

# Essential Physics I

英語で物理学の  
エッセンス I

Lecture 2: 25-04-16



The 12th "Sci-Tech Talk in English"  
新渡戸カレッジ応援イベント

# ASTRONOMY in Ancient Egypt

Dr. Sarah Symons  
(McMaster U/U of Tokyo)

Astronomical knowledge from ancient Egypt has survived to the present day carved and painted in tombs and temples along the Nile. This talk describes some of the earliest written records of star movements and explains the "star maps" still visible in the Valley of the Kings today. It also outlines some of the recent advances in our understanding of tables of star movements made four thousand years ago.  
※日経サイエンス(2016.2)掲載

**Apr. 26, 2016**  
17:30 - 18:30

Hokkaido University Library, Media Court  
北海道大学附属図書館 メディアコート

問合せ先 / Inquiry : 理学研究院国際化支援室 / Office for International Academic Support (OIAS) Ext. 2916

Organizer: Office for International Academic Support, Faculty of Science  
In collaboration with Hokkaido University Library

## Tomorrow!

### Public science talk in English

### On star tables found in Egyptian coffins

### Central library, media court 5:30 - 6:30 pm

<http://www.sci.hokudai.ac.jp/international/project/sci-tech-talk/>

# Course

If you see this...

Please enter the Course ID provided by your instructor:  
EP12016TASKER

**!** Your subscription to Mastering does not include access to the book for this course: Essential University Physics, 1e, Wolfson.

Please check the Course ID and try again. If you continue to get this message, please [contact support](#) for more information.

Course ID: EP12016TASKER3e

(And check the webpage)

## News

[22-04-2016] If you purchased the new 3rd edition of "Essential University Physics" textbook, you may see this error on the MasteringPhysics system:  
「Essential University Physics」の教科書（3版）を買いましたら、「MasteringPhysics」でたぶんエラーメッセージを受けるでしょう。

Welcome to MasteringPhysics

### Join Your Online Course

Did you receive a Course ID from your instructor?

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If so, please try course ID: EP12016TASKER3e.

違う「Course ID」を使おうとしてください：EP12016TASKER3e

# Course

On <http://www.masteringphysics.com>

If you see this error:

You must configure your browser to allow popup windows for this site

# Course

MasteringPhysics: Course Home - Windows Internet Explorer

http://session.masteringphysics.com/myct/courseHome?start=1

MasteringPHYSICS

Logged in as Chris Pearson | Help | Log Out


## Welcome!

Return to Introduction to MasteringPhysics

Chris Pearson

Mastering presents homework items assigned by your instructor and works with you to answer them. Homework items typically have an introduction, possibly figures, and one or more parts for you to answer.

Part A Part B



**Type of help offered**

- Mastering tells you immediately whether or not your answers are correct. Usually, you will have multiple chances to arrive at the correct answer. Your instructor will determine how many tries you have available.
- In many items, hints are available to help you if you get stuck. If you don't need the hints to solve the problem, you can still use them for review later on.
- If you submit an incorrect answer, Mastering often responds with specific, helpful feedback.
- Mastering is forgiving of many typos and formatting mistakes. If it can't figure out what you entered, it will let you know and give you another chance.

These exercises were chosen specifically to lead you through the key features of Mastering and are not intended to test your knowledge of any specific subject material. Therefore, on this item you will not be penalized for using hints and submitting incorrect answers. In fact, you should submit incorrect answers and use the hints to see what happens!

**Part A**

How many squares are in this  $2 \times 2$  grid (Part A figure) ? Note that the figure link lets you know that a figure goes along with this part. This figure is available to the left.

Enter your answer as a number in the box below and then submit your answer by clicking Submit.

Number of squares =

submit my answers give up review part

**Grading**

See the help file available by clicking the Help tab in the top right corner, if you want to know more about how grading works. Here is the most important information you'll need.

popup window

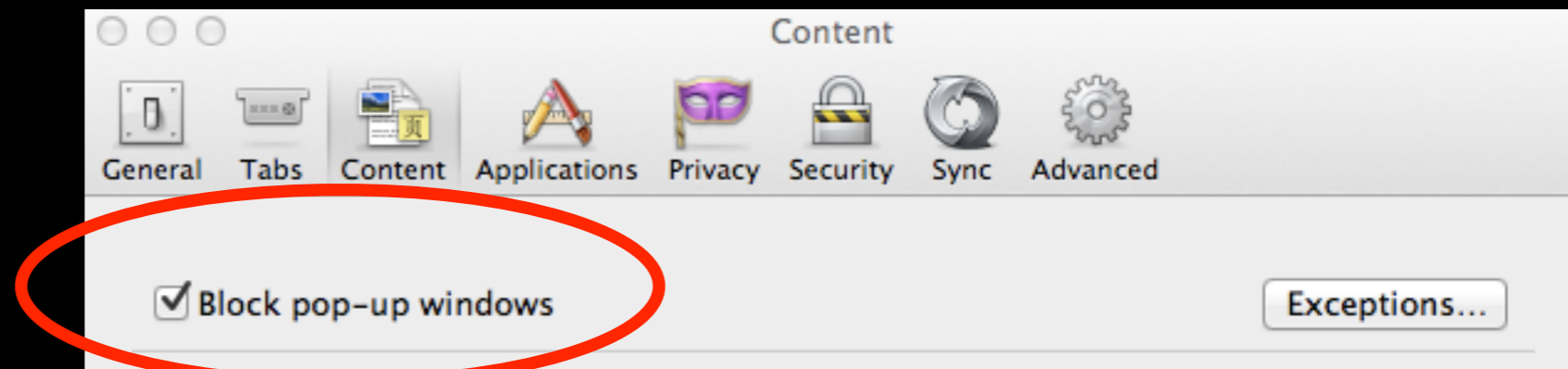
# Course

On <http://www.masteringphysics.com>

If you see this error:

You must configure your browser to allow popup windows for this site

Look in your web browser's preferences

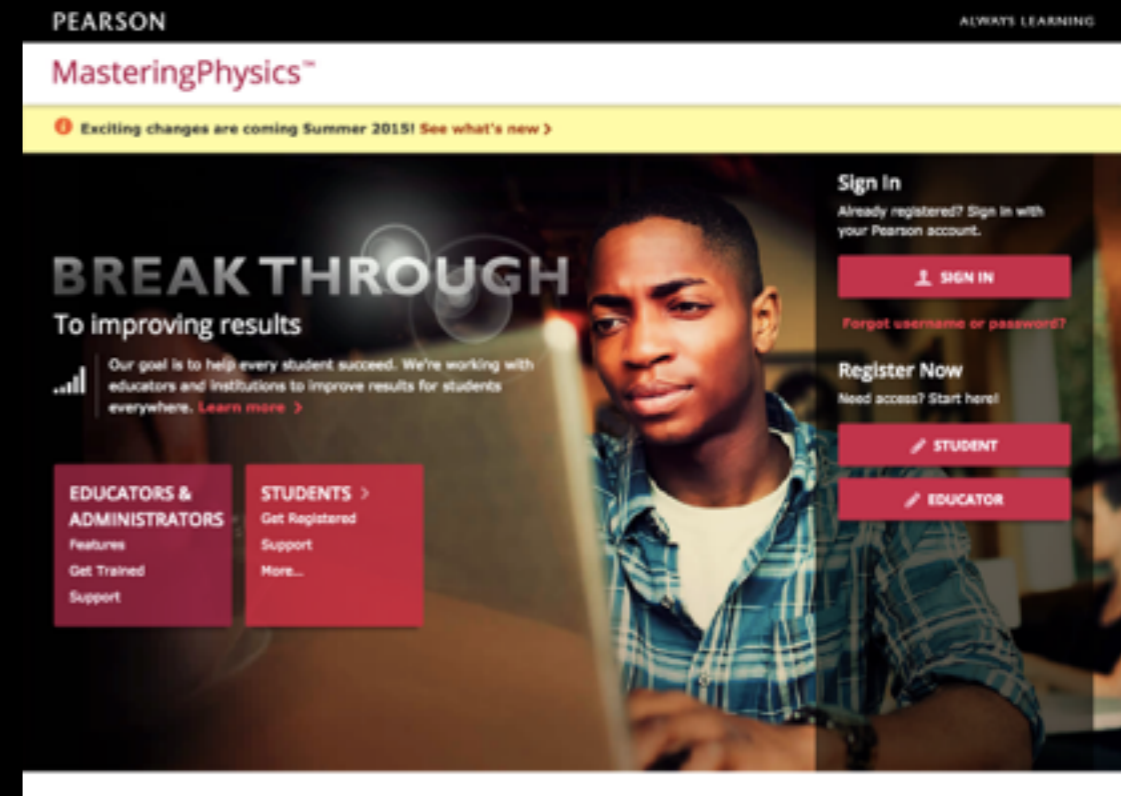
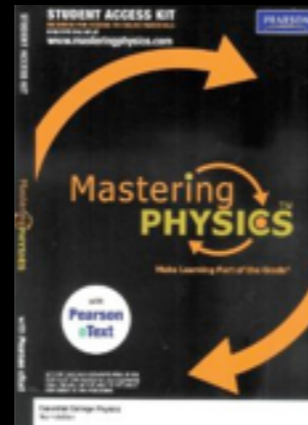


e.g. firefox

# Course

Homework on  
<http://www.masteringphysics.com>

Access code  
(from textbook)  
needed



Not here last week?

See me after class.



# Course

Usually,  
Homework is due the  
following lecture.

(1 week)

But, because last  
week was the 1st  
week...

1st homework is due  
2016/05/02


SUN	MON	TUE	WED	THU	FRI	SAT
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

www.free-printable-calendar.com



SUN	MON	TUE	WED	THU	FRI	SAT
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

www.free-printable-calendar.com



SUN	MON	TUE	WED	THU	FRI
1	2	3	4	5	6
8	9	10	11	12	13
15	16	17	18	19	20
22	23	24	25	26	27
29	30	31			

www.free-printable-calendar.com





# Course

This week's homework is **also** due next lecture...

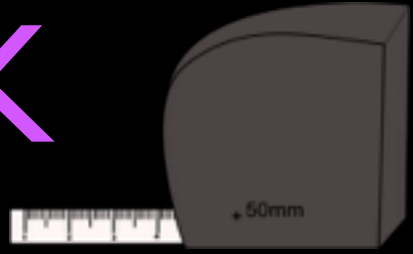
2016 MAY						
SUN	MON	TUE	WED	THU	FRI	SAT
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

week 1 and week 2  
homework due next week

来週に宿題1と宿題2  
を出してください。

# This lecture

K



m



kg

s

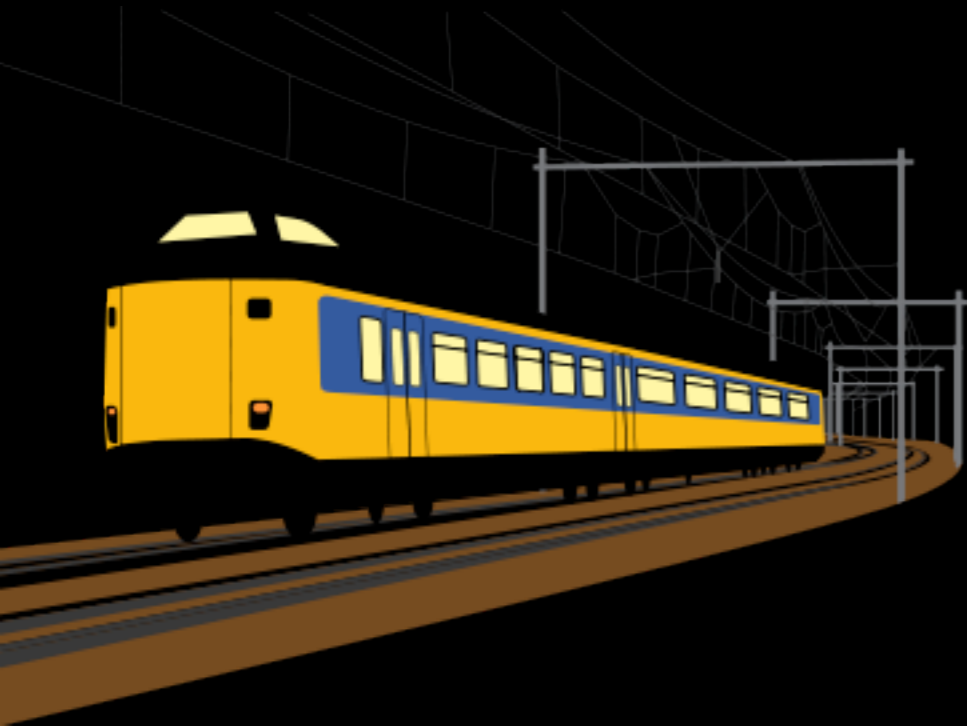


A



cm

## Units



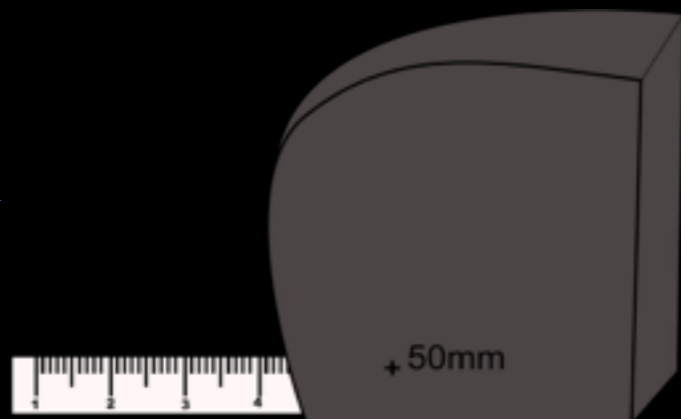
## Motion in 1D

# This lecture

- Know why units are important
- Convert (change) between units
- Use **scientific notation** for large and small numbers
- Understand the difference between **speed** and **velocity**
- Calculate **average** and **instantaneous velocity** and **acceleration**
- Use equations for **constant acceleration**

# Units

K



m



kg

s



A



cm

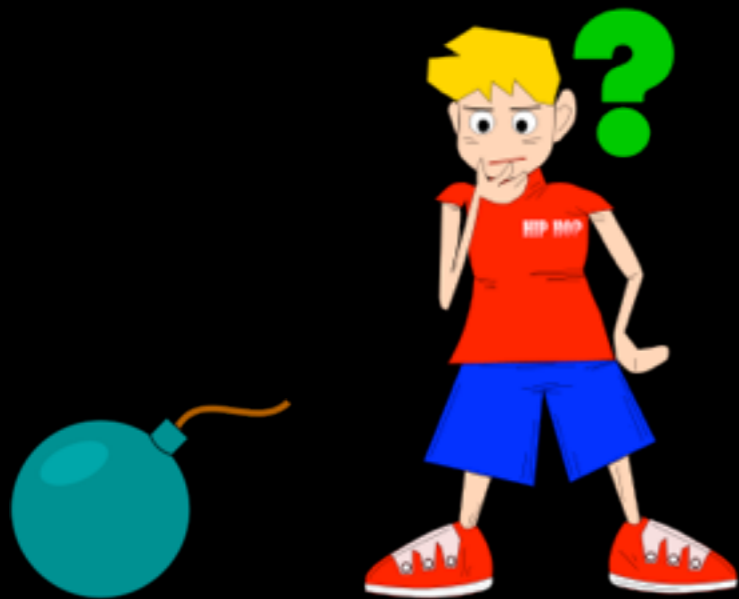
# Units



Why are units important?

If we don't use units... numbers have no meaning

e.g. This bomb will explode in...



Answer = 3



Answer = 3 seconds

and the wrong units can be a disaster!

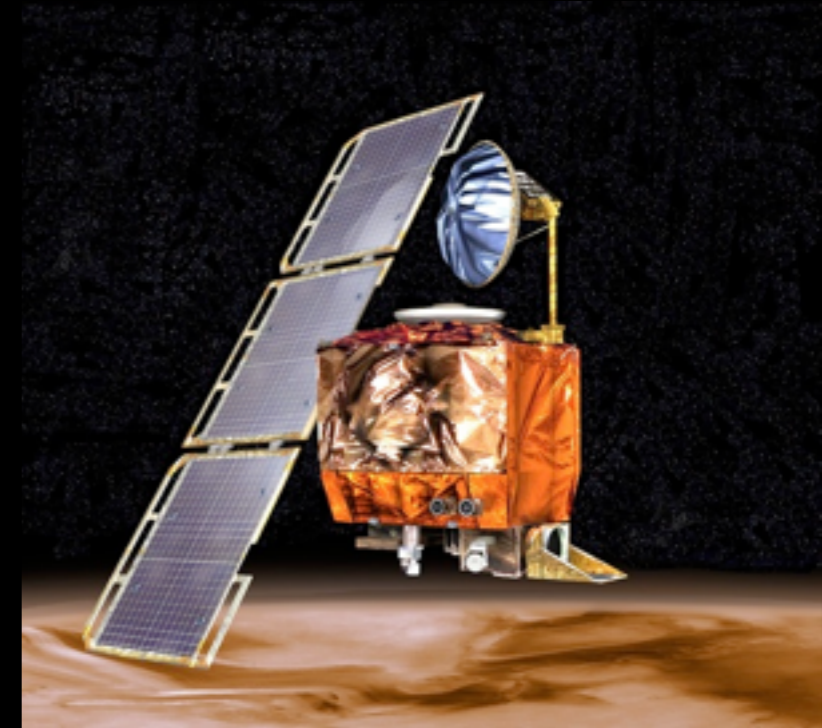
# Units



Why are units important?

1998-12-11: NASA launched the  
'The Mars Climate Orbiter'

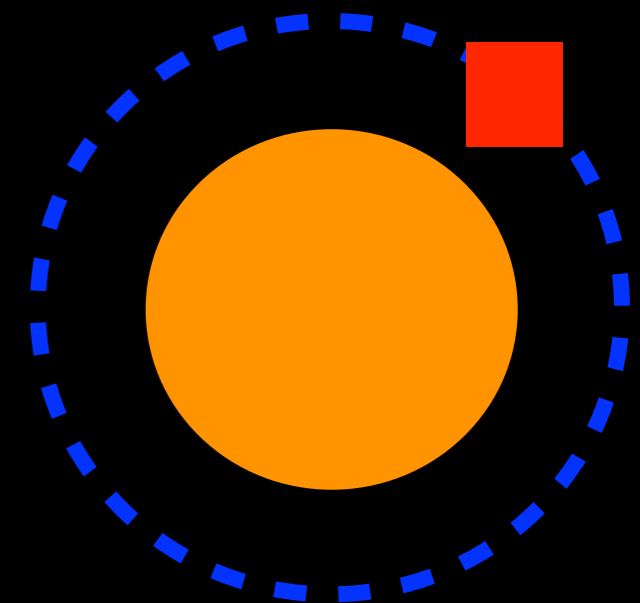
Aim: Study Mars's climate while orbiting  
above the planet.



It travelled for 9 months...

...travelled 415,000,000 miles to reach Mars.

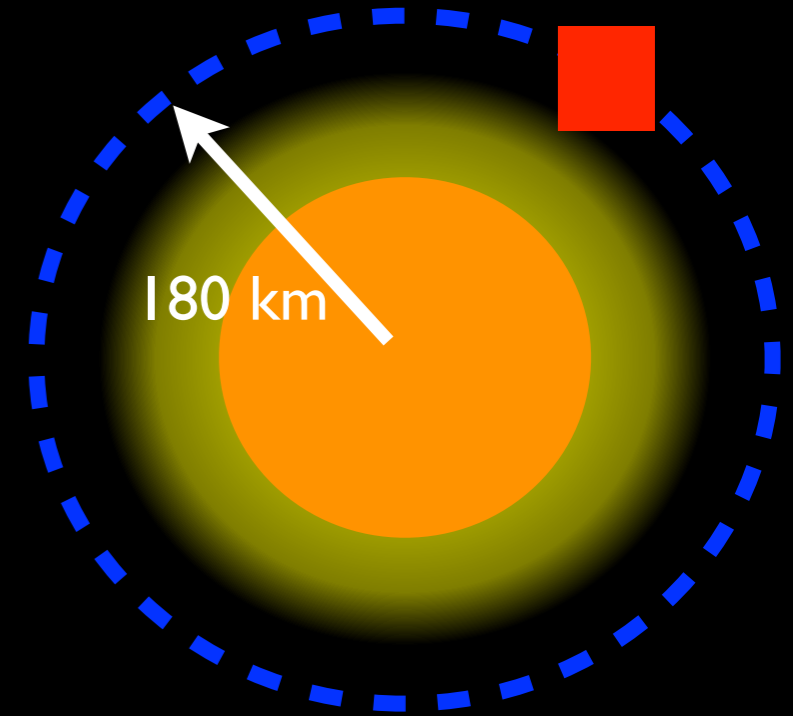
Then, something went wrong....



# Units

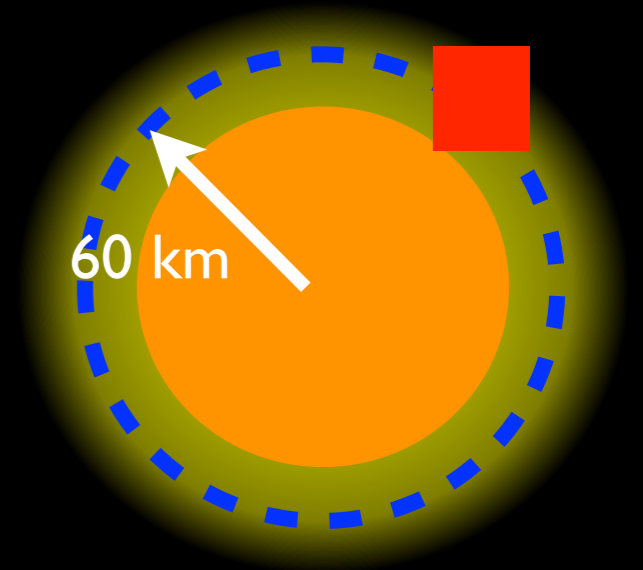


Why are units important?



Instead of orbiting Mars at height of 180 km...

... it orbited at only 60 km ...



... and entered Mars's atmosphere ...

... which it wasn't built to do.



The spacecraft was burned & destroyed.

# Units



Why are units important?

What went wrong?

The spacecraft's direction was changed by the force (F) from its engines.



The engineers measured this force in **pounds (lb)**

... but the flight controllers measured the force in **Newtons (N)**.

Since  $1 \text{ lb} = 4.45 \text{ Newtons}$ ,  $4.45 \times$  the correct force was used.



# Units



W

What went

The spacec  
force (F) fro

The engine

... but the f



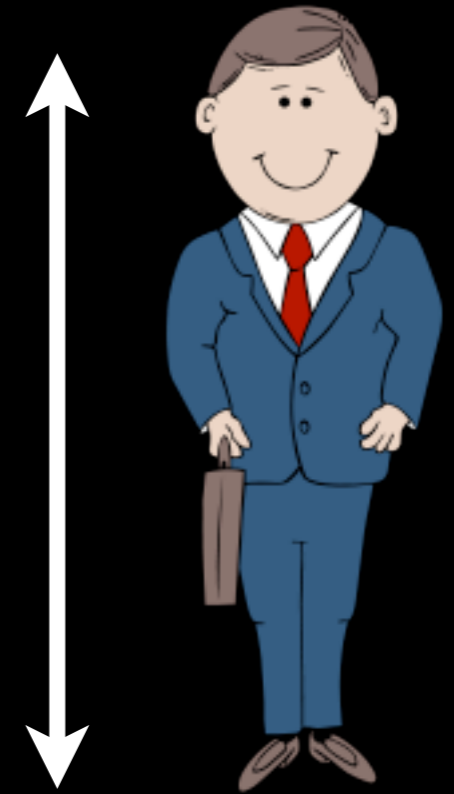
tons (N).

Since 1 lb = 4.45 Newtons, 4.45 x the correct force was used.

