Essential Physics I

英語で物理学の

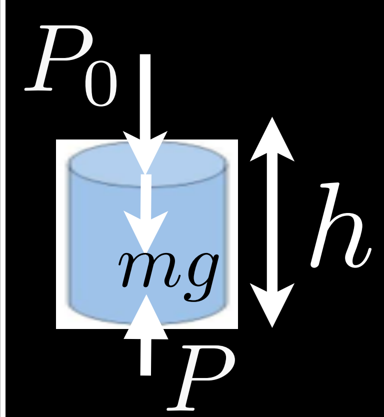
エッセンス I

Lecture 12:   04-07-16

Last lecture: review

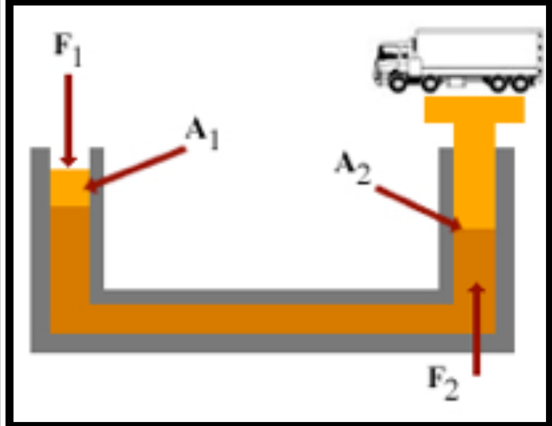
Fluids

Hydrostatic equilibrium



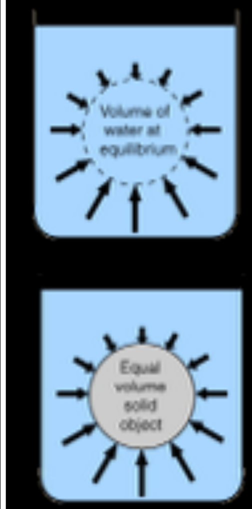
$$P = P_0 + \rho g \Delta h$$

for liquid (constant ρ)



$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

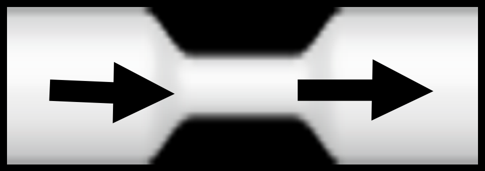
Archimedes' Principal



Buoyancy: pressure force on object

= weight of fluid displaced

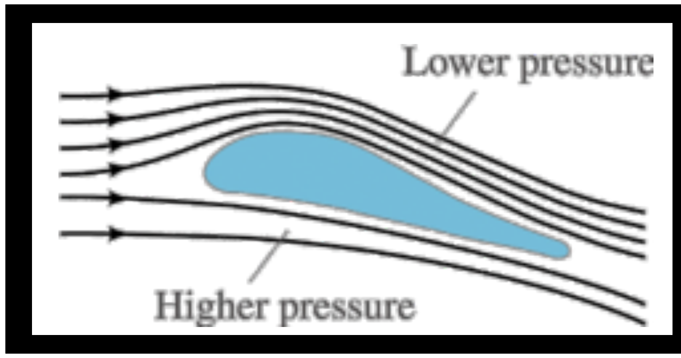
volume flow rate



$Av =$ constant

for liquid (constant ρ)

Bernoulli's equation

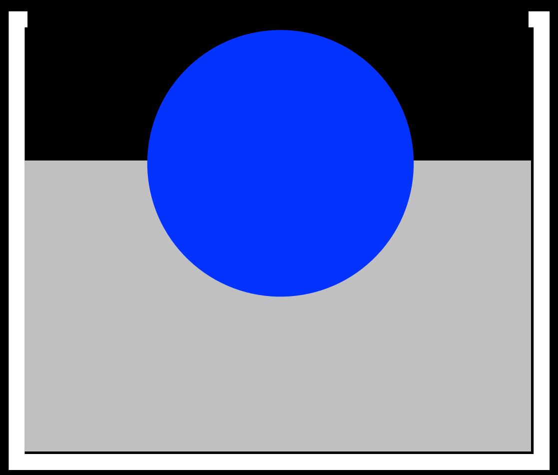


$$P + \rho g y + \frac{1}{2}\rho v^2 = \text{ constant}$$

Last lecture: review

A lead ball ($\rho = 11.3\,\mathrm{g/cm^3}$) enters a tub of mercury ($\rho = 13.6\,\mathrm{g/cm^3}$).

What happens?



(A) Lead ball will float with ~ 83% of volume below mercury surface

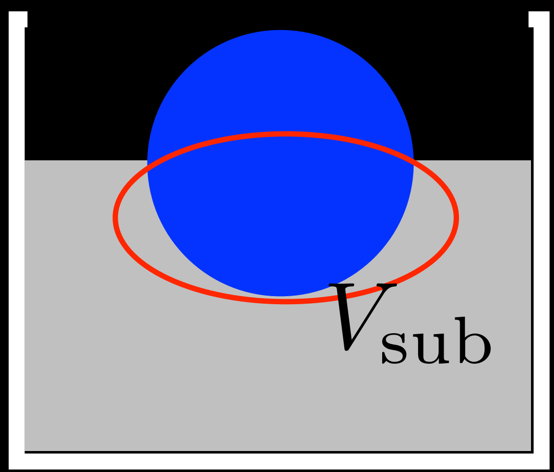
(B) Lead ball will float with 100% of volume below mercury surface

(C) Lead ball will float with ~17% of volume below mercury surface

(D) Lead ball will sink

Last lecture: review

A lead ball ($\rho = 11.3\,\mathrm{g/cm^3}$) enters a tub of mercury ($\rho = 13.6\,\mathrm{g/cm^3}$).



What happens?

(A) Lead ball will float with ~ 83% of volume below mercury surface

Archimedes' Principal

Buoyancy force = weight of mercury displaced $= \rho_{\mathrm{Hg}} V_{\mathrm{sub}} g$

= weight of lead ball $= \rho_{\mathrm{Fe}} V_{\mathrm{tot}} g$

$$\frac{\rho_{\mathrm{Fe}}}{\rho_{\mathrm{Hg}}} = \frac{V_{\mathrm{sub}}}{V_{\mathrm{tot}}} = 0.83$$

Last lecture: review

You are driving a convertible car at 65 mph.

The soft roof and windows are closed.

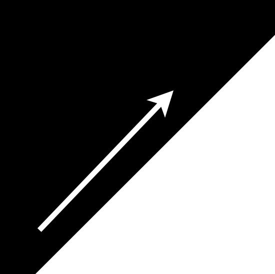
The roof...

(A) bows inward

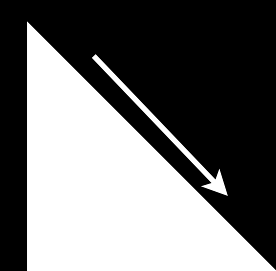
(B) Same as when car is stopped

(C) bows outward

(D) bows inward only when driving uphill



(E) bows inward only when driving downhill



Last lecture: review

You are driving a convertible car at 65 mph.

The soft roof and windows are closed.

The roof...

(C) bows outward

Bernoulli's equation $P + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$

lower constant higher

→ lower pressure

→ pressure gradient, ΔP

→ ↑ force

Last lecture: review

To provide the lift force needed for flight, aeroplane wings *must* be designed so that...:

(A) Air molecules will be deflected downward when they flow past the wing

(B) Air molecules will be deflected upward when they flow past the wing

(C) Air molecules will move faster over the upper surface of the wing than the lower surface

(D) Air molecules will move slower over the upper surface of the wing than the lower surface

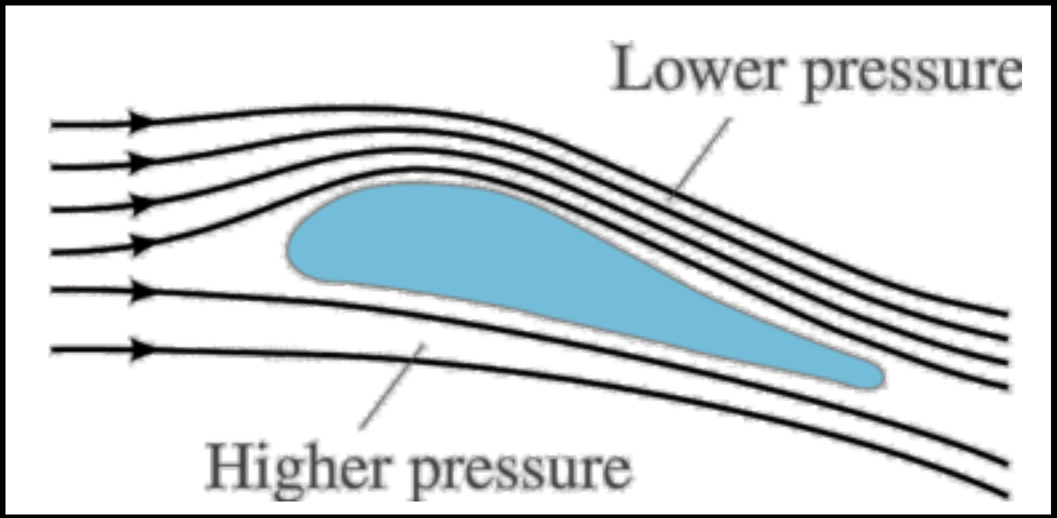
Last lecture: review

To provide the lift force needed for flight, aeroplane wings *must* be designed so that...:

(C) Air molecules will move faster over the upper surface of the wing than the lower surface

Bernoulli's equation $P + \rho g y + \frac{1}{2}\rho v^2 =$ constant

Long path ⟶ faster gas ⟶ lower pressure



force = lift

Short path

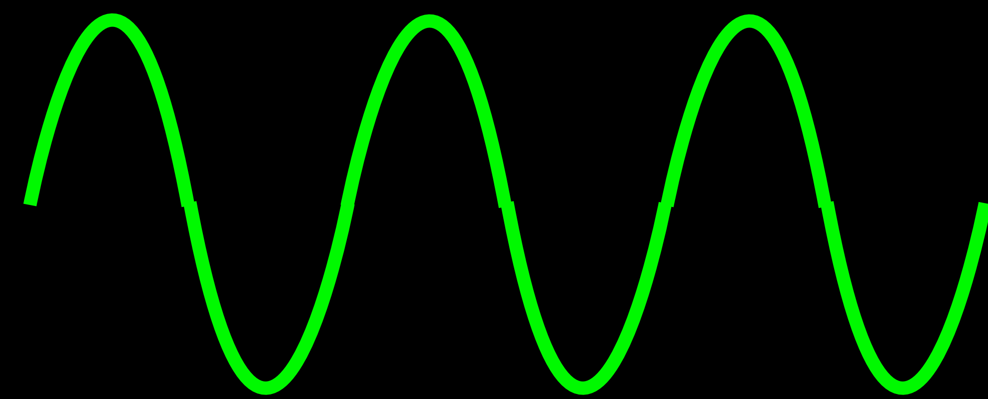
= time

Optics

Reflection & Refraction

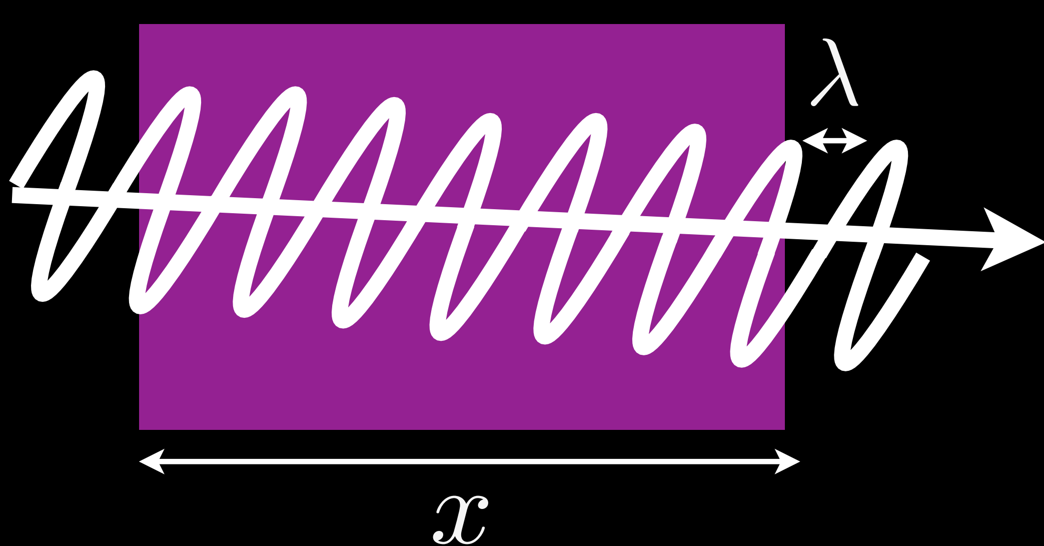
Optics

Light is an wave:



$$y(x,t) = A\cos(kx \pm \omega t)$$

But, if light is interacting with an object much bigger than its wavelength:



$$\lambda << x$$

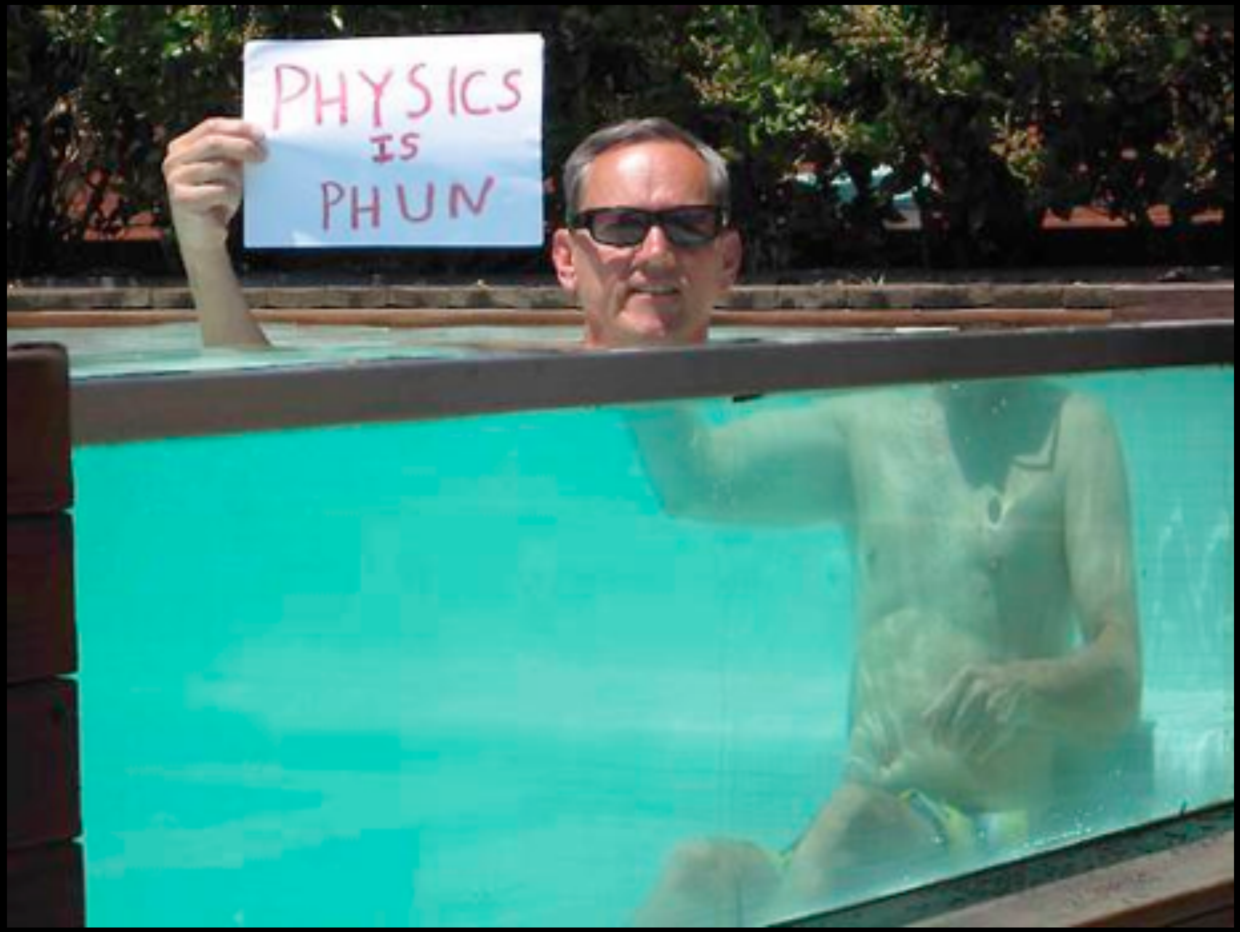
Assume:

