m})(0.096\,\mathrm{m})^2$$

$$= -0.32\,\mathrm{mJ}$$

work done by fly: $0.32\,\mathrm{mJ}$

Work

Quiz

A force $\bar{F} = 2x + 5$ N acts on a particle. Find the work done by the force as the particle moves from x = 0 m to x = 2 m.

(a) 14 J

(b) 9 J

(c) 18 J

(d) 8 J

$$W = \int_0^{2\,m} \bar{F}(x)dx$$

$$= \int_0^{2\,m} (2x + 5)dx$$

$$= \left[\frac{2x^2}{2} + 5x\right]_0^{2\,\mathrm{m}}$$

$$= 2^2 + 5 \times 2 = 14\,\mathrm{J}$$

Energy

We have seen: $$W = \int F dx$$

Therefore: $$W_{\mathrm{net}} = \int F_{\mathrm{net}} dx$$

But we also know: $$F_{\mathrm{net}} = ma \quad = m\frac{dv}{dt}$$

Therefore: $$W_{\mathrm{net}} = \int m\frac{dv}{dt}dx \quad = \int m dv \frac{dx}{dt}$$

Energy

But:

$$\frac{dx}{dt} = v$$

Therefore:

$$W_{\text{net}} = \int mvdv$$

Suppose an object starts at speed v_1 and ends at v_2 then:

$$W_{\text{net}} = \int_{v_1}^{v_2} mvdv \;= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$K = \frac{1}{2}mv^2$$ Kinetic energy

Energy

The change in an object's kinetic energy is equal to the net work done on the object:

Work-energy theorem:

$$\Delta K = W_{\text{net}}$$

Energy

Quiz

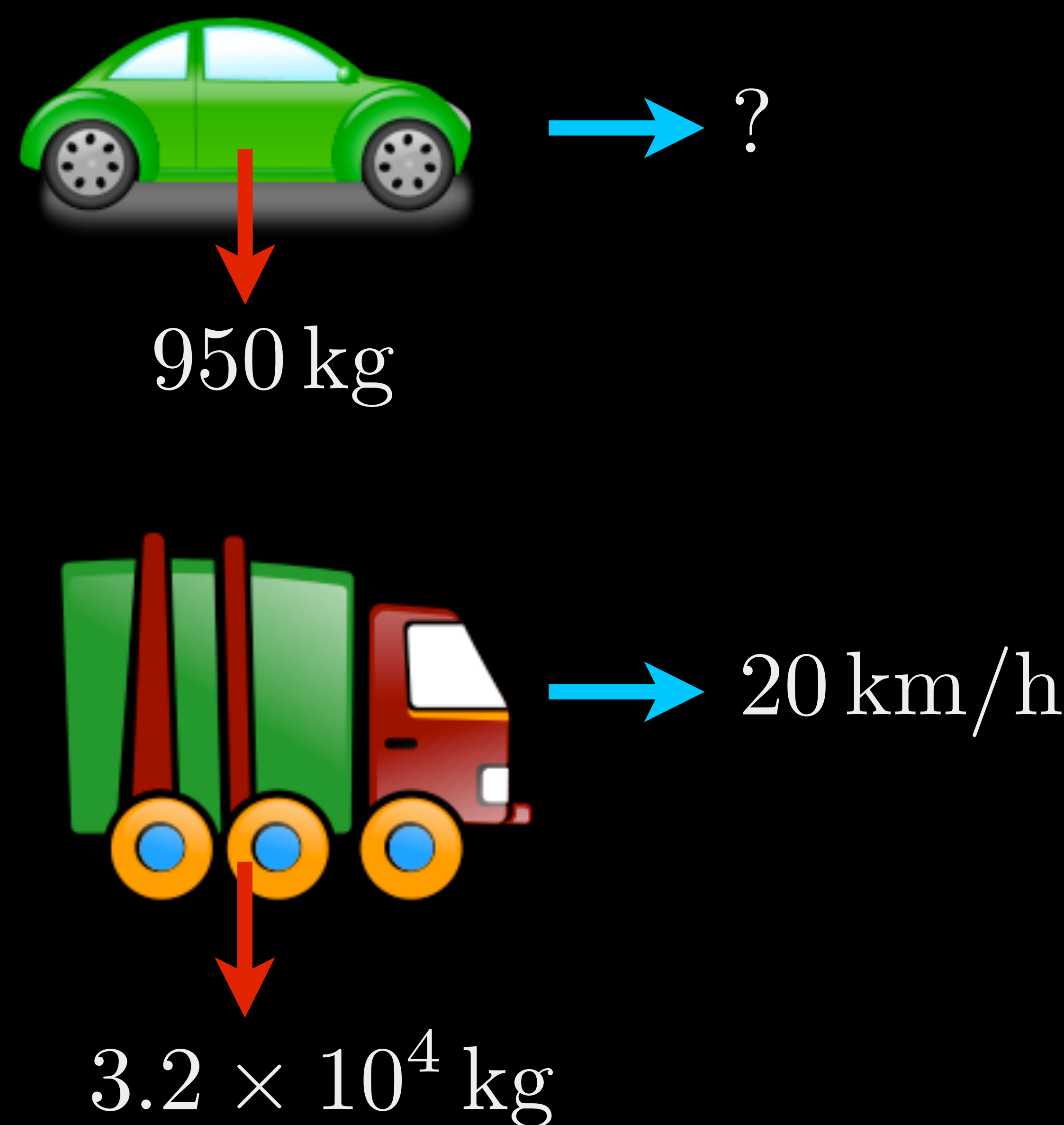
At what speed must a 950 kg car be moving to have the same kinetic energy as a $3.2 \times 10^4 \mathrm{kg}$ truck going at 20 km/h?