# Units

Average man's  
height in Japan

67.4 inches



Tokyo Skytree 634 metres



# Units

Bacteria 2 micrometers



Average distance to the moon from Earth

238,855 miles



Whitney Mountain, USA

4418 metres



CN Tower, Toronto

1,815 feet



Radius of the Hydrogen atom

0.53 angstrom

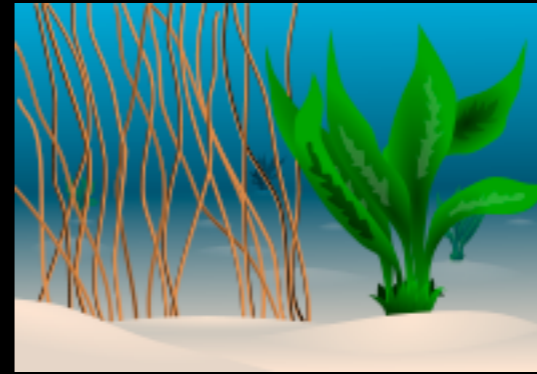


# Units

Mount Fuji 12,388 feet



Deepest point in the ocean 5966 fathom



Odori Park 1,640 yards



# Units

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# Units

# Quiz

Which is the 3rd tallest?

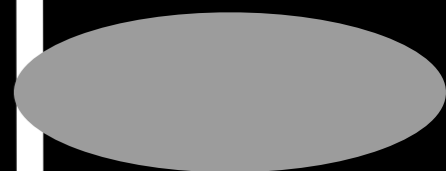
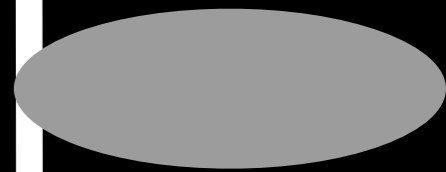
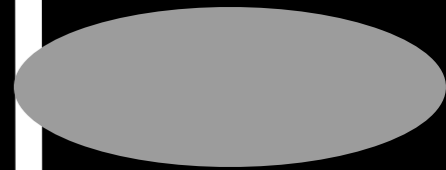
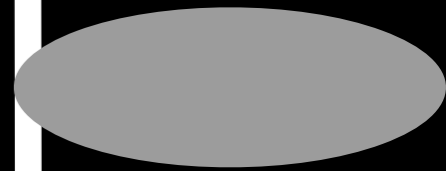
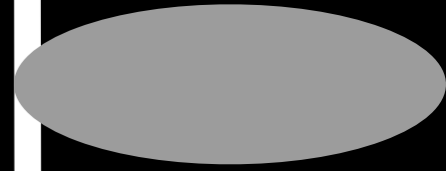
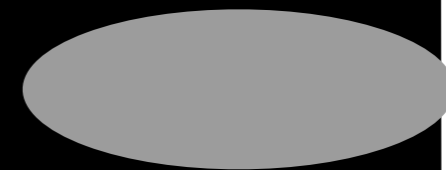
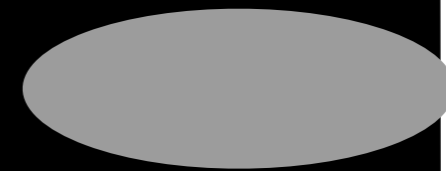
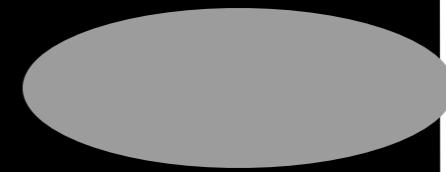
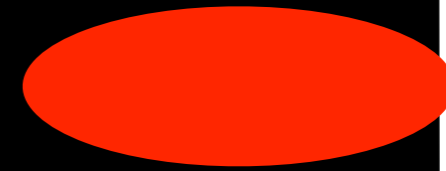
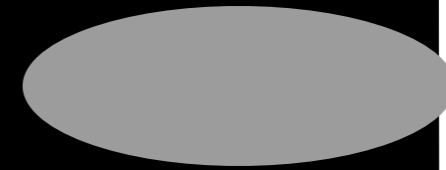
(A) Mount Fuji

(B) Deepest point in the ocean

(C) Tokyo Skytree

(D) Whitney Mountain

(E) Odori Park



# Units

Average man's height in Japan 67.4 inches

Bacteria 2 micrometers

Whitney Mountain, USA 4418 metres

Radius of the Hydrogen atom 0.53 angstrom

Deepest point in the ocean 5966 fathom

Tokyo Skytree 634 metres

Average distance to the moon from Earth 238,855 miles

CN Tower, Toronto 1,815 feet

Mount Fuji 12,388 feet

Odori Park 1,640 yards

# Units

Average distance to the moon from Earth 238,855 miles

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# Units

# Quiz

Which is the 3rd tallest?

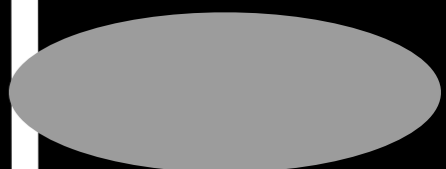
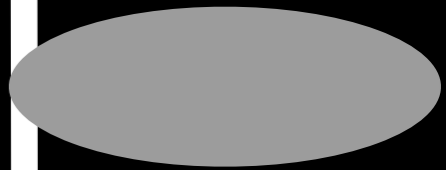
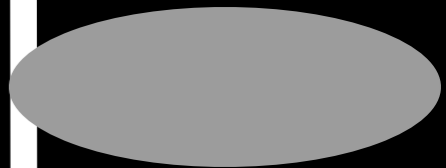
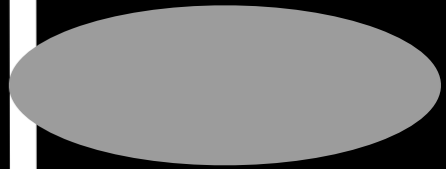
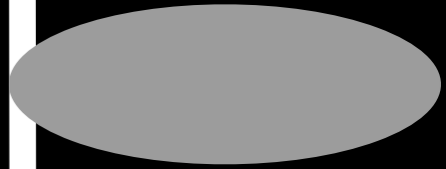
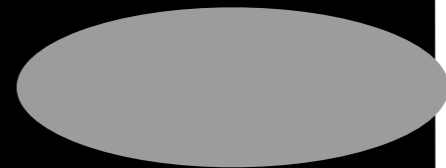
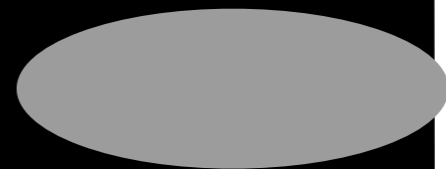
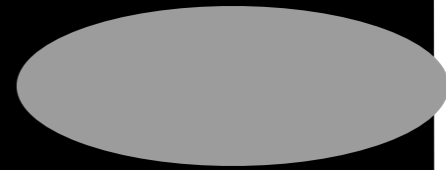
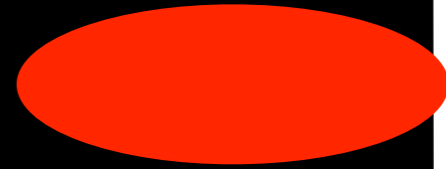
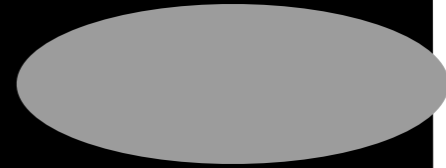
(A) Mount Fuji

(B) Deepest point in the ocean

(C) Tokyo Skytree

(D) Whitney Mountain

(E) Odori Park

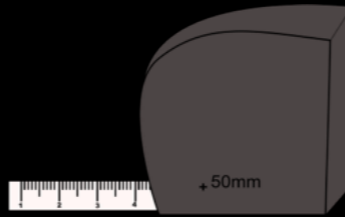


# Units

MUCH easier if we all use the **same units**

The 'Systeme International d'Unites'  
(International System of Units): SI

**Length**



**metres [m]**

**Time**



**seconds [s]**

**Mass**



**kilograms [kg]**

**Temperature**



**kelvin [K]**

# Units

Average distance to the moon from Earth 238,855 miles

Deepest point in the ocean 5966 fathom

Whitney Mountain, USA 4418 metres

Mount Fuji 12,388 feet

Odori Park 1,640 yards

Tokyo Skytree 634 metres

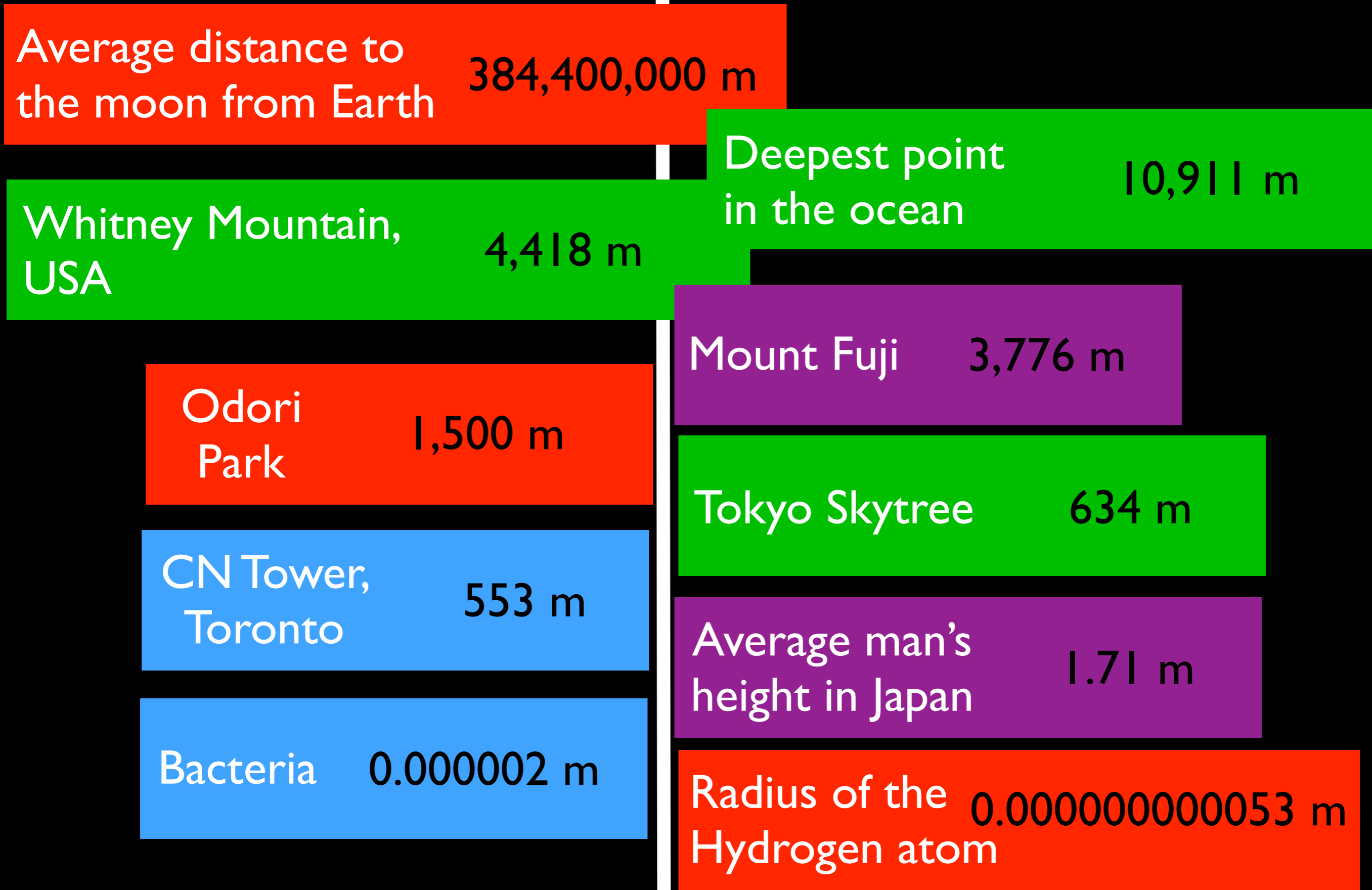
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Average man's height in Japan 67.4 inches

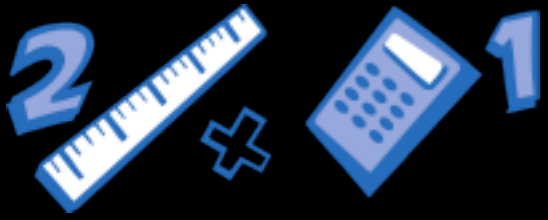
Bacteria 2 micrometers

Radius of the Hydrogen atom 0.53 angstrom

# Units



# Units



## Calculating units

Units can be calculated like algebra:

$$x = v \times t$$

distance = velocity x time



→ 80km/h

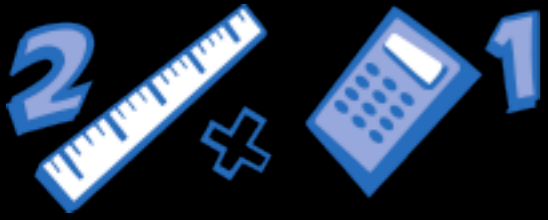


3h

$$= \left( \frac{80\text{km}}{\text{h}} \right) \times 3\text{h}$$

$$= 240 \text{ ?}$$

# Units



## Calculating units

Units can be calculated like algebra:

$$x = v \times t$$

distance = velocity x time



→ 80km/h



3h

$$= \left( \frac{80\text{km}}{\cancel{\text{h}}} \right) \times \cancel{3\text{h}}$$

$$= 240 \text{ km}$$



## Calculating units

Calculate:  $a = \frac{v}{t}$  for:  $v = 6\text{m/s}$   
 $t = 3\text{s}$

(A)  $a = 2\text{m}$

(B)  $a = 2\text{m/s}^2$

(C)  $a = 2\text{m/s}$

(D)  $a = 2\text{s}$

$$a = \frac{6\text{ m/s}}{3\text{ s}}$$

$$= \frac{6\text{ m}}{3\text{ s} \times \text{s}} = 2\frac{\text{m}}{\text{s}^2}$$



## Calculating units

Calculate:  $E = \frac{1}{2}mv^2$  for:  $m = 10\text{kg}$   
 $v = 2\text{m/s}$

(A)  $E = 20 \text{ kg s}^2$

(B)  $E = 20 \text{ kg s}^2/\text{m}^2$

(C)  $E = 20 \text{ kg m/s}$

(D)  $E = 20 \text{ kg m}^2/\text{s}^2$

$$E = \frac{1}{2} (10 \text{ kg}) (2 \text{ m/s})^2$$
$$= 20 \frac{\text{kgm}^2}{\text{s}^2}$$

Also called the 'joule' : J



# Units



## Changing units

$$x = v \times t = 240 \text{ km}$$



→ 80km/h



3h

But what if we wanted miles?

$$1\text{mi} = 1.61\text{km}$$

$$\left( \frac{1\text{mi}}{1.61\text{km}} \right) = \left( \frac{\cancel{1.61\text{km}}}{\cancel{1.61\text{km}}} \right)$$

÷ 1.61km

$$\left( \frac{1\text{mi}}{1.61\text{km}} \right) = 1$$

... we can x 1 by anything

# Units



## Changing units

$$x = v \times t = 240 \text{ km}$$



→ 80km/h



3h

But what if we wanted miles?

$$x = v \times t$$

$$= \left( \frac{80 \text{ km}}{\text{h}} \right) \times 3 \text{ h}$$

= 1

$$= 240 \text{ km} \times \left( \frac{1 \text{ mi}}{1.61 \text{ km}} \right)$$

# Units



## Changing units

$$x = v \times t = 240 \text{ km}$$



→ 80km/h



3h

But what if we wanted miles?

$$x = v \times t$$

$$= \left( \frac{80 \text{ km}}{\text{h}} \right) \times 3 \text{ h}$$

$$= 240 \text{ km} \times \left( \frac{1 \text{ mi}}{1.61 \text{ km}} \right) = \left( \frac{240 \times 1 \text{ mi}}{1.61} \right) = 149 \text{ mi}$$



## Changing units

Change **65 miles / hour** to **m / s**

(A) 29 m/s

(B) 104.65 m/s

(C) 40.4 m/s

(D) 1.74 m/s

1 mile = 1.61 km



## Changing units

Change **65 miles / hour** to **m / s**

$$1 \text{ mile} = 1.61 \text{ km}$$

$$\frac{1 \text{ mile}}{1 \text{ mile}} = \frac{1.61 \text{ km}}{1 \text{ mile}}$$

÷ 1 mile

$$1 = \frac{1.61 \text{ km}}{1 \text{ mile}}$$

$$65 \frac{\cancel{\text{mile}}}{\text{hour}} \times \left( \frac{1.61 \text{ km}}{\cancel{1 \text{ mile}}} \right) = 104.65 \text{ km/hour}$$



## Changing units

Change **65 miles / hour** to **m / s**

$$1 \text{ km} = 1000 \text{ m}$$

$$\frac{1 \text{ km}}{1 \text{ km}} = \frac{1000 \text{ m}}{1 \text{ km}}$$



$$\div 1 \text{ km}$$

$$1 = \frac{1000 \text{ m}}{1 \text{ km}}$$

$$104.65 \frac{\cancel{\text{km}}}{\text{hour}} \times \left( \frac{1000 \text{ m}}{\cancel{1 \text{ km}}} \right) = 104650 \text{ m/hour}$$



## Changing units

Change **65 miles / hour** to **m / s**

$$1 \text{ hour} = 3600 \text{ s}$$

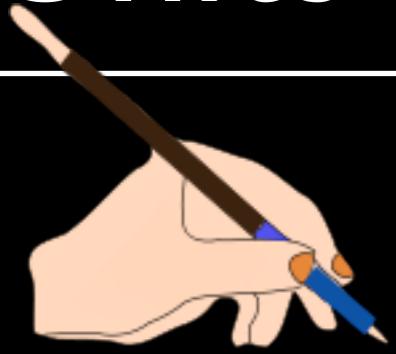
$$\frac{1 \text{ hour}}{3600 \text{ s}} = \frac{3600 \text{ s}}{3600 \text{ s}}$$

$$\div 3600 \text{ s}$$

$$\frac{1 \text{ hour}}{3600 \text{ s}} = 1$$

$$104650 \frac{\text{m}}{\cancel{\text{hour}}} \times \left( \frac{\cancel{1 \text{ hour}}}{3600 \text{ s}} \right) = 29 \text{ m/s}$$

# Units



## Scientific notation

But... if we use a single unit set, the numbers can get very big:



Radius of the observable Universe:

100,000,000,000,000,000,000,000,000 m

26 zeros

$$= 1 \times 10^{26} \text{ m}$$

or very small:



Size of a proton:

0.0000000000000001 m

15 zeros

$$= 1 \times 10^{-15} \text{ m}$$



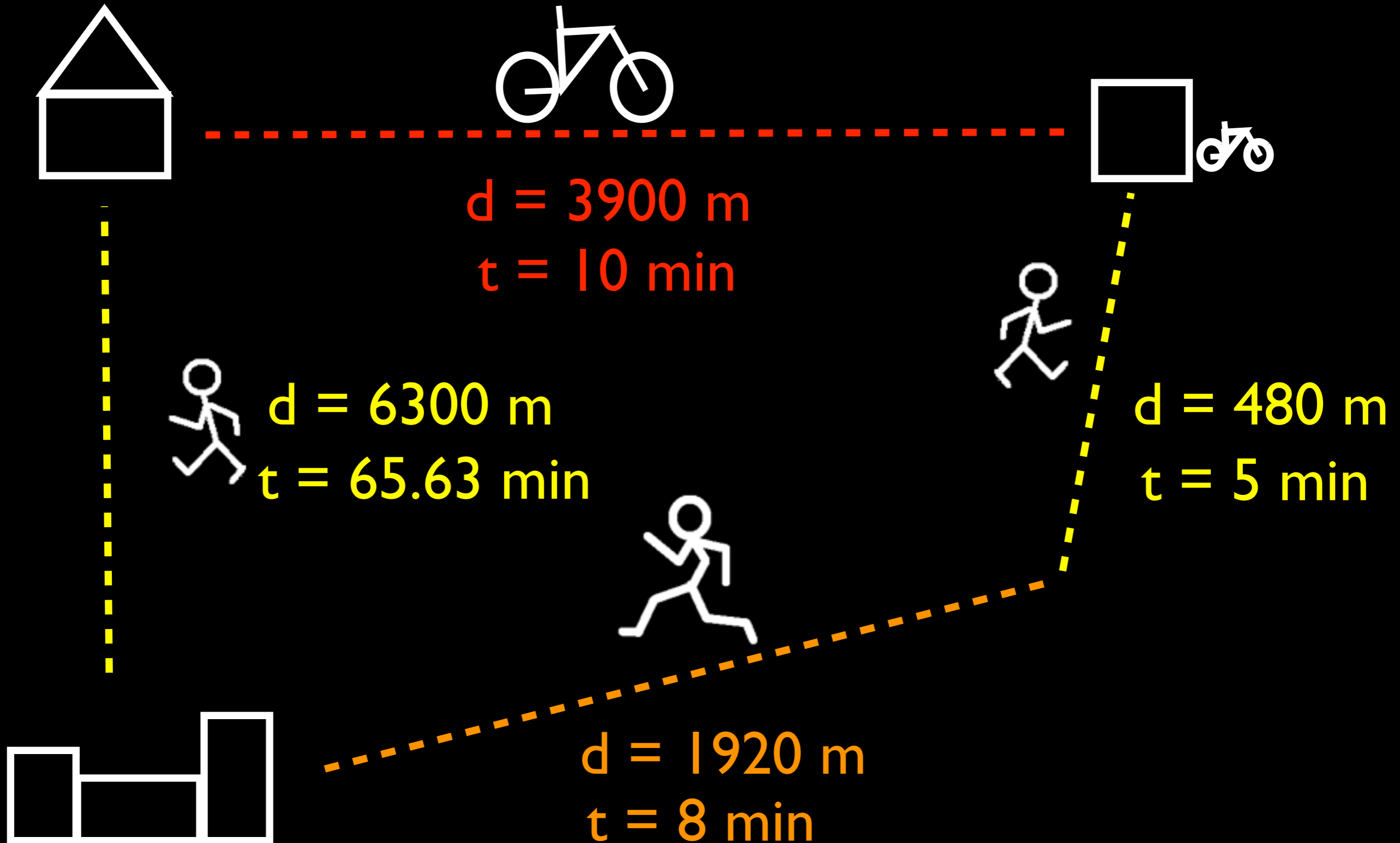
# Motion in 1D



# Motion in 1D



## Speed & Velocity

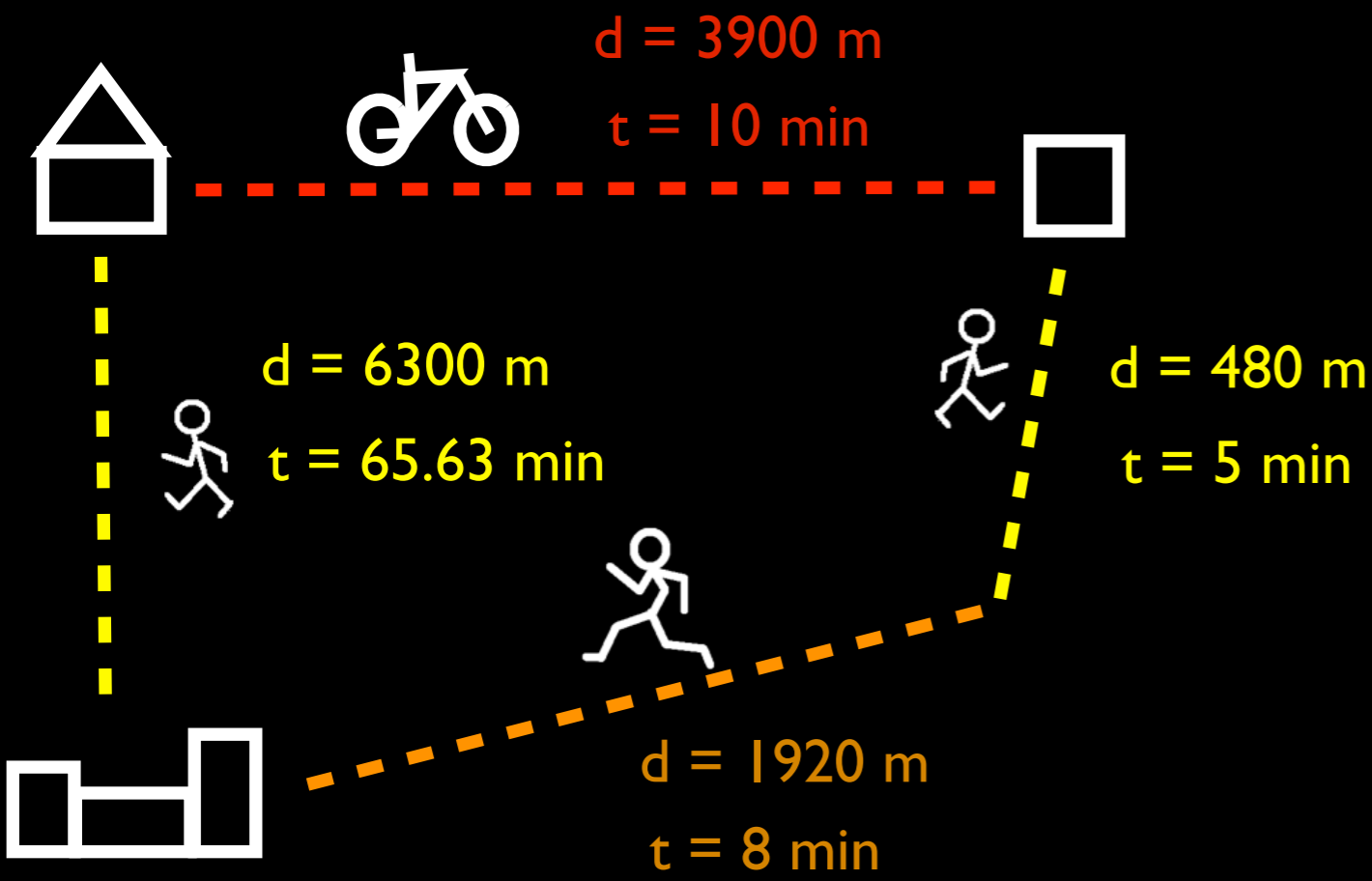


# Motion in 1D



## Speed & Velocity

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$



$$\begin{aligned} \text{Total distance} &= 3900 \text{ m} \\ &+ 480 \text{ m} + 1920 \text{ m} + 6300 \text{ m} \\ &= 12,600 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Total time} &= 10 \text{ min} \\ &+ 5 \text{ min} + 8 \text{ min} + 65.63 \text{ min} \\ &= 88.63 \text{ min} \end{aligned}$$

$$\text{Average speed} = \frac{12,600 \text{ m}}{88.63 \text{ min}} = 142 \text{ m/min} = 2.4 \text{ m/s}$$