Light travels in a straight line: a ray

Geometrical optics

Reflection & Refraction

Reflection



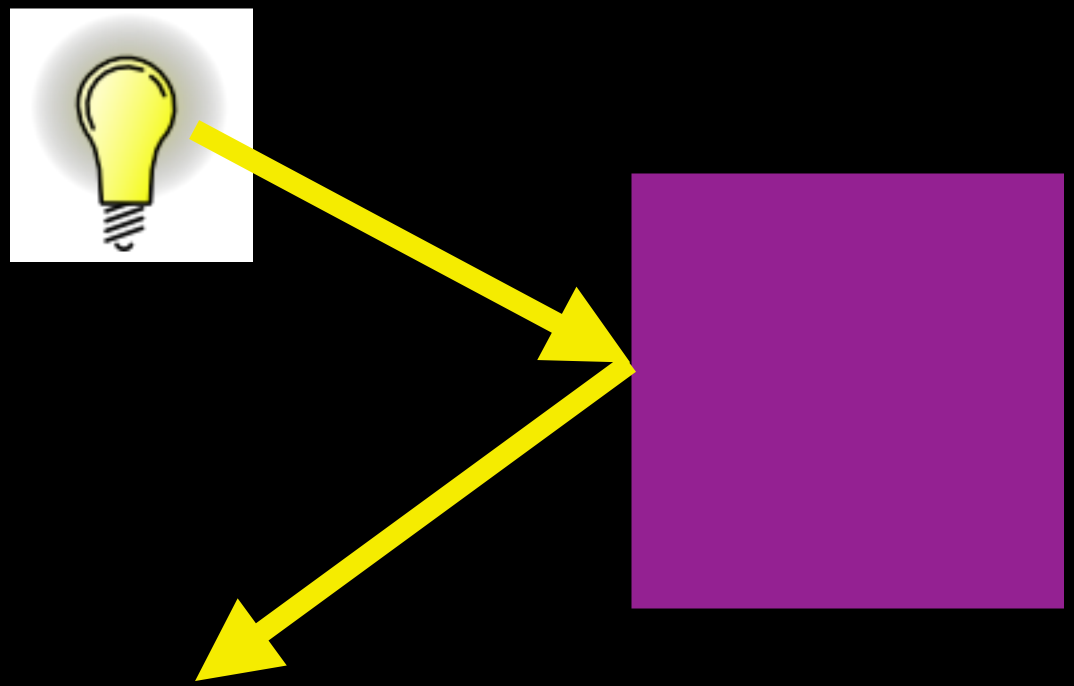
Refraction

Reflection & Refraction

Reflection:

Light ray hits surface

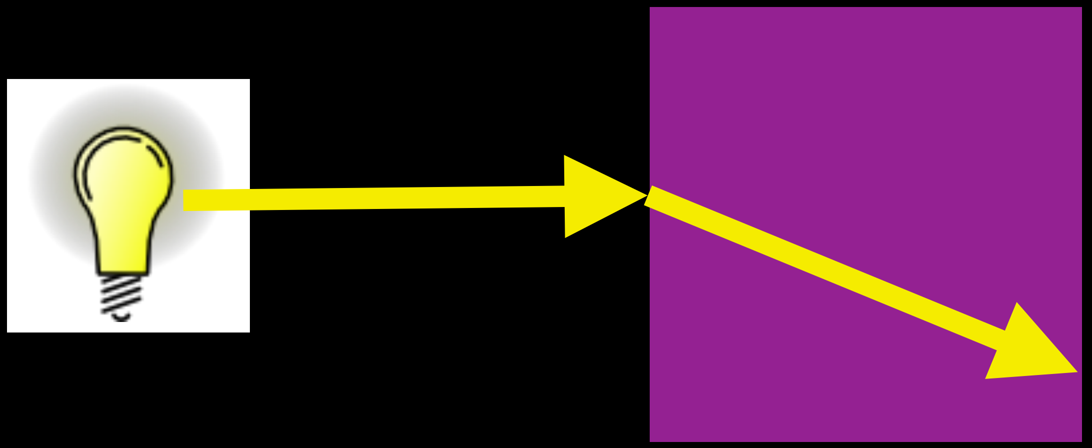
Ray moves away from surface



Refraction:

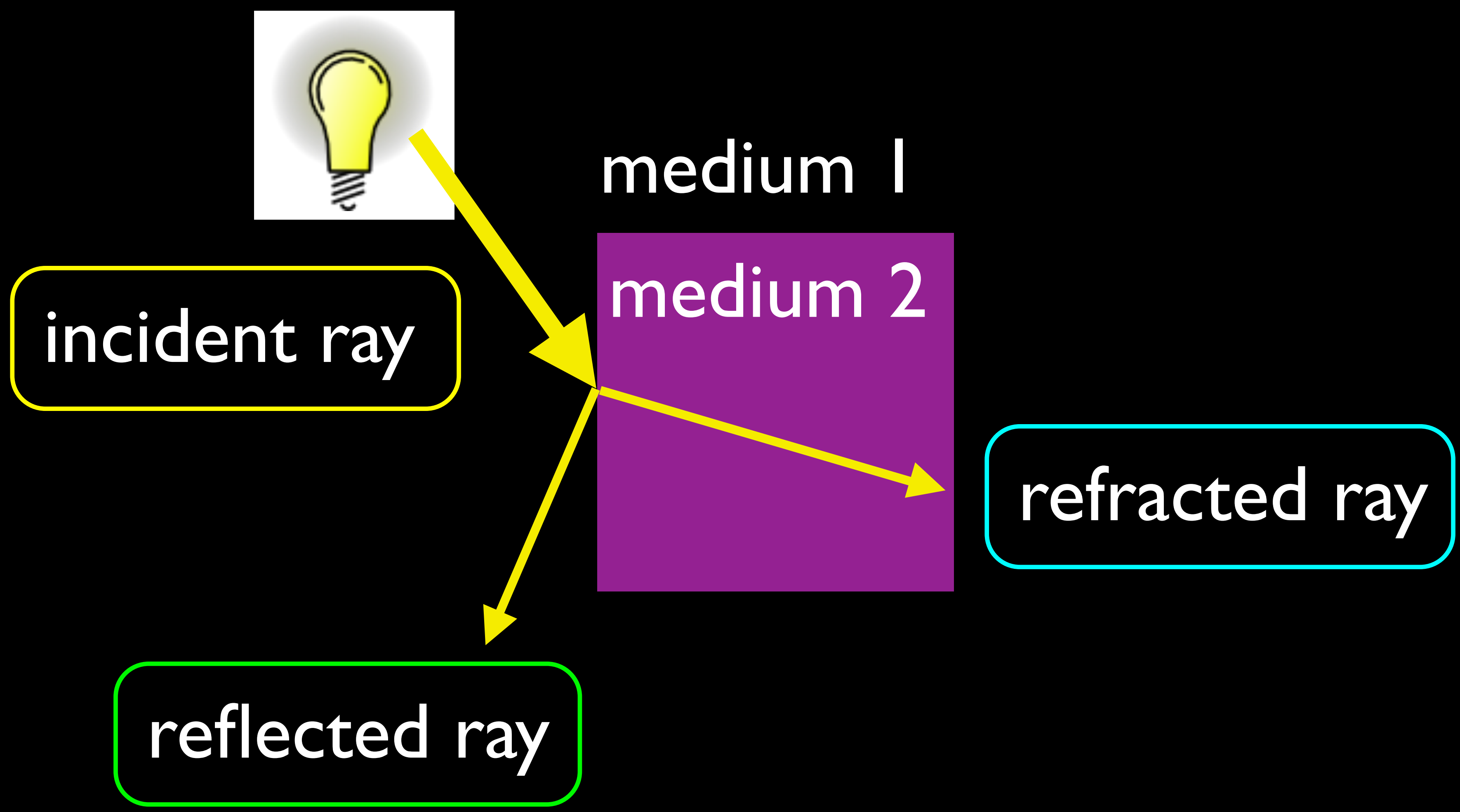
Light ray hits surface

Ray enters object and changes direction.



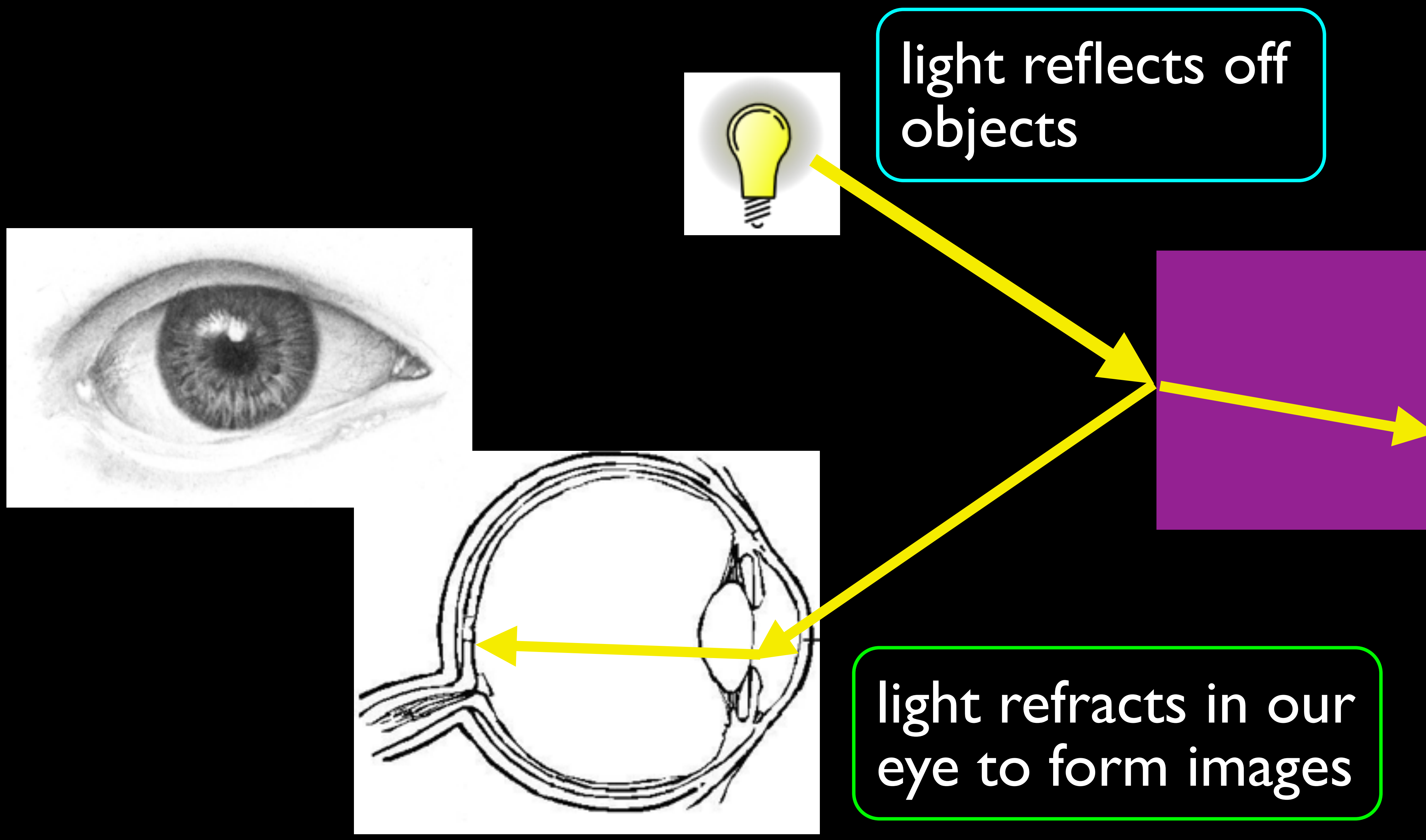
Reflection & Refraction

Usually, a ray is both reflected and refracted.

