

Essential Physics I

英語で物理学の
エッセンス I

Lecture 14: 25-07-16

Exam

Next Lecture! (1 week)

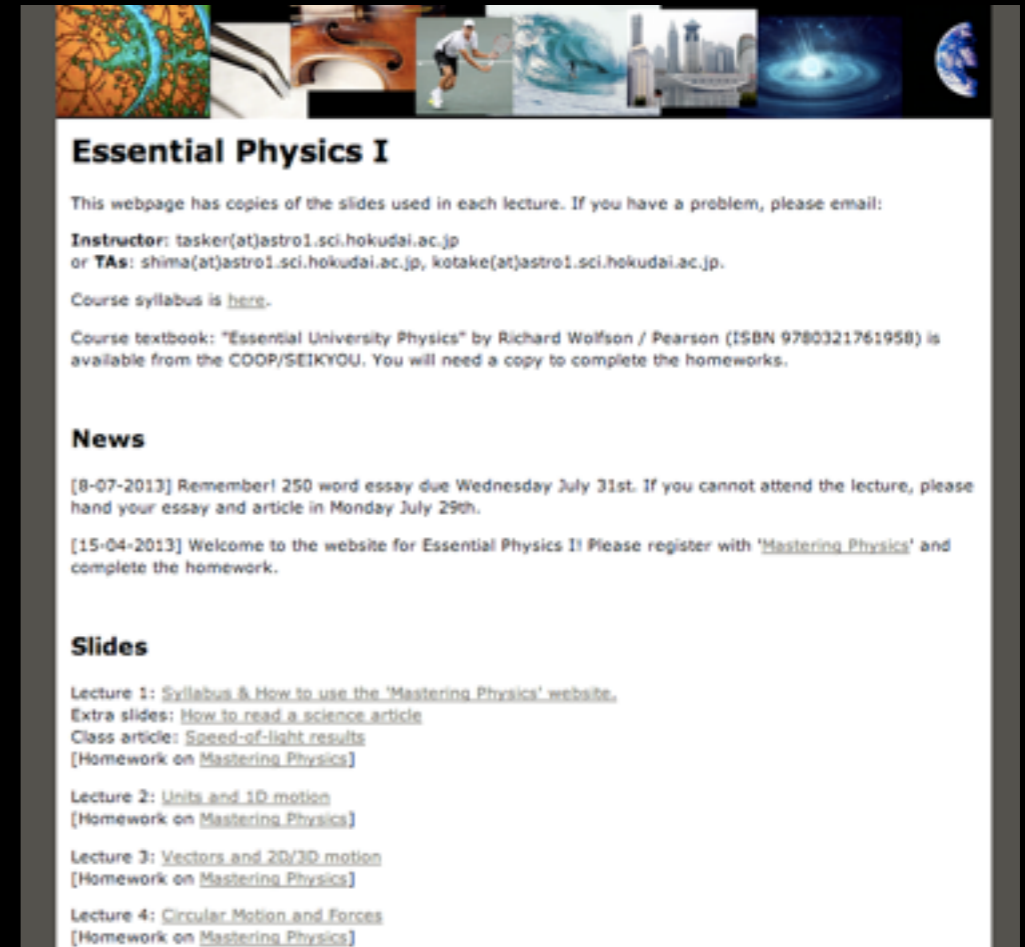
All lecture slides on course website:

<http://astro3.sci.hokudai.ac.jp/~tasker/teaching/ep1>

Remember your calculator!



Dictionary is OK!
(Phone is not)



Essential Physics I

This webpage has copies of the slides used in each lecture. If you have a problem, please email:

Instructor: tasker@astro1.sci.hokudai.ac.jp
or **TAs:** shima@astro1.sci.hokudai.ac.jp, kotake@astro1.sci.hokudai.ac.jp.

Course syllabus is [here](#).

Course textbook: "Essential University Physics" by Richard Wolfson / Pearson (ISBN 9780321761958) is available from the COOP/SEIKYOU. You will need a copy to complete the homeworks.

News

[8-07-2013] Remember! 250 word essay due Wednesday July 31st. If you cannot attend the lecture, please hand your essay and article in Monday July 29th.

[15-04-2013] Welcome to the website for Essential Physics I! Please register with 'Mastering Physics' and complete the homework.

Slides

Lecture 1: [Syllabus & How to use the 'Mastering Physics' website](#).
Extra slides: [How to read a science article](#)
Class article: [Speed-of-light results](#)
[Homework on [Mastering Physics](#)]

Lecture 2: [Units and 1D motion](#)
[Homework on [Mastering Physics](#)]

Lecture 3: [Vectors and 2D/3D motion](#)
[Homework on [Mastering Physics](#)]

Lecture 4: [Circular Motion and Forces](#)
[Homework on [Mastering Physics](#)]

SHOW ALL WORKING!

Next week's exam

10 multiple choice questions (A)
(B)
(C)
(D)

- ➔ ~6 classical mechanics
- ➔ ~2 oscillations, waves & fluids
- ➔ ~2 optics

Homework	40 %
Attendance / clickers	20 %
Exam	40 %

Total 100 %

➔ Pass > 60 %

Next week's exam

This is a question.

(A)

(B)

(C)

(D)



If this is wrong....

You will get marks!

SHOW YOUR WORKING!

$$x = x_0 + ut + \frac{1}{2}at^2$$

$$t = \frac{-u \pm \sqrt{u^2 - 4(a/2)(x_0 - x)}}{a}$$

$$= \frac{-12 \pm \sqrt{144 - 4(9.81/2)(17)}}{9.81}$$



But parts of this are right...

Last week:

Mirrors & Lenses

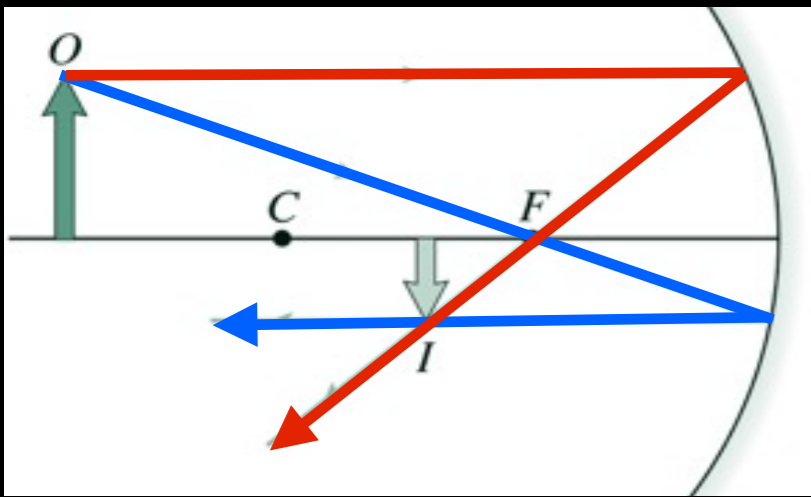


Image position can be found by drawing 2 light rays from points on the object.

Rays touch a **real image**

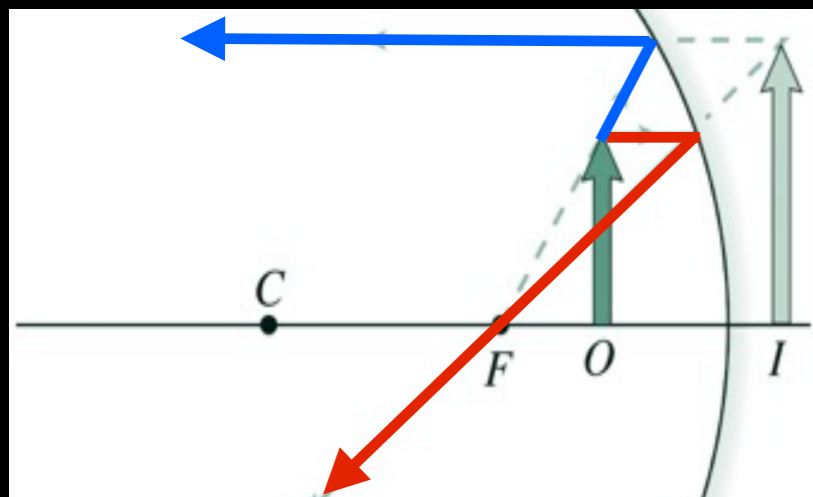
Last week:

Mirrors & Lenses

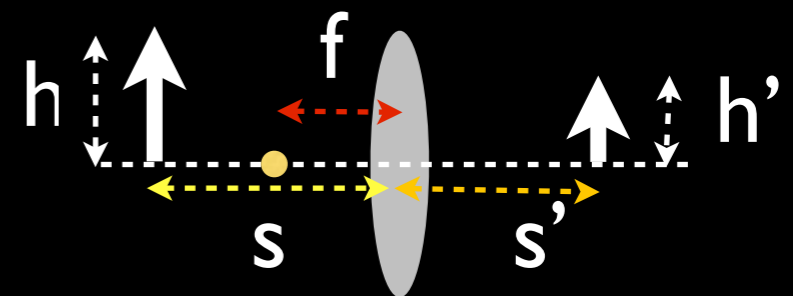
Image position can be found by drawing 2 light rays from points on the object.

Rays touch a **real image**

Rays only appear to touch a **virtual image**

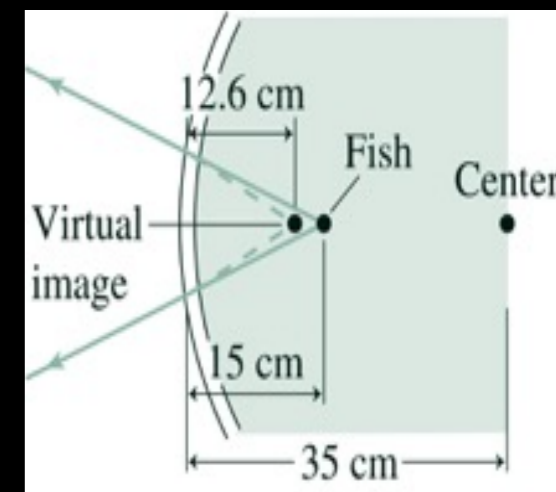
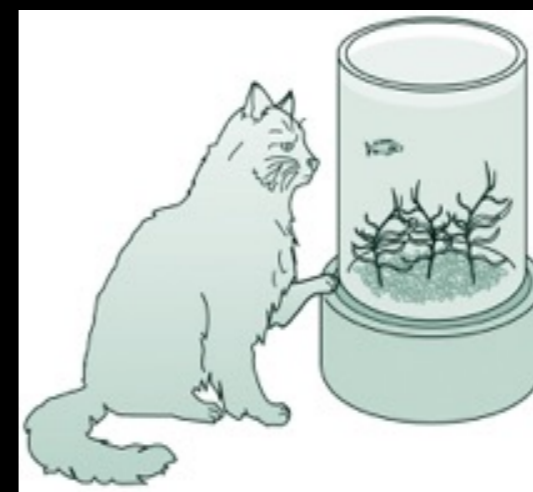


Magnification: $M = \frac{h'}{h} = -\frac{s'}{s}$



Mirror/Lens equation:
(thin lenses) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