# Motion in 1D



## Speed & Velocity

Speed is different to **velocity** which includes **direction**

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time}}$$

change in position



$$\text{Average velocity} = v_{\text{av}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

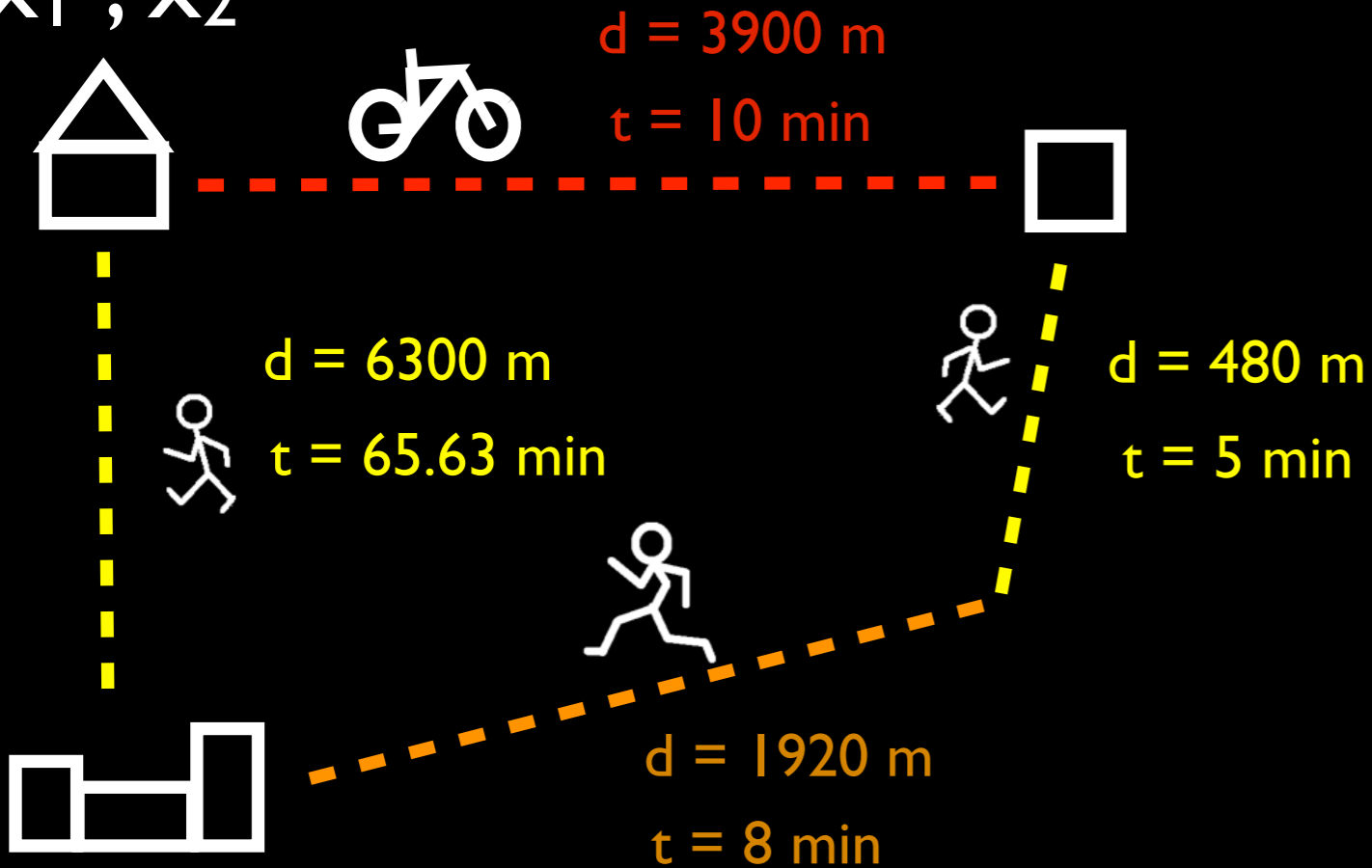
# Motion in 1D



## Speed & Velocity

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time}} = \frac{\Delta x}{\Delta t}$$

$x_1, x_2$



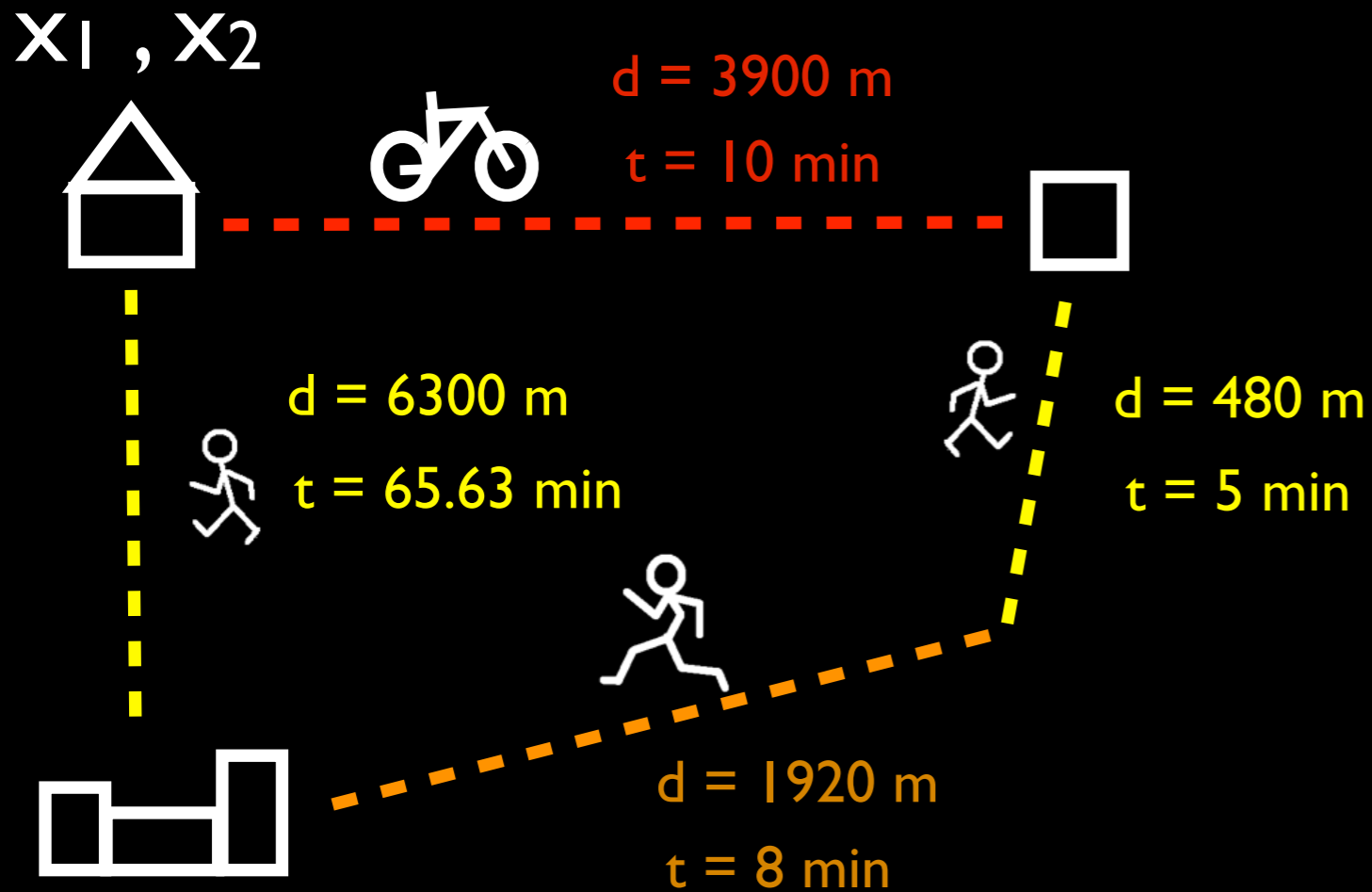
$$\begin{aligned} \text{Total displacement} &= x_2 - x_1 \\ &= 0 \end{aligned}$$

$$\text{Average velocity} = v_{\text{av}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0}{88.63 \text{ min}} = 0$$

# Motion in 1D



## Speed & Velocity

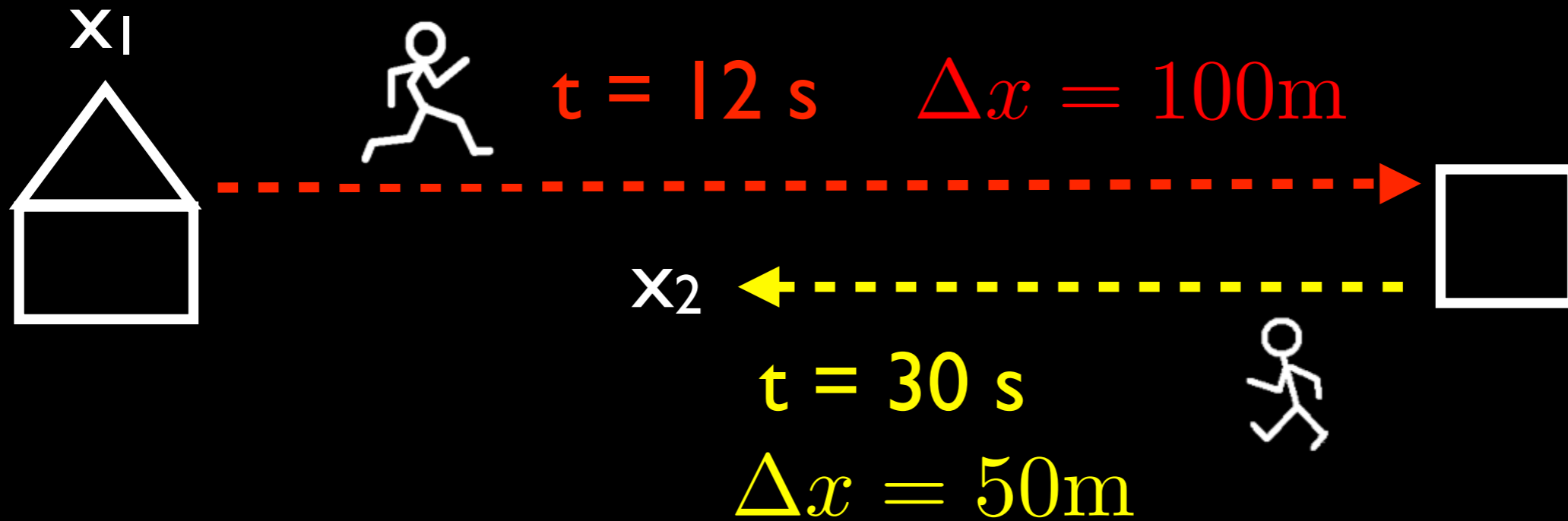


Total displacement = 0

Total distance = 12,600 m

Average speed = 2.4 m/s

Average velocity = 0



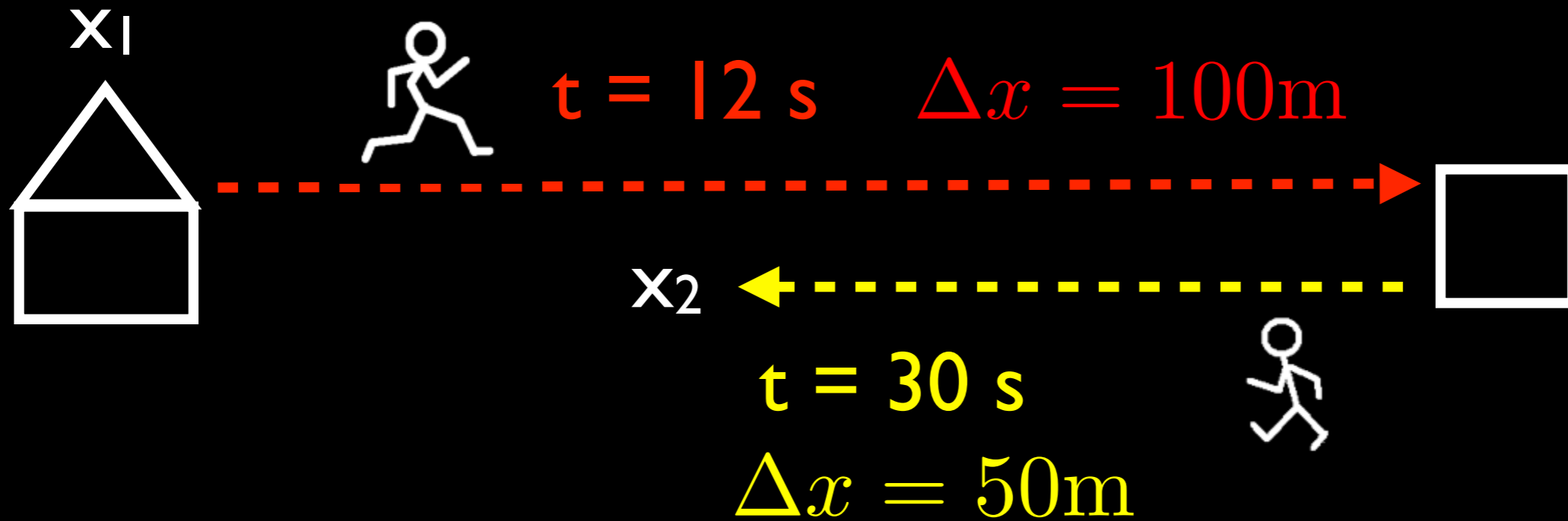
What is the average **speed** for the total trip?

(A) 1.2 m/s

(C) 3.6 m/s

(B) 5 m/s

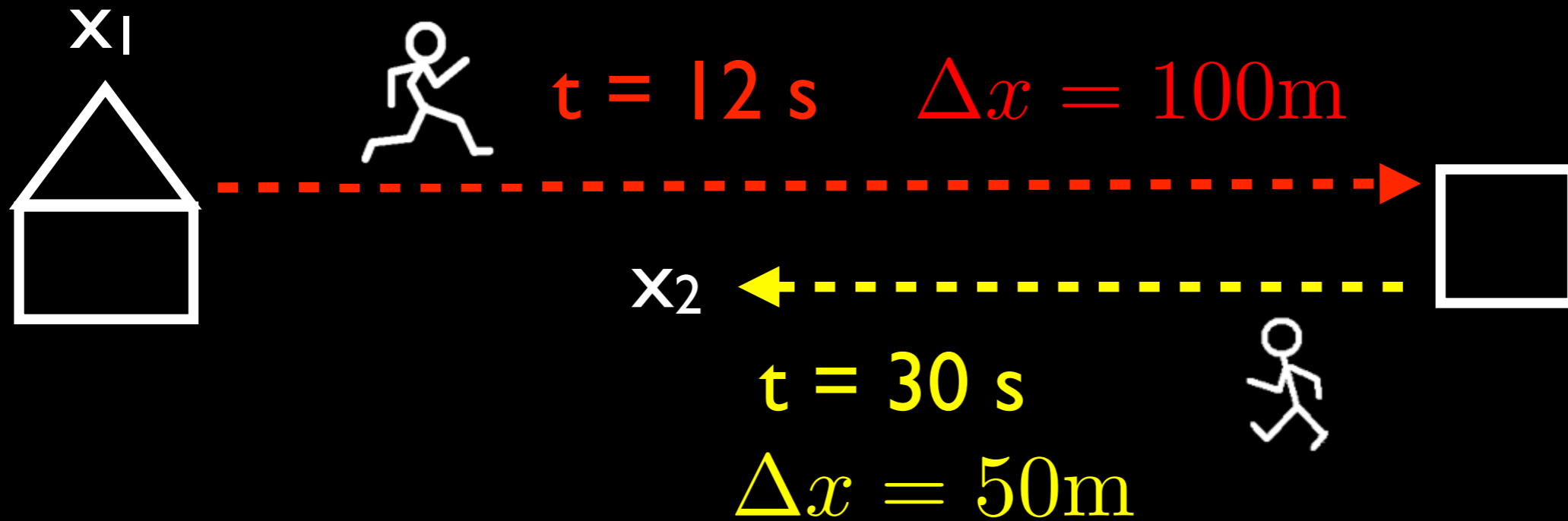
(D) 2.8 m/s



What is the average **speed** for the total trip?

$$\begin{aligned} \text{Average speed} &= \frac{\text{Total distance}}{\text{Total time}} = \frac{100\text{ m} + 50\text{ m}}{12\text{ s} + 30\text{ s}} \\ &= 3.6\text{ m/s} \end{aligned}$$





What is the average **velocity** for the total trip?

(A)  $1.2\text{ m/s}$

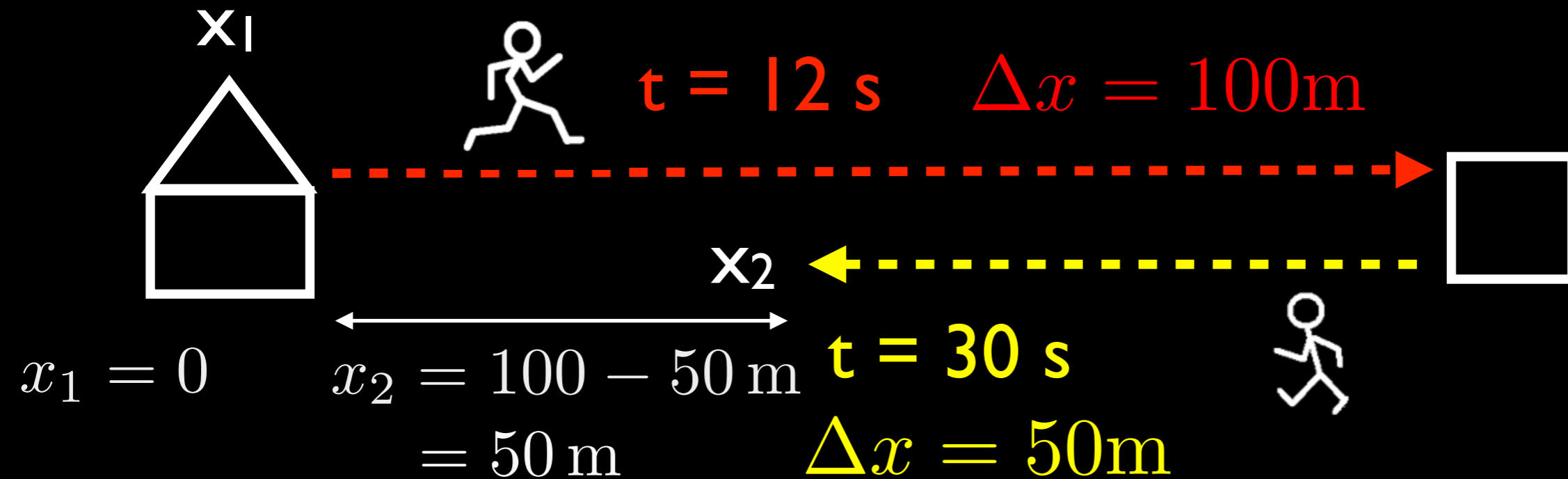
(C)  $3.6\text{ m/s}$

(B)  $5\text{ m/s}$

(D)  $2.8\text{ m/s}$

# Motion in 1D

# Quiz



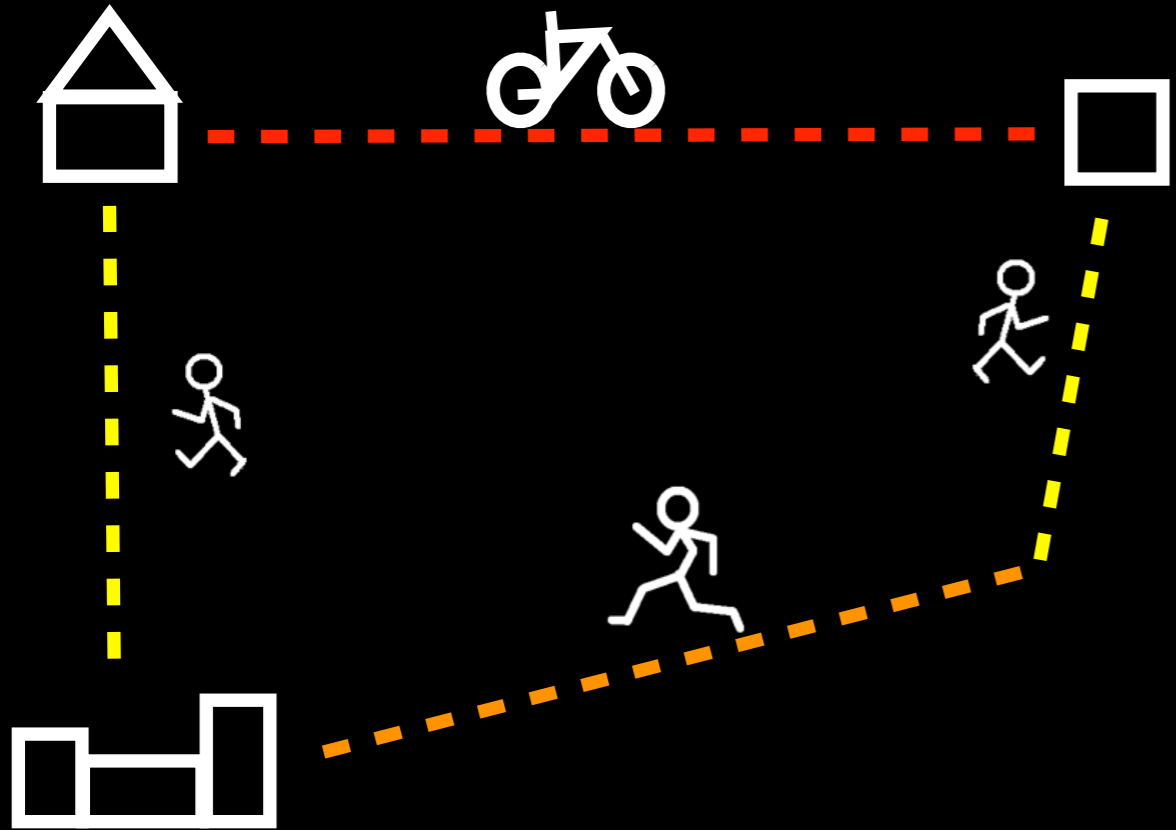
What is the average **velocity** for the total trip?

$$\begin{aligned} \text{Average velocity} &= \frac{\text{total displacement}}{\text{total time}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_1 + t_2} \\ &= \frac{50 \text{ m} - 0}{12 \text{ s} + 30 \text{ s}} = 1.2 \text{ m/s} \end{aligned}$$

# Motion in 1D



## Average & Instantaneous Velocity



Problem:

Average velocity does not tell the details of a motion

Did the person speed up?