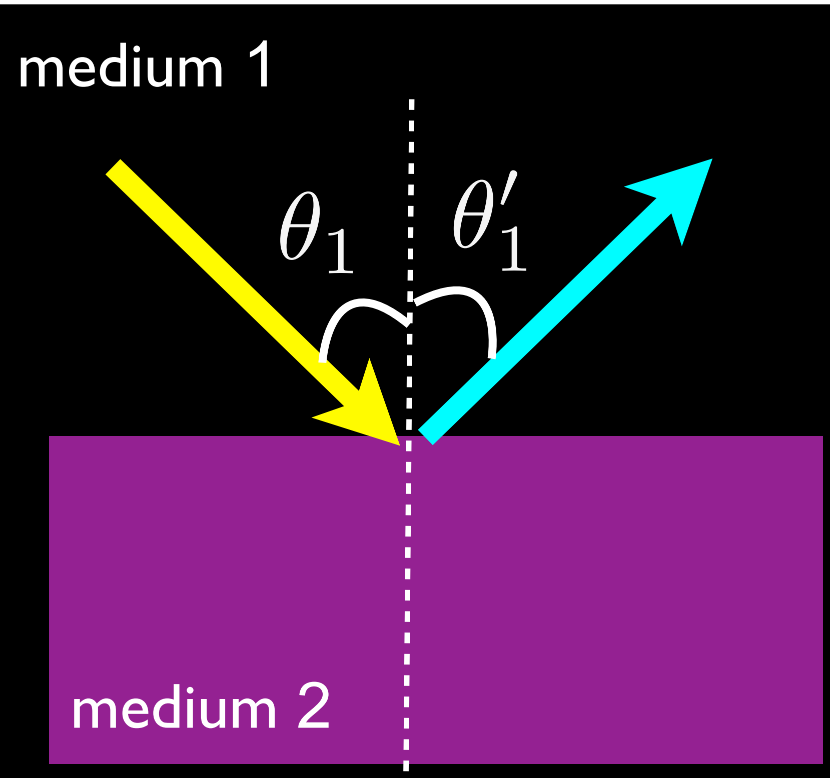


Reflection & Refraction

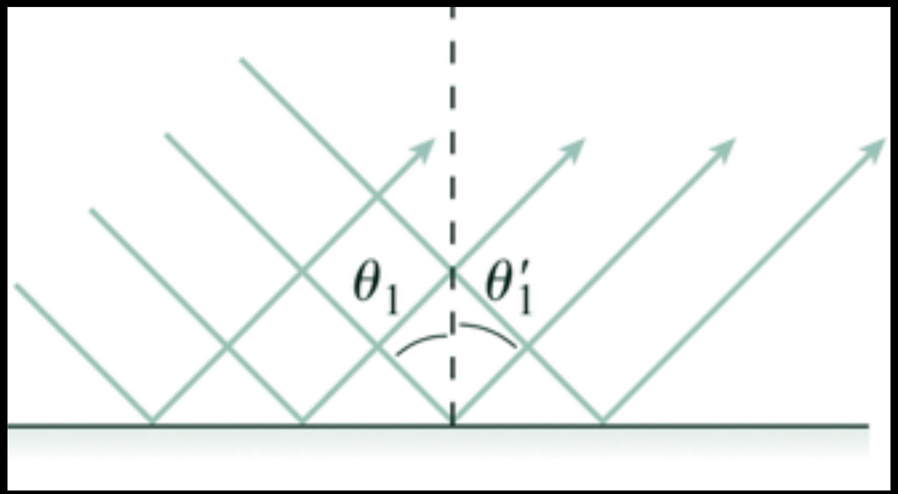
Your eye used both reflection and refraction:



Reflection



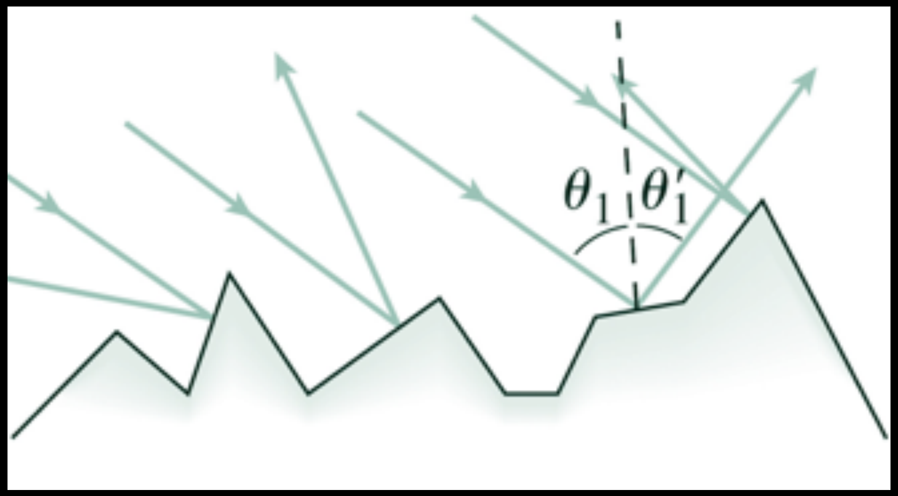
$$\theta_1' = \theta_1$$



Specular reflection

Parallel rays, smooth surface

Rays reflected without distortion.

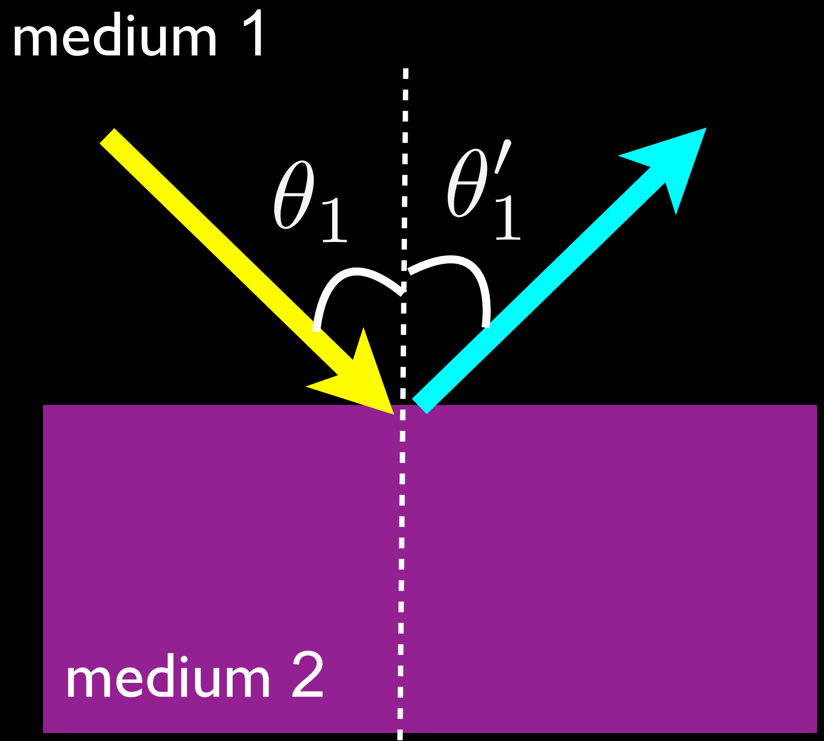


Diffuse reflection

Rough surface

Rays reflected in different directions

Reflection



$$\theta_1' = \theta_1$$

e.g. Au foil

Specular reflection

Parallel rays, smooth surface

Rays reflected without distortion.

e.g. white paper

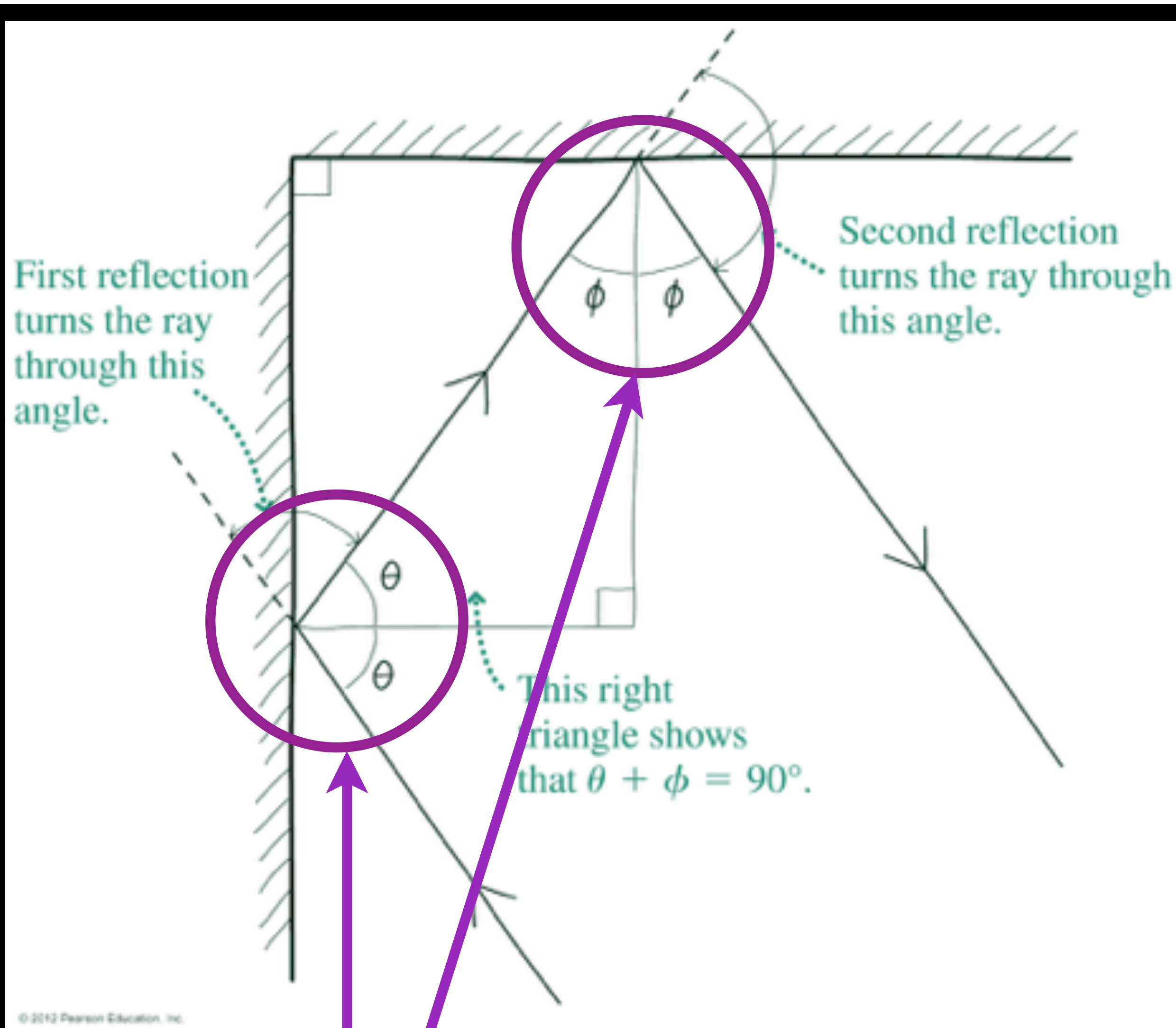
Diffuse reflection

Rough surface

Rays reflected in different directions

Reflection

Example



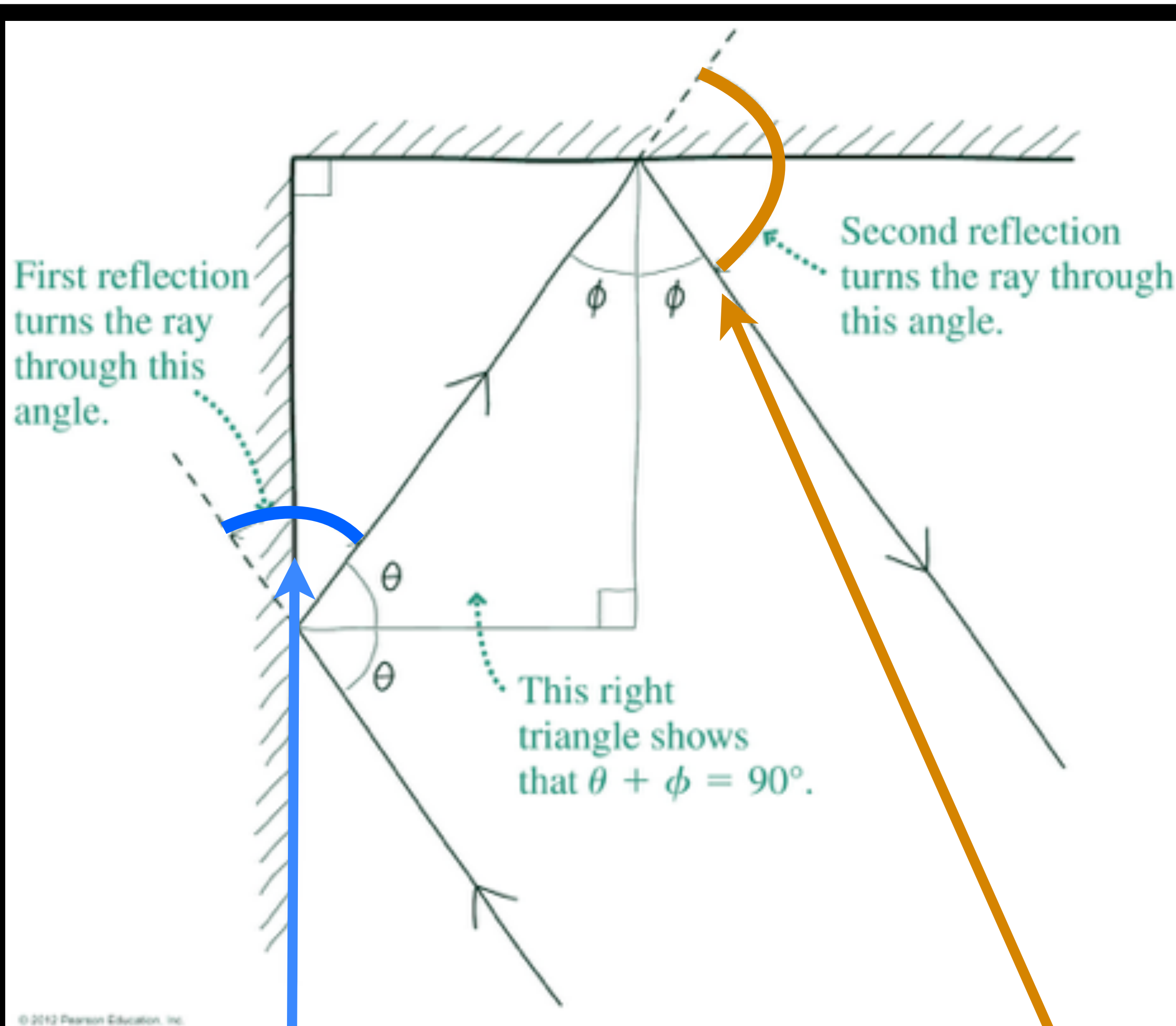
since $\theta_1' = \theta_1$

2 mirrors joined at right angles (perpendicular)

Show any incident light ray will return anti-parallel (opposite direction)

turn 180°

Reflection



2 mirrors joined at right angles (perpendicular)