Inside a lens: $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$

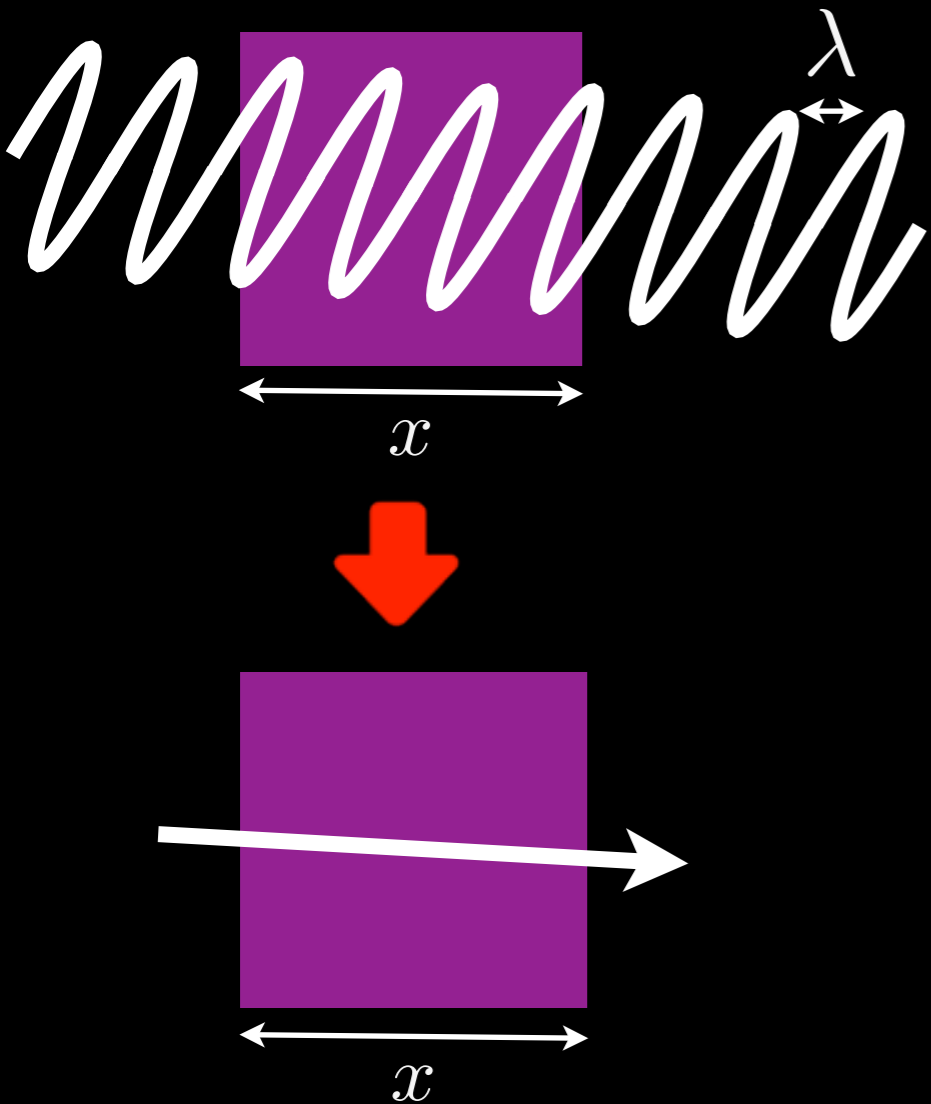


Optics

Interference & Diffraction



So far...



We assumed:

Light travels in a straight line: a **ray**
(Geometrical optics)

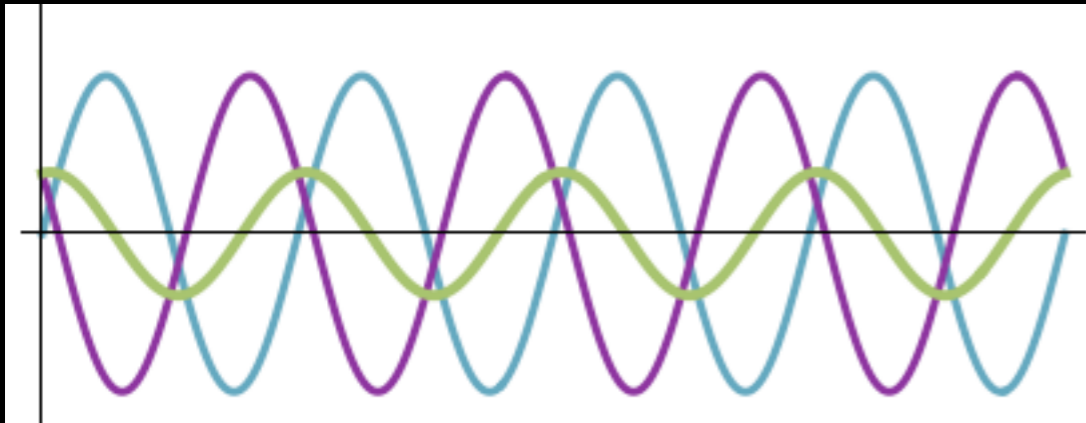
$$\lambda \ll x$$



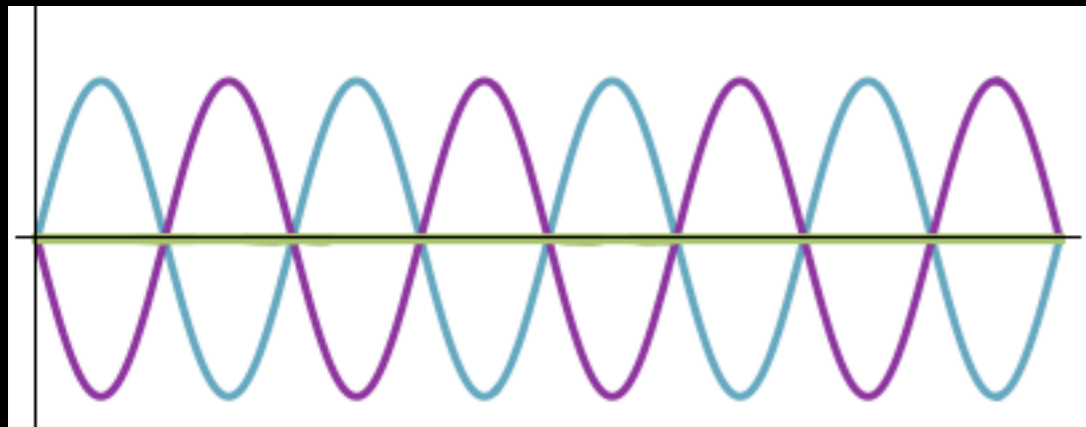
But, sometimes the wave nature of light is very important.

Interference and **diffraction**

Lecture 10: waves add

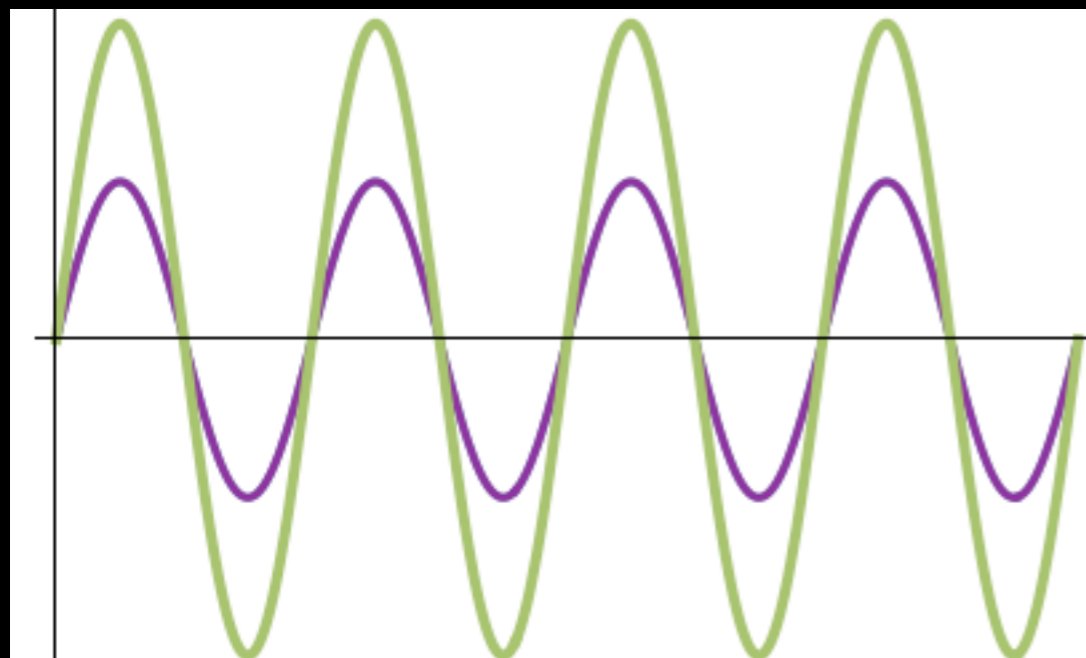


wave 1 + wave 2 = resulting wave



wave 1 + wave 2 cancel

Destructive interference



waves coincide

Constructive interference

Light waves are the same.

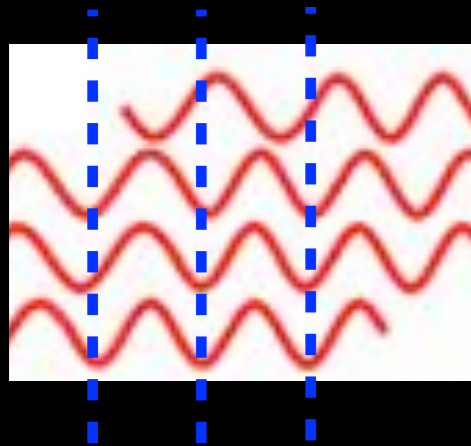
Coherence

For **continuous interference**, waves must be **coherent**.

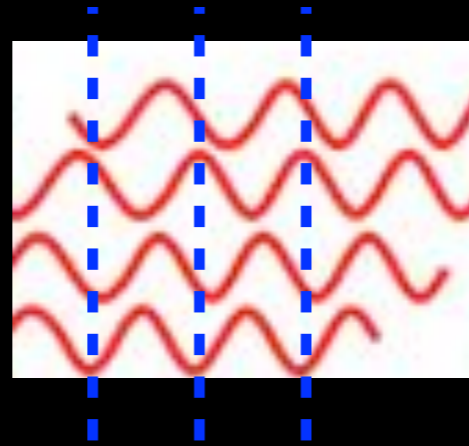
e.g. always constructive or
 always destructive or
 always

Coherence

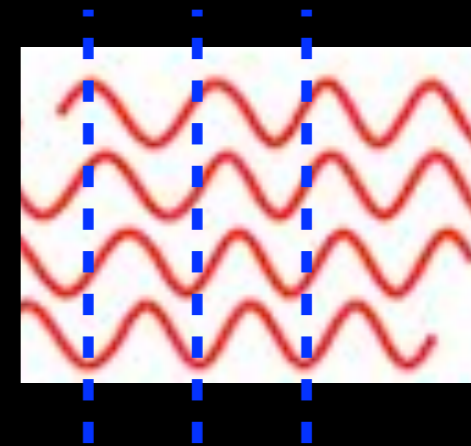
For continuous interference, waves must be **coherent**.



t_1



t_2

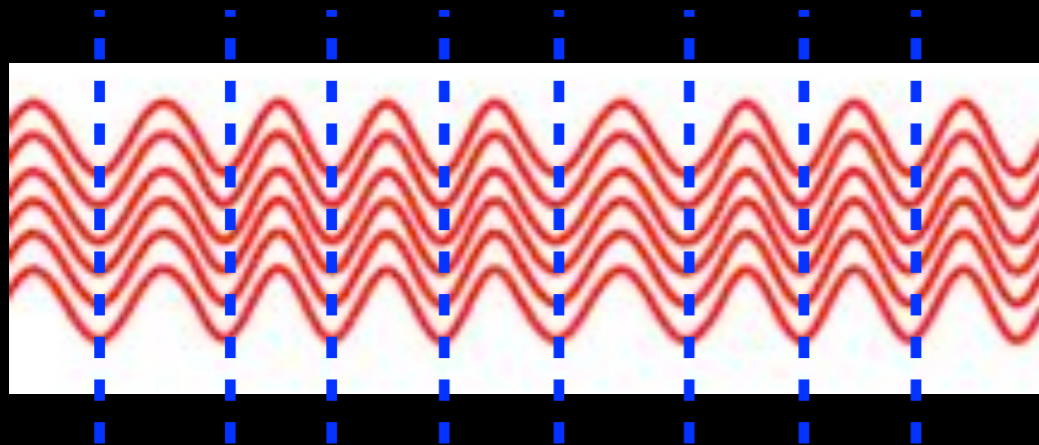


t_3

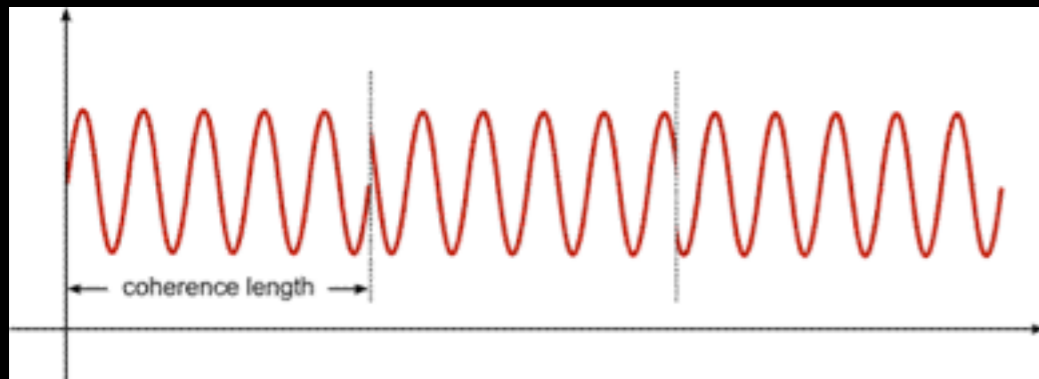
If phase between waves changes, wave sum changes