Slow down?



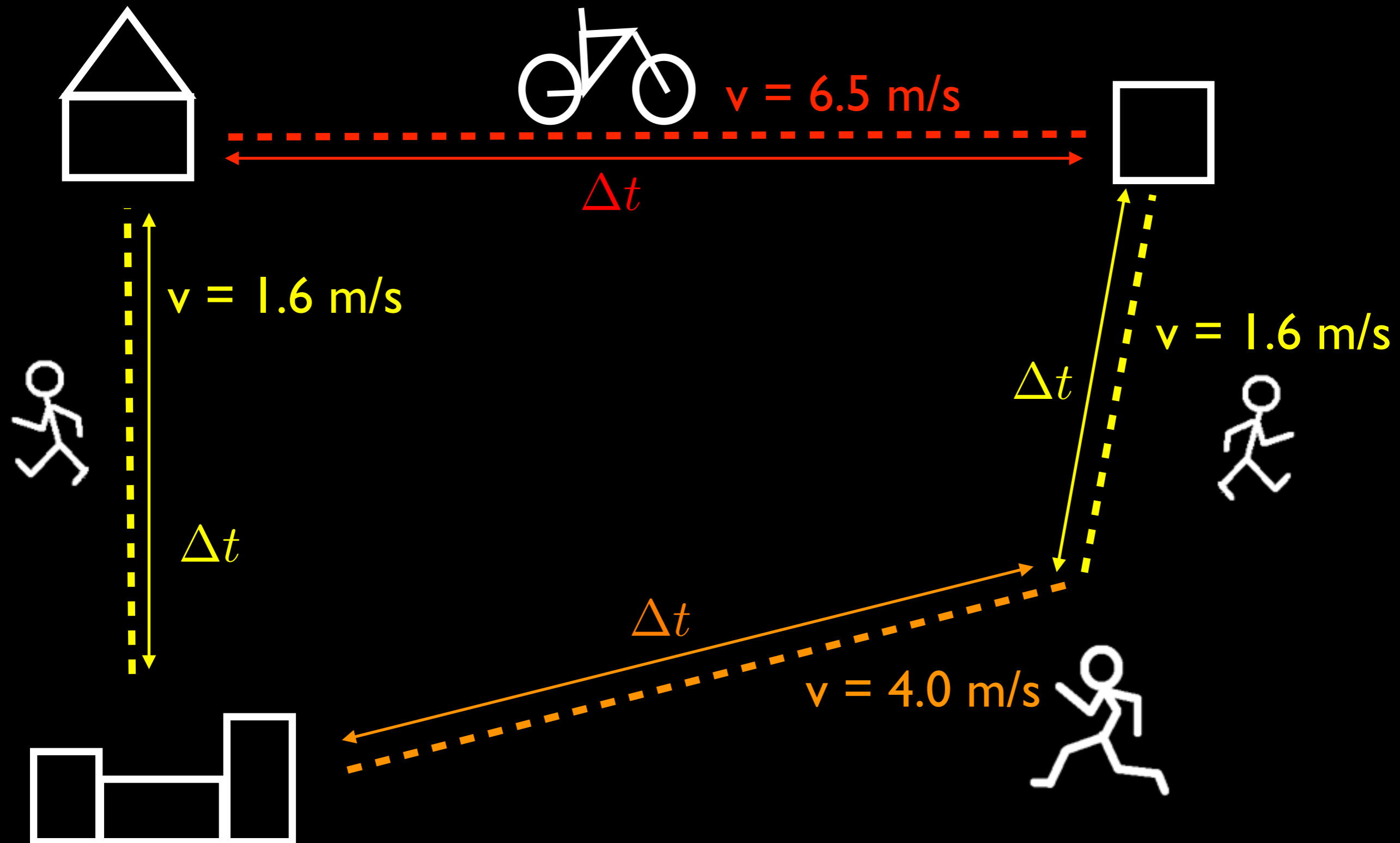
Stop?



# Motion in 1D



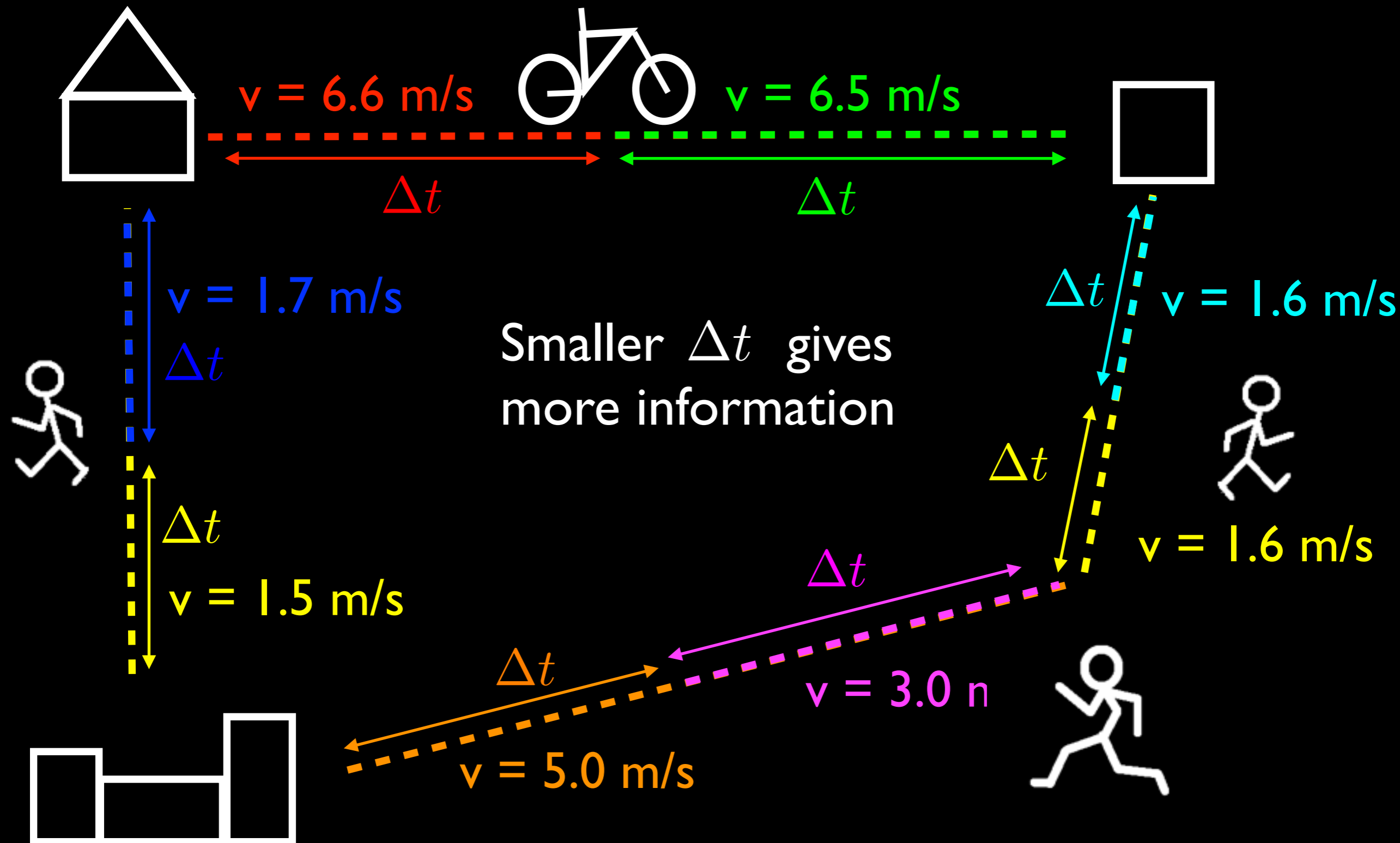
## Average & Instantaneous Velocity



# Motion in 1D



## Average & Instantaneous Velocity



# Motion in 1D

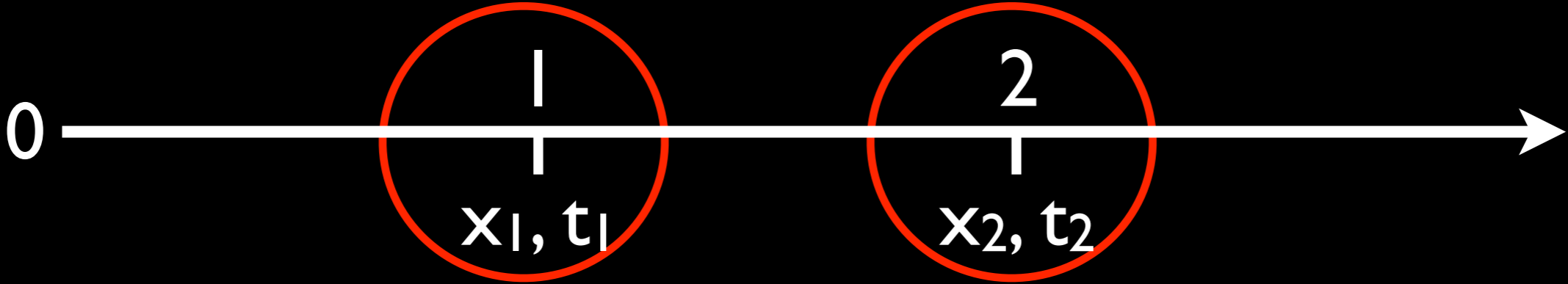


## Average & Instantaneous Velocity

Can we measure velocity at only one time,  $t_1$ ?

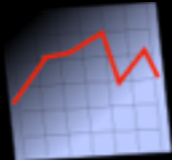


With observations, no!



Need 2 points to get velocity

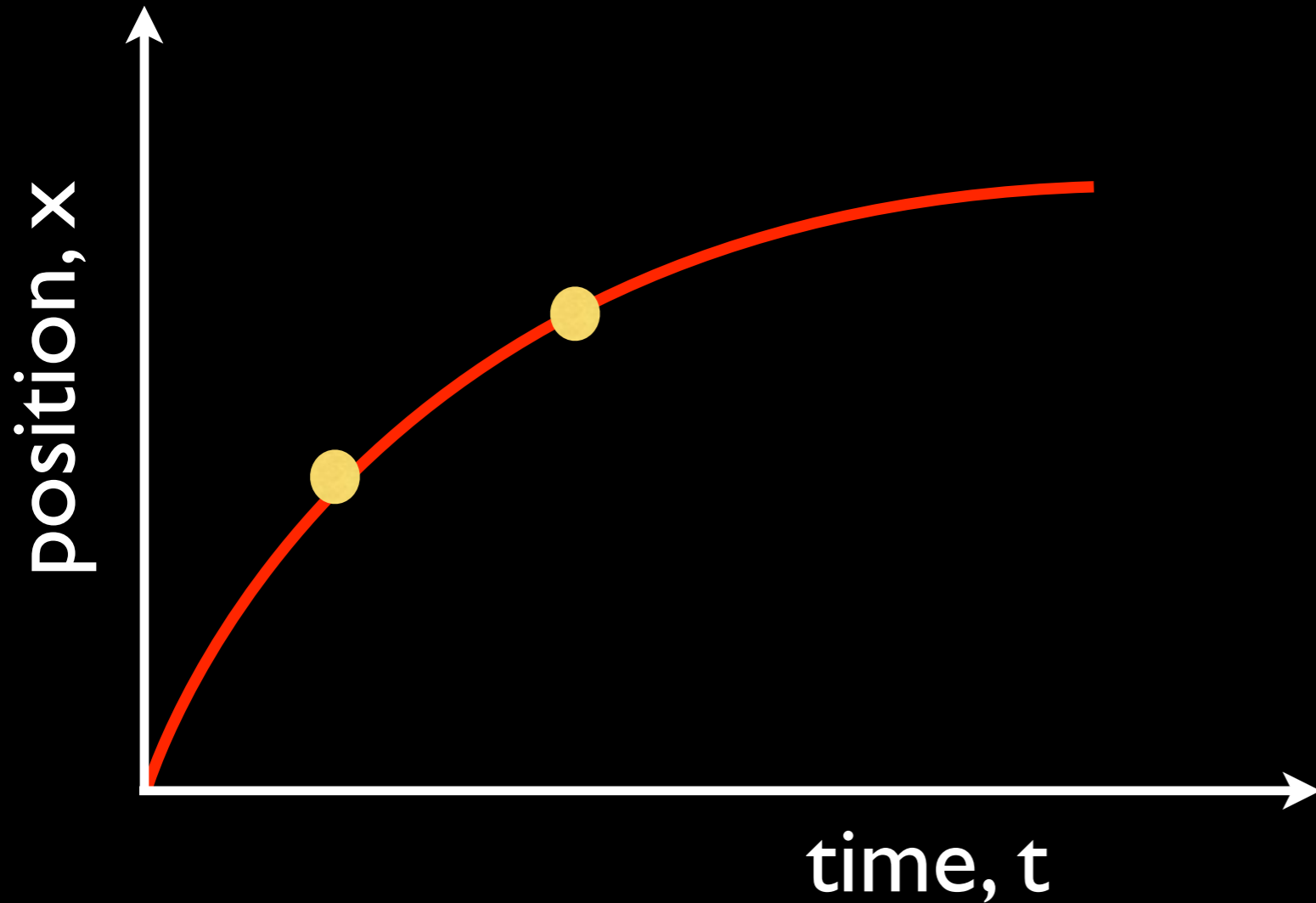
But ... let's look at a graph...



# Motion in 1D



## Average & Instantaneous Velocity



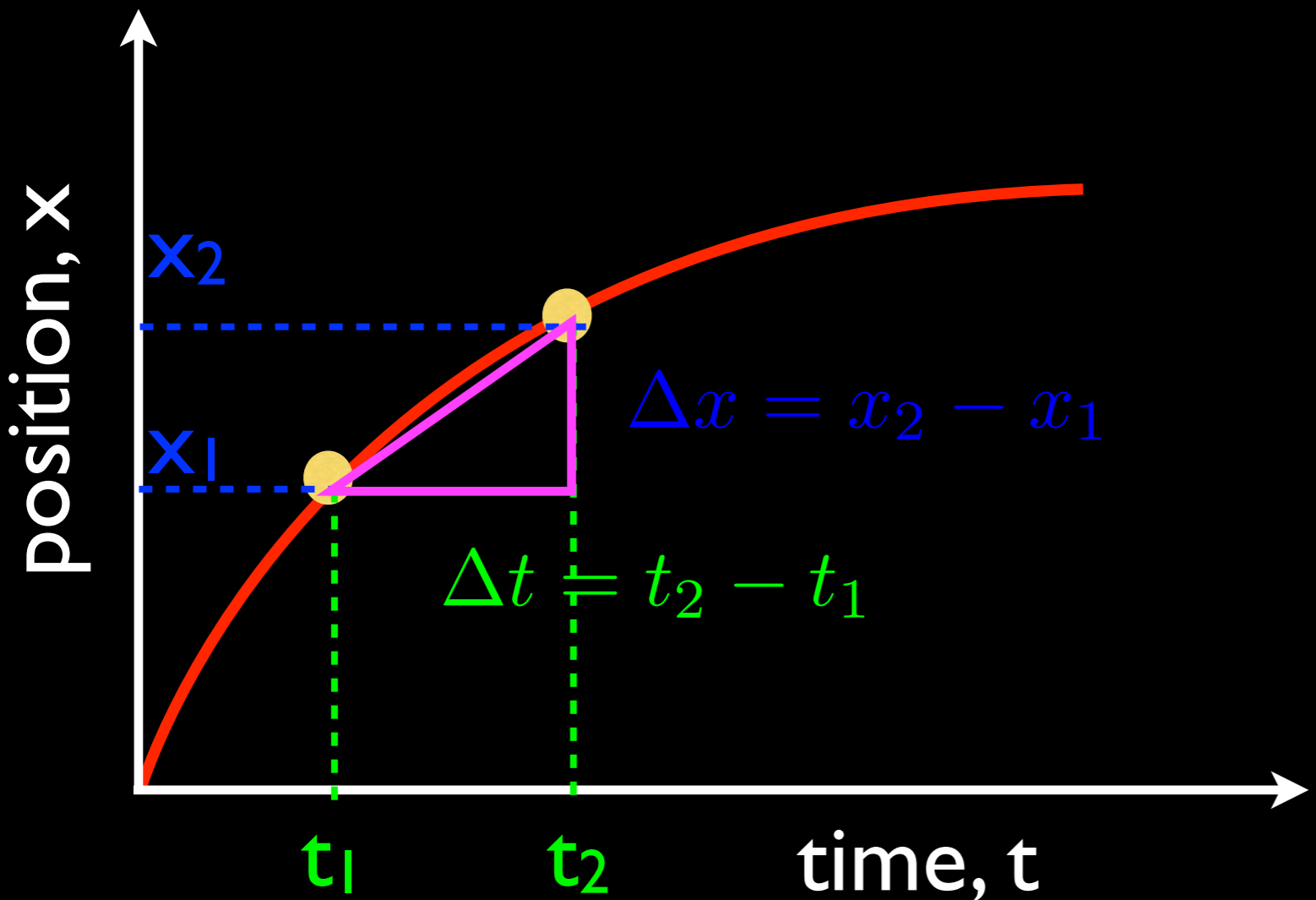
$$v_{\text{av}} = \frac{\Delta x}{\Delta t}$$

Position-versus-time graph

# Motion in 1D



## Average & Instantaneous Velocity



$$v_{av} = \frac{\Delta x}{\Delta t}$$

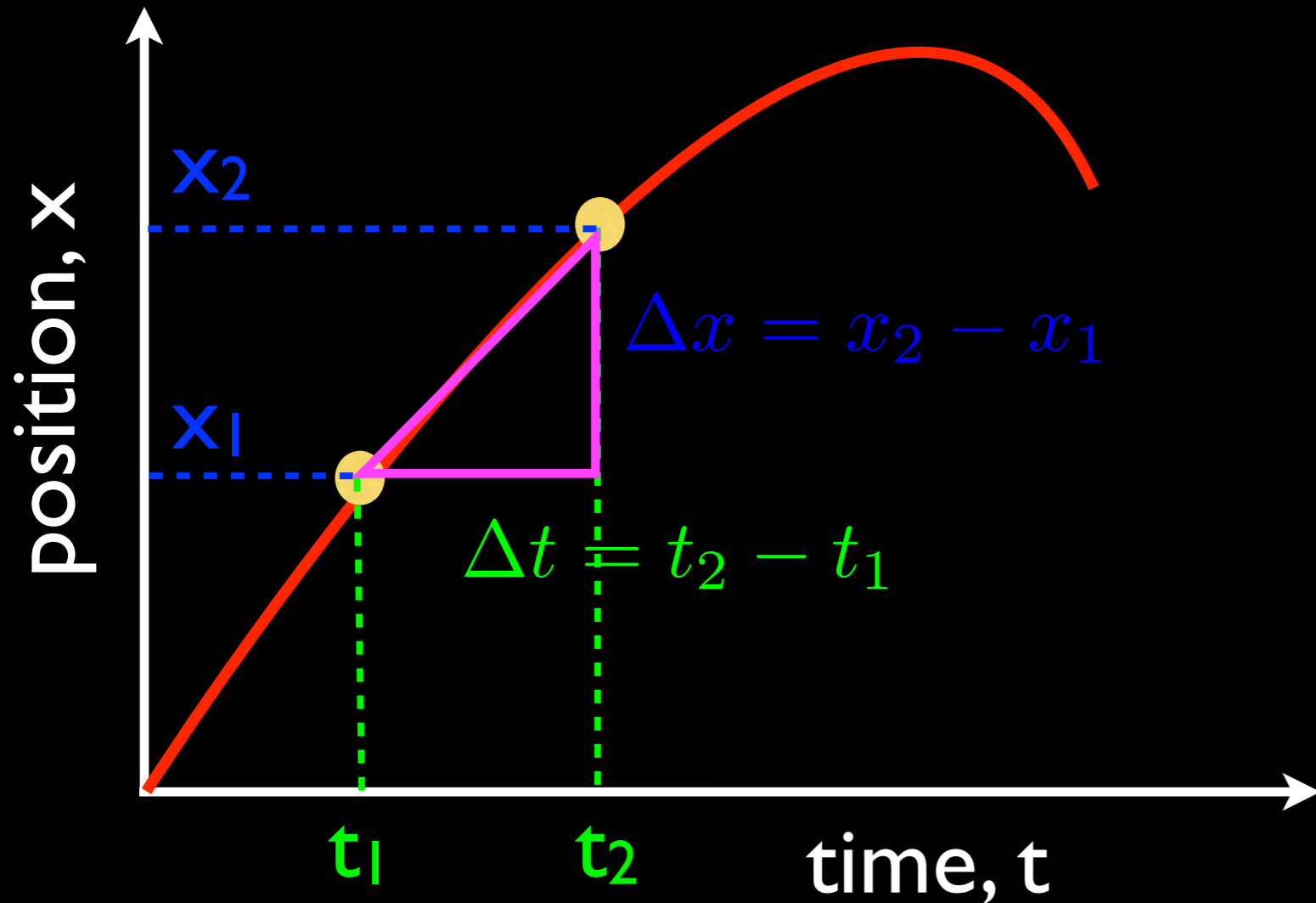
Position-versus-time graph



# Motion in 1D



## Average & Instantaneous Velocity



$$v_{\text{av}} = \frac{\Delta x}{\Delta t}$$

$\Delta x$  has increased  $\uparrow$   
 $\Delta t$  is the same  $=$

We have moved further in the same time.

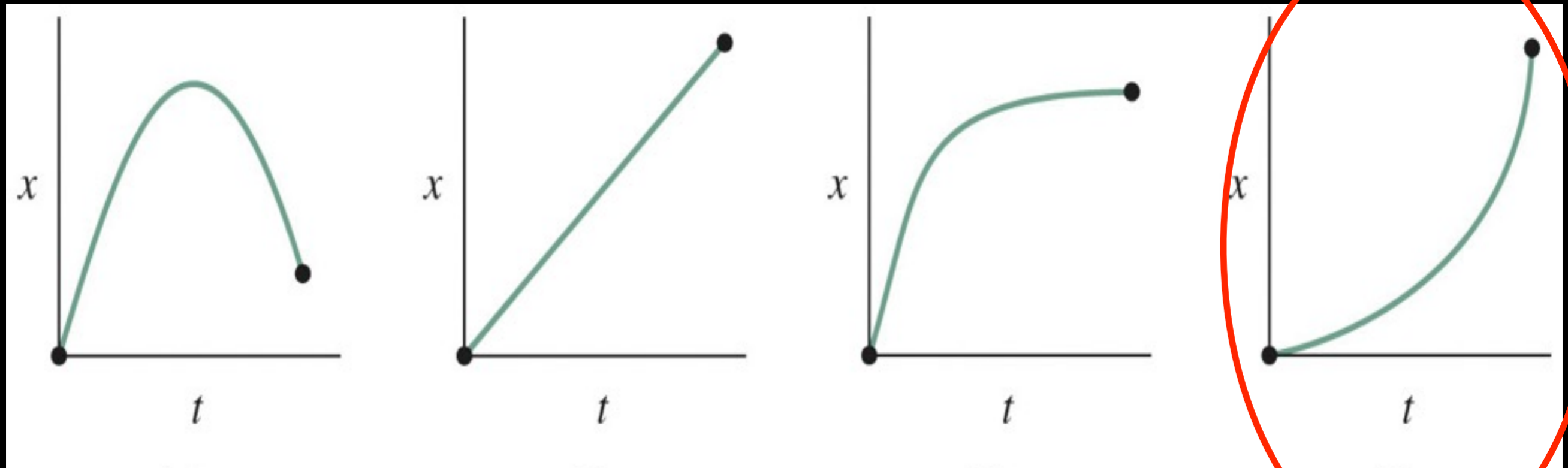
➔ Velocity is higher.