Show any incident light ray will return anti-parallel (opposite direction)

turn $180°$

light ray turned by

$$180° - 2\theta$$

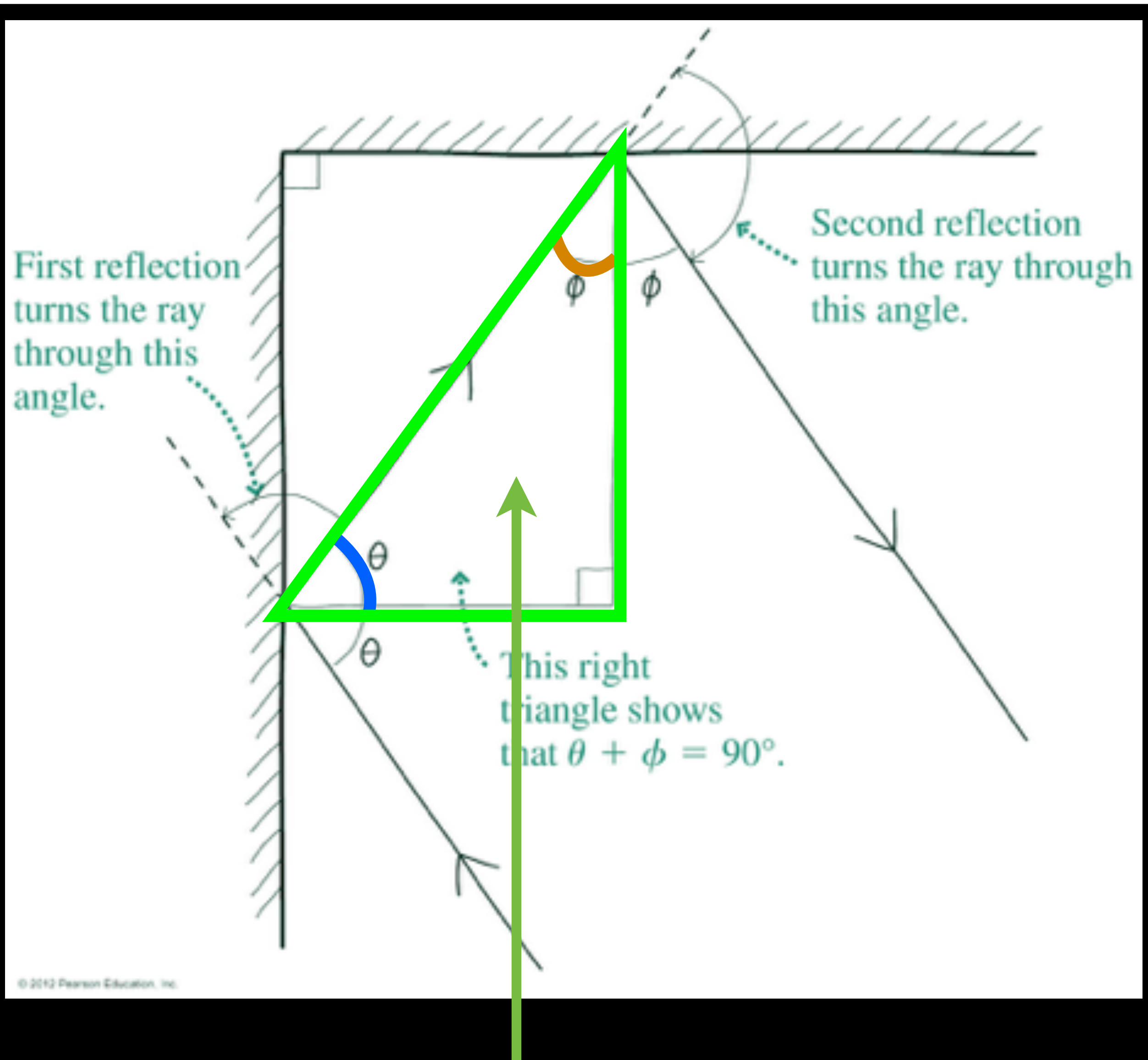
light ray turned by

$$180° - 2\phi$$

Total turning angle: $(180° - 2\theta) + (180° - 2\phi) = 360° - 2(\theta + \phi)$

Reflection

Example



$$\theta + \phi = 90^\circ$$

2 mirrors joined at right angles (perpendicular)

Show any incident light ray will return anti-parallel (opposite direction)

turn 180°

Total turning angle: $(180^\circ - 2\theta) + (180^\circ - 2\phi) = 360^\circ - 2(\theta + \phi)$

$$360^\circ - 2(90^\circ) = 180^\circ$$

Reflection

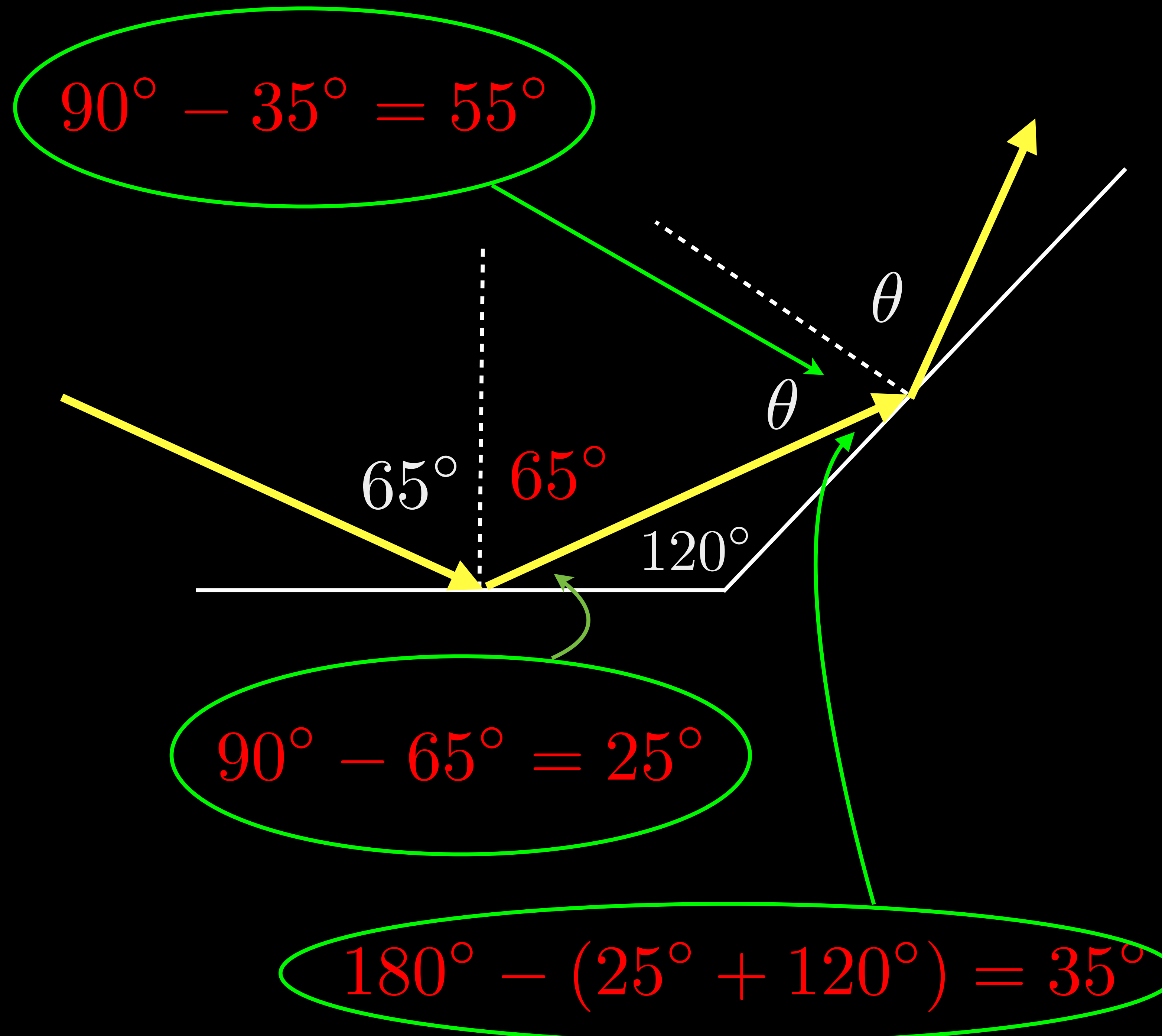
What is the angle θ ?

(A) 120°

(B) 25°

(C) 65°

(D) 55°



Reflection

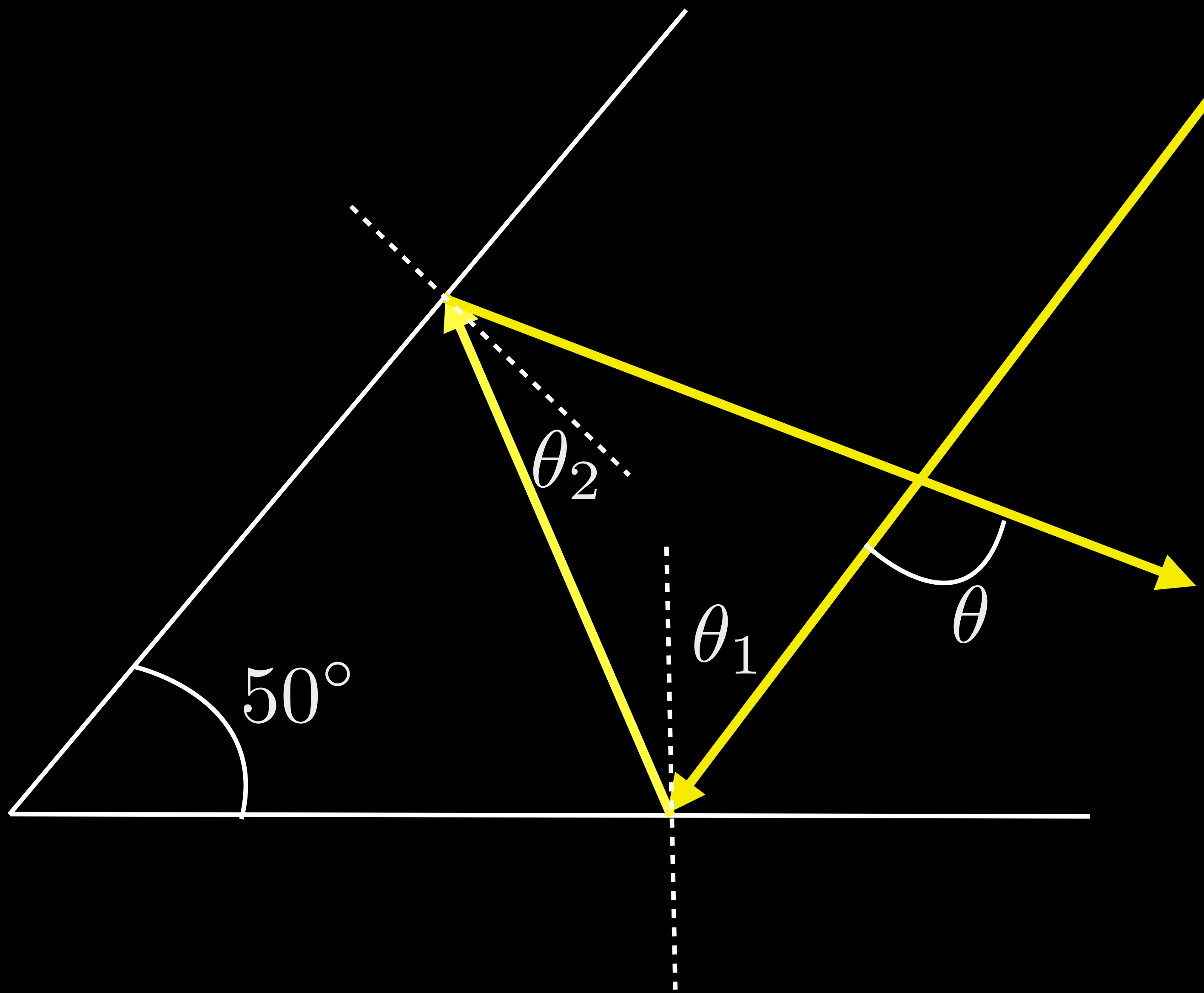
What is the angle θ ?

(A) 100°

(B) 50°

(C) 95°

(D) 120°

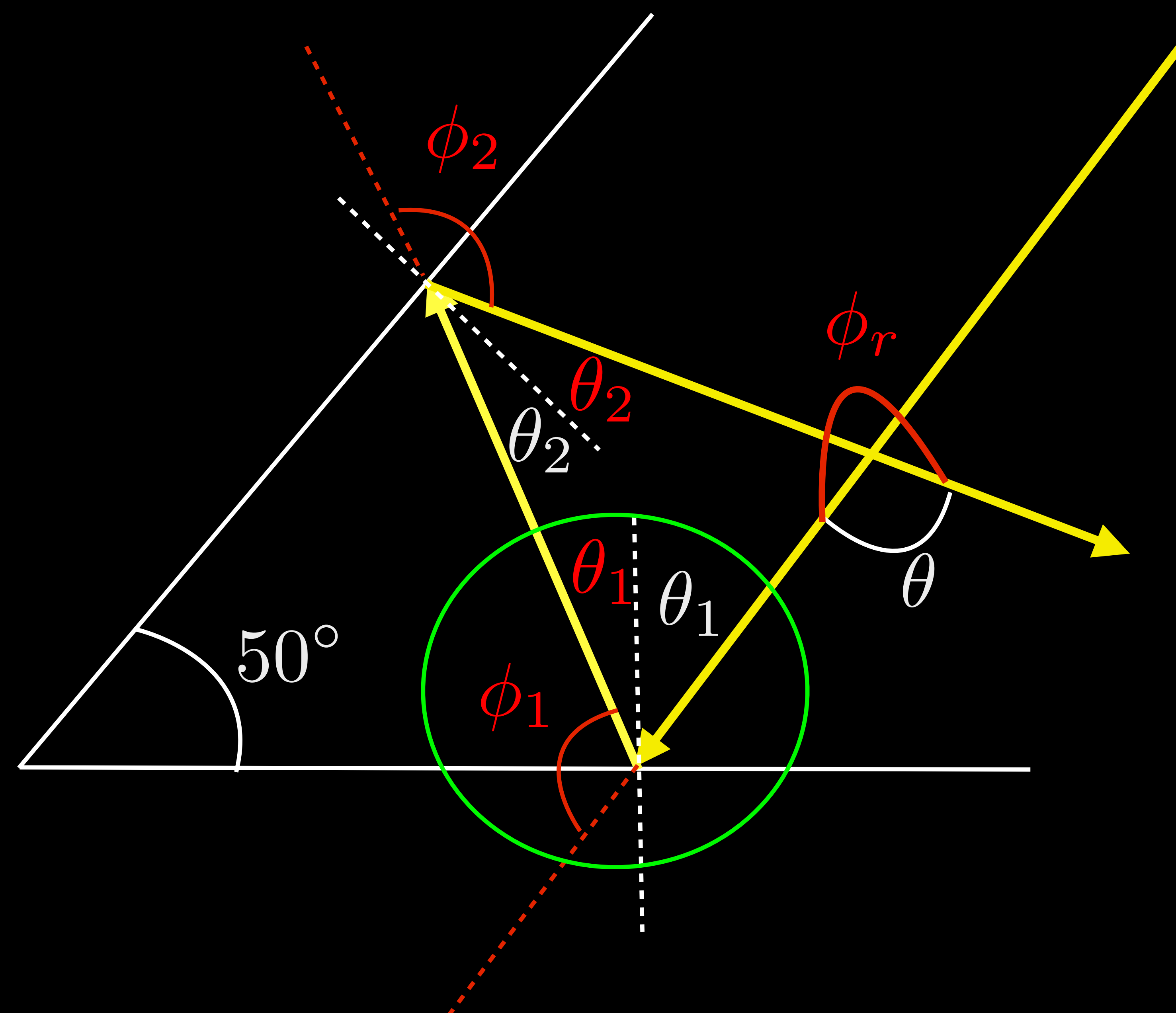


Reflection

What is the angle θ ?