(a) 116 km/h

(b) 3.4 km/h

(c) 58 km/h

(d) 673 km/h



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$$\mathrm{KE} = \frac{1}{2} m_T v_T^2 \qquad = \frac{1}{2} m_c v_c^2$$

$$v_c = \pm v_T \sqrt{\frac{m_T}{m_c}}$$

$$= \pm 20\,\mathrm{km/h} \sqrt{\frac{3.2 \times 10^4\,\mathrm{kg}}{950\,\mathrm{kg}}}$$

$$= \pm 116\,\mathrm{km/h}$$

Energy

If the speed of a car is increased by 2, by what factor will the minimum stopping distance be increased?

(assuming breaking force is constant)

$v \rightarrow 2v$

$$\Delta K = W_{\text{net}}$$

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = W_{\text{net}} = \bar{F} \cdot \bar{d} = F \times d$$ (1 dimension)

$\frac{1}{2}mv_2^2 \rightarrow 0$

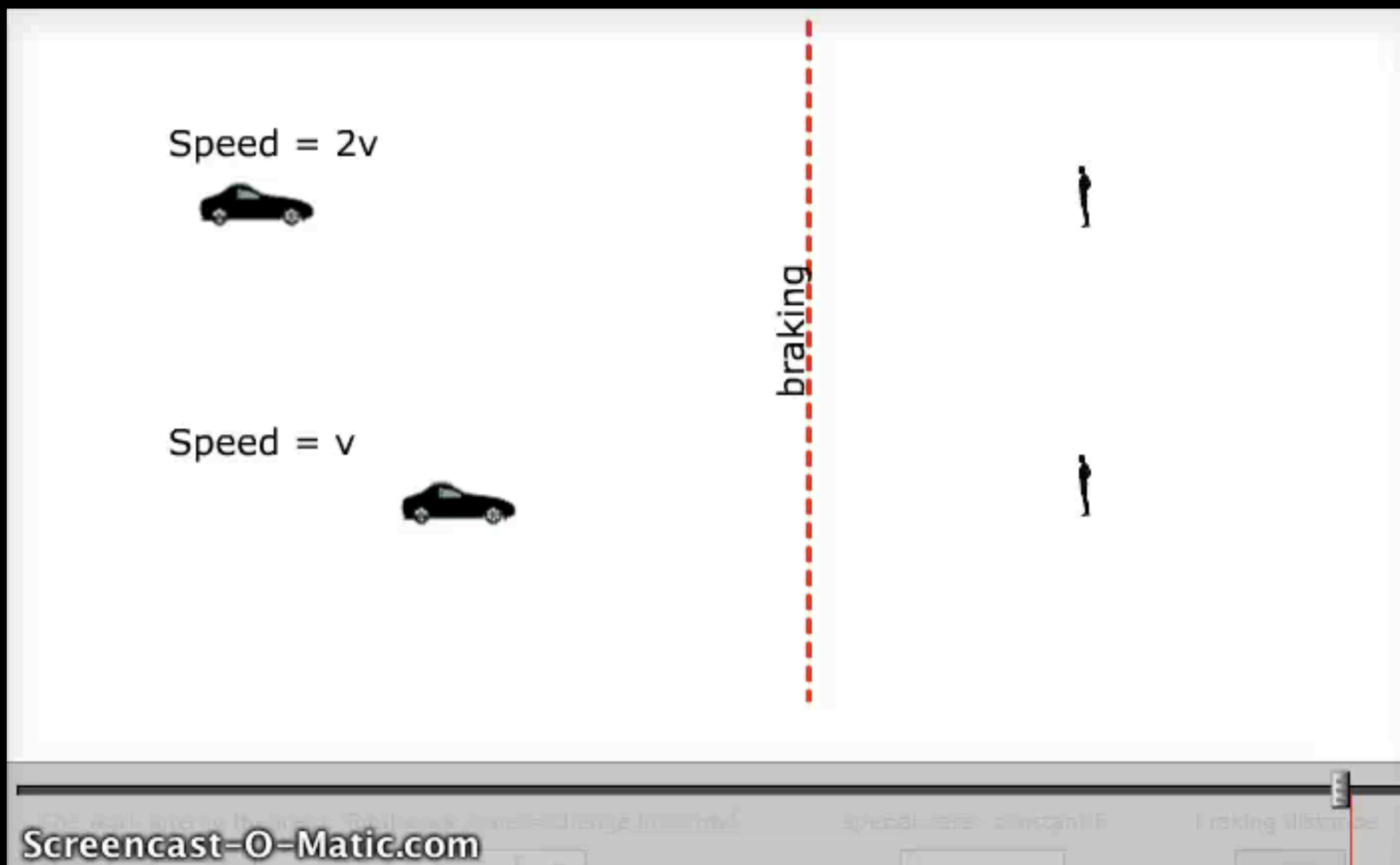
$$-\frac{1}{2}mv_1^2 = F \times d_1$$

$$-\frac{1}{2}m(2v_1)^2 = F \times d_2$$

$$\frac{d_1}{d_2} = \frac{1}{4} \rightarrow d_2 = 4d_1$$