incoherent



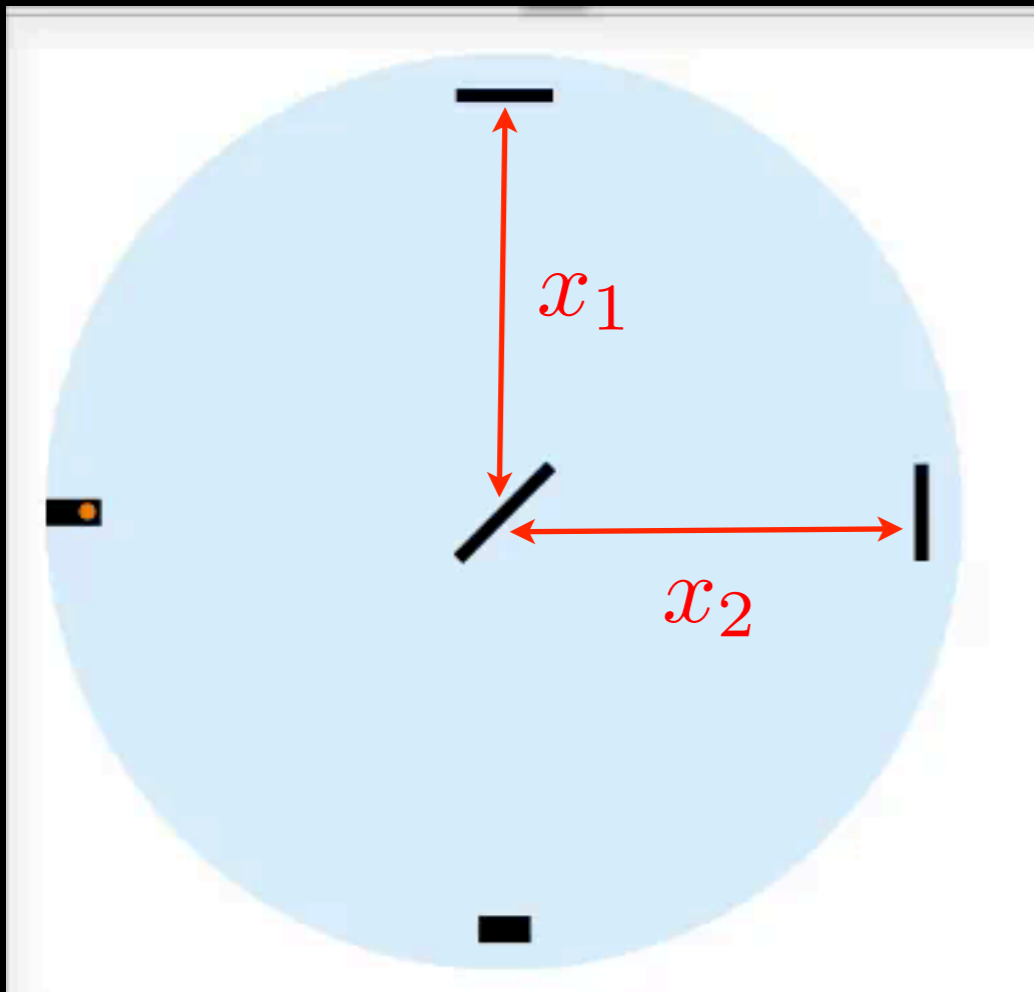
Coherent waves



Length where the wave phase is constant: **coherence length**

Lasers: long coherence length

Destructive and constructive



Even for lasers, hard to keep coherence



Split a single light source



Light travels different paths

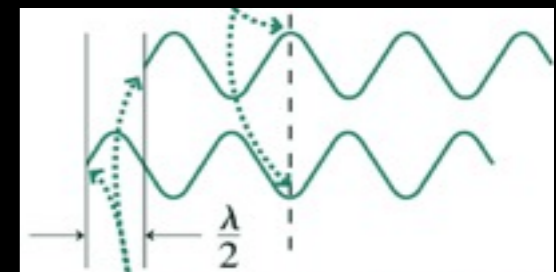
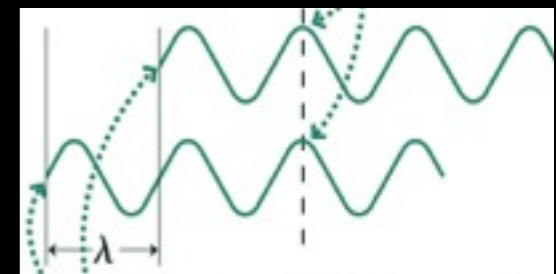


Recombine

If $x_1 = (m\lambda)x_2$, constructive interference.

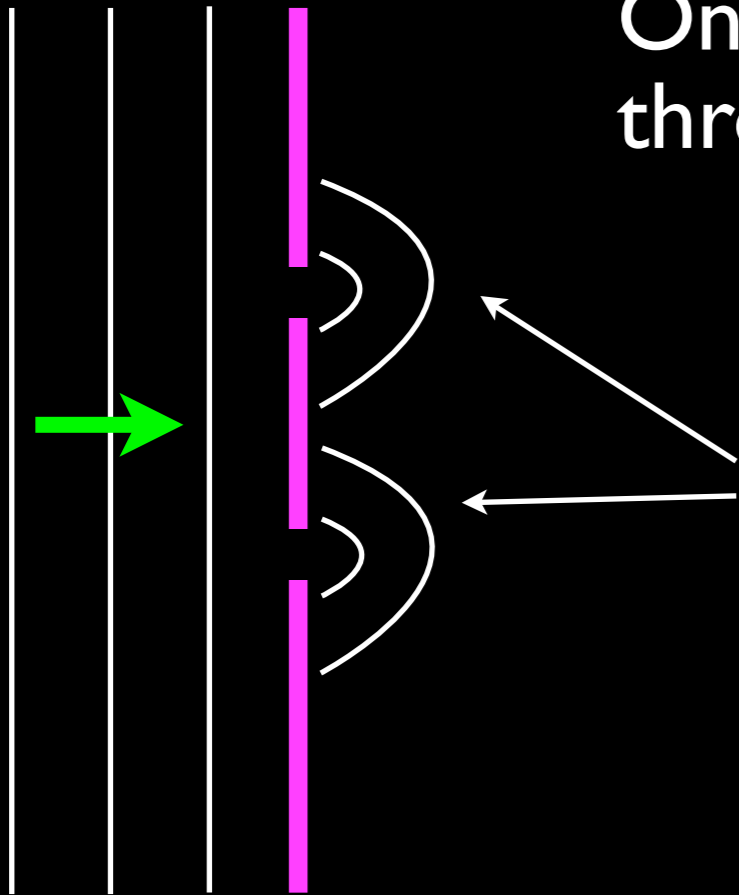
If $x_1 = \left(m + \frac{1}{2}\right)\lambda x_2$,

destructive interference patterns.



Double-Slit interference

One method of splitting a source is to pass through two narrow slits.

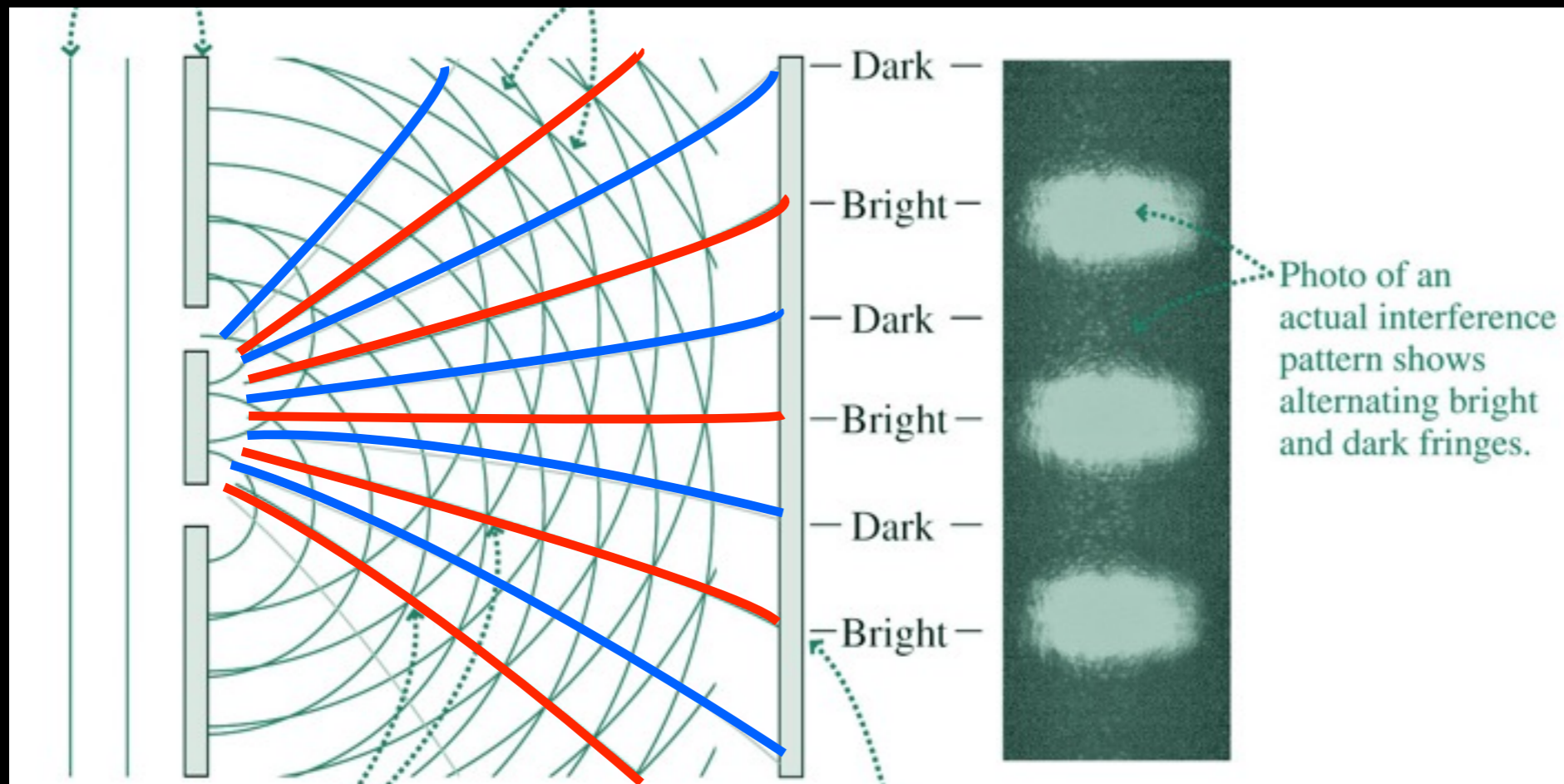


Produces 2 coherent sources.