➔ **Gradient** (slope) is steeper.

same  
meaning

# Motion in 1D

# Quiz



A.

B.

C.

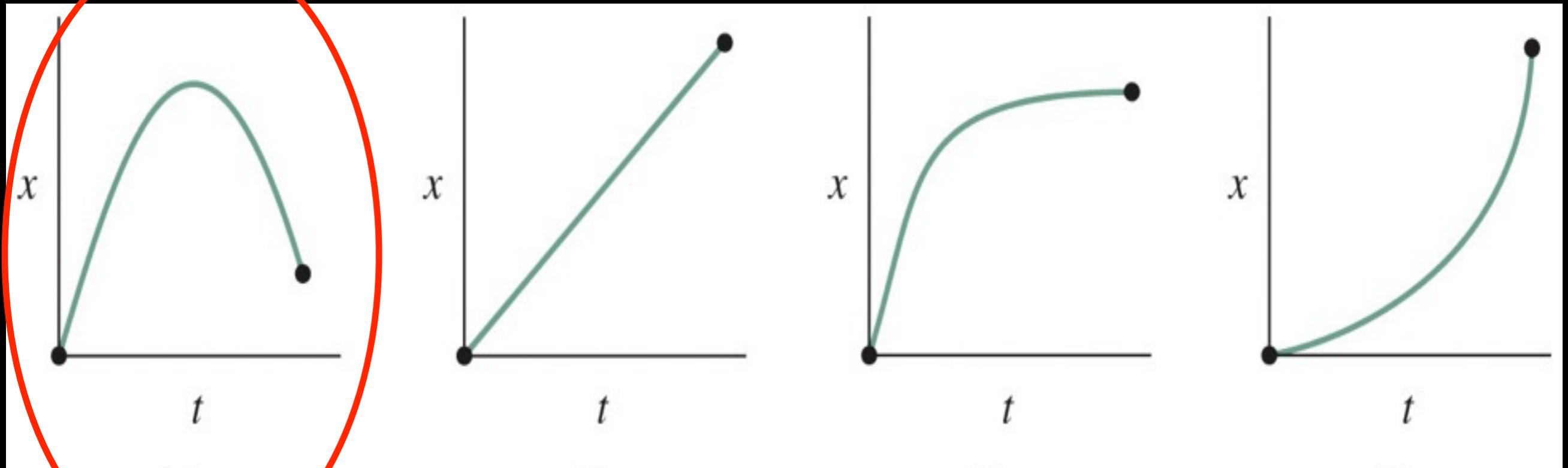
D.

Which graph shows an object starting slowly, then becoming faster?

Gradient increases ➔ Velocity is higher.

# Motion in 1D

# Quiz



A.

B.

C.

D.

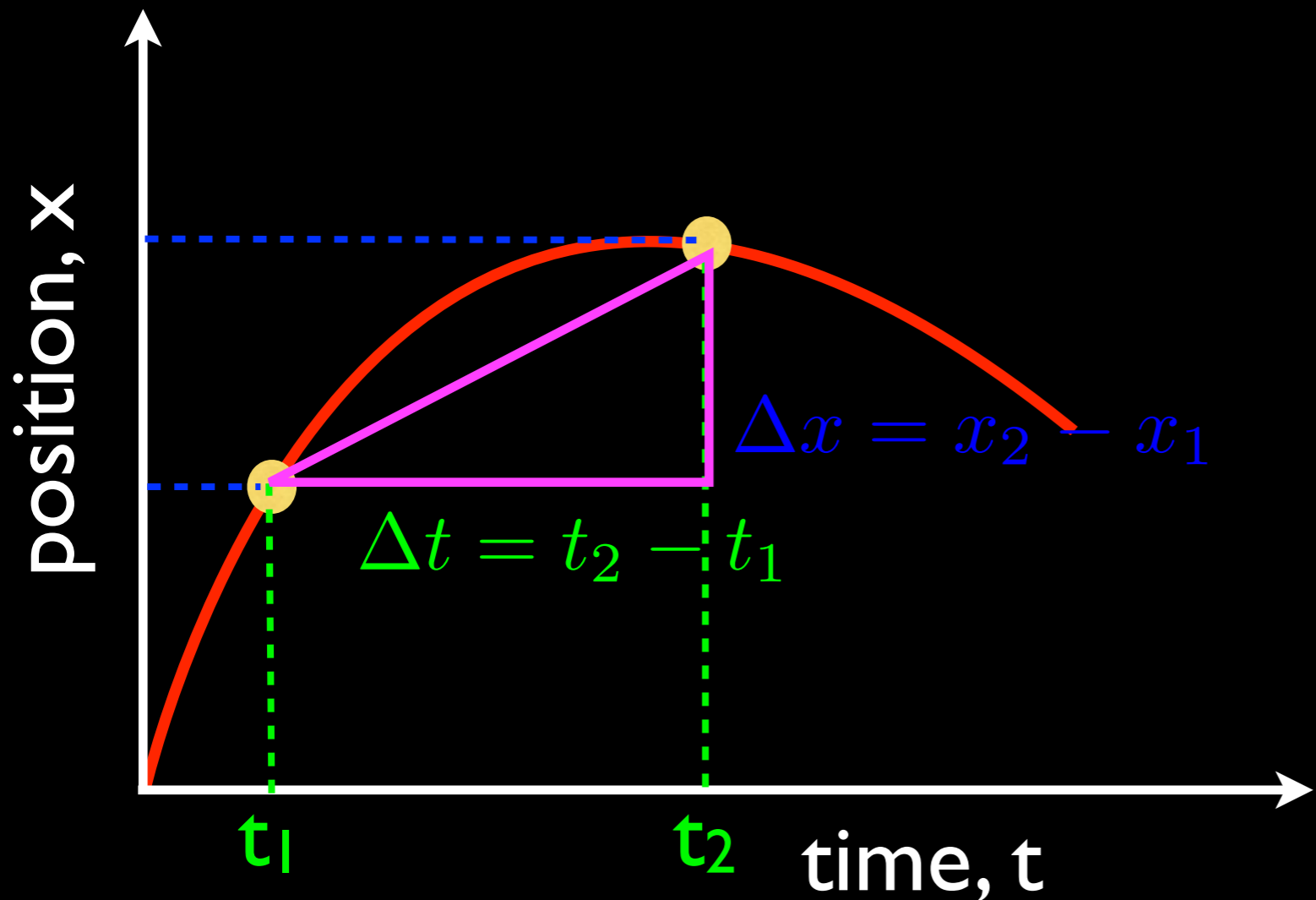
Which graph shows an object changing direction?



# Motion in 1D



## Average & Instantaneous Velocity

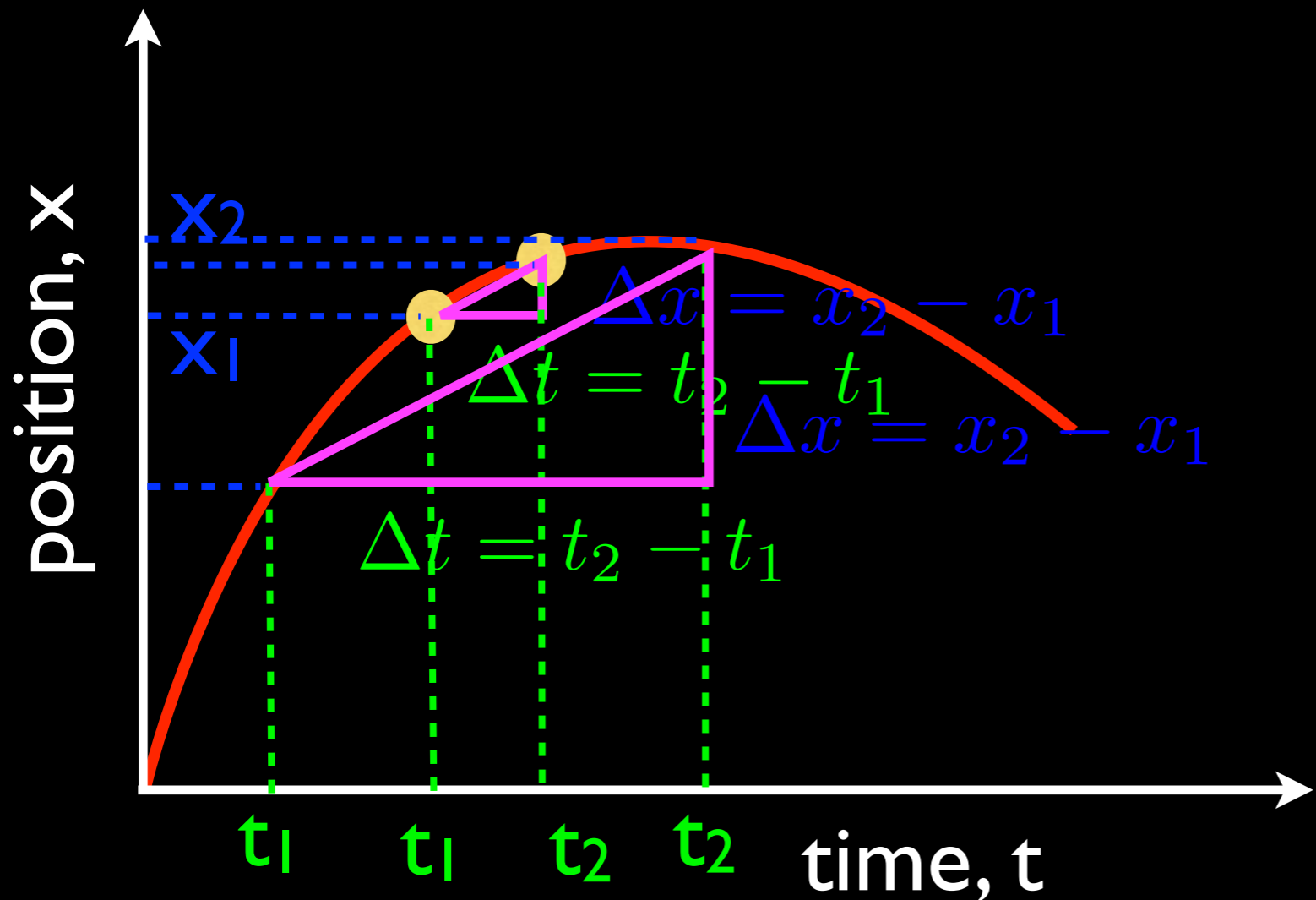


$$v_{av} = \frac{\Delta x}{\Delta t}$$

# Motion in 1D



## Average & Instantaneous Velocity

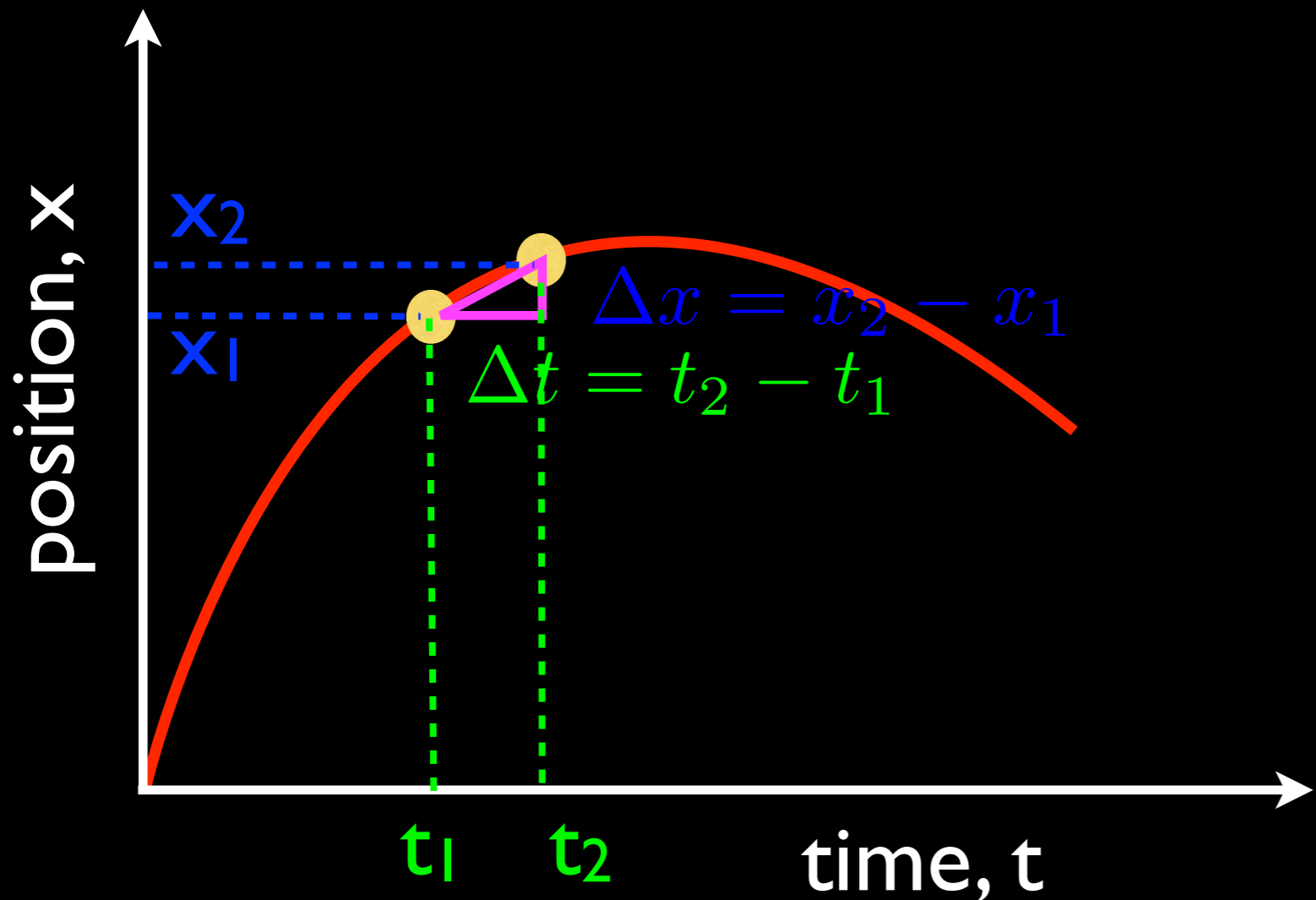


$$v_{\text{av}} = \frac{\Delta x}{\Delta t}$$


# Motion in 1D




## Average & Instantaneous Velocity



$$v_{av} = \frac{\Delta x}{\Delta t}$$

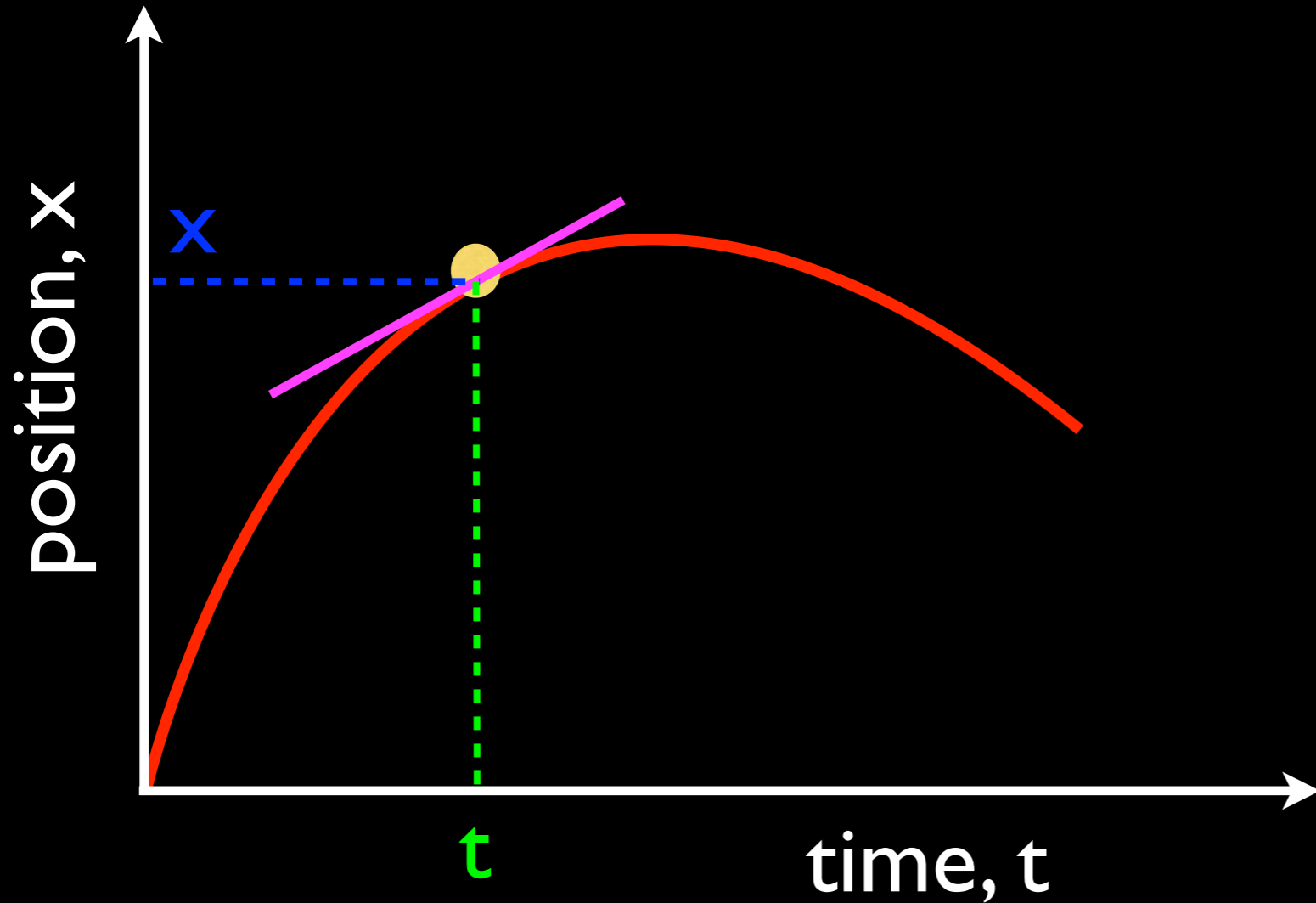
As the points get closer:   $\Delta t$  gets smaller

average velocity  instantaneous velocity

# Motion in 1D



## Average & Instantaneous Velocity



$$v = \frac{dx}{dt}$$

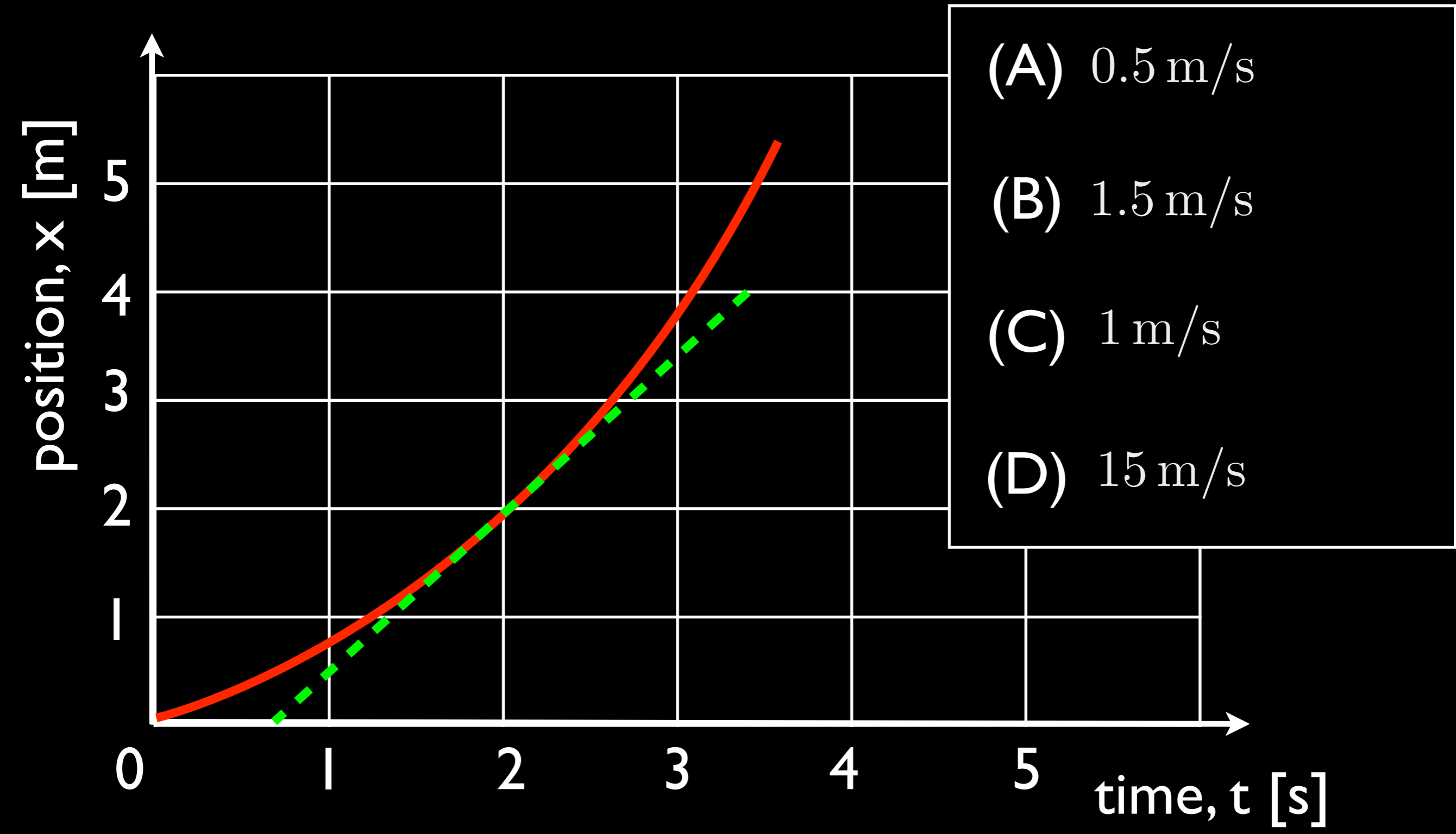
Instantaneous velocity = gradient at  $(x,t)$