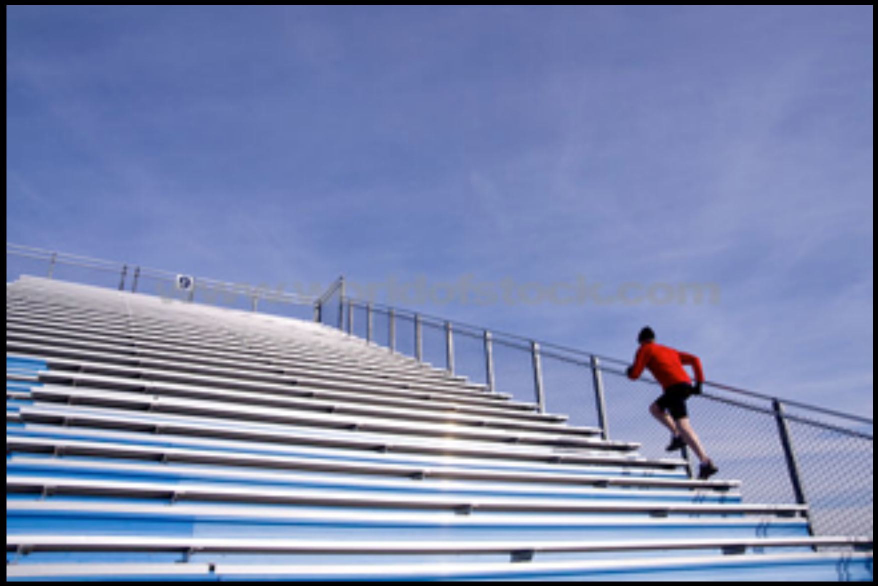
Energy

If the speed of a car is increased by 2, by what factor will the minimum stopping distance be increased?



Power

Since $W = F_x \Delta x$, it takes the same amount of work to run up a flight of stairs and to walk up.



But isn't running harder?

Yes! Because the rate at which you do the work has increased.

Power: $$P_{\text{average}} = \frac{\Delta W}{\Delta t}$$ Unit: W

$\Delta t \to 0$

$$P = \frac{dW}{dt}$$

Power

Since: $W = \bar{F} \cdot \Delta \bar{r}$

We can write for a small change in $\Delta \bar{r}$:

$\Delta \bar{r} \to d\bar{r}$: $$dW = \bar{F} \cdot d\bar{r}$$

Divide both sides by dt :

$$P = \frac{dW}{dt} = \bar{F} \cdot \frac{d\bar{r}}{dt} = \bar{F} \cdot \bar{v}$$

$$\boxed{P = \bar{F} \cdot \bar{v}}$$

Power

Example

Which consumes more energy:

(a) 1.2 kW hair drier used for 10 minutes?

(b) 7 W night light left on for 24 hours?