cylindrical wavefronts interfere

Double-Slit interference



↑
illuminating screen

Constructive interference



bright on screen

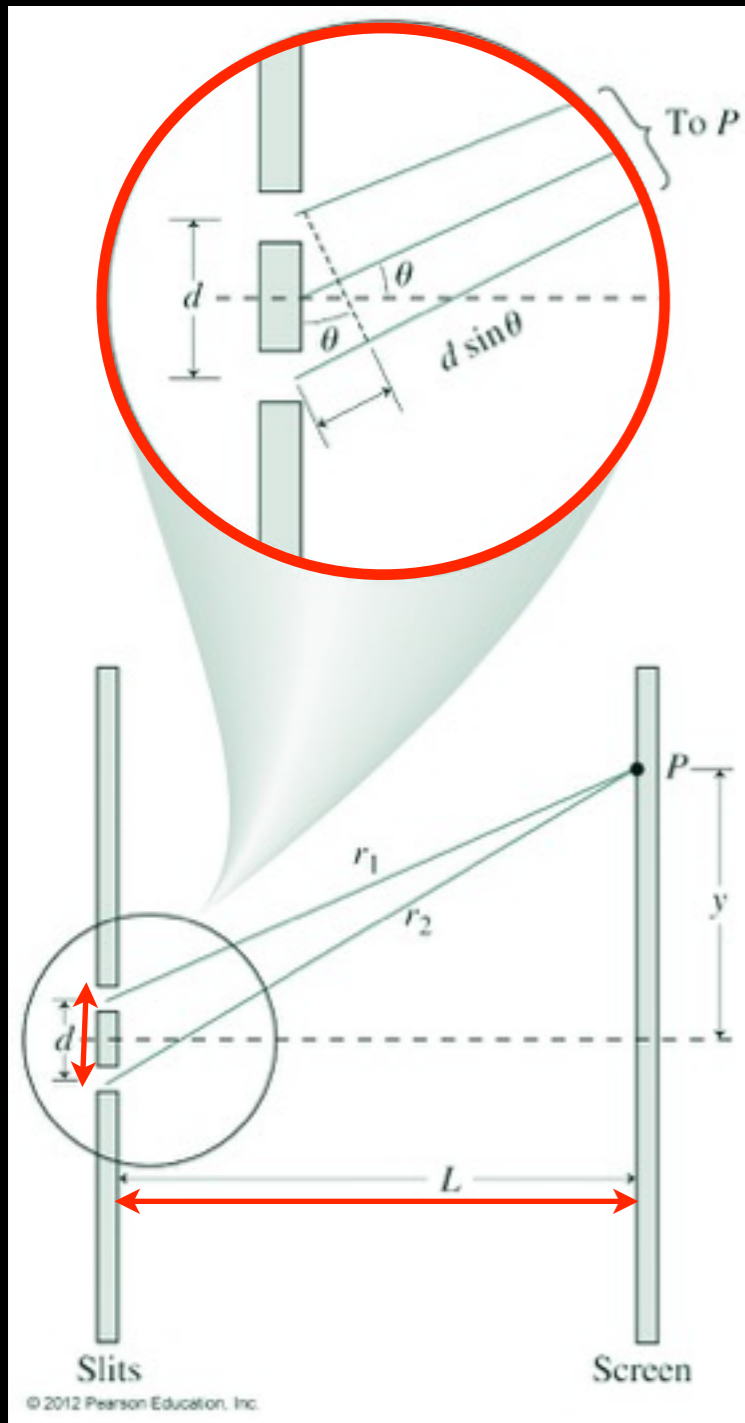
Destructive interference



dark on screen

interference fringes

Double-Slit interference



Constructive interference



Difference in path: $m\lambda$

If $L > d$, rays \sim parallel



Difference in path: $d \sin \theta$

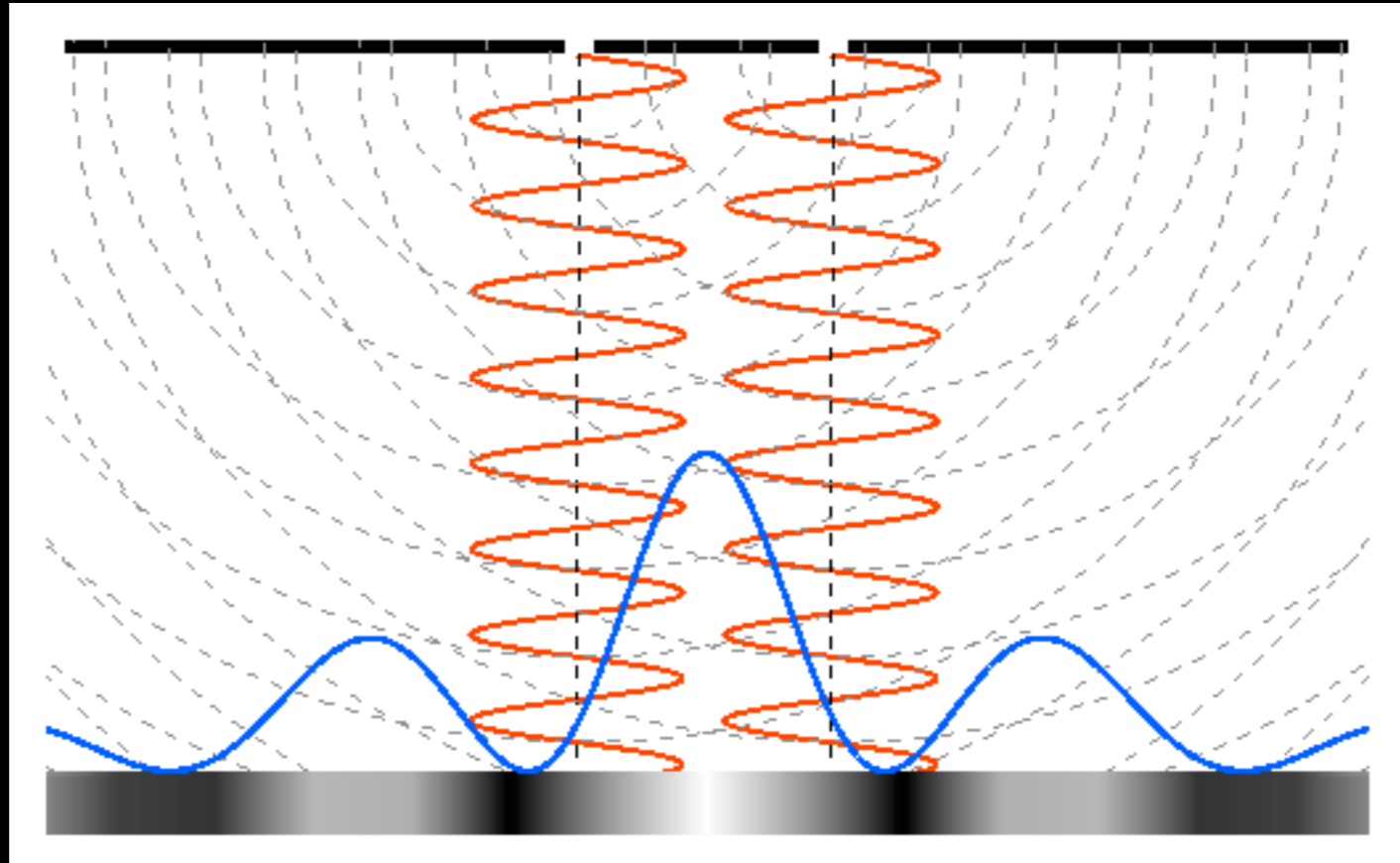
Bright fringes (constructive interference):

$$d \sin \theta = m\lambda$$

$$m = 0, 1, 2, \dots$$

fringe order

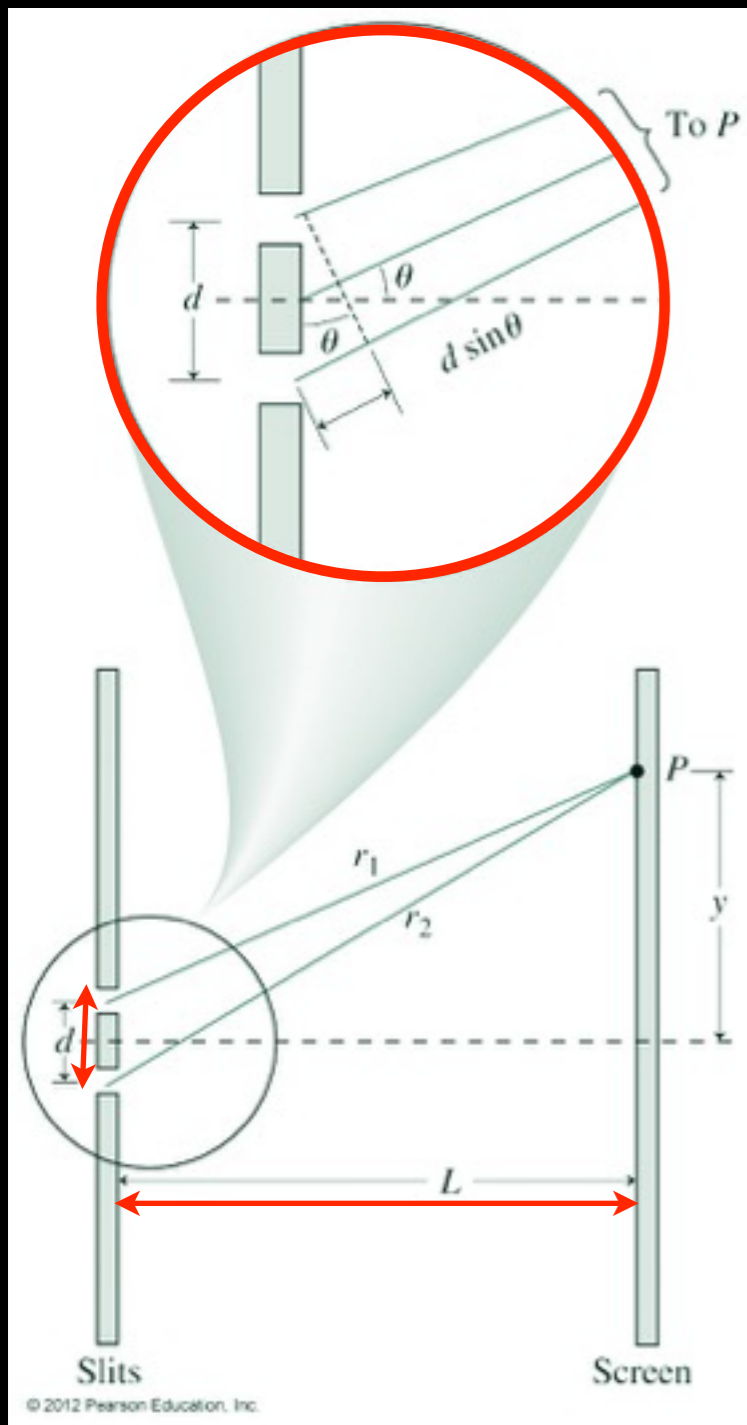
Double-Slit interference



$m = 1$ $m = 0$ $m = 1$

Bright fringes

Double-Slit interference



Destructive interference



Difference in path: $\left(m + \frac{1}{2}\right) \lambda$

dark fringes:

$$d \sin \theta = \left(m + \frac{1}{2}\right) \lambda \quad m = 0, 1, 2, \dots$$

Typically $L \sim 1\text{m}$, $d < 1\text{mm}$ and $\lambda < 1\mu\text{m}$

Therefore: $\sin \theta \simeq \tan \theta = \frac{y}{L}$



$$y_{\text{bright}} = m \frac{\lambda L}{d} \quad y_{\text{dark}} = \left(m + \frac{1}{2}\right) \frac{\lambda L}{d}$$

Double-Slit interference

Quiz

Constructive interference of 2 coherent waves will occur if their path difference is...

(A) λ

(B) $\frac{3\lambda}{2}$

(C) $\frac{5\lambda}{2}$

(D) $\frac{\lambda}{2}$

Double-Slit interference

Example

2 slits 0.075 mm apart are located 1.5 m from a screen.

The 3rd-order bright fringe is 3.8 cm from the screen centre.

What is λ ?

$$m = 3$$

Use: $y_{\text{bright}} = m \frac{\lambda L}{d}$



$$\lambda = \frac{y_{\text{bright}} d}{m L}$$

$$= \frac{(0.038\text{m})(0.075 \times 10^{-3}\text{m})}{(3)(1.5\text{m})}$$

$$= 633\text{nm}$$

Double-Slit interference

Quiz

If you increase the slit separation in a 2 slit system, how does the spacing of the interference fringes change?

- (A) They become further apart
- (B) The spacing does not change
- (C) They become closer together

$$y_{\text{bright}} = m \frac{\lambda L}{d}$$

$$y_{\text{dark}} = \left(m + \frac{1}{2} \right) \frac{\lambda L}{d}$$

Double-Slit interference

Quiz

A double-slit experiment has a slit spacing 0.12 mm.

If the bright fringes are 5 mm apart when the slits are illuminated with 633-nm wavelength laser light...

What is the slit-to-screen distance, L ?