$$v = \frac{dx}{dt}$$

# Motion in 1D

# Quiz

Find the average velocity between  $t = 0$  and  $t = 2\text{s}$

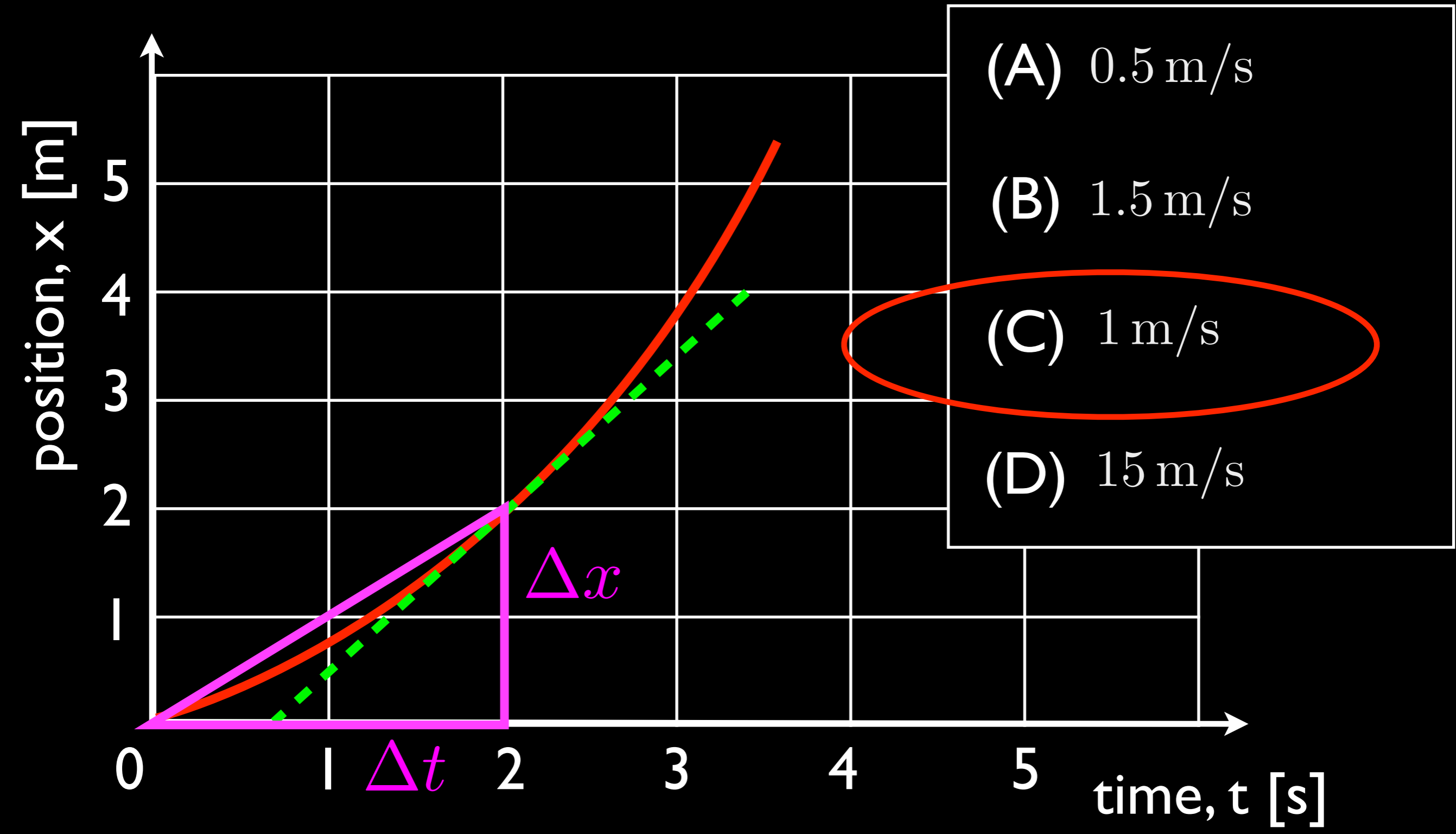




# Motion in 1D

# Quiz

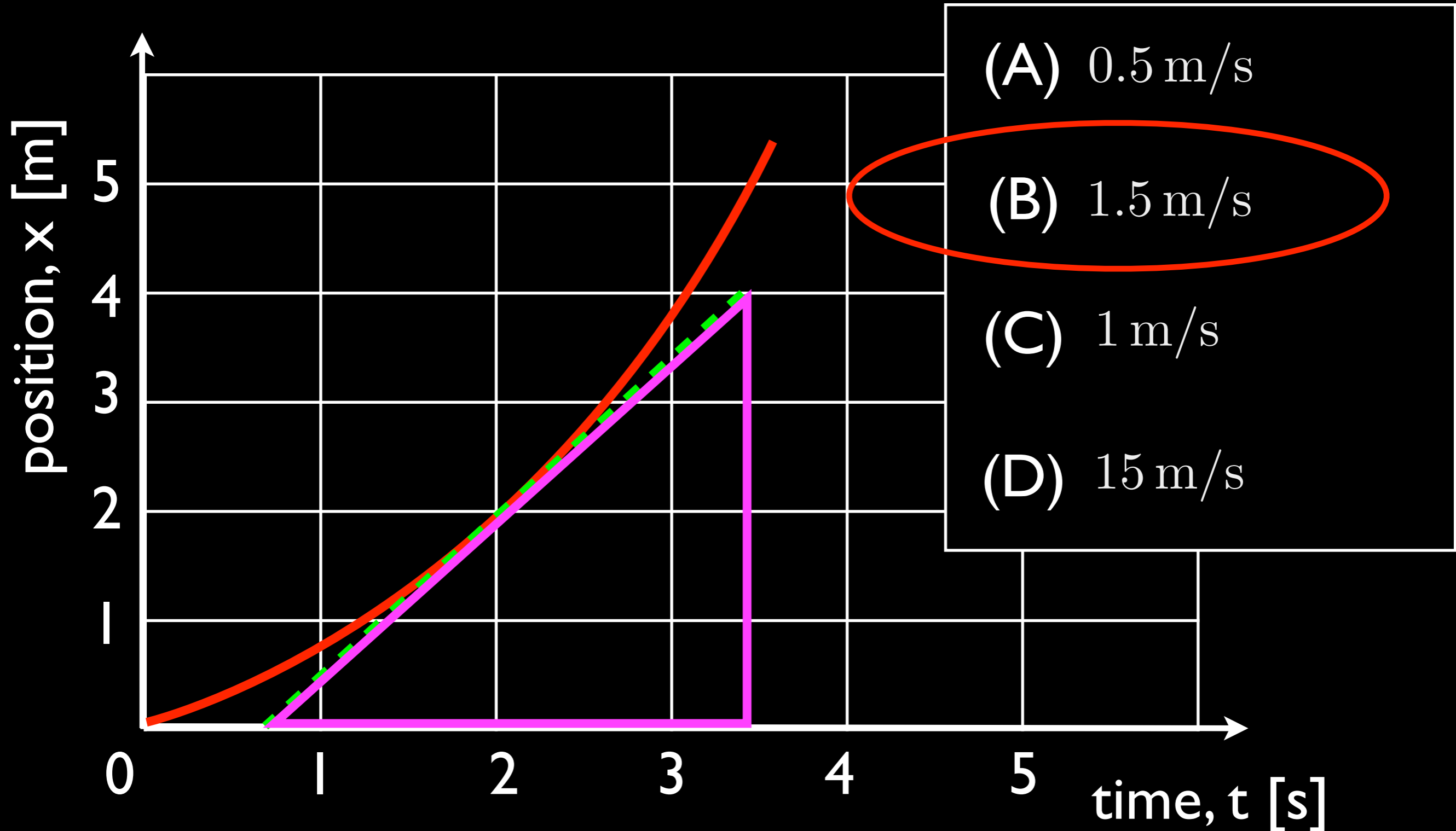
Find the average velocity between  $t = 0$  and  $t = 2\text{ s}$



# Motion in 1D

# Quiz

Find the instantaneous velocity at  $t = 2\text{ s}$



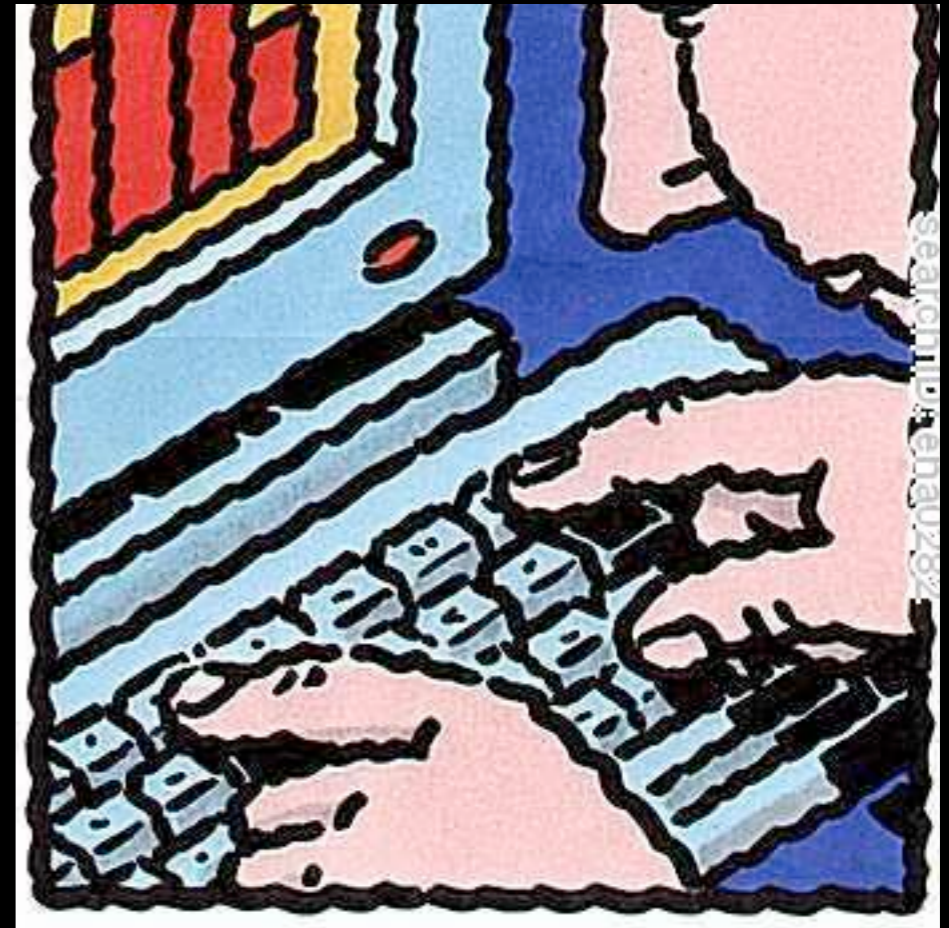
# Motion in 1D



## Average & Instantaneous Velocity

We can find the instantaneous velocity on a graph...

... but this is slow.

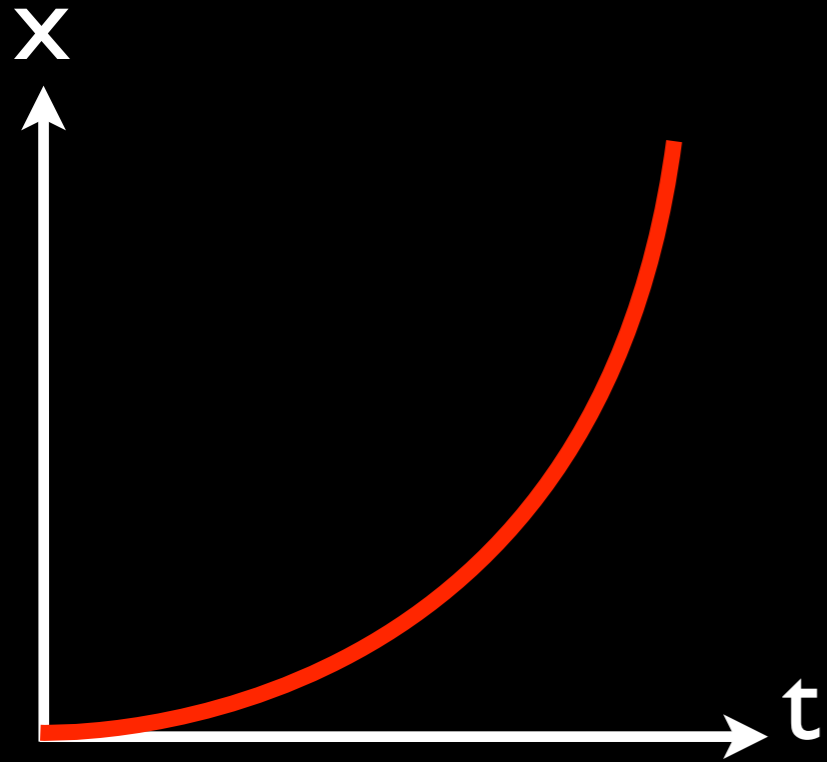


Can we find the instantaneous velocity mathematically?

# Motion in 1D

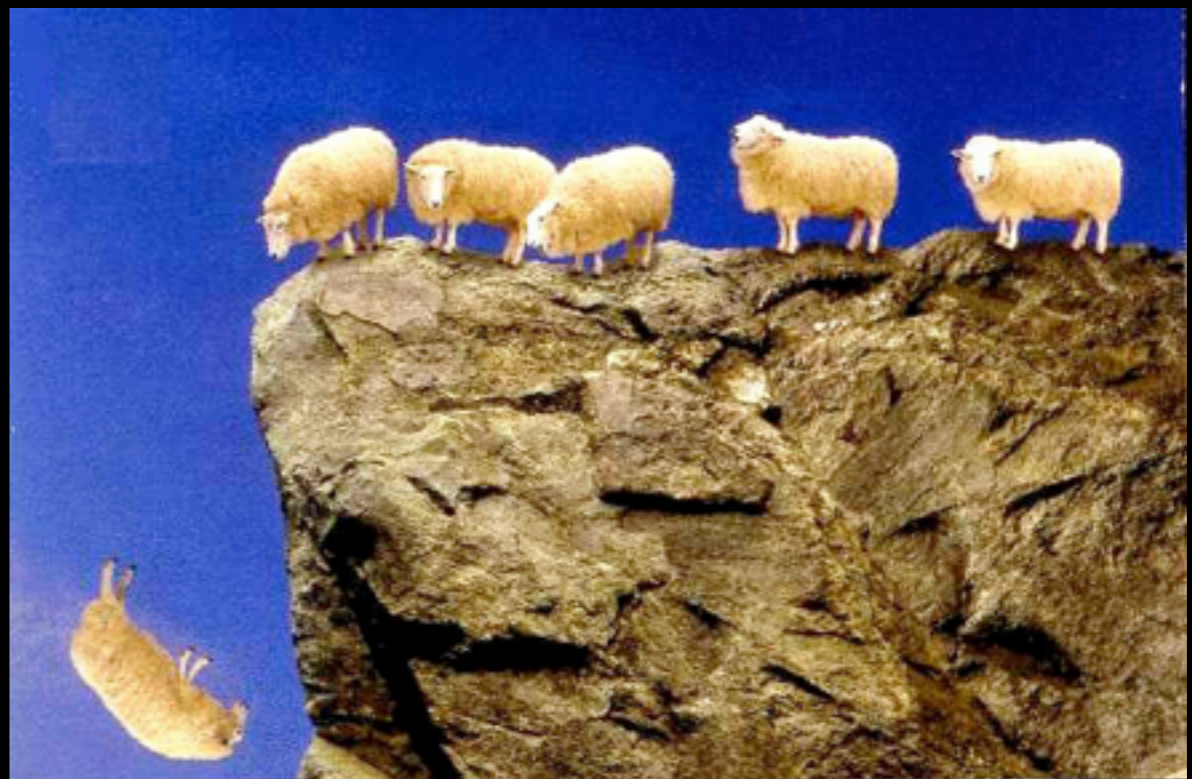


## Average & Instantaneous Velocity



A sheep is dropped from a cliff

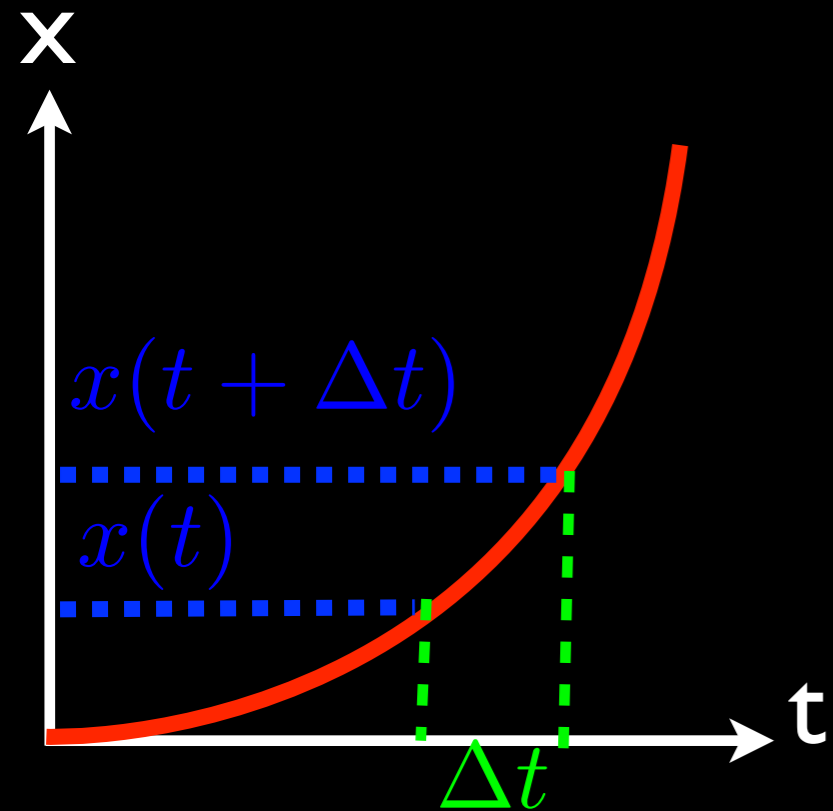
$$x = 5t^2$$



# Motion in 1D



## Average & Instantaneous Velocity



A sheep is dropped from a cliff

$$x = 5t^2$$

Find the velocity at later time

$$t + \Delta t$$

At  $t + \Delta t$ :

$$x(t + \Delta t) = 5(t + \Delta t)^2$$

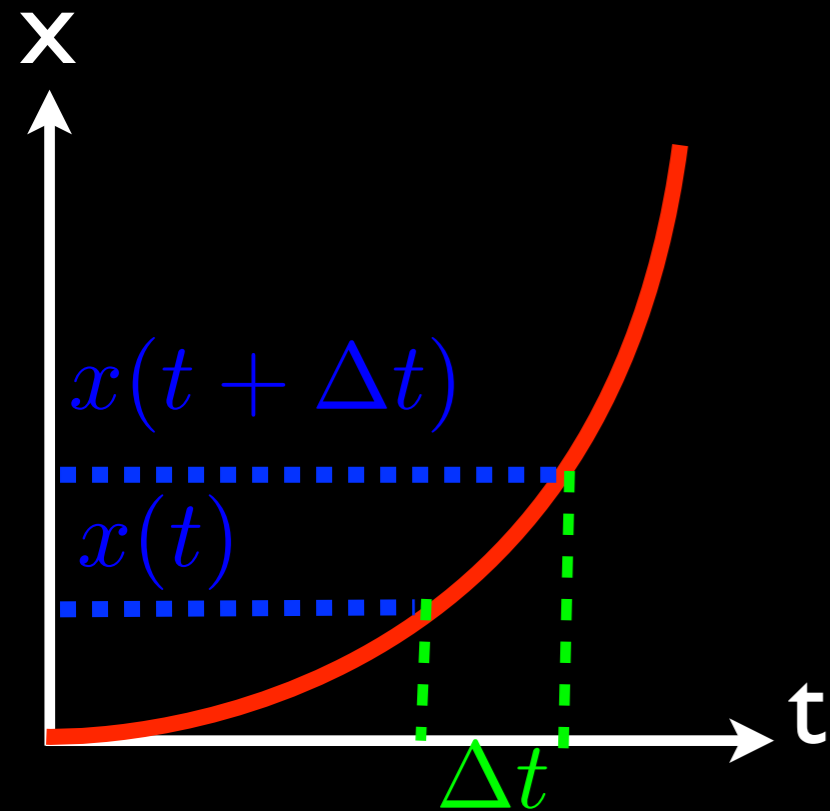
$$= 5[t^2 + 2t\Delta t + (\Delta t)^2]$$

$$= 5t^2 + 10t\Delta t + 5(\Delta t)^2$$

# Motion in 1D



## Average & Instantaneous Velocity



A sheep is dropped from a cliff

$$x = 5t^2$$

Find the velocity at later time

$$t + \Delta t$$

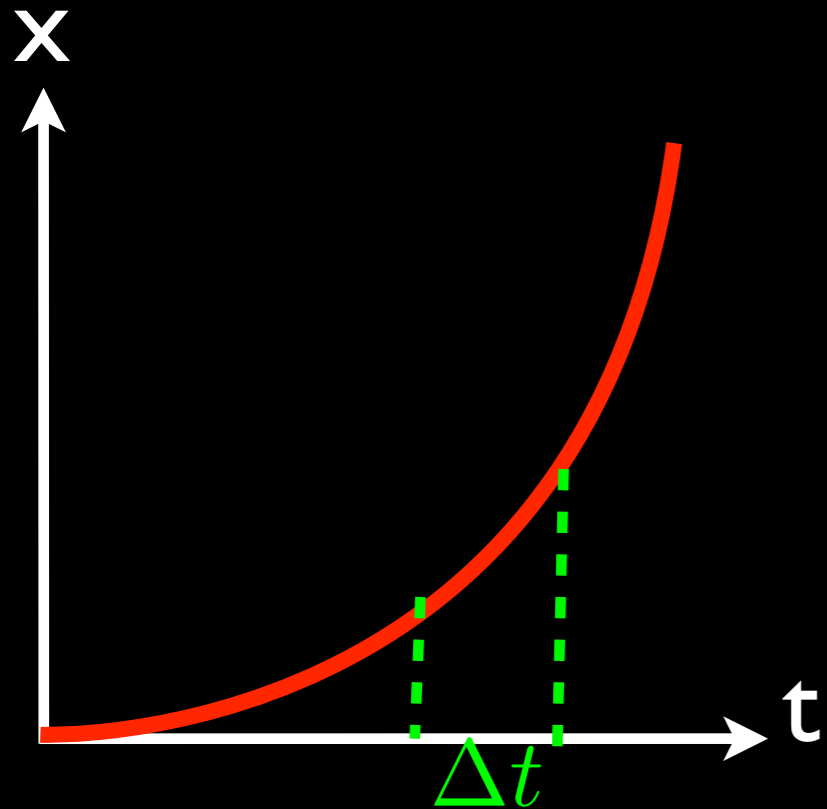
Displacement:

$$\begin{aligned}\Delta x &= x(t + \Delta t) - x(t) \\ &= [5t^2 + 10t\Delta t + 5(\Delta t)^2] - 5t^2 \\ &= 10t\Delta t + 5(\Delta t)^2\end{aligned}$$

# Motion in 1D



## Average & Instantaneous Velocity



A sheep is dropped from a cliff

$$x = 5t^2$$

Find the velocity at later time

$$t + \Delta t$$

Average velocity:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{10t\Delta t + 5(\Delta t)^2}{\Delta t} = 10t + 5\Delta t$$

$$\Delta t \rightarrow 0 \quad v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = 10t = \frac{dx}{dt}$$

# Motion in 1D



## Average & Instantaneous Velocity

Instantaneous velocity:

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Velocity is a **rate of change**

e.g. position,  
velocity....



The **differential** gives the rate of change of a quantity at a single point in time.