Average power: $P_{\mathrm{av}} = \dfrac{\Delta W}{\Delta t}$

Hair drier: $\Delta W = P_{\mathrm{av}} \Delta t \;\; = (1.2\,\mathrm{kW})(10\,\mathrm{min} \times \dfrac{60\,\mathrm{s}}{1\,\mathrm{min}}) = 720\,\mathrm{kJ}$

Light: $(7\,\mathrm{W})(24\,\mathrm{hr} \times \dfrac{3600\,\mathrm{s}}{1\,\mathrm{hr}}) \;\; = 605\,\mathrm{kJ}$

$\longrightarrow$  (but close!)

Power

Quiz

A sprinter completes a 100 m run in 10.6 s, doing 22.4 kJ of work. What is her average power output?

(a) 2.1 W

(b) 4.2 kW

(c) 2.1 kW

(d) 1.05 kW

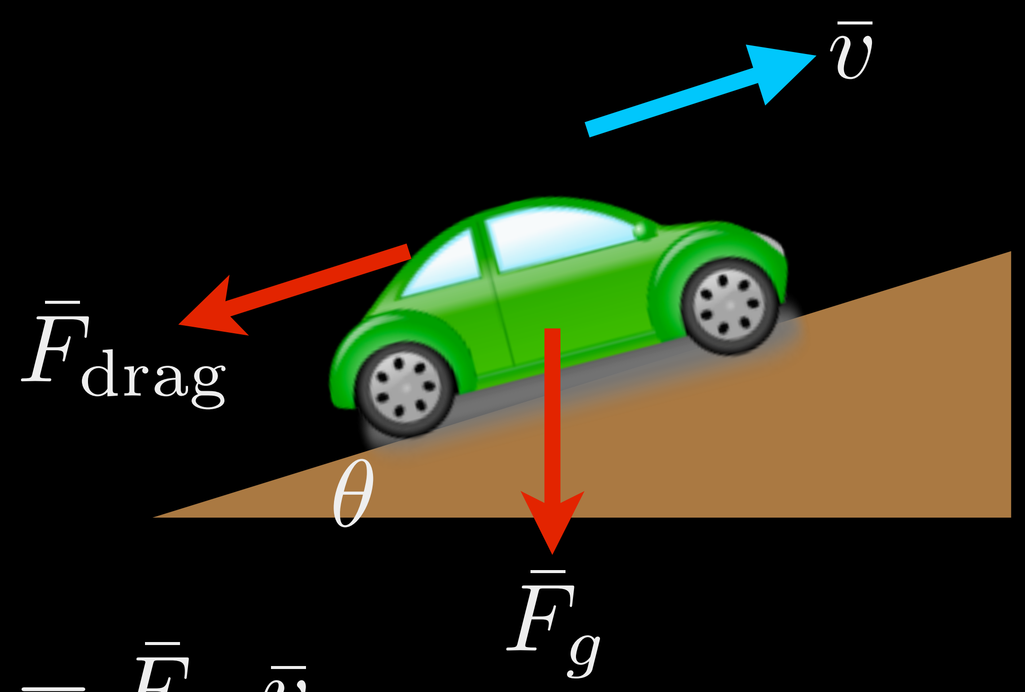
$$P_{\mathrm{av}} = \frac{\Delta W}{\Delta t} = \frac{22.4\,\mathrm{kJ}}{10.6\,\mathrm{s}} = 2.1\,\mathrm{kW}$$

Putting it together

Example

A 1400 kg car ascends a mountain road at a steady 60 km/h against a 450 N force of air resistance.

If the engine supplies energy to the wheels at a rate of 38 kW, what is the slope angle of the road?