(A) 0.95cm

(B) 95cm

(C) 150cm

(D) 15cm

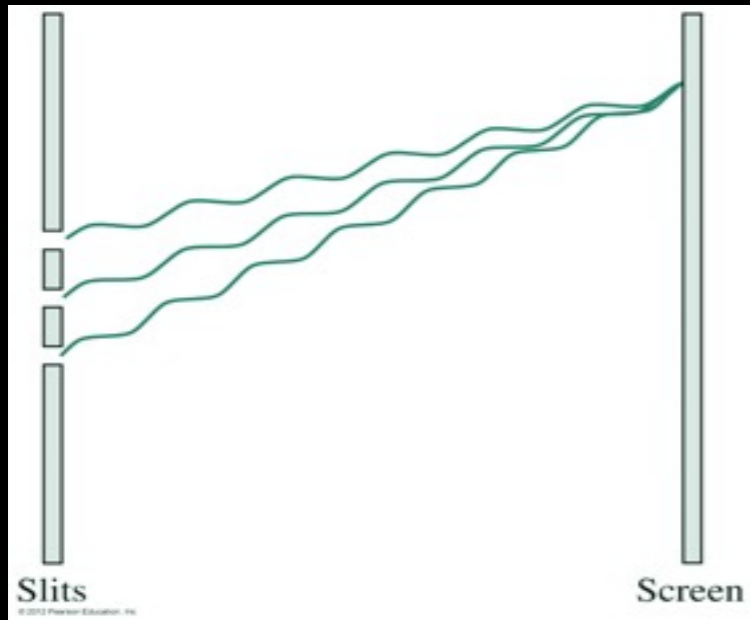
Use: $y_{\text{bright}} = m \frac{\lambda L}{d}$

$$\Delta y = 1 \frac{\lambda L}{d} - 0 \frac{\lambda L}{d} = 5\text{mm}$$

$$L = \frac{\Delta y d}{\lambda} = \frac{(0.12\text{mm})(5.0\text{mm})}{633\text{nm}} = 95\text{cm}$$

Multiple-Slit interference

What if we add more slits?



Constructive interference

All 3 waves must be in phase

Difference in paths: $m\lambda$

For evenly spaced slits: $d \sin \theta = m\lambda$
(same as for 2 slits)

Destructive interference ... more complicated

All waves must add to zero.

For 3 waves, each wave must out of phase with by $1/3$.

$$d \sin \theta = \begin{pmatrix} \left(m + \frac{1}{3}\right) \lambda \\ \left(m + \frac{2}{3}\right) \lambda \end{pmatrix}$$

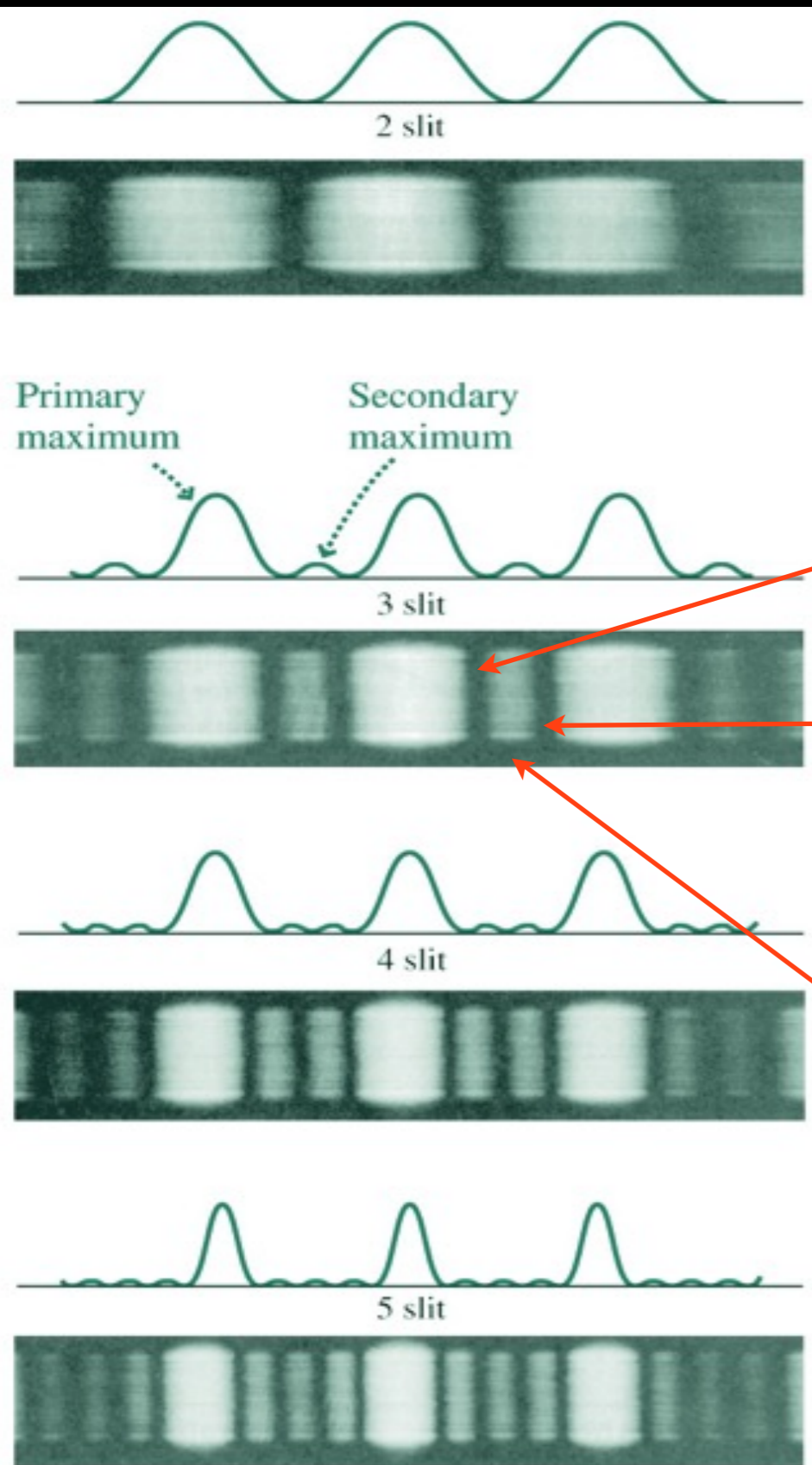
general 

$$d \sin \theta = \frac{m}{N} \lambda$$

slit no. 

m is integer but not integer multiple of N

Multiple-Slit interference



$$\left(m + \frac{1}{3}\right) \lambda$$

2 minima between primary maxima $m\lambda$ and $(m + 1)\lambda$

$$\left(m + \frac{2}{3}\right) \lambda$$

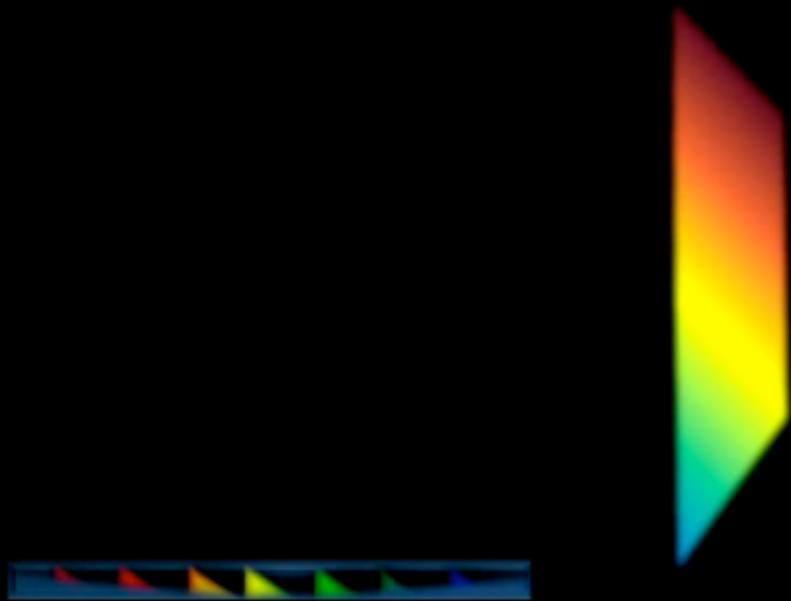
secondary maxima

not fully constructive or fully destructive interference.

Diffraction grating

What if we add *even more* slits?

Diffraction grating: ~ 1000 s slits / cm



If using multi-wavelength light:

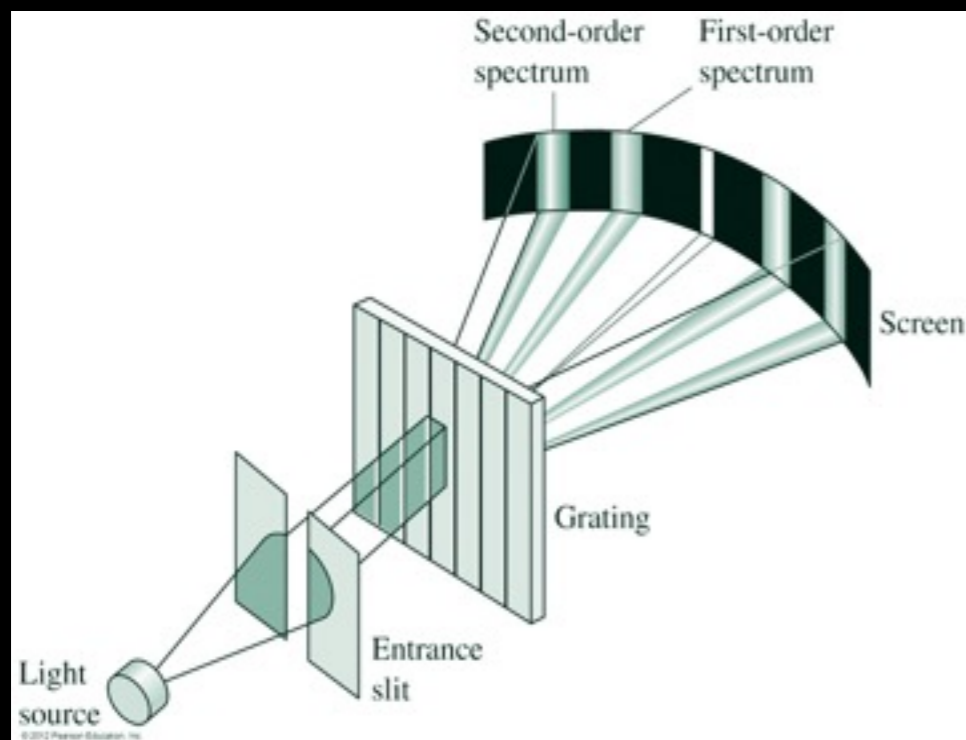
since: $d \sin \theta = m\lambda$

for $m > 0$, angular position θ depends on λ



maxima occur in different places for different wavelengths

Many slits gives very precise locations for the wavelengths (clear separations)



Diffraction grating

Example

Light from glowing Hydrogen contains $H\alpha$ (hydrogen-alpha) 656.3nm and $H\beta$ (hydrogen-beta) 486.1 nm.

Find the 1st order angular separation between these wavelengths when using a grating of 6000 slits /cm.

$$m = 1$$

$$6000 \text{ slits / cm} \rightarrow d = \frac{1}{6000} \text{ cm} = 1.667 \mu\text{m}$$

$$\theta_{\alpha} = \sin^{-1} \left(\frac{\lambda}{d} \right) = \sin^{-1} \left(\frac{0.6563 \mu\text{m}}{1.667 \mu\text{m}} \right) = 23.2^{\circ}$$
$$\theta_{\beta} = \sin^{-1} \left(\frac{\lambda}{d} \right) = \sin^{-1} \left(\frac{0.4861 \mu\text{m}}{1.667 \mu\text{m}} \right) = 17^{\circ}$$

$\Delta\theta = 6.2^{\circ}$

Diffraction grating

Quiz

Green light at 520 nm is diffracted by a grating with 3000 lines / cm.

Through what angle is the light diffracted in 5th order?