Total rotation: $\phi_1 + \phi_2 = \phi_r$

$$\phi_1 = 180° - 2\theta_1$$



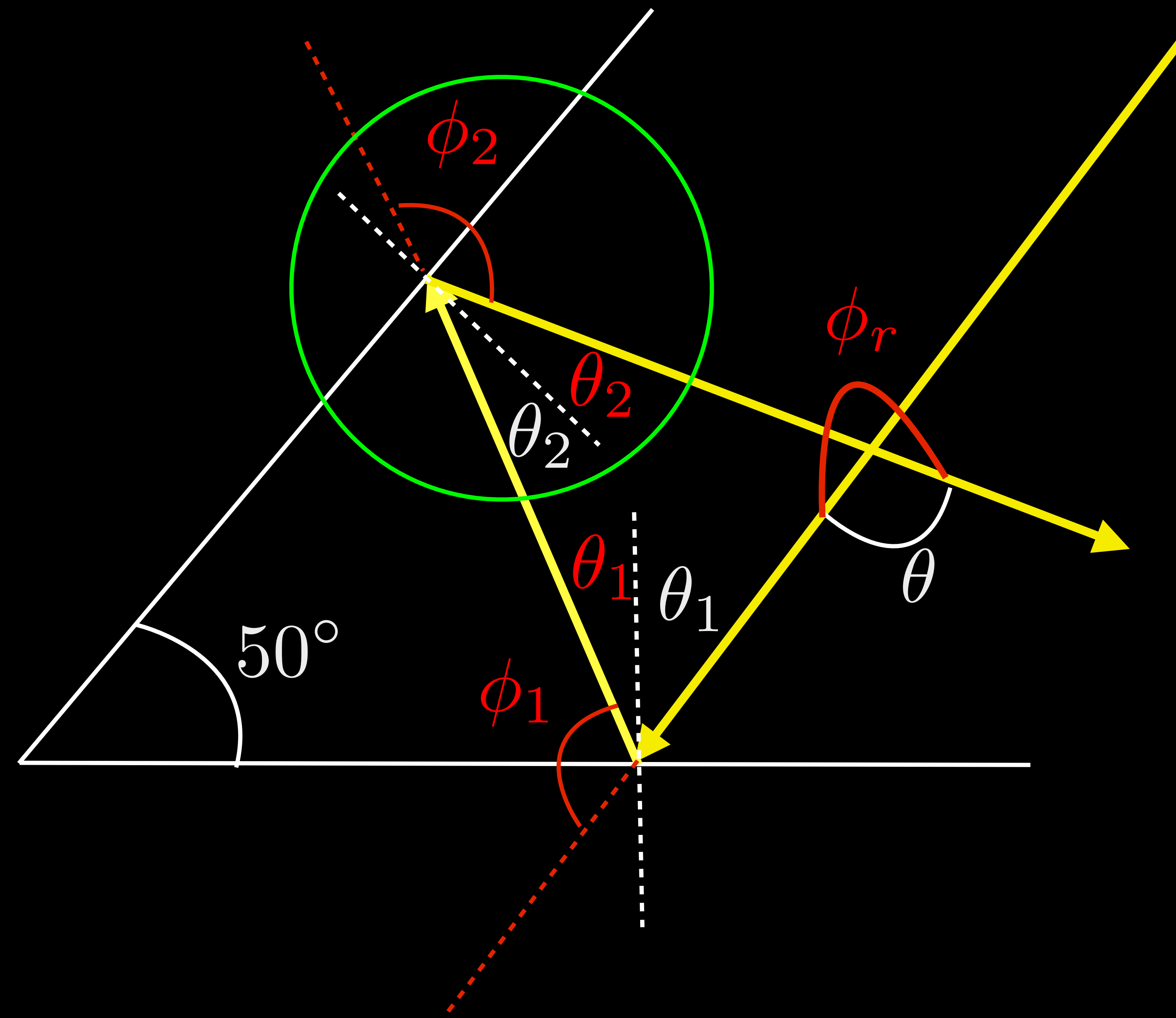
Reflection

What is the angle θ ?

Total rotation: $\phi_1 + \phi_2 = \phi_r$

$$\phi_1 = 180^\circ - 2\theta_1$$

$$\phi_2 = 180^\circ - 2\theta_2$$



Reflection

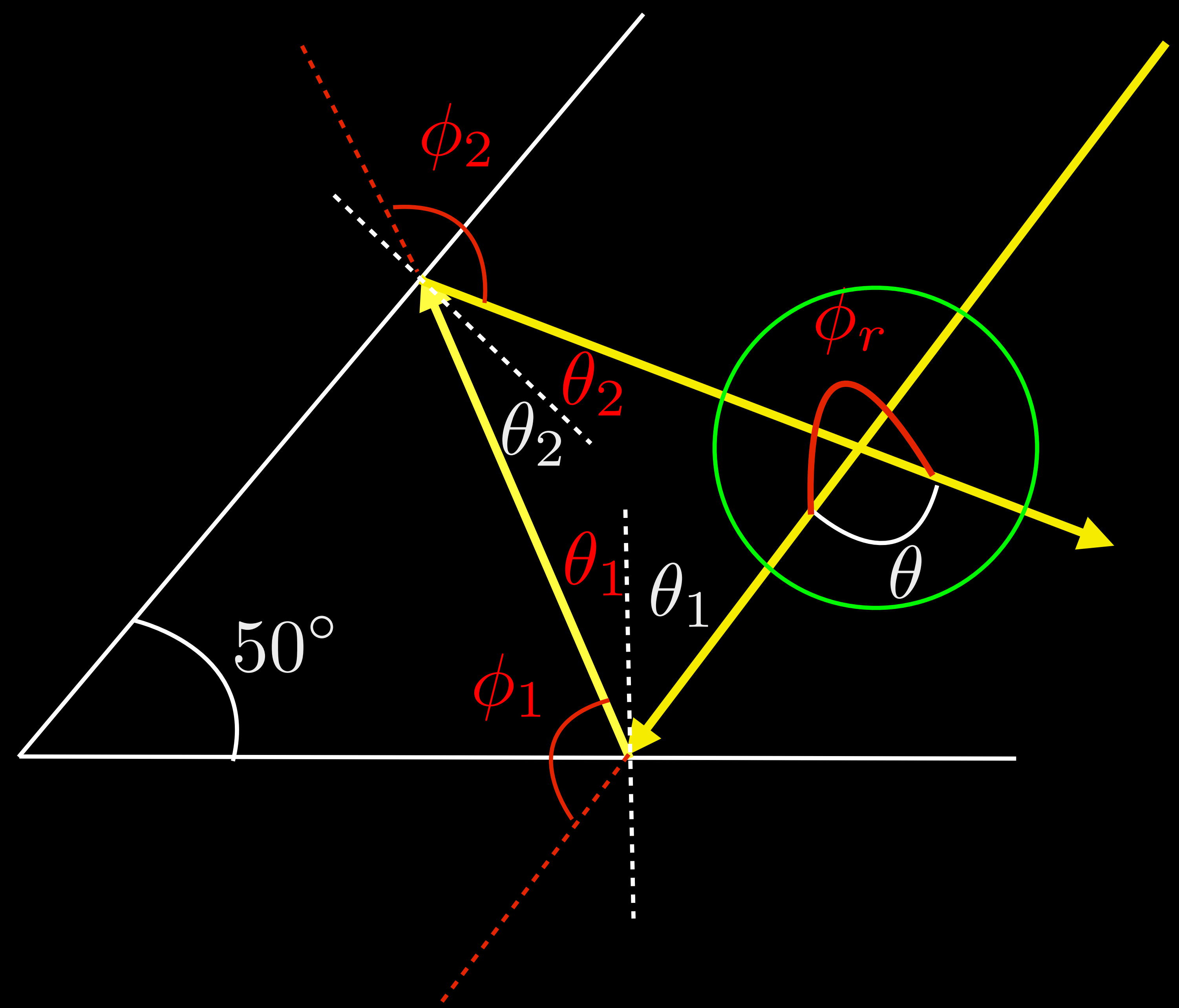
What is the angle θ ?

Total rotation: $\phi_1 + \phi_2 = \phi_r$

$$\phi_1 = 180^\circ - 2\theta_1$$

$$\phi_2 = 180^\circ - 2\theta_2$$

$$\theta = 360^\circ - \phi_r$$



Reflection

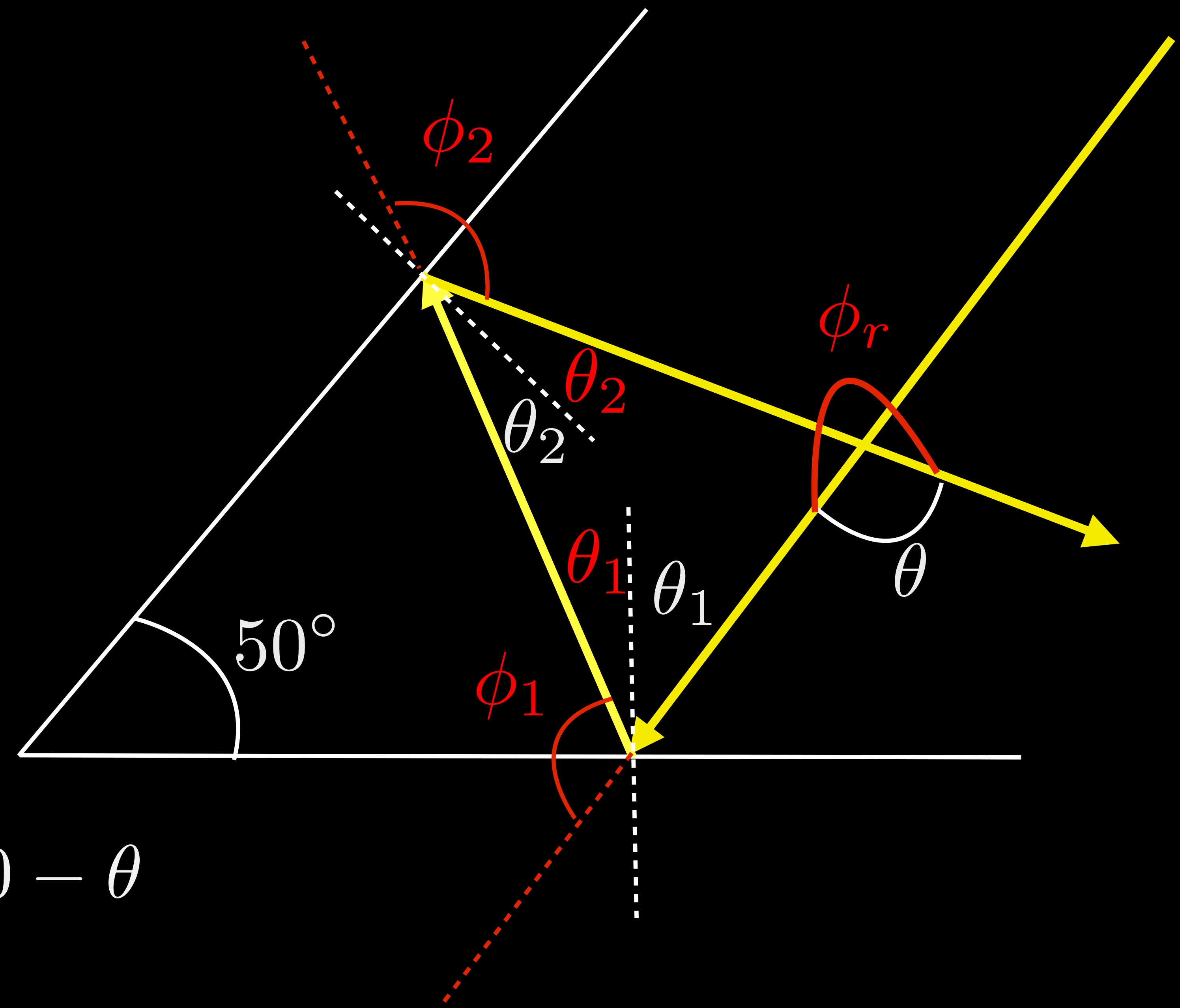
What is the angle θ ?