constant velocity: $\Delta \text{KE} = 0$

↓

work done on car $W = 0$

Power from engine = Power used on gravity and drag $= \bar{F} \cdot \bar{v}$

$P_g = \bar{F}_g \cdot \bar{v} \;= -mg\sin\theta \times v$ Power used on gravity

$P_{\text{drag}} = \bar{F}_{\text{drag}} \cdot \bar{v}$ Power used on drag

Putting it together

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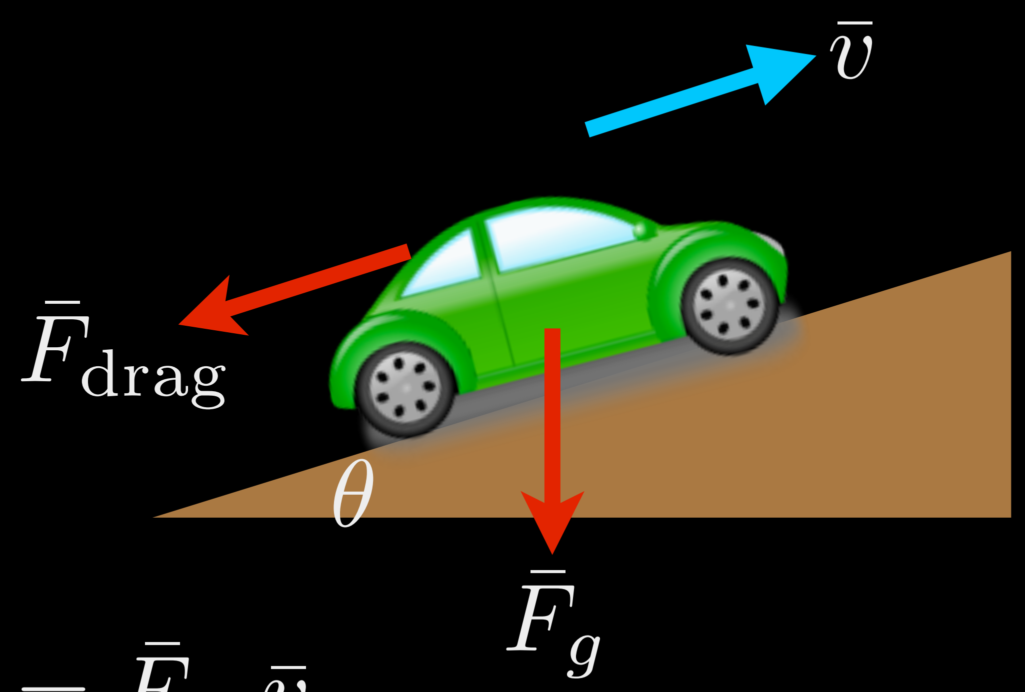

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$P_g = \bar{F}_g \cdot \bar{v} = -mg\sin\theta \times v$ Power used on gravity

$P_{\text{drag}} = \bar{F}_{\text{drag}} \cdot \bar{v} = -F_{\text{drag}} \times v$ Power used on drag

Putting it together

Example

$$P_{\text{tot}} = P_{\text{engine}} + P_g + P_{\text{drag}} = 0$$

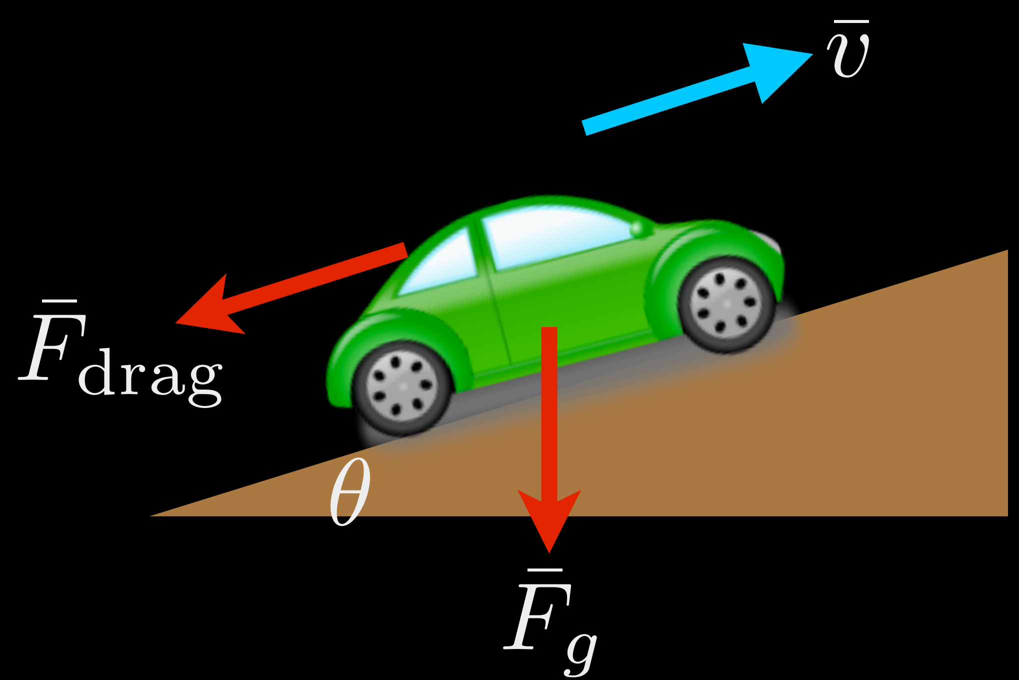
$$P_{\text{engine}} - mgv\sin\theta - F_{\text{drag}}v = 0$$

$$\theta = \sin^{-1}\left(\frac{P_{\text{engine}} - F_{\text{drag}}v}{mgv}\right)$$

$$= \sin^{-1}\left(\frac{38,000\,\text{W} - (450\,\text{N})(16.7\,\text{m/s})}{(1400\,\text{kg})(9.81\,\text{m/s})(16.7\,\text{m/s})}\right)$$

$$(v = 60\,\text{km/h} = 16.7\,\text{m/s})$$

$$= 7.7^\circ$$



Putting it together

Quiz

A tractor tows a plane from its airport gate, doing 8.7 MJ of work.