(A) 0.07°

maxima at: $\theta = \sin^{-1} \left(\frac{m\lambda}{d} \right)$

(B) 9.0°

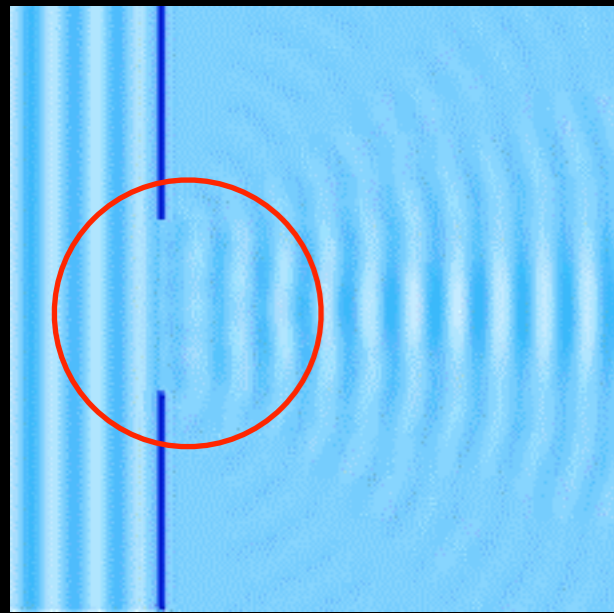
$$d = \frac{1}{3000\text{cm}^{-1}} = 3.33\mu\text{m}$$

(C) 51°

$$\theta = \sin^{-1} \left(\frac{5 \times 520\text{nm}}{3.33\mu\text{m}} \right) = 51^\circ$$

(D) 0.05°

Diffraction



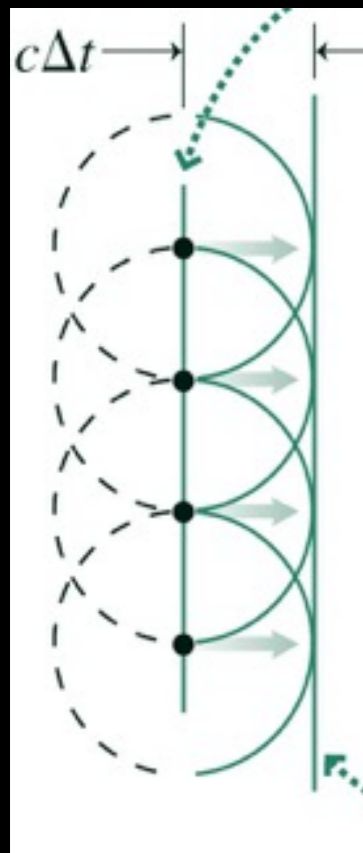
Why does the slit cause cylindrical waves?



light bending as it passes object

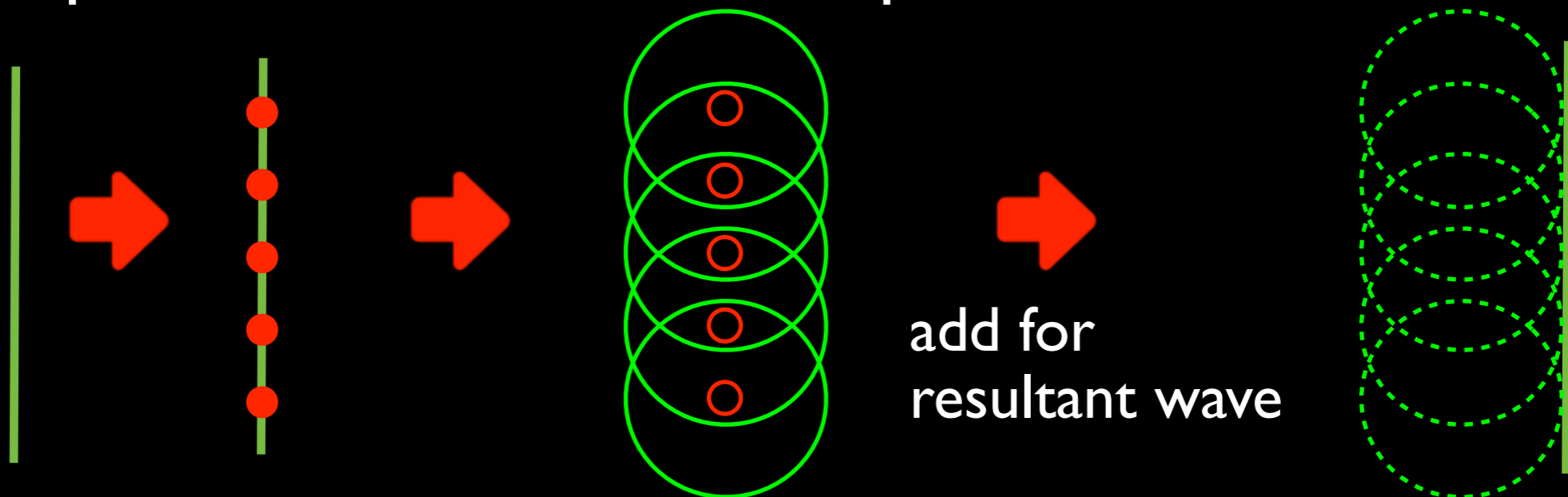
Diffraction

Really need Maxwell's equations but...



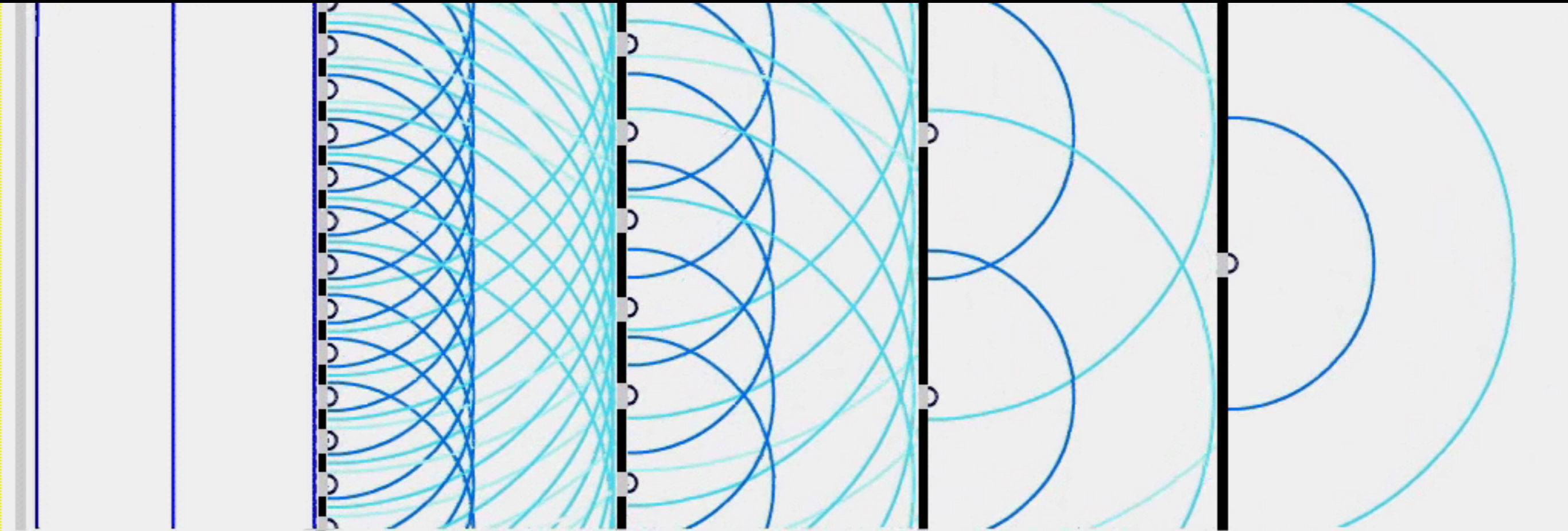
Huygens' Principal: (pronounce: her-genz)

All points on wave front are spherical wave sources.



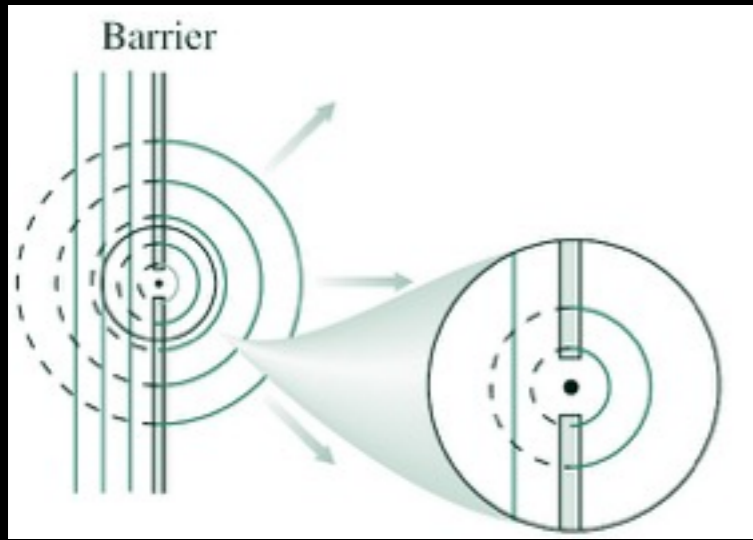
Diffraction

Spherical waves blocked by the barrier; only some pass through slit.



Sum of the remaining spherical waves → wave bends (diffraction)

Diffraction



If the slit width \sim wavelength



slit = single wave source

But....

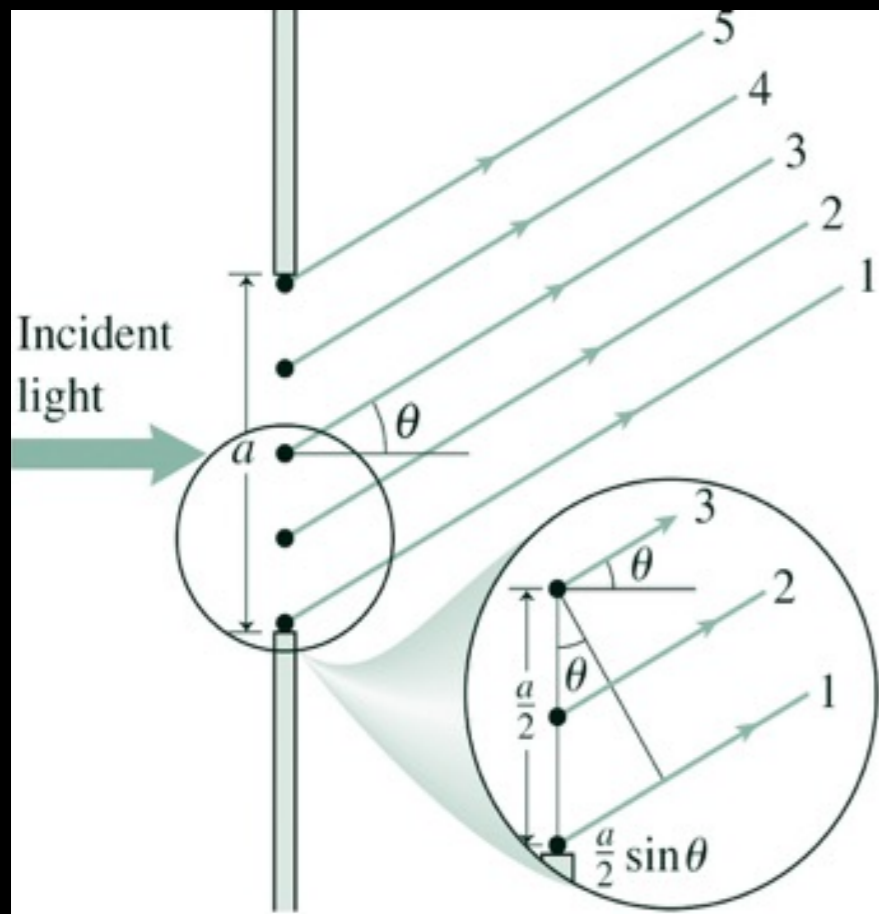
If slit width is *larger*



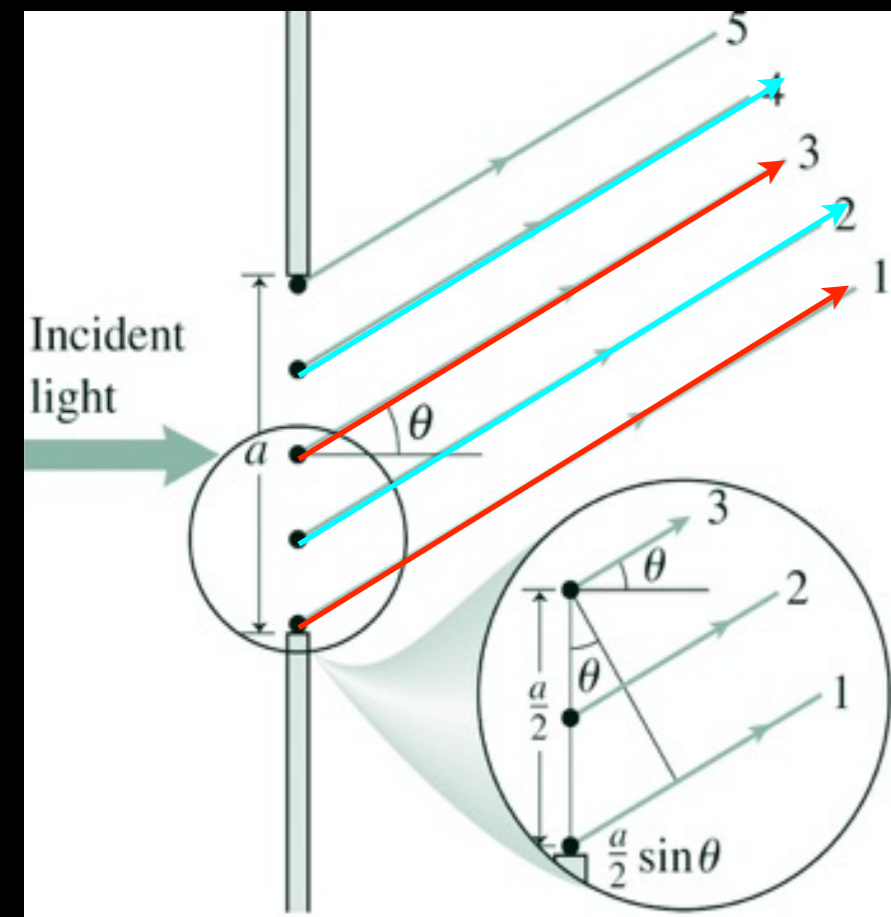
Each point in slit is a wave source



Interference between waves in slit



Diffraction



Consider 5 equal spaced sources:

Path length for ray 1 and 3 differ by $\frac{1}{2} a \sin \theta$

Destructive interference if

$$\frac{1}{2} a \sin \theta = \frac{1}{2} \lambda$$

or

$$a \sin \theta = \lambda$$

But if ray 1 and 3 interfere destructively, so do 2 and 4...