Total rotation: $\phi_1 + \phi_2 = \phi_r$

$$\phi_1 = 180^\circ - 2\theta_1$$

$$\phi_2 = 180^\circ - 2\theta_2$$

$$\theta = 360^\circ - \phi_r$$



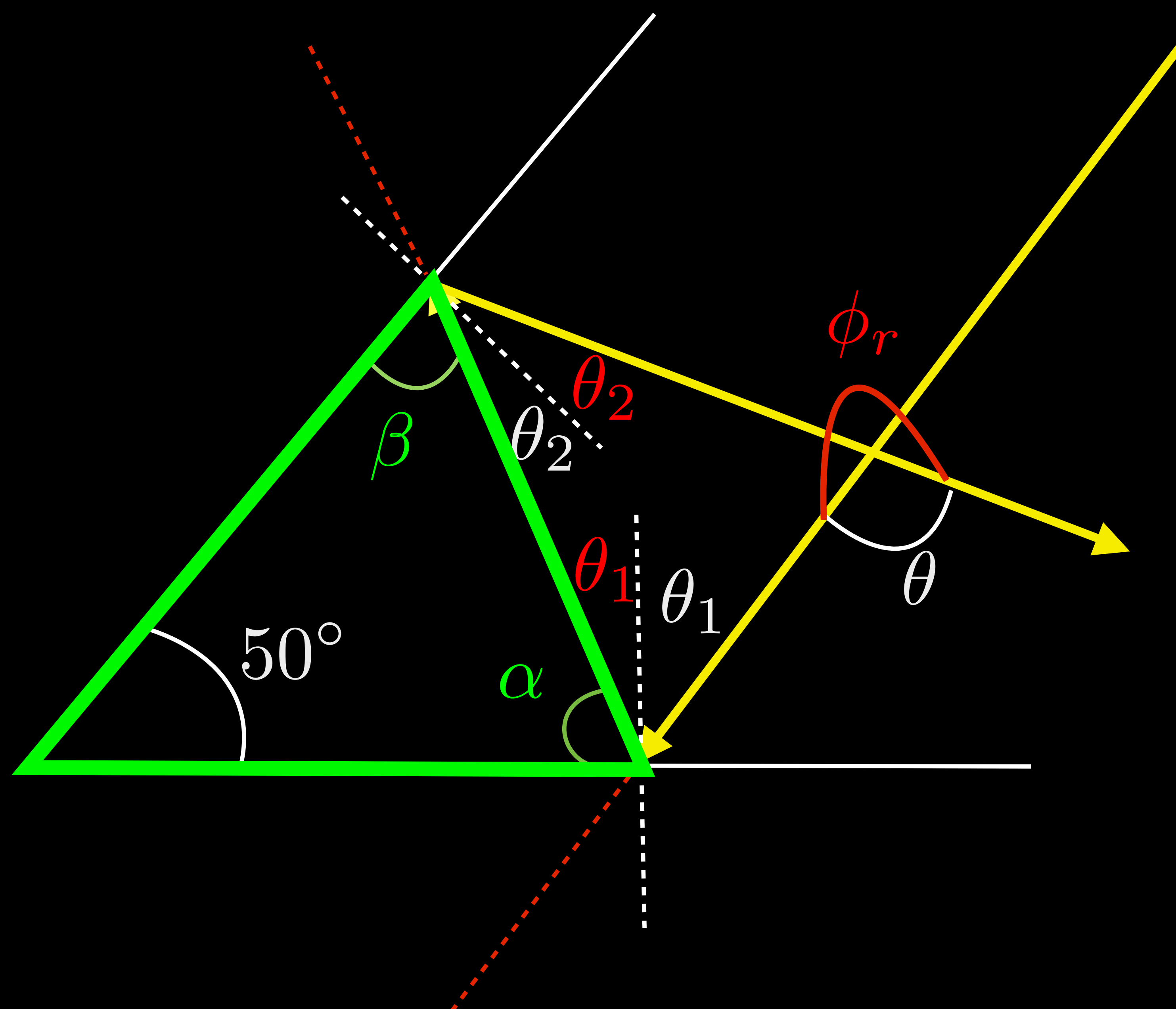
$$180 - 2\theta_1 + 180 - 2\theta_2 = 360 - \theta$$

$$2\theta_1 + 2\theta_2 = \theta$$

Reflection

What is the angle θ ?

$\alpha + \beta + 50^\circ = 180^\circ$ ➡ $\alpha + \beta = 130^\circ$

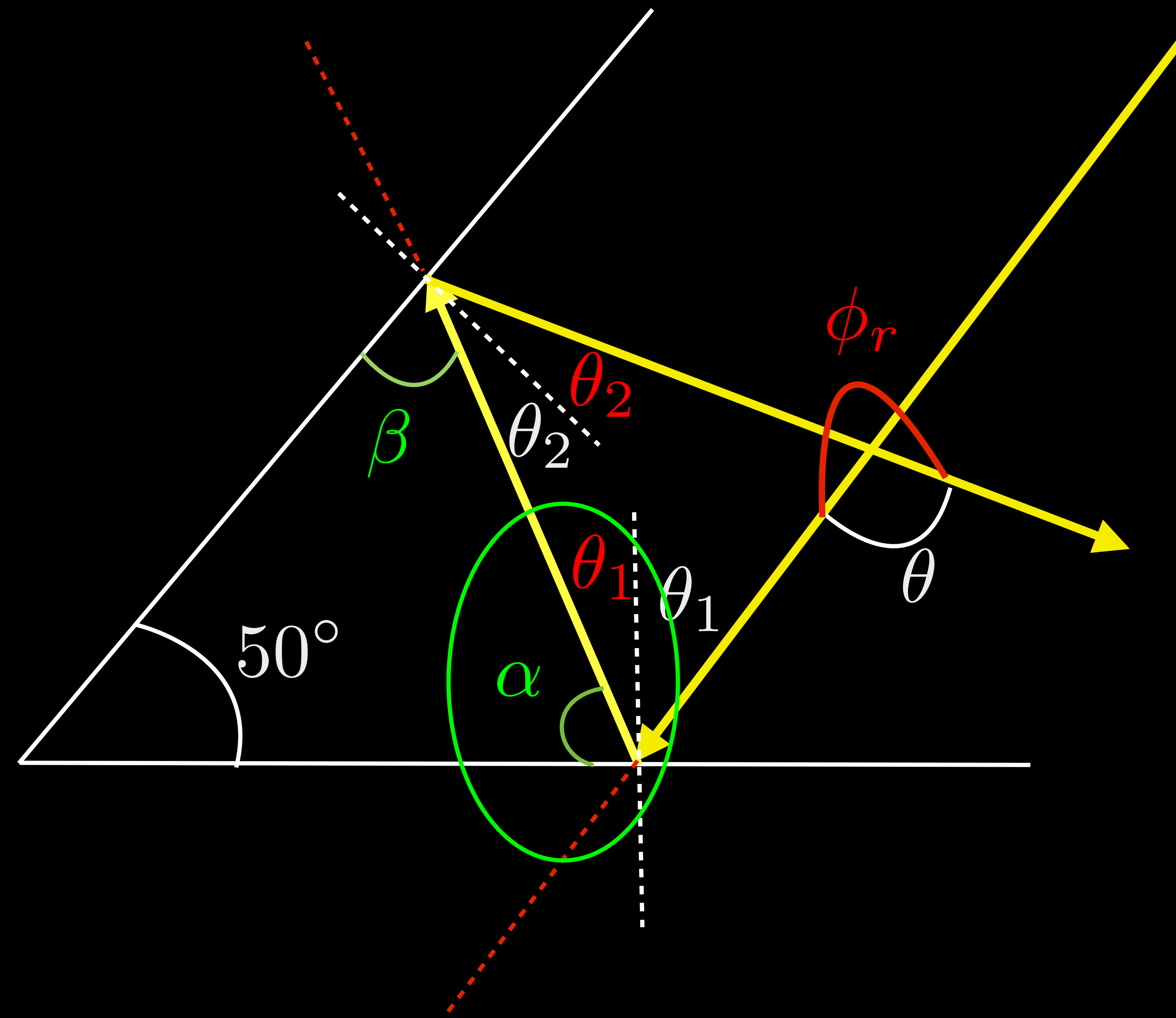


Reflection

What is the angle θ ?

$$\alpha + \beta + 50^\circ = 180^\circ \quad \Rightarrow \quad \alpha + \beta = 130^\circ$$

$$\alpha + \theta_1 = 90^\circ$$

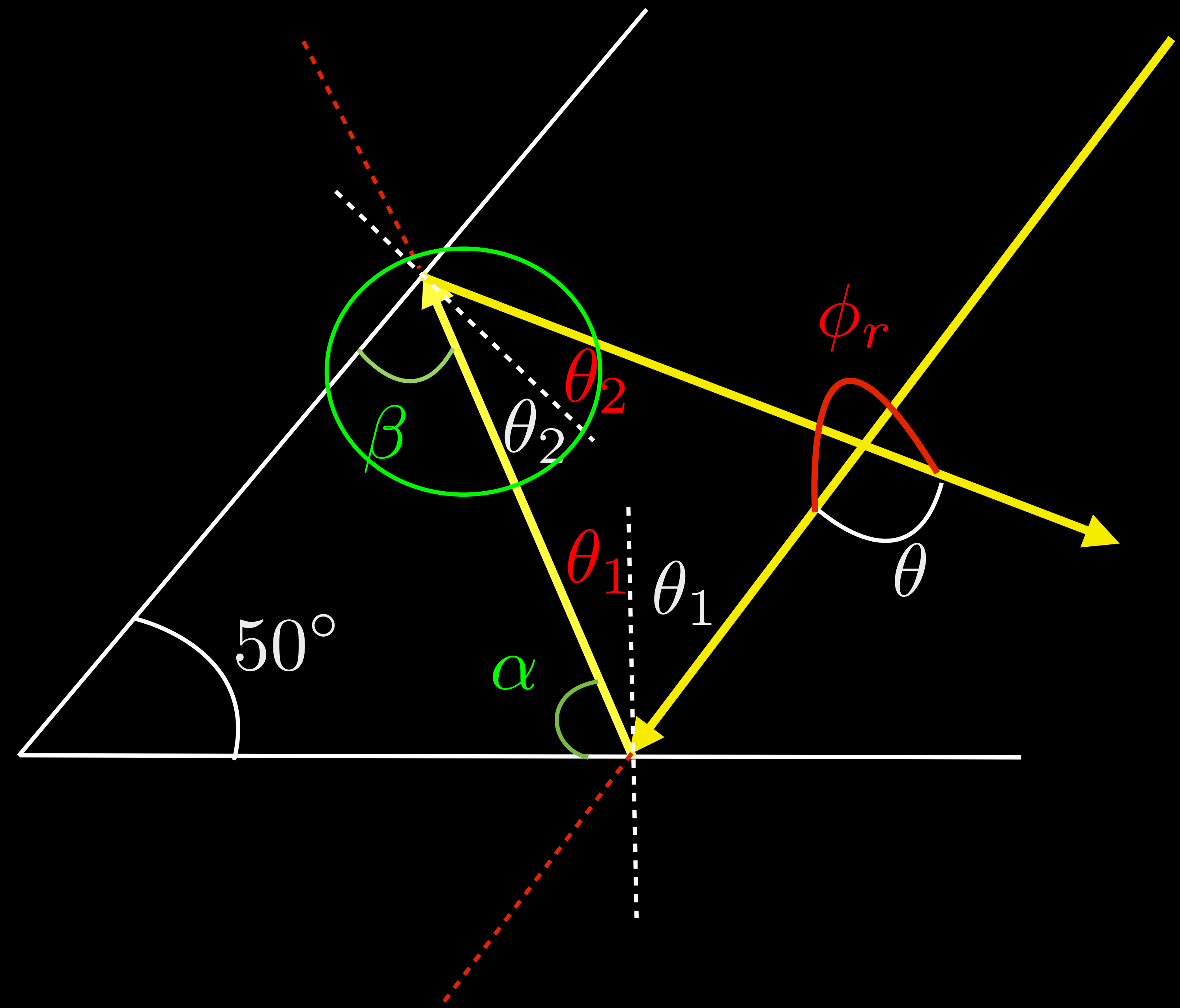


What is the angle θ ?

$$\alpha + \beta + 50^\circ = 180^\circ \Rightarrow \alpha + \beta = 130^\circ$$

$$\alpha + \theta_1 = 90^\circ$$

$$\beta + \theta_2 = 90^\circ$$



Reflection

What is the angle θ ?

$$\alpha + \beta + 50^\circ = 180^\circ \quad \Rightarrow \quad \alpha + \beta = 130^\circ$$

$$\alpha + \theta_1 = 90^\circ$$

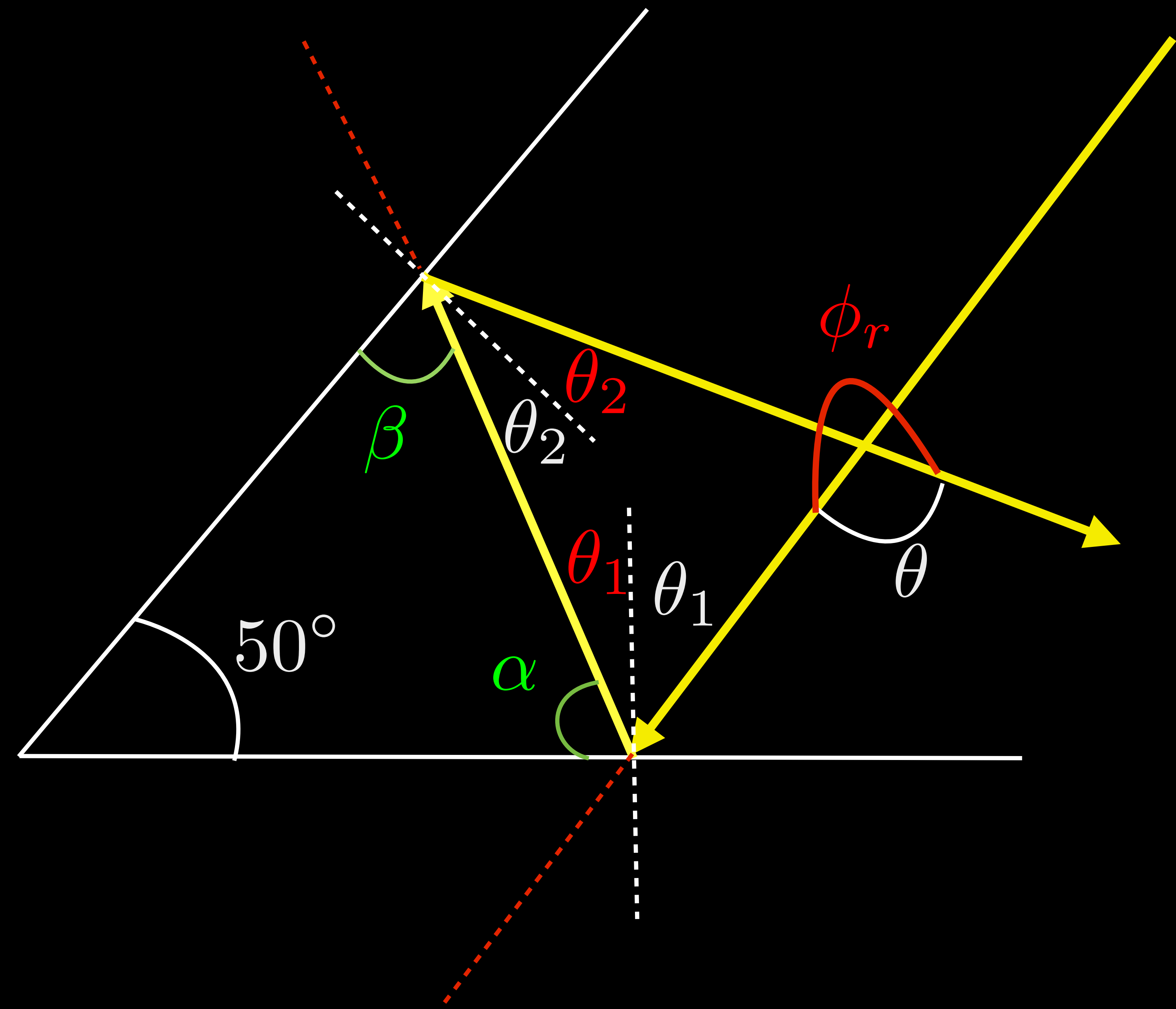
$$\beta + \theta_2 = 90^\circ$$

$$(\alpha + \theta_1) + (\beta + \theta_2) = 180^\circ$$

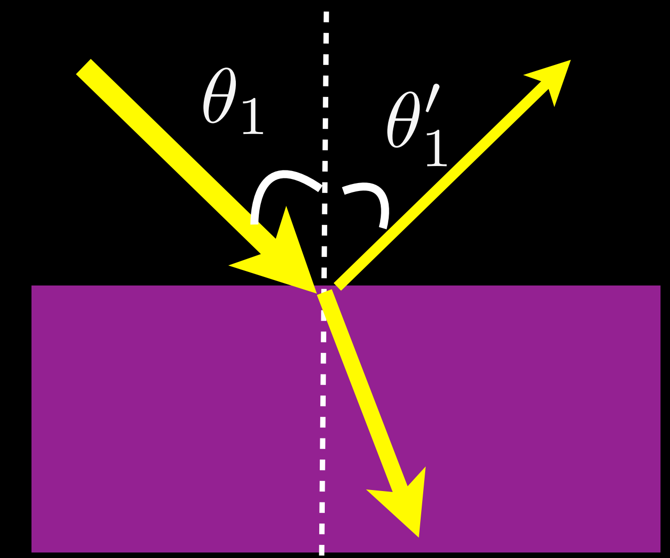
$$(\alpha + \beta) + (\theta_1 + \theta_2) = 180^\circ$$