The link between the plane to the tractor makes a 22 degree angle with the plane's motion, and the tension in the link is 0.41 MN.

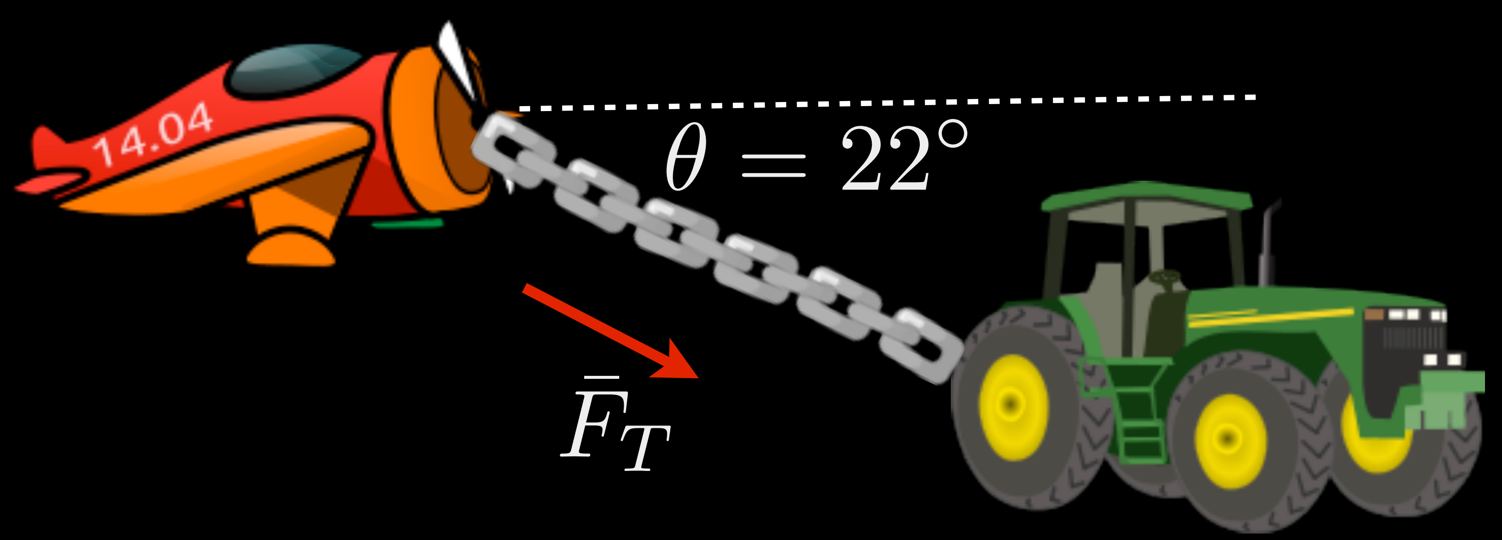
How far does the tractor move the plane?