# Motion in 1D



## Average & Instantaneous Velocity

### Differentiation

If:  $x = bt^n$        $b, n = \text{constants}$

Then:  $\frac{dx}{dt} = nbt^{n-1}$



## Average & Instantaneous Velocity

The height of a rocket:  $x = 2.9t^2$  m

Find the (instantaneous) velocity at 20s

(A) 1160 m/s

(C) 58 m/s

(B) 116 m/s

(D) 20 m/s



## Average & Instantaneous Velocity

The height of a rocket:  $x = 2.9t^2$  m

Find the (instantaneous) velocity at 20s

$$v = \frac{dx}{dt} = 2.9 \times 2 \times t$$

$$v = 5.8 \times 20 = 116\text{m/s}$$

# Motion in 1D



## Acceleration

If  $v = \frac{dx}{dt}$  then what is  $\frac{dv}{dt} = ?$

A change in velocity is **acceleration**

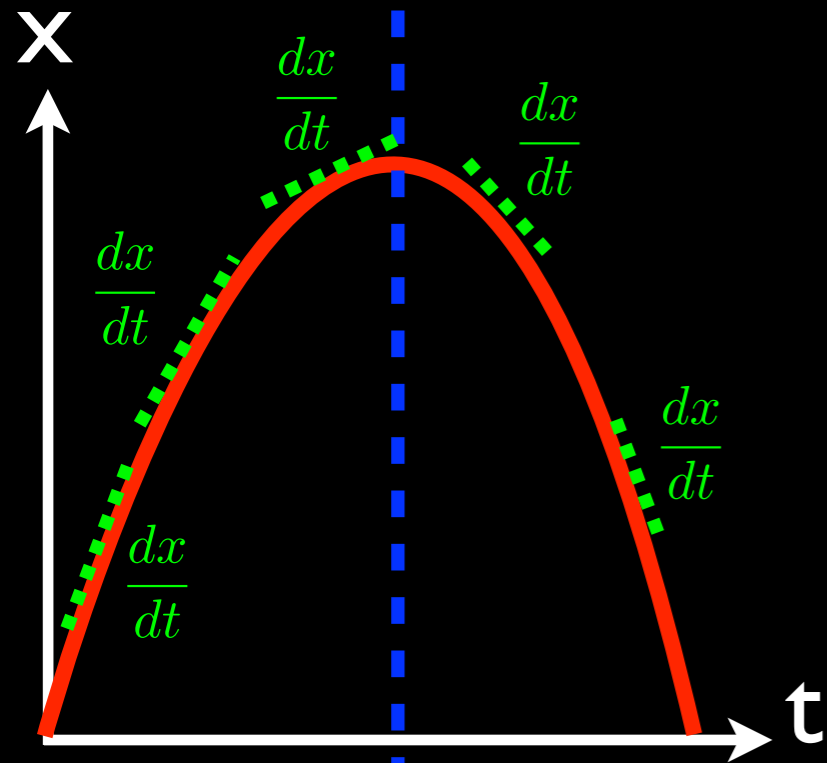
$$\text{average acceleration} = \frac{\Delta v}{\Delta t}$$

$$\text{(instantaneous) acceleration} = \frac{dv}{dt}$$

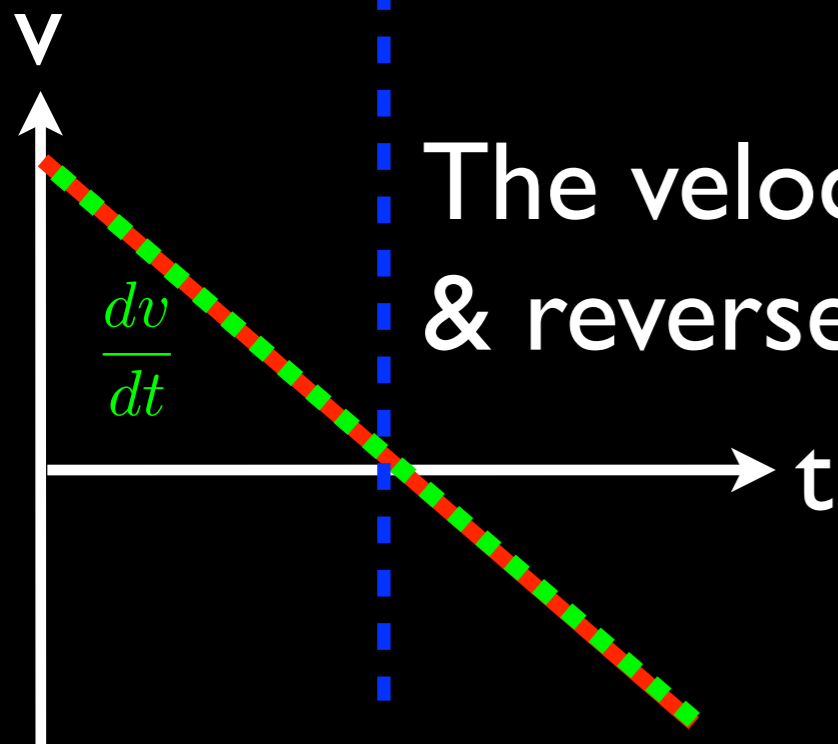
# Motion in 1D



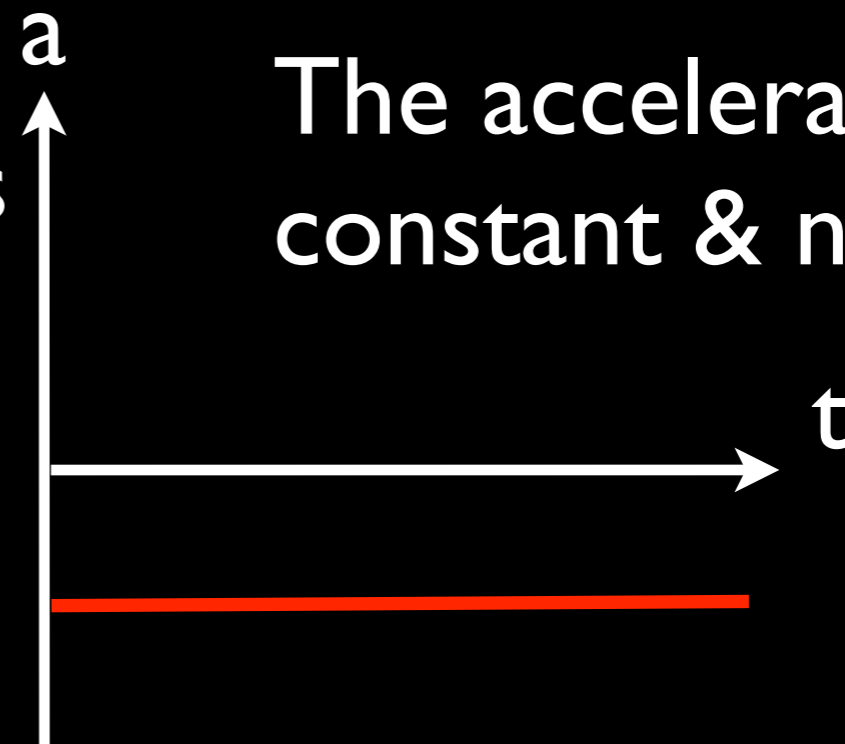
## Acceleration



A ball is thrown vertically into the air



The velocity decreases & reverses direction



The acceleration is constant & negative

# Motion in 1D

# Quiz

A Subaru Impreza does  
0 - 60 mph in 5.2 s

What is its average  
acceleration?



(60 mph = 100 km/h)

(A)  $5.34 \text{ m/s}^2$

(B)  $19.2 \text{ m/s}^2$

(C)  $11.5 \text{ m/s}^2$

(D)  $19231 \text{ m/s}^2$

# Motion in 1D

A Subaru Impreza does  
0 - 60 mph in 5.2 s

What is its average  
acceleration?



(60 mph = 100 km/h)

$$60 \text{ mph} = 100 \text{ km/h} = 100 \left( \frac{\text{km}}{\text{h}} \right) \times \left( \frac{1000 \text{ m}}{\text{km}} \right) \times \left( \frac{\text{h}}{3600 \text{ s}} \right)$$
$$= 27.8 \text{ m/s}$$

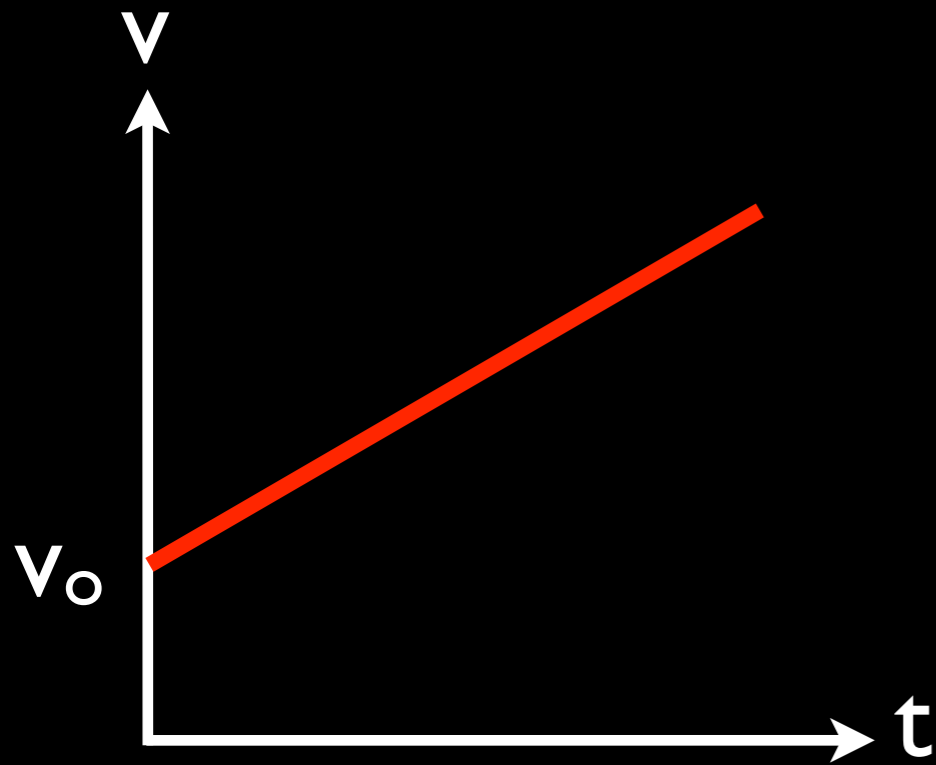
$$a_{\text{av}} = \frac{\Delta v}{\Delta t} = \frac{27.8 \text{ m/s}}{5.2 \text{ s}} = 5.34 \text{ m/s}^2$$

# Motion in 1D



## Acceleration

When the acceleration is constant, the **equations for motion** are simple



$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0}$$

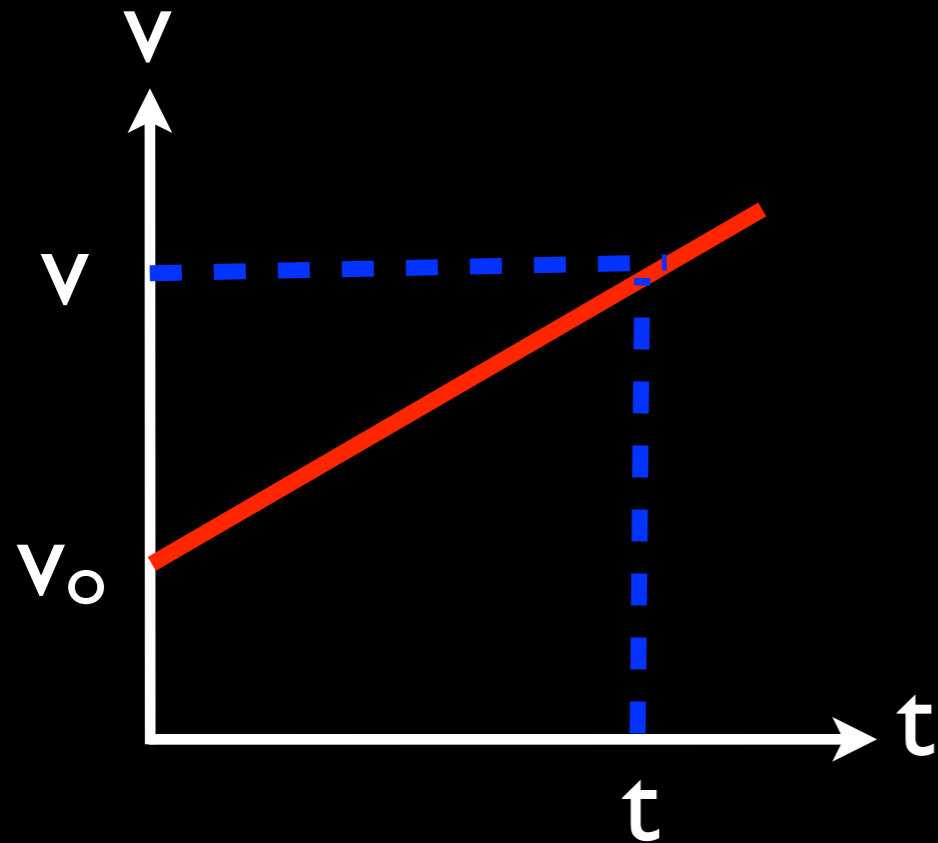

$$v = v_0 + at$$



# Motion in 1D



## Acceleration



Average velocity over time,  $t$ :

$$v_{\text{av}} = \frac{1}{2}(v_0 + v)$$

But, we know:

$$v_{\text{av}} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0}$$

Therefore:  $x = x_0 + \frac{1}{2}(v_0 + v)t$

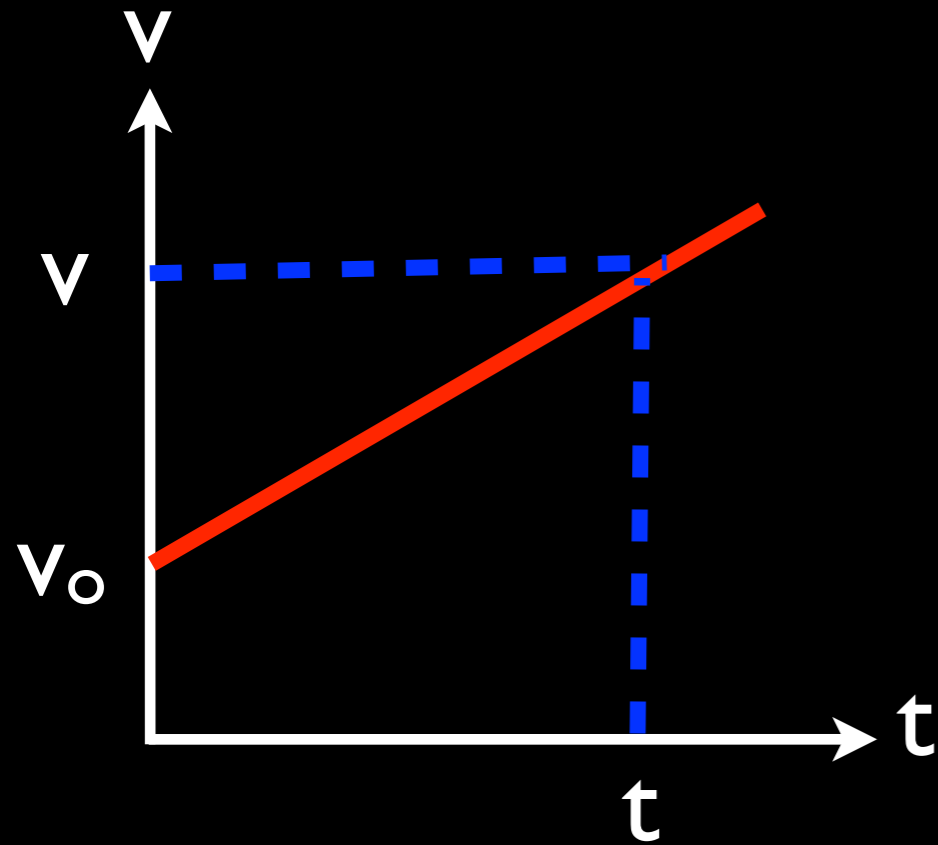
But:  $v = v_0 + at$

So:  $x = x_0 + v_0 t + \frac{1}{2}at^2$

# Motion in 1D



## Acceleration



Finally, since:

$$v = v_0 + at$$

Then:

$$t = \frac{v - v_0}{a}$$

and since:

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

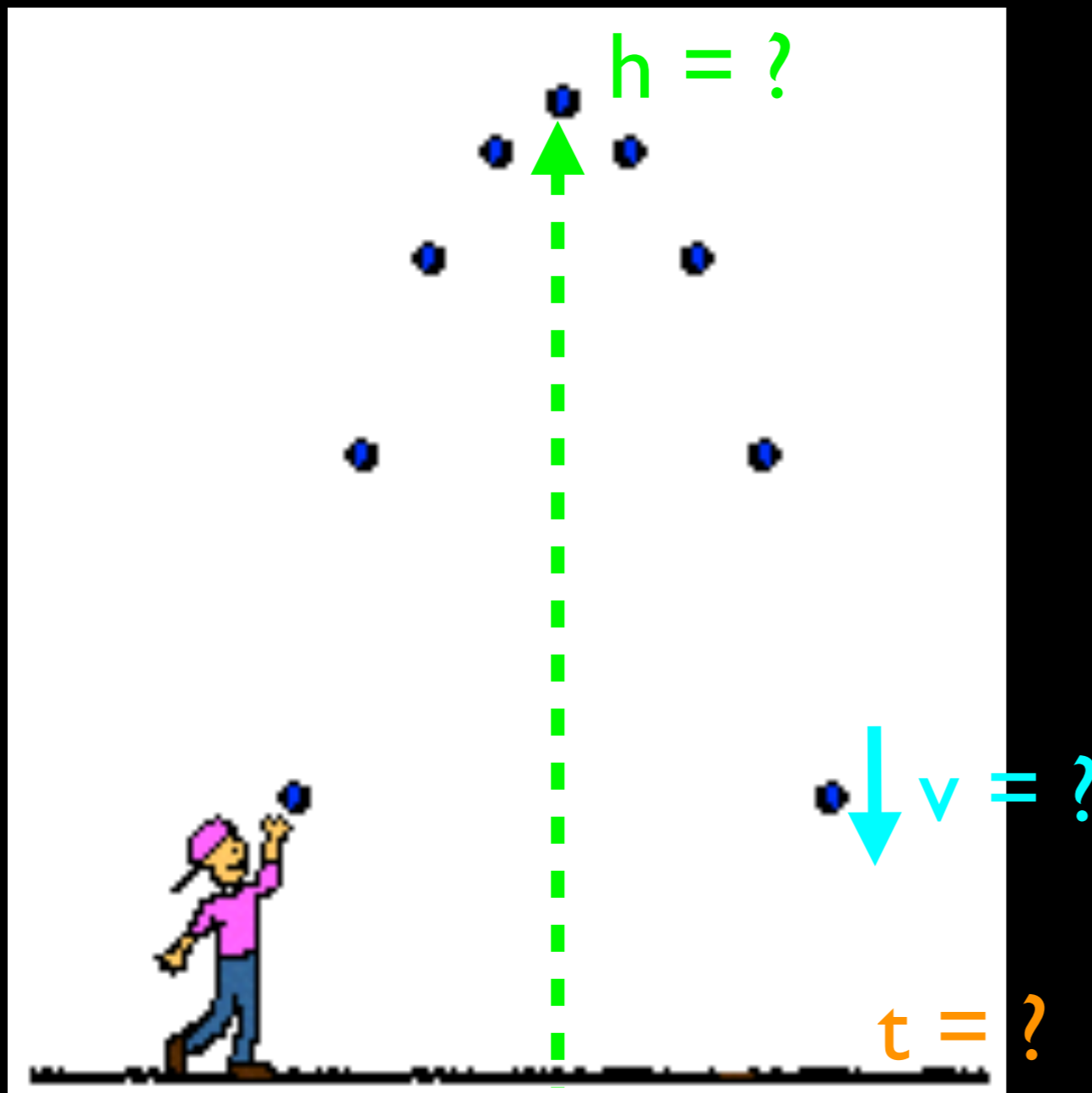
So:

$$v^2 = v_0^2 + 2a(x - x_0)$$

# Motion in 1D

## Example

You throw a ball with an initial velocity  $v_0 = 7.3 \text{ m/s}$ , at a height of  $1.5 \text{ m}$ , with  $a = 9.8 \text{ m/s}^2$  (g) downward.



What is the maximum height?

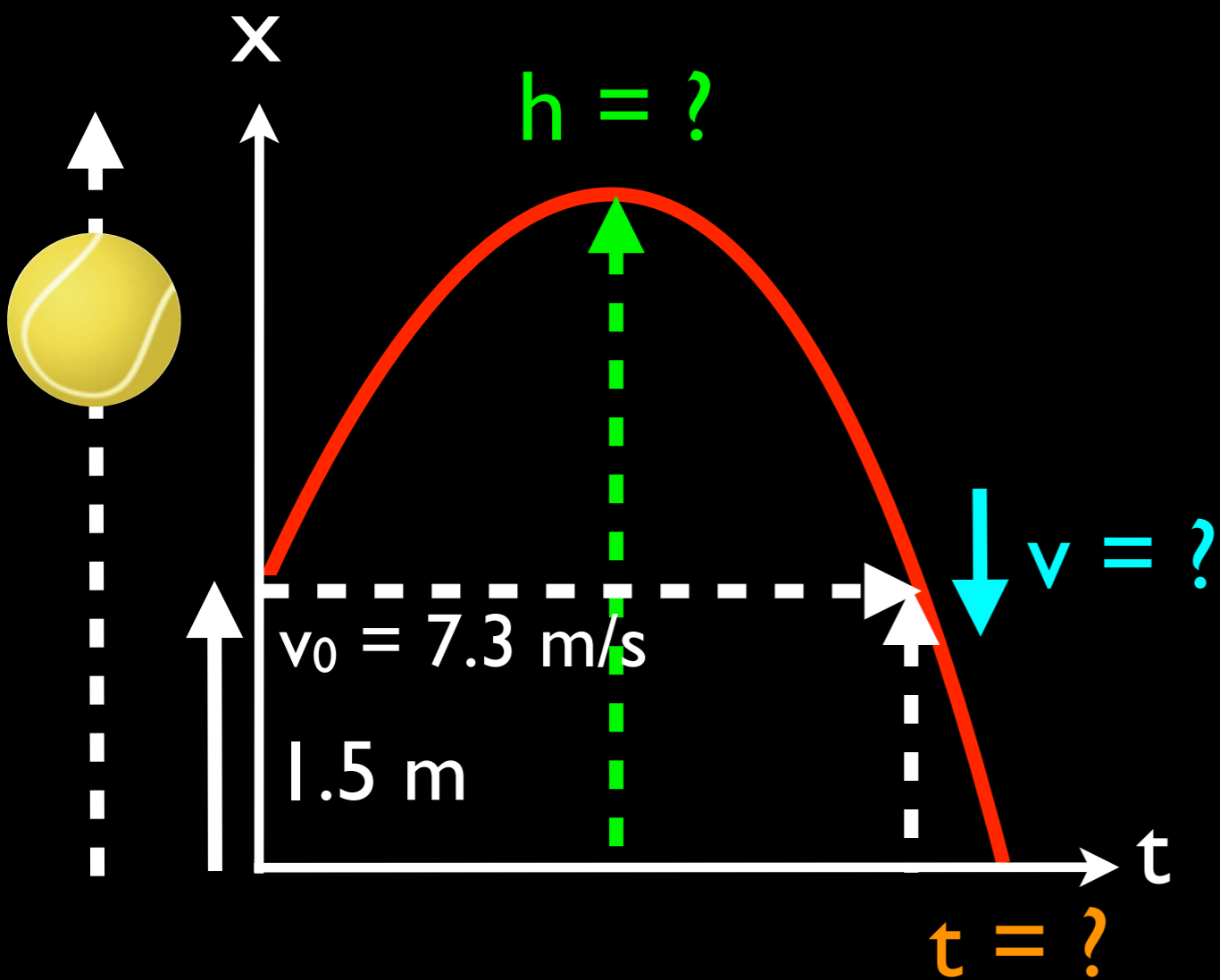
What is the speed when it passes your hand again?

When does it hit the floor?

# Motion in 1D

## Example

You throw a ball with an initial velocity  $v_0 = 7.3 \text{ m/s}$ , at a height of  $1.5 \text{ m}$ , with  $a = 9.8 \text{ m/s}^2$  ( $g$ ) downward.