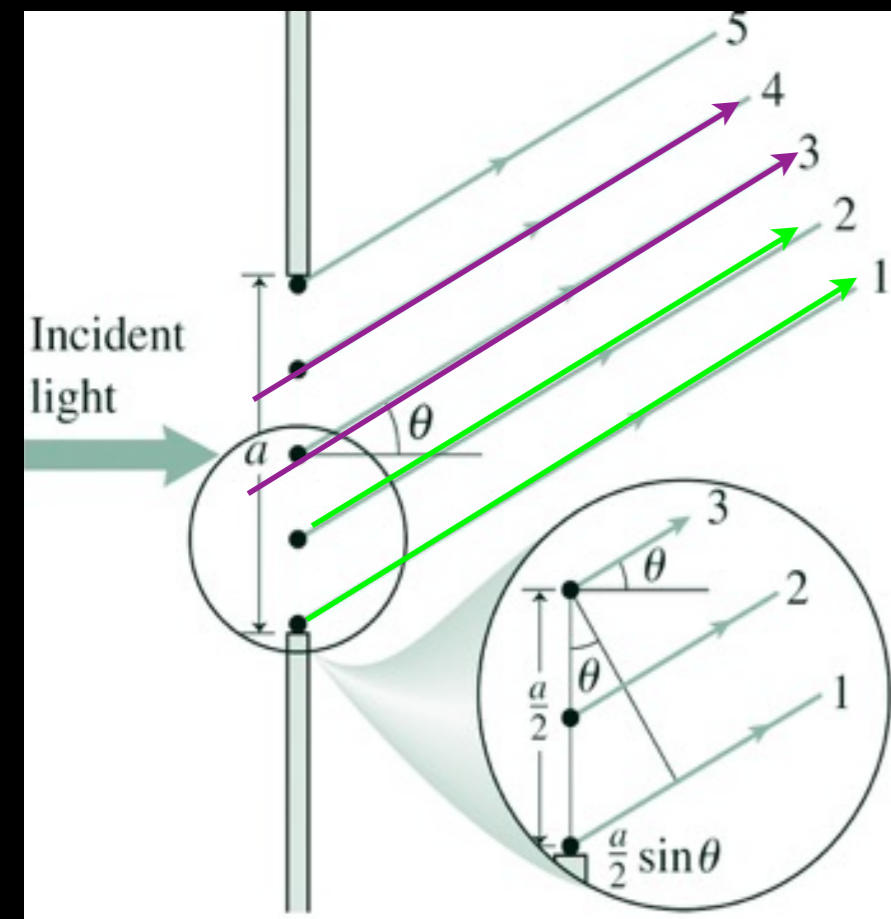


All rays in lower half of slit will interfere destructively with ray $\frac{a}{2}$ above it.



If you look at the slit at θ such that $a \sin \theta = \lambda$, you will see no light

Diffraction



Similarly

Ray 1 and 2 differ by $\frac{1}{4} a \sin \theta$

Destructive interference if

$$\frac{1}{4} a \sin \theta = \frac{1}{2} \lambda$$

or

$$a \sin \theta = 2\lambda$$

But is ray 1 and 2 interfere destructively, so do 3 and 4...

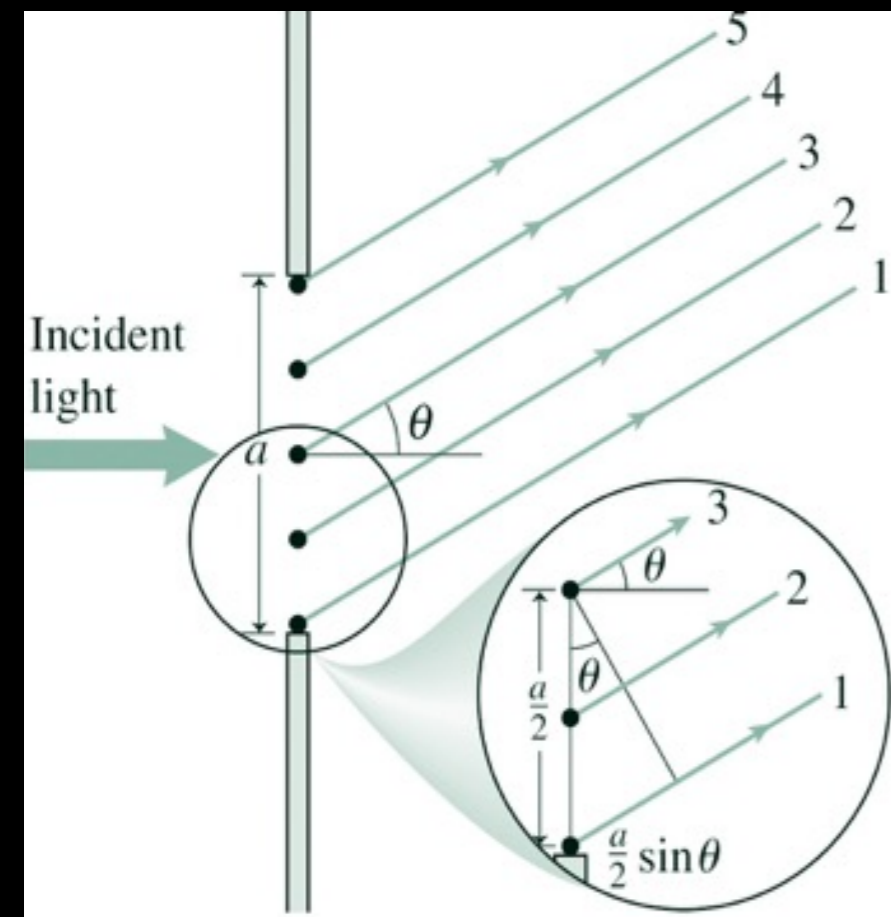


All rays in lower 3/4 of slit will interfere destructively with ray $\frac{a}{4}$ above it.



If you look at the slit at θ such that $a \sin \theta = 2\lambda$, you will see no light

Diffraction



Generally:
(divide slit into 6, 7, 8 etc sources)

$$a \sin \theta = m\lambda$$

Destructive interference, single-slit diffraction

for $m = 1, 2, 3, \dots$

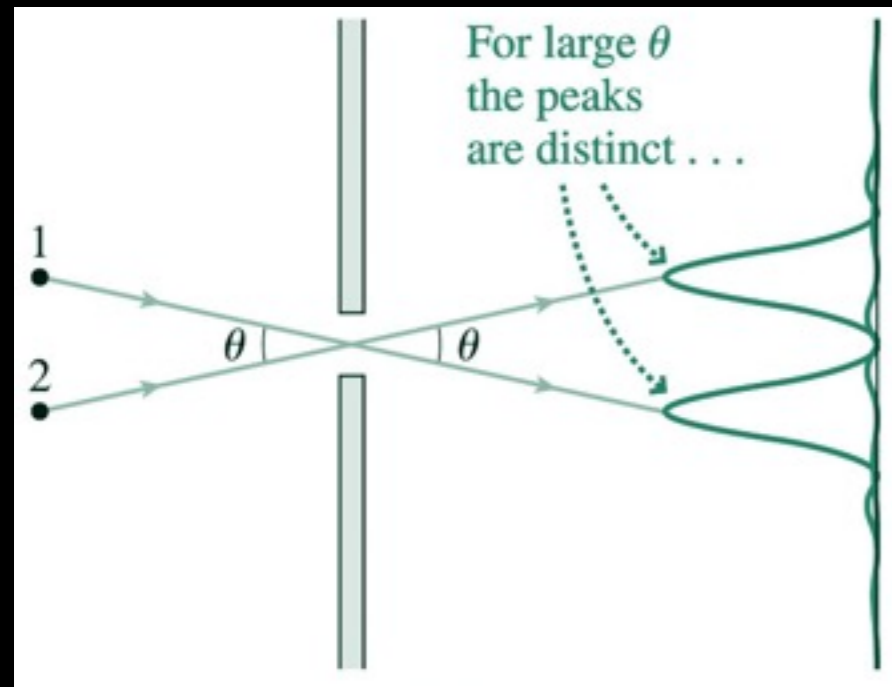


for $m = 0$ no destructive interference
at the central maximum.

Diffraction Limit

Diffraction creates a limit for how close two objects can be...

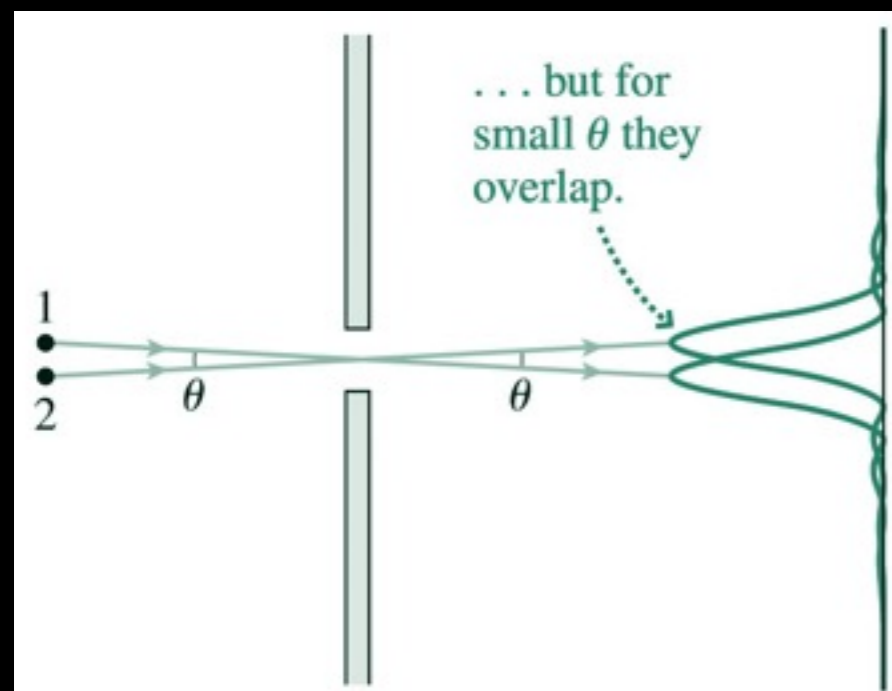
... and still be resolved.