(a) 2.3 m

(b) 20.1 m

(c) 10 m

(d) 22.9 m



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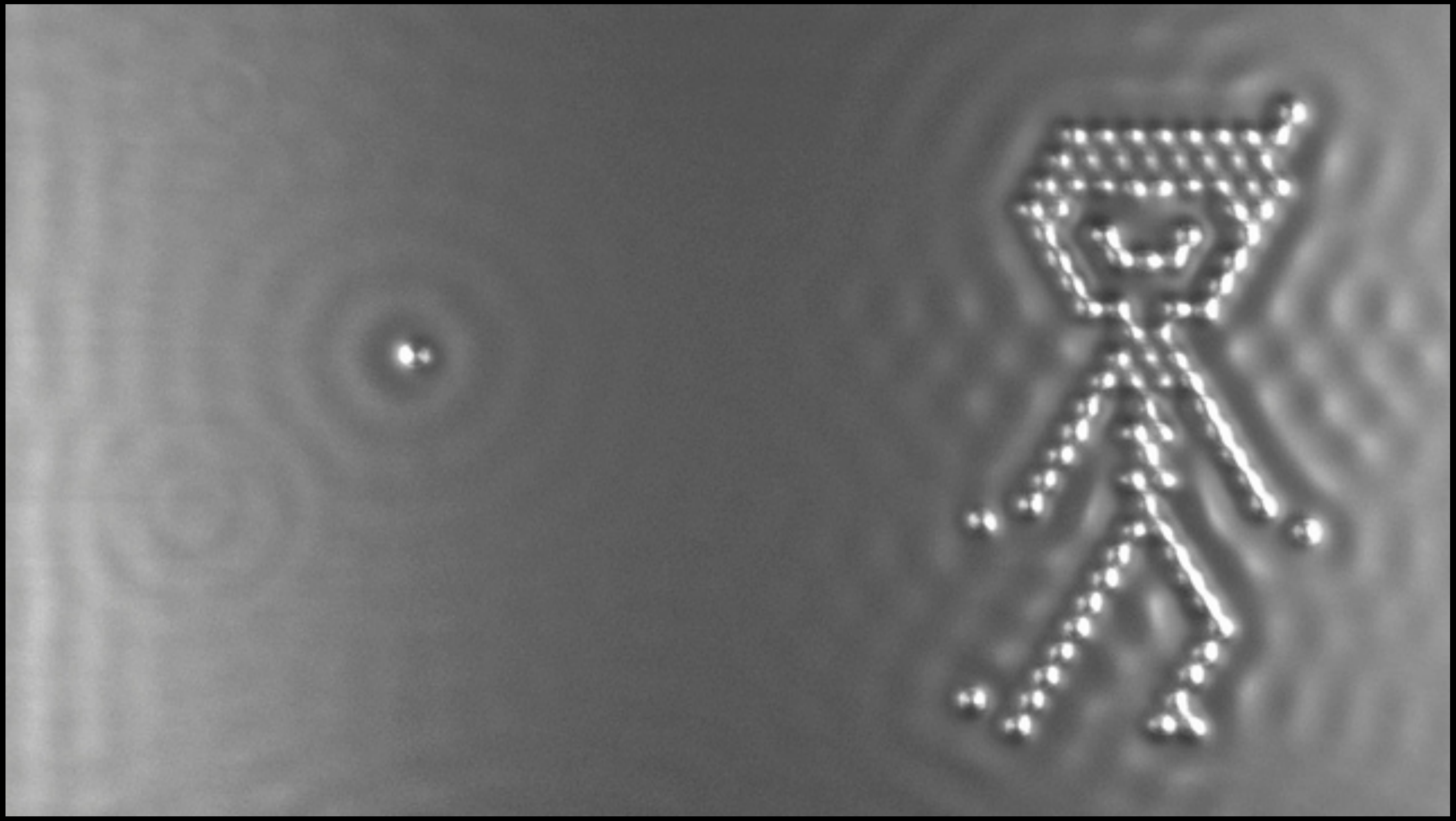
$$W = \bar{F} \cdot \Delta \bar{r} = F_T \cos\theta \Delta r$$

$$\Delta r = \frac{W}{F_T \cos\theta} = \frac{8.7 \times 10^6 \text{ J}}{4.1 \times 10^5 \text{ N} \cos 22^\circ}$$

$$= 22.9 \text{ m}$$

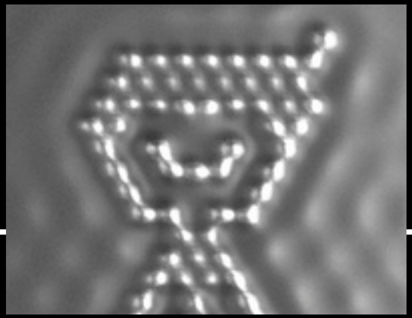
Science in the news



The world's smallest movie

Moving atoms

Quiz



What were IBM trying to do?

(a) Make a movie using a computer

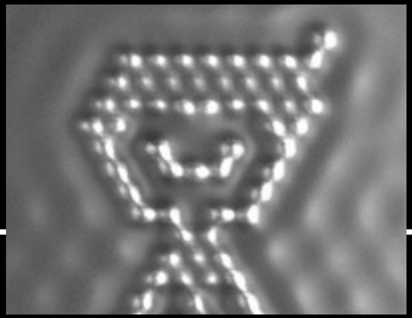
(b) Make a movie using atoms

(c) Try a new way to move atoms

(d) Move the biggest number of atoms

Moving atoms

Quiz



What was the science question?

(a) Can we make a faster, smaller computer?