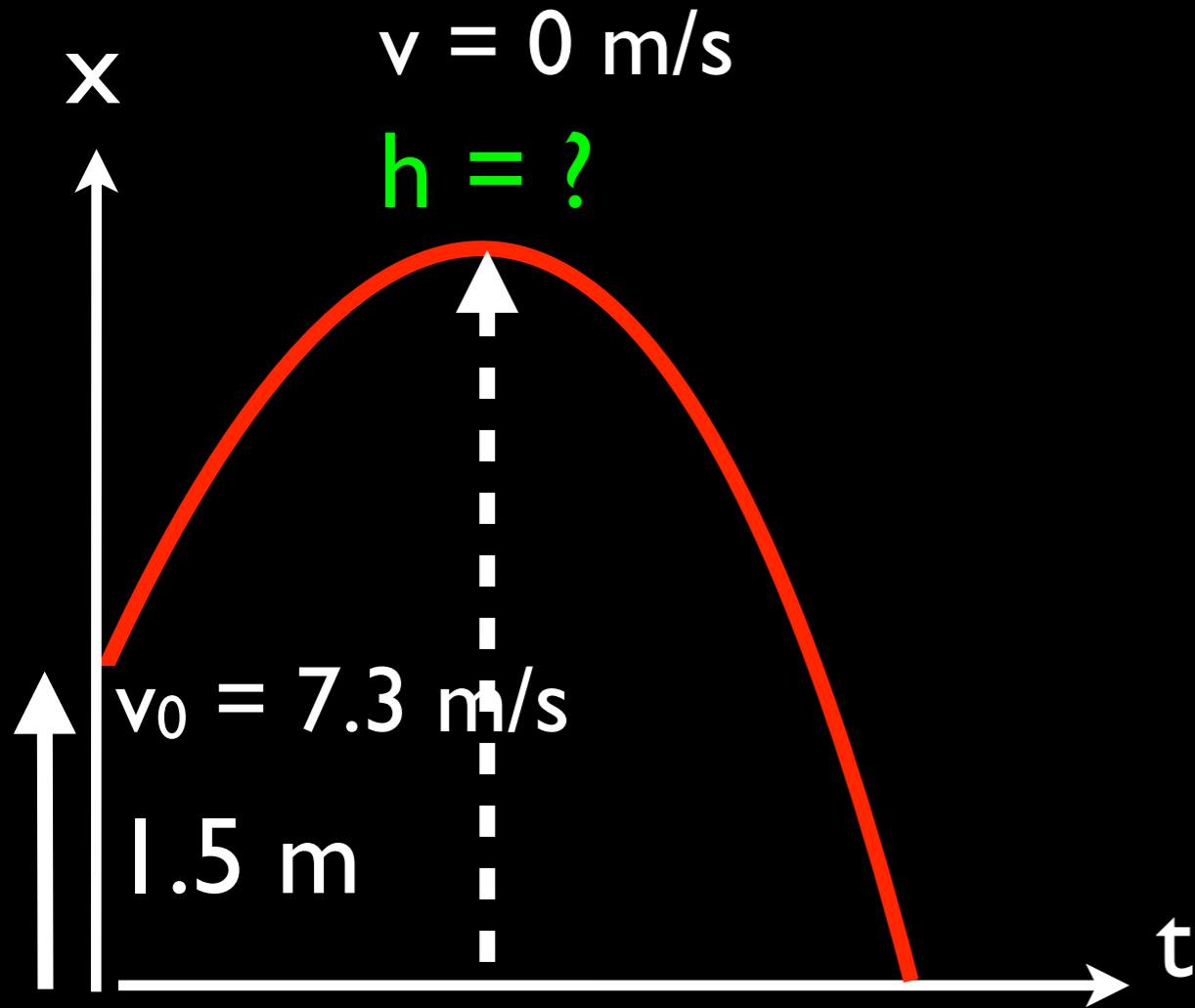
What is the maximum height?

What is the speed when it passes your hand again?

When does it hit the floor?

# Motion in 1D

## Example



What is the maximum height?



At maximum height,  $v = 0$

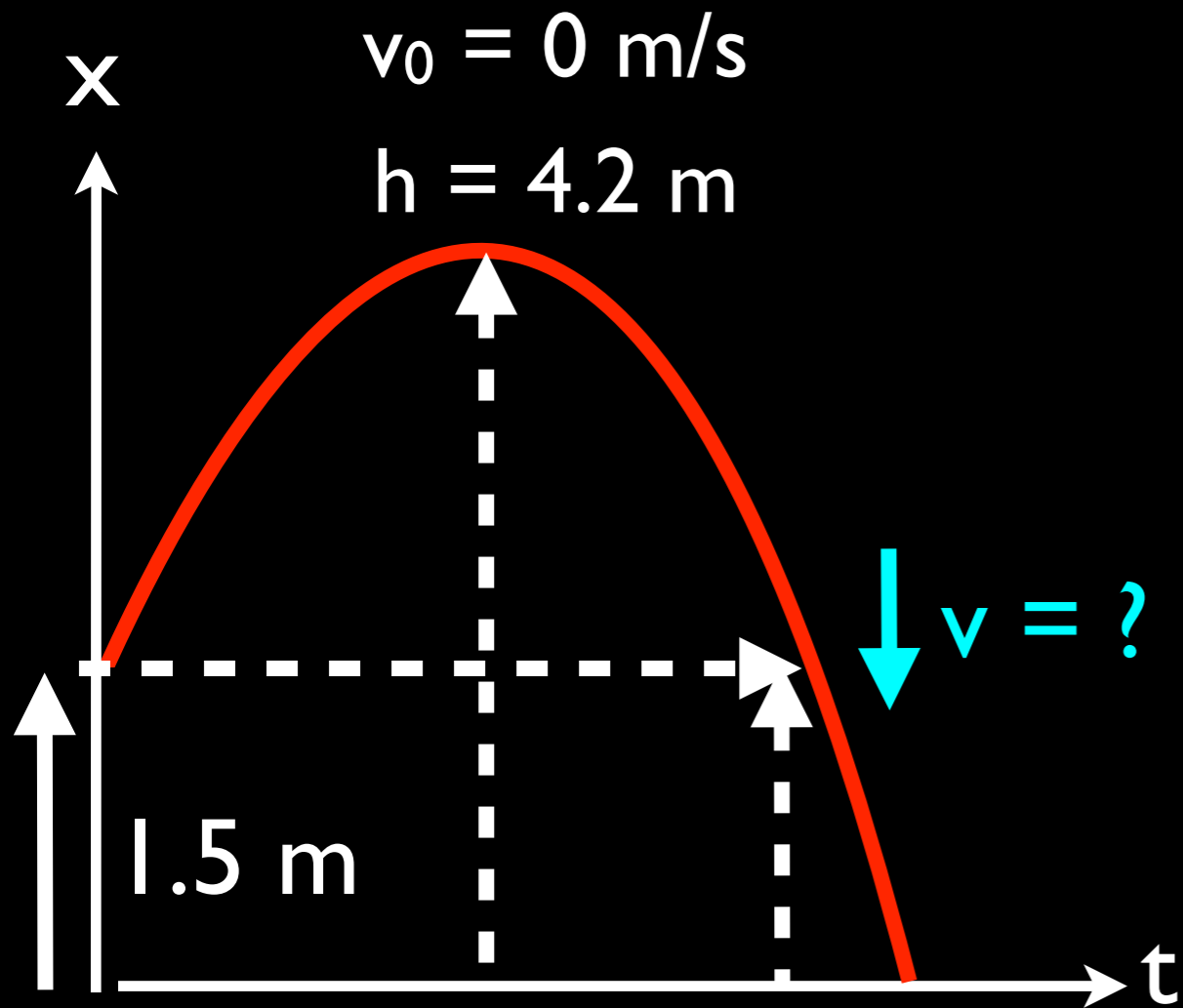
$$v^2 = v_0^2 + 2a(x - x_0)$$

$$0 = (7.3 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(h - 1.5 \text{ m})$$

➔  $h = 4.2 \text{ m}$

# Motion in 1D

## Example



What is the speed when it passes your hand again?

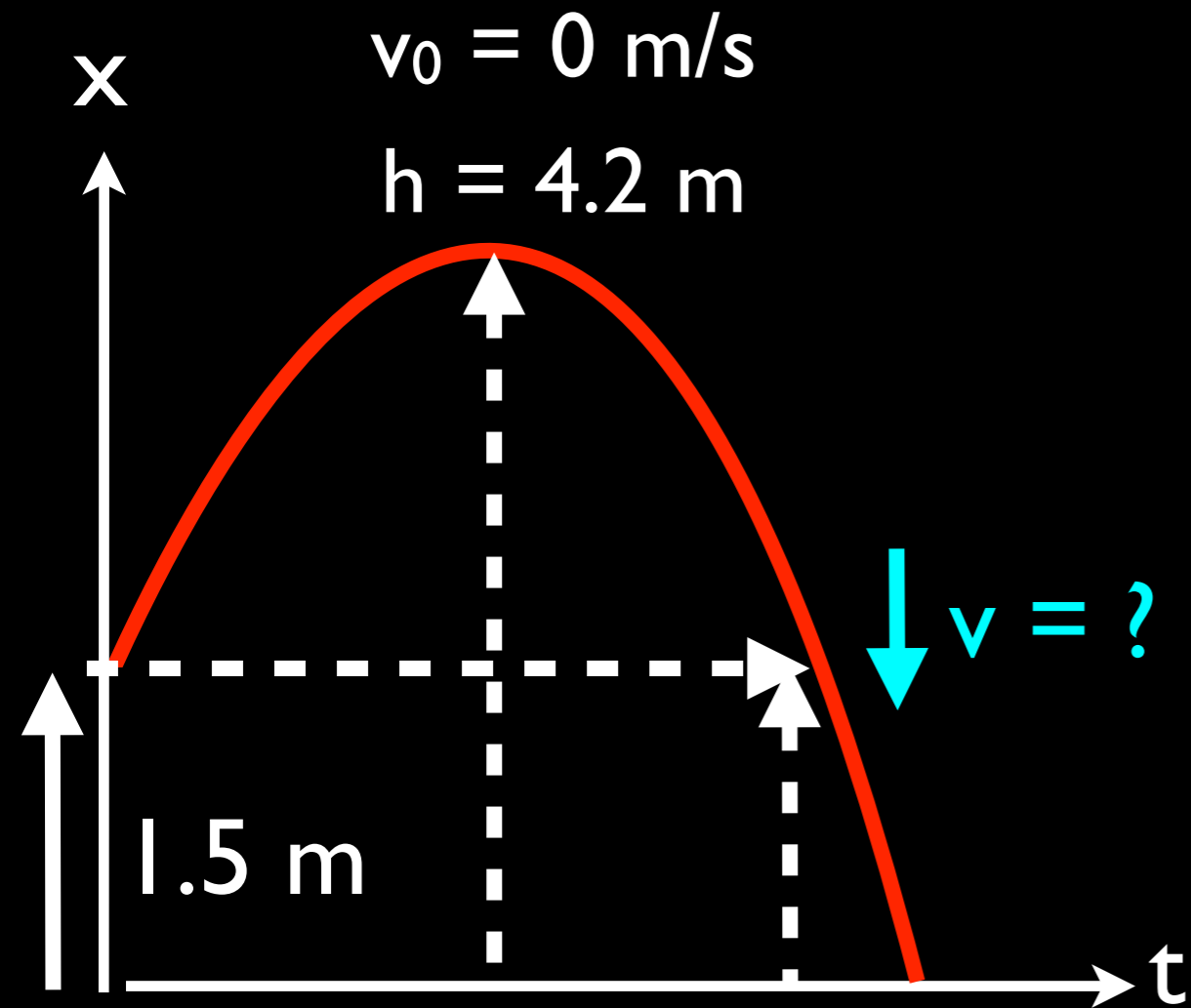
$$v^2 = v_o^2 + 2a(x - x_o)$$

$$= 2(-9.8 \text{ m/s}^2)(1.5 \text{ m} - 4.2 \text{ m})$$

$$= \pm 7.3 \text{ m/s}$$

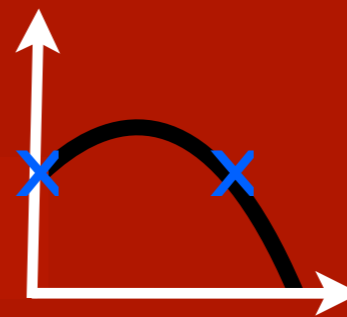
# Motion in 1D

## Example



What is the speed when it passes your hand again?

Why 2 answers?



Because ball is at  $1.5 \text{ m}$  twice!

Why are they the same?

Lecture 6!

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$= 2(-9.8 \text{ m/s}^2)(1.5 \text{ m} - 4.2 \text{ m})$$

$$= \pm 7.3 \text{ m/s}$$

# Motion in 1D

# Example

---

Quadratic equation:

$$at^2 + bt + c = 0$$

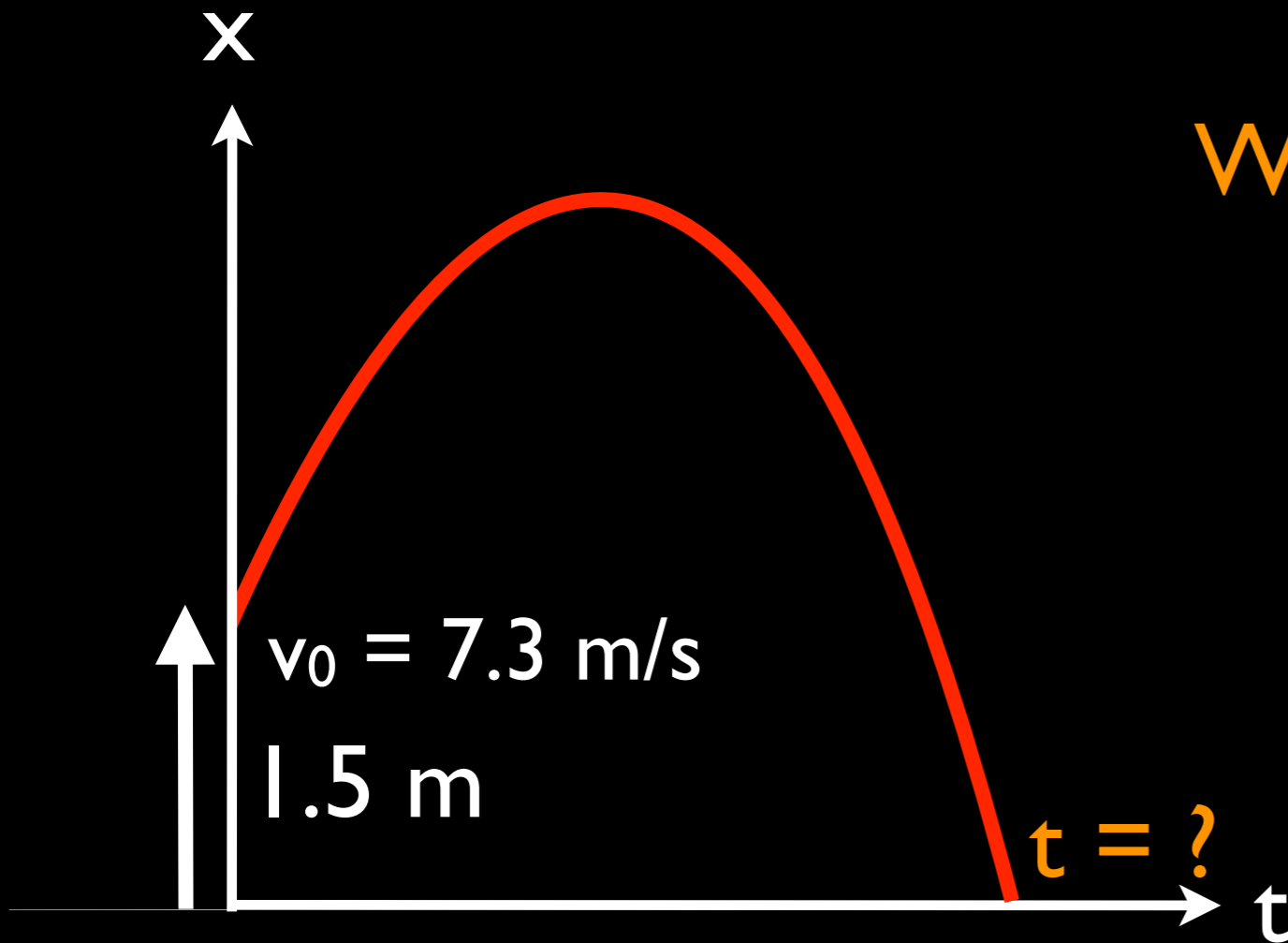
Has 2 solutions:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



# Motion in 1D

## Example



When does it hit the floor?



At floor,  $x = 0$

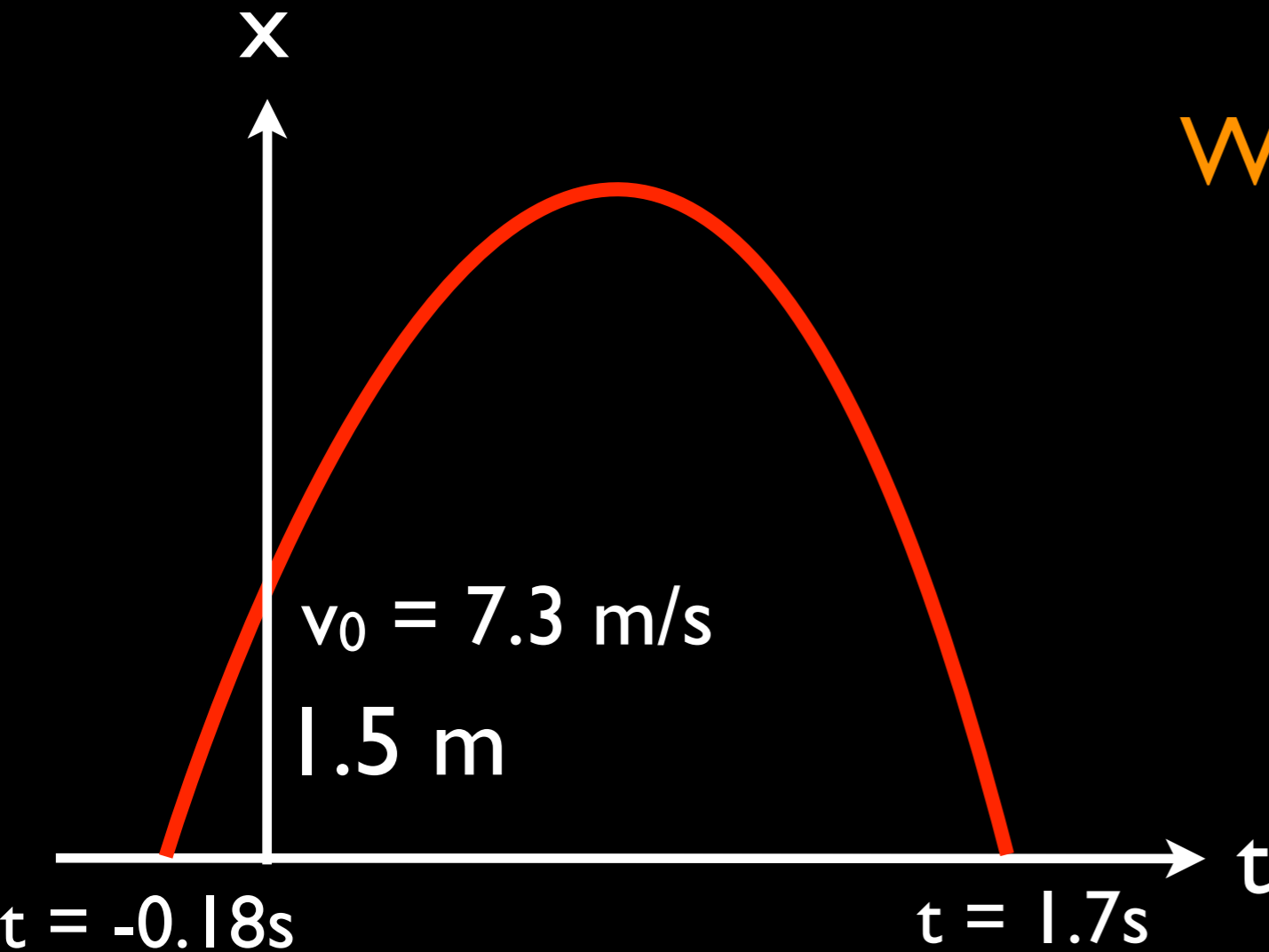
(Quadratic equation)

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \quad \rightarrow \quad t = \frac{-v_0 \pm \sqrt{v_0^2 - 4 \left(\frac{1}{2} a\right) x_0}}{2 \left(\frac{1}{2} a\right)}$$

$$t = \frac{7.8 \pm \sqrt{60.84 + 2(9.8)1.5}}{9.8} = 1.7 \text{ s}$$

# Motion in 1D

# Example



When does it hit the floor?



At floor,  $x = 0$

(Quadratic equation)

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$



$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4 \left(\frac{1}{2}a\right) x_0}}{2 \left(\frac{1}{2}a\right)}$$

$$t = \frac{7.8 \pm \sqrt{60.84 + 2(9.8)1.5}}{9.8}$$

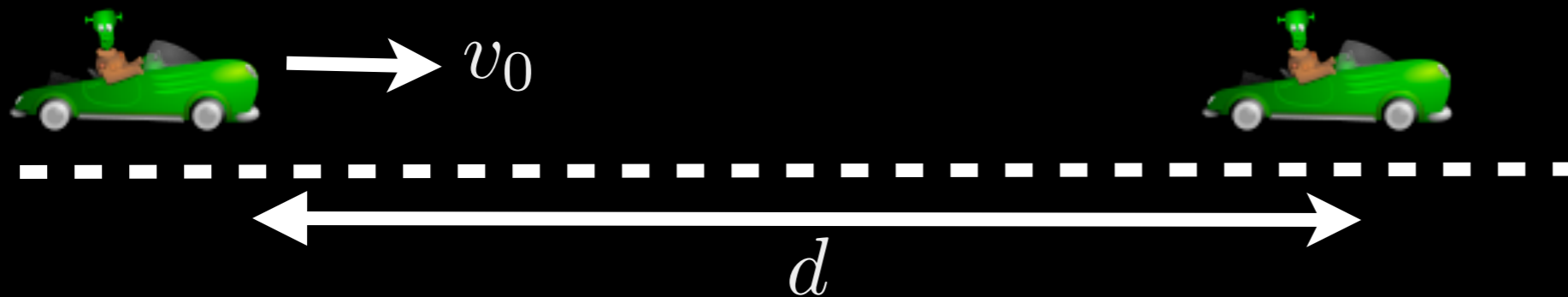
$$= 1.7 \text{ s} \quad \text{— (or = 0.18 s) —}$$

# Motion in 1D

# Quiz

A car is moving at initial speed,  $v_0$

... breaks and comes to a stop in distance  $d$ .



If the car is now moves at initial speed,  $2v_0$

... what is the distance before stopping?

(braking force the same)

(A)  $2d$

(B)  $4d$

(C)  $d$

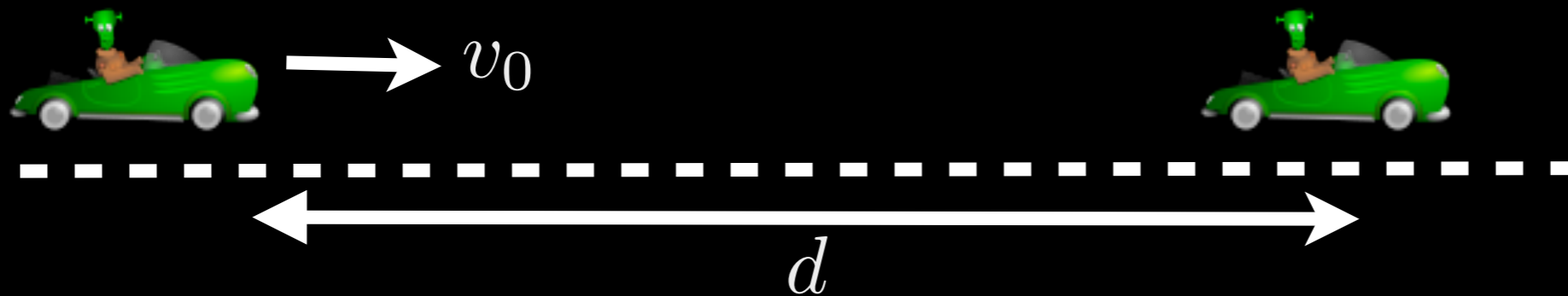
(D)  $d/2$

# Motion in 1D

# Quiz

A car is moving at initial speed,  $v_0$

... breaks and comes to a stop in distance  $d$ .



If the car is now moves at initial speed,  $2v_0$

... what is the distance before stopping?

$$v^2 = v_0^2 + 2a(x - x_0) \Rightarrow 0 = (v_0)^2 + 2(-a)d \Rightarrow d = \frac{v_0^2}{2a}$$

$$0 = (2v_0)^2 + 2(-a)d_2 \Rightarrow d_2 = \frac{4v_0^2}{2a} = 4d$$

# This lecture

- Know why units are important
- Convert (change) between units
- Use **scientific notation** for large and small numbers
- Understand the difference between **speed** and **velocity**
- Calculate **average** and **instantaneous velocity** and **acceleration**
- Use equations for **constant acceleration**