130° $\quad$ $\theta/2$

$$\theta = 100^\circ$$

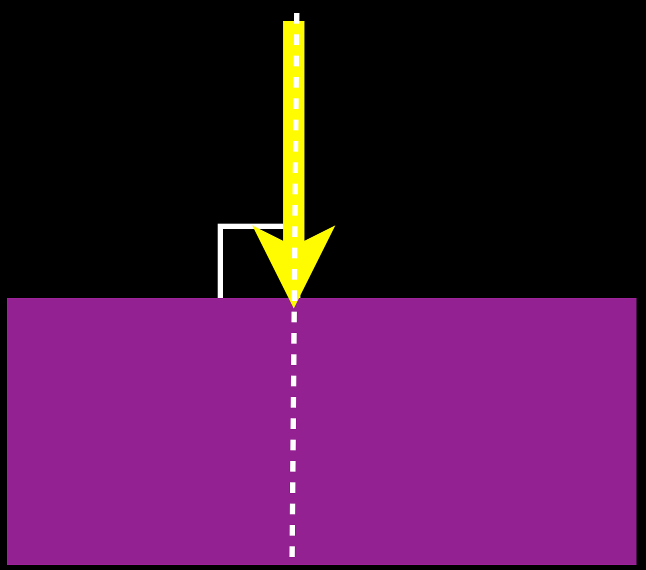


Partial Reflection



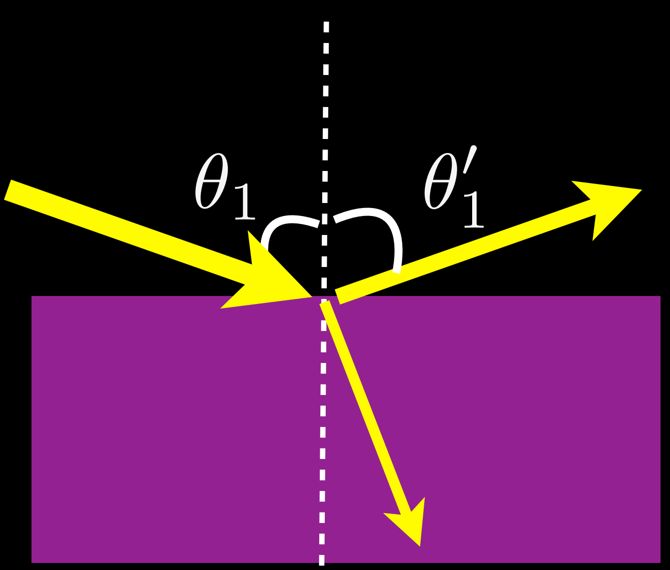
Some reflection always occurs

... even in glass



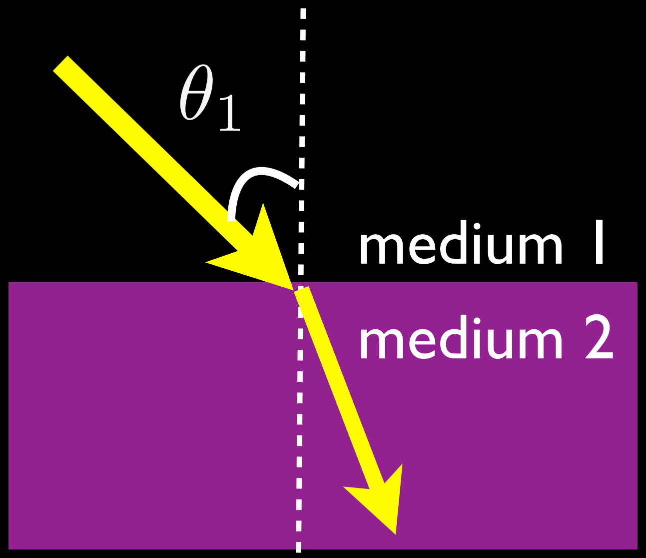
Smallest reflection for an incident ray normal (90°) to surface.

Glass ~ 4% reflected light



Larger angle results in more reflection

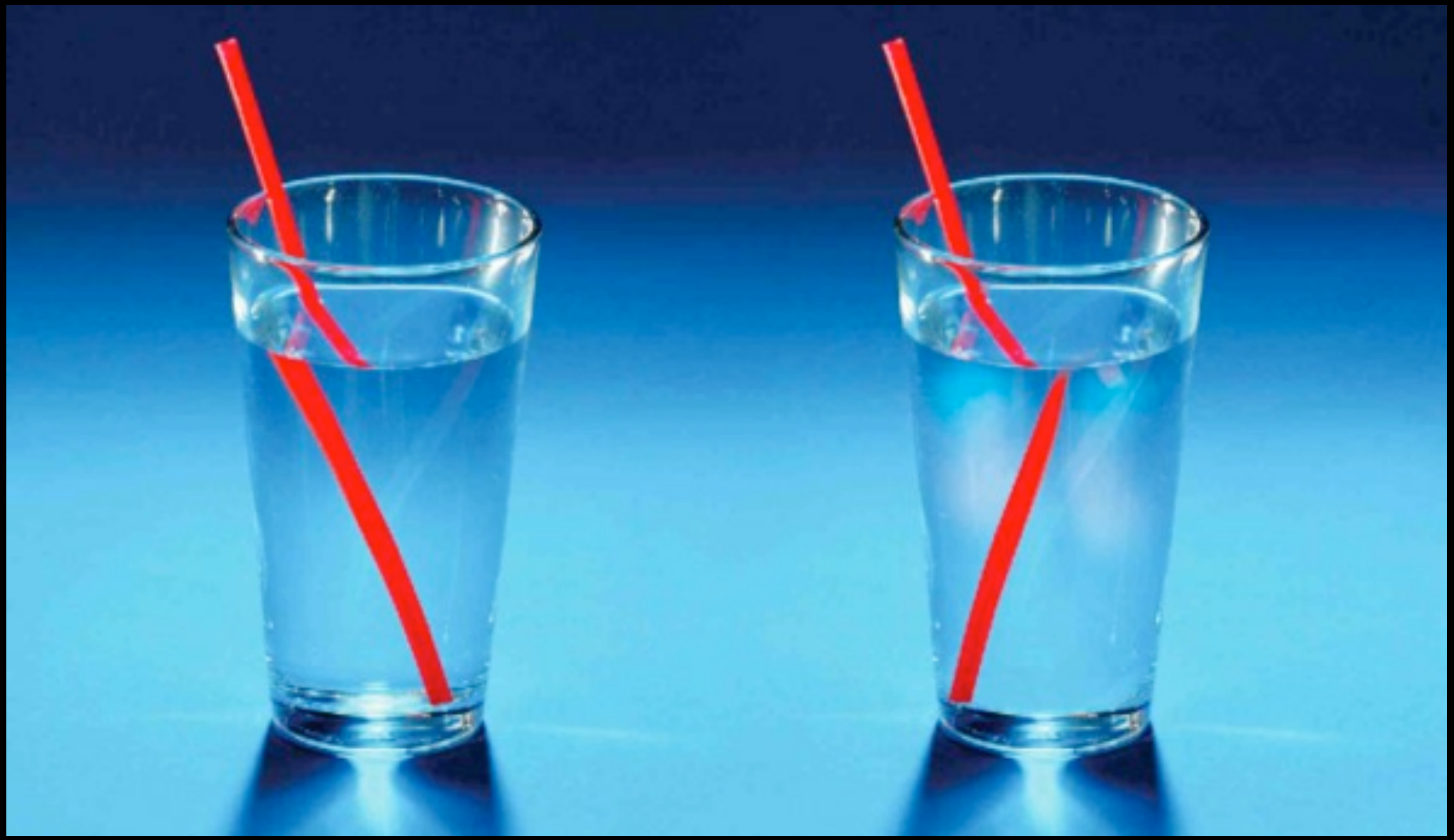
Refraction



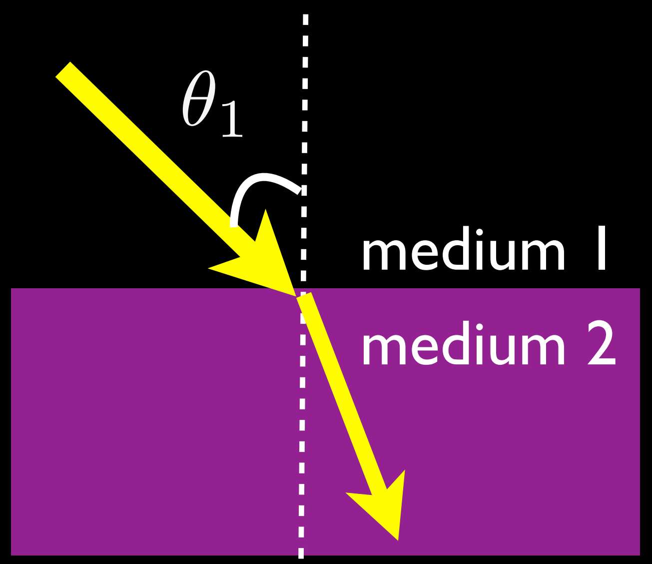
Refraction occurs when a light ray enters a new medium.

It changes speed

and direction



Refraction



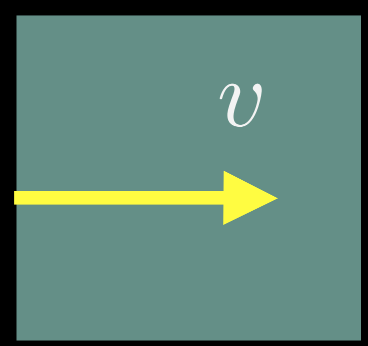
Refraction occurs when a light ray enters a new medium.

It changes speed

and direction

The speed of light in a transparent medium is lower than in a vacuum.

vacuum



transparent media

e.g. glass

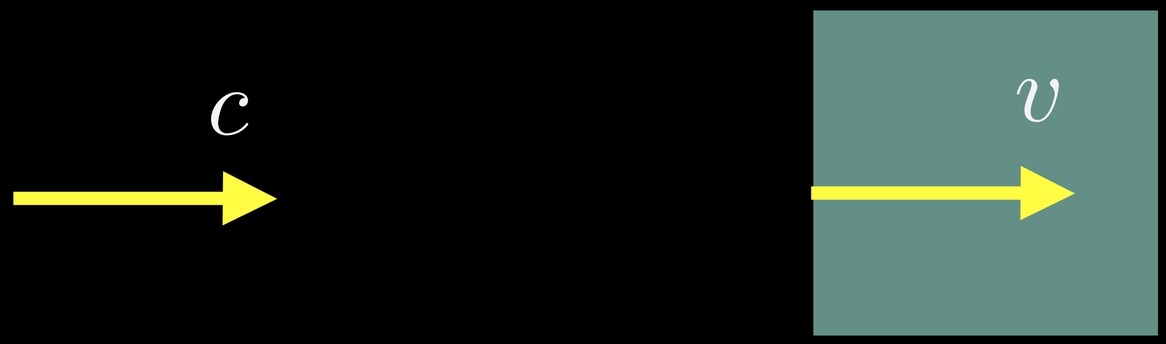
$$v < c$$

$$c = 3 \times 10^8 \, \mathrm{m/s}$$

speed of light in a vacuum

(index of refraction) $$n = \frac{c}{v}$$

Refraction



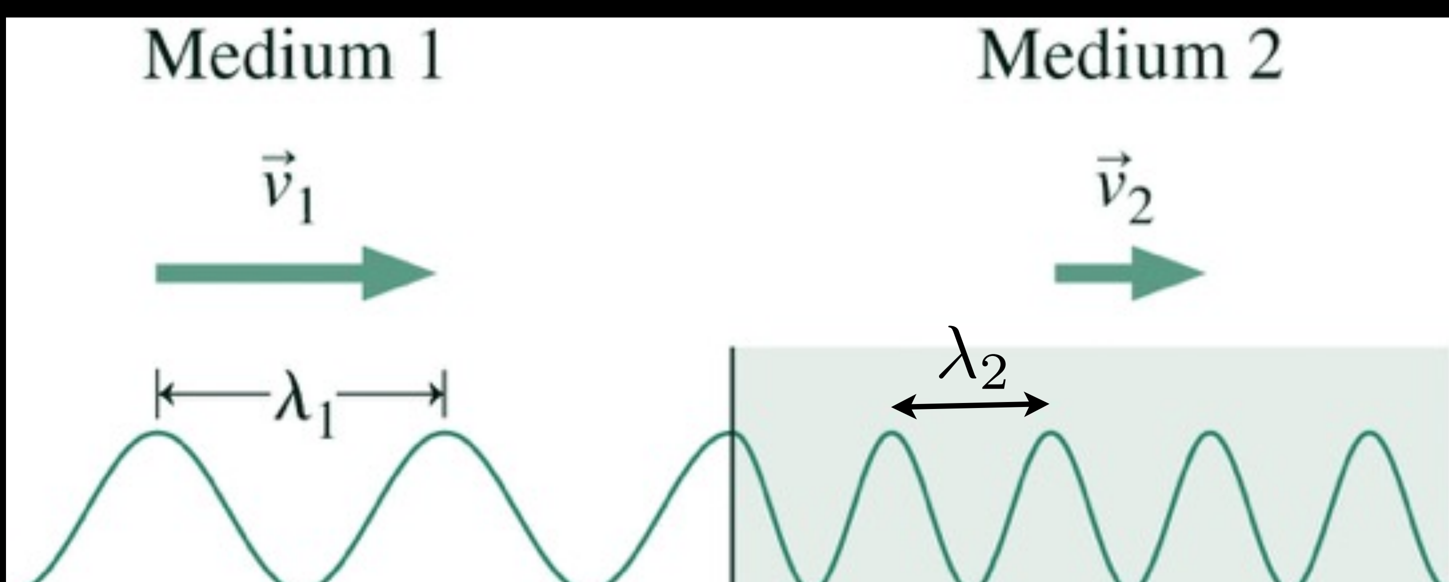
vacuum

transparent media

e.g. glass

$$n = \frac{c}{v}$$

$$v = f\lambda \quad \Rightarrow \quad \lambda = \frac{v}{f} = \frac{c}{nf}$$



Neither c nor f change.