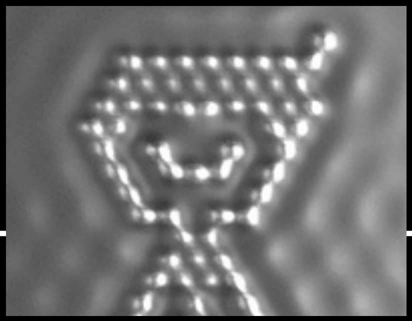
(b) Can we make screens for iPhones to play movies?

(c) How small can you make a magnet for data storage?

(d) No science purpose: advertising campaign

Moving atoms

Quiz



How many atoms did they move?

(a) 500

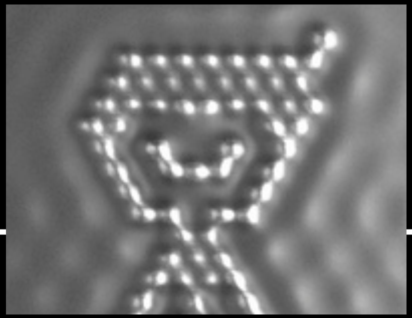
(b) 5,000

(c) 10,000

(d) 1,000,000

Moving atoms

Quiz



What type of molecules (atoms) were used?

(a) Carbon monoxide (CO)

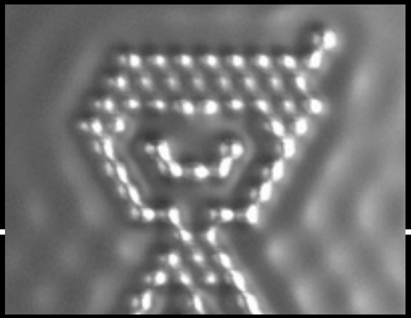
(b) Nitrogen (N)

(c) Carbon (C)

(d) Zinc (Zn)

Moving atoms

Quiz



How does the 'scanning tunnelling microscope' arrange atoms?

(a) The tip of the microscope physically pushes the atoms on the surface

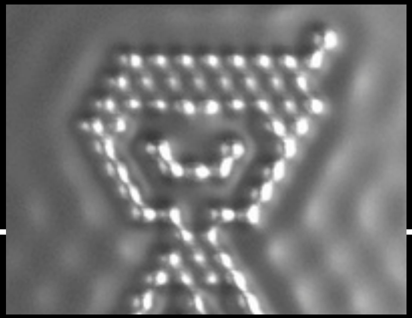
(b) A laser from the microscope drags the surface atoms

(c) The microscope drops its atoms on the surface

(d) A reaction between atoms on microscope and surface allows atoms to be dragged

Moving atoms

Quiz



What is the smallest number of atoms needed to store data?

(a) 1

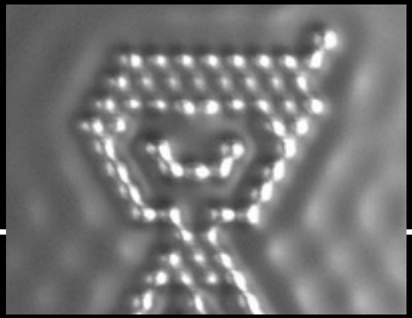
(b) 12

(c) 1300

(d) 1,000,000

Moving atoms

Quiz



With such a device, how many movies could your iPhone hold?

(a) 2

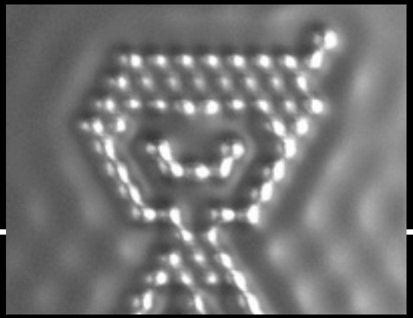
(b) 500

(c) 10,000

(d) All movies ever made

Moving atoms

Quiz



If the atom was the size of an orange, how big would the orange look through the microscope?

(a) The size of a watermelon

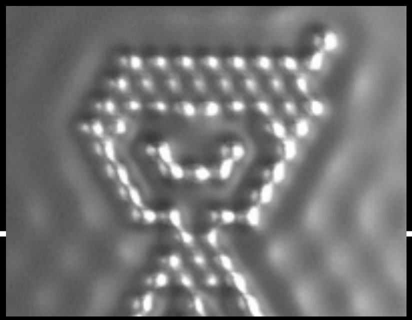
(b) The size of the moon

(c) The size of the Earth

(d) The size of the Sun

Moving atoms

Quiz



What would the scientists like this movie to achieve?

(a) Encourage 1,000 children to do science

(b) Get more funding for science

(c) Allow them to present at an international conference

(d) Make people remember IBM