2 sources illuminate the slit

Their waves reach the slit at different angles

(assume sources are incoherent, no regular interference pattern)



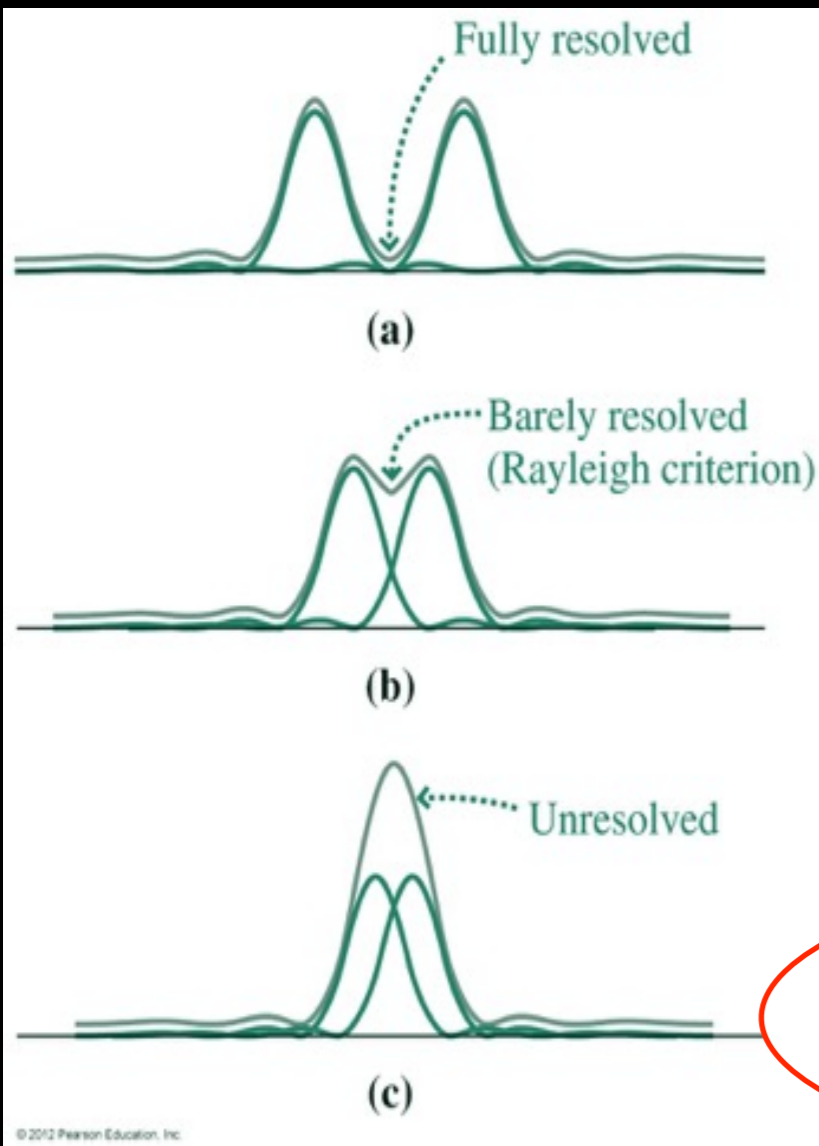
2 single slit diffraction patterns

As the sources get closer



Central maxima begins to overlap

Diffraction Limit



Diffraction patterns merge



2 peak structure disappears

2 peak visible if

Central *maxima*
of peak 1

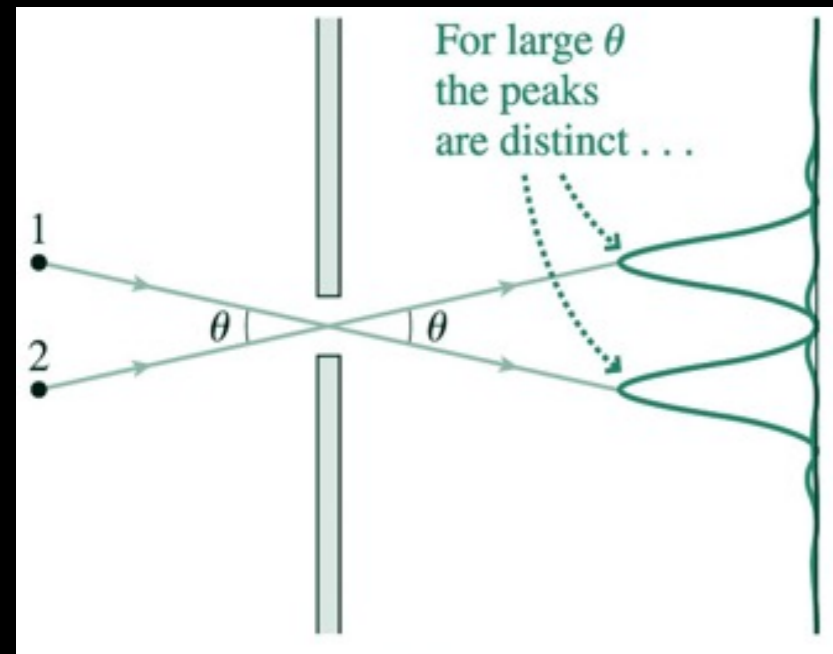
=

1st *minima*
of peak 2

Rayleigh criterion

when true, 2 sources are *just resolved*

Diffraction Limit



The angular separation, θ , between the peaks

=

The angular separation, θ , between the sources

1st minima: $\sin \theta = \frac{\lambda}{a}$

since $\lambda \ll a$ \rightarrow small angle approximation: $\sin \theta \simeq \theta$

$\rightarrow \theta_{\min} = \frac{\lambda}{a}$ (Rayleigh criterion, slit)

If using a circular aperture, not slit (e.g. a camera):

$\rightarrow \theta_{\min} = \frac{1.22\lambda}{D}$ (Rayleigh criterion, circular aperture)
 D \leftarrow aperture diameter

Diffraction Limit

Example

An astroid 20×10^6 km away appears on a collision course with the Earth!

What is the minimum size for the astroid that could be resolved with the 2.4 m diameter **diffraction-limited** Hubble Space **Telescope**?

diffraction is what prevents the resolved imaged
(not atmosphere ... etc)



circular aperture

Diffraction Limit

Example

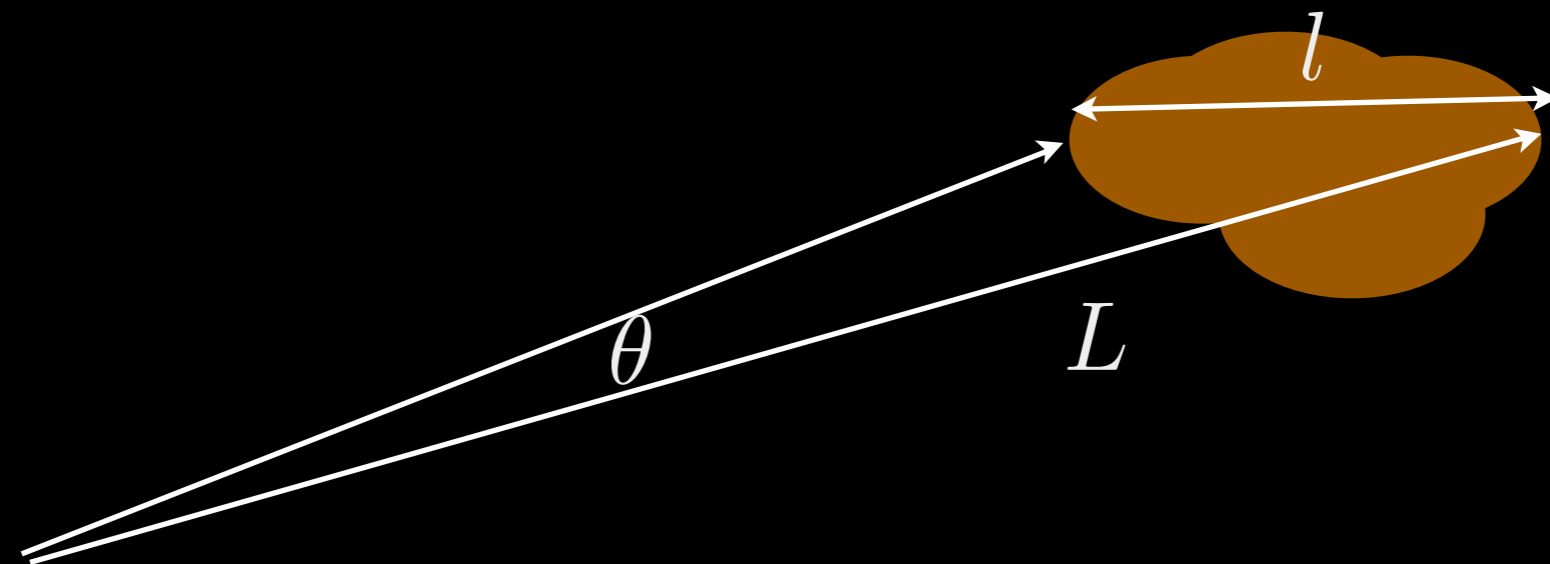
An asteroid $20 \times 10^6 \text{ km}$ away appears on a collision course with the Earth!

What is the minimum size for the asteroid that could be resolved with the 2.4 m diameter diffraction-limited Hubble Space Telescope?

$$\theta_{\min} = \frac{1.22\lambda}{D}$$

$$\frac{l}{L} = \frac{1.22\lambda}{D}$$

$$l = \frac{1.22\lambda L}{D} = 5.6 \text{ km}$$



Opposite ends of the asteroid = 2 peaks

$$\theta \simeq \frac{l}{L} \quad (\text{small angle approximation})$$

Diffraction Limit

Example

An astroid $20 \times 10^6 \text{ km}$ away appears on a collision course with the Earth!

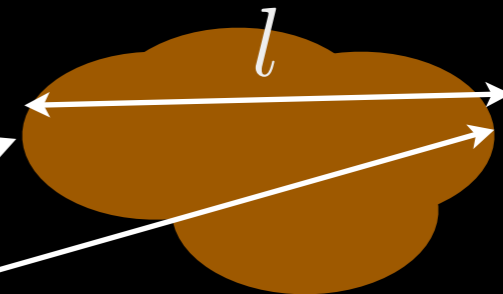
What is the minimum size for the astroid that could be resolved with the 2.4 m diameter diffraction-limited Hubble Space Telescope?

$$\theta_{\min} = \frac{1.22\lambda}{D}$$

$$\frac{l}{L} = \frac{1.22\lambda}{D}$$

$$l = \frac{1.22\lambda L}{D} = 5.6 \text{ km}$$

big problem!



Op

θ

bid = 2 peaks

(approximation)

Diffraction Limit

Quiz

What is the longest wavelength you could use to resolve an object with angular diameter 0.44 mrad, using a microscope with aperture 1.2 mm in diameter?

(A) 528nm

$$\theta_{\min} = \frac{1.22\lambda}{D}$$

(B) 220nm

$$\lambda = \frac{\theta_{\min} D}{1.22} = \frac{(0.44\text{mrad})(1.2\text{mm})}{1.22} = 430\text{nm}$$

(C) 130nm

(D) 430nm

Man-made star shines
in the Southern Sky

What has been created at the VLBA (Very Large Telescope Array)?

- (A) The biggest mirror on a telescope
- (B) An artificial star
- (C) A new type of laser
- (D) A defense shield

Why is this useful?

- (A) The laser reflects off stars to find their distance
- (B) We can communicate with extra-terrestrial life
- (C) Allows the 'adaptive optics' technique to be used anywhere in the sky.
- (D) It tracks communication satellites around the world

How big is the mirror on the telescope that the laser is launched from?

(A) 10 m

(B) 8.2 m

(C) 3 m

(D) 5 m

What height is the “star” created?

(A) 90 km

(B) 10 km

(C) The same as our nearest star

(D) The same as the moon

How faint (weak / hard to see) is the artificial star?

- (A) 20 x fainter than the faintest star that can be seen with the telescopes
- (B) 20 x fainter than the brightest star that can be seen by eye
- (C) 20 x fainter than the faintest star that can be seen by eye
- (D) 20 x fainter than the brightest star that can be seen by eye

What limits a ground-based telescope's image sharpness?

(how good the image is)

(A) Diffraction

(B) Atmospheric turbulence

(C) Moon light

(D) Sun light

How does **adaptive optics** solve this?

- (A) It uses a bigger mirror
- (B) The laser finds the object's location more accurately
- (C) **A flexible mirror corrects the image**
- (D) The laser freezes the image

Adaptive optics needs reference star. Why isn't a real star used?

- (A) A suitable star isn't always in the sky area you want to observe
- (B) A real star's light is too variable (it twinkles)
- (C) The reference star must be red
- (D) The moon is too bright

How does the laser make the star?

- (A) Shoots a beam of light into the air
- (B) Causes sodium atoms in the atmosphere to glow
- (C) The laser reflects off a real star, making it brighter
- (D) The laser reflects off the moon

What do scientists hope to observe with this new technique?

- (A) Black holes forming in other galaxies
- (B) 'high redshift' (young) galaxies
- (C) Our galactic centre
- (D) All the above