Only λ and n .

$$\lambda \propto \frac{1}{n}$$

Refraction

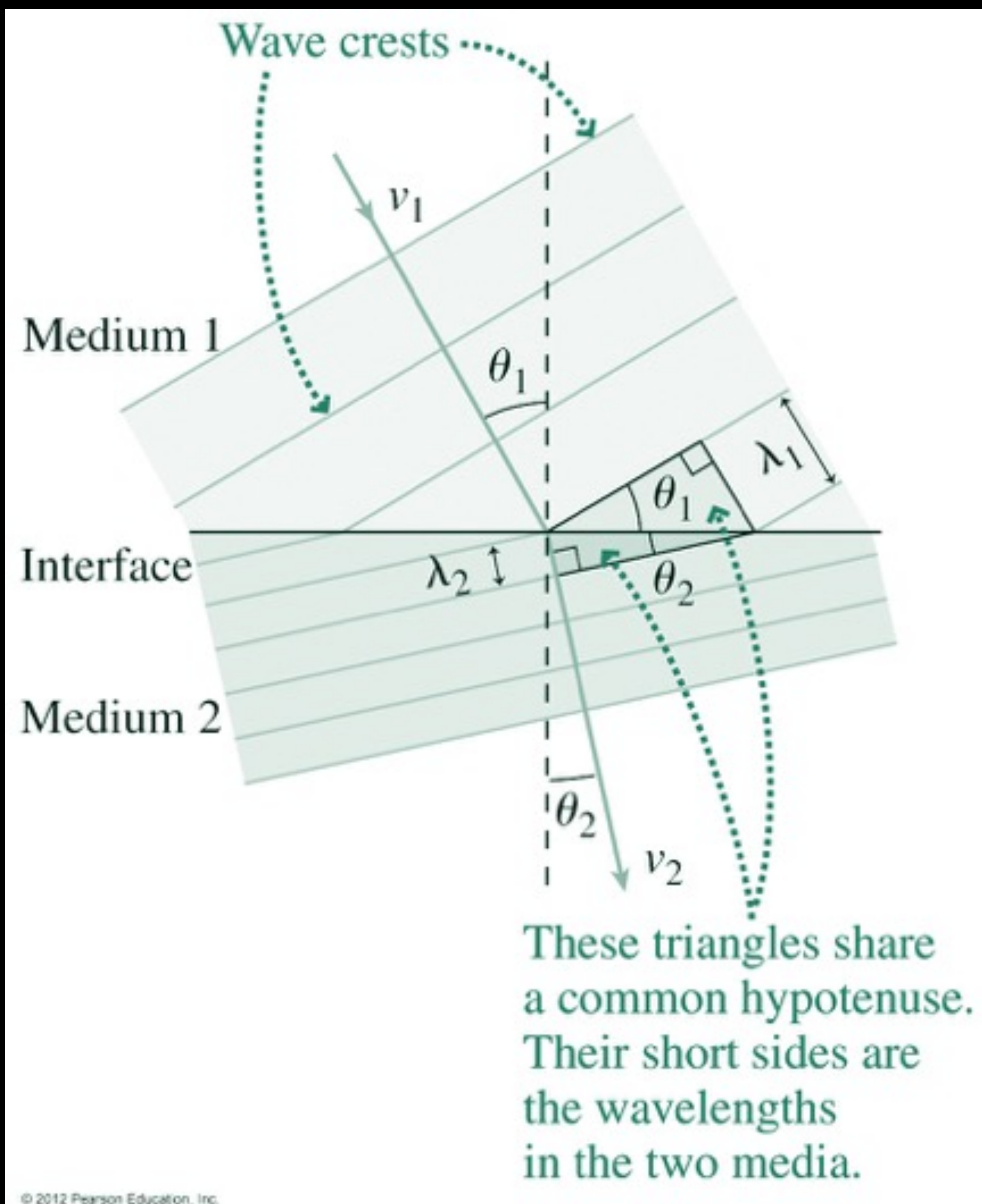
Table 30.1 Indices of Refraction*

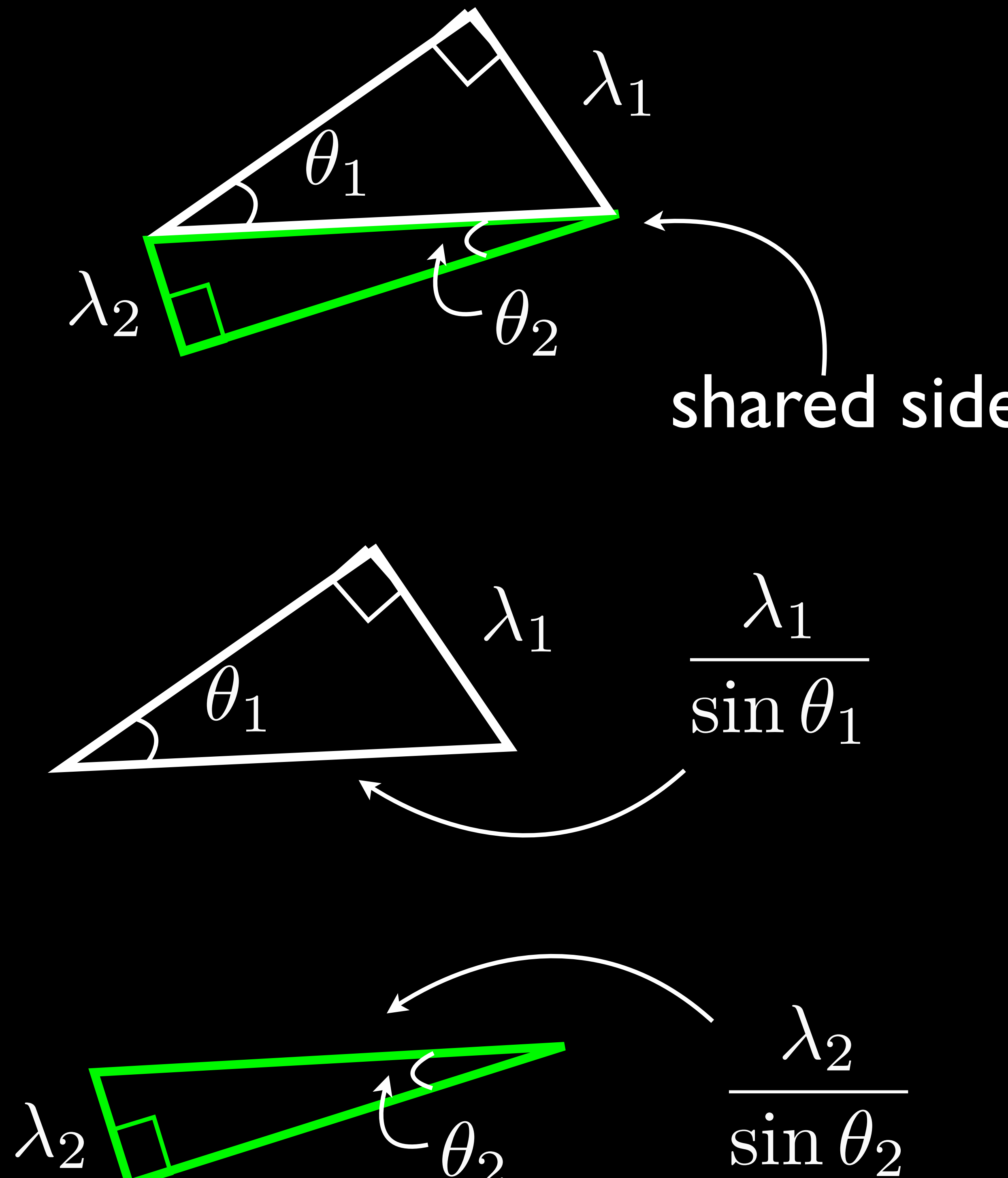
Substance	Index of Refraction, *n*
Gases	
Air	1.000293
Carbon dioxide	1.00045
Liquids	
Water	1.333
Ethyl alcohol	1.361
Glycerine	1.473
Benzene	1.501
Diiodomethane	1.738
Solids	
Ice (H_2O)	1.309
Polystyrene	1.49
Glass	1.5–1.9
Sodium chloride (NaCl)	1.544
Diamond (C)	2.419
Rutile (TiO_2)	2.62

*At 1 atm pressure and temperatures ranging from 0°C to 20°C, measured at a wavelength of 589 nm (the yellow line of sodium).

Refraction

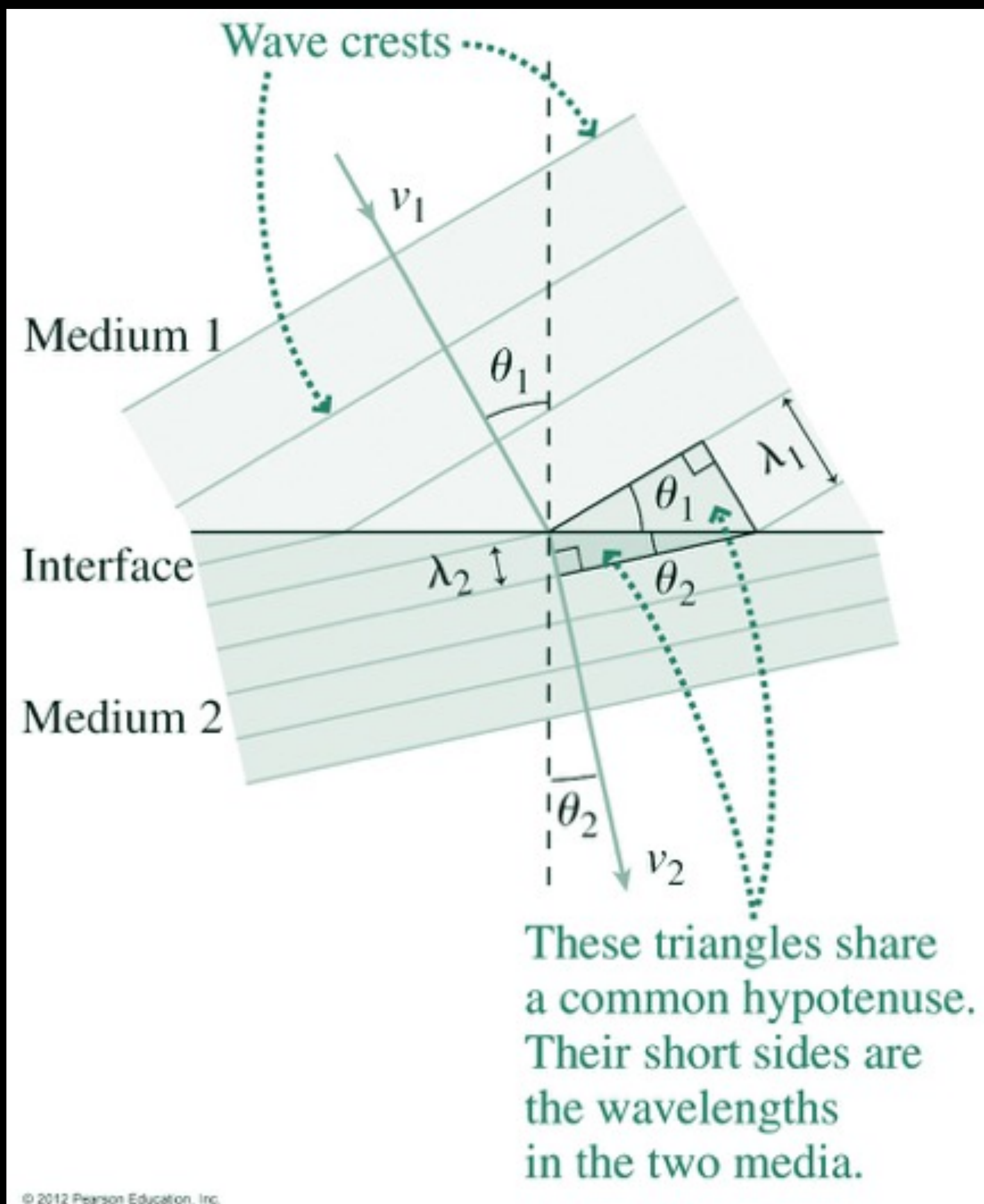
The change in λ causes the direction to change:

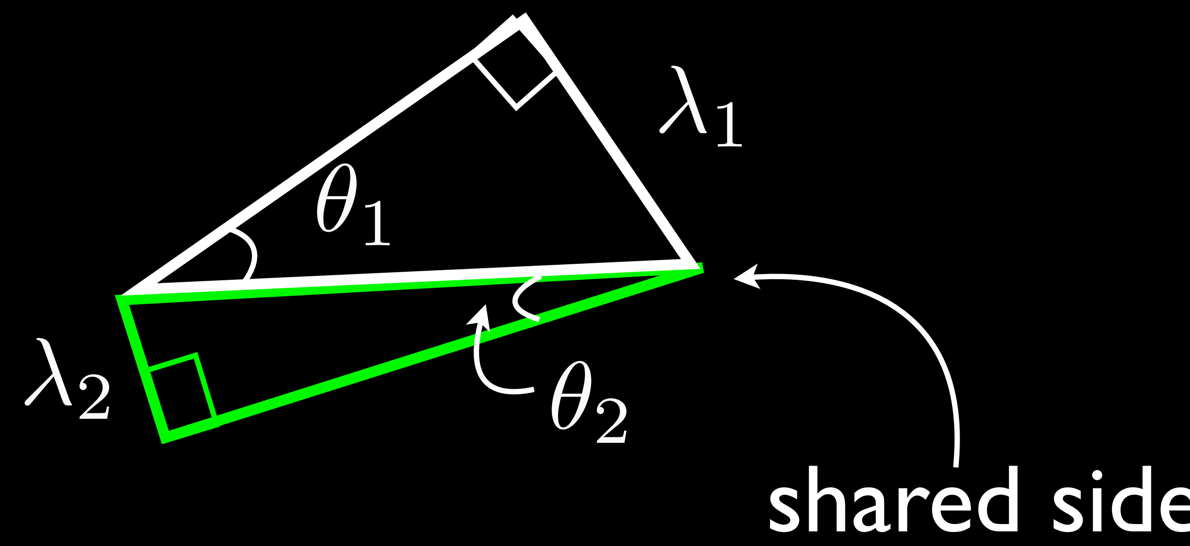




Refraction

The change in λ causes the direction to change:





$$\frac{\lambda_1}{\sin\theta_1} = \frac{\lambda_2}{\sin\theta_2}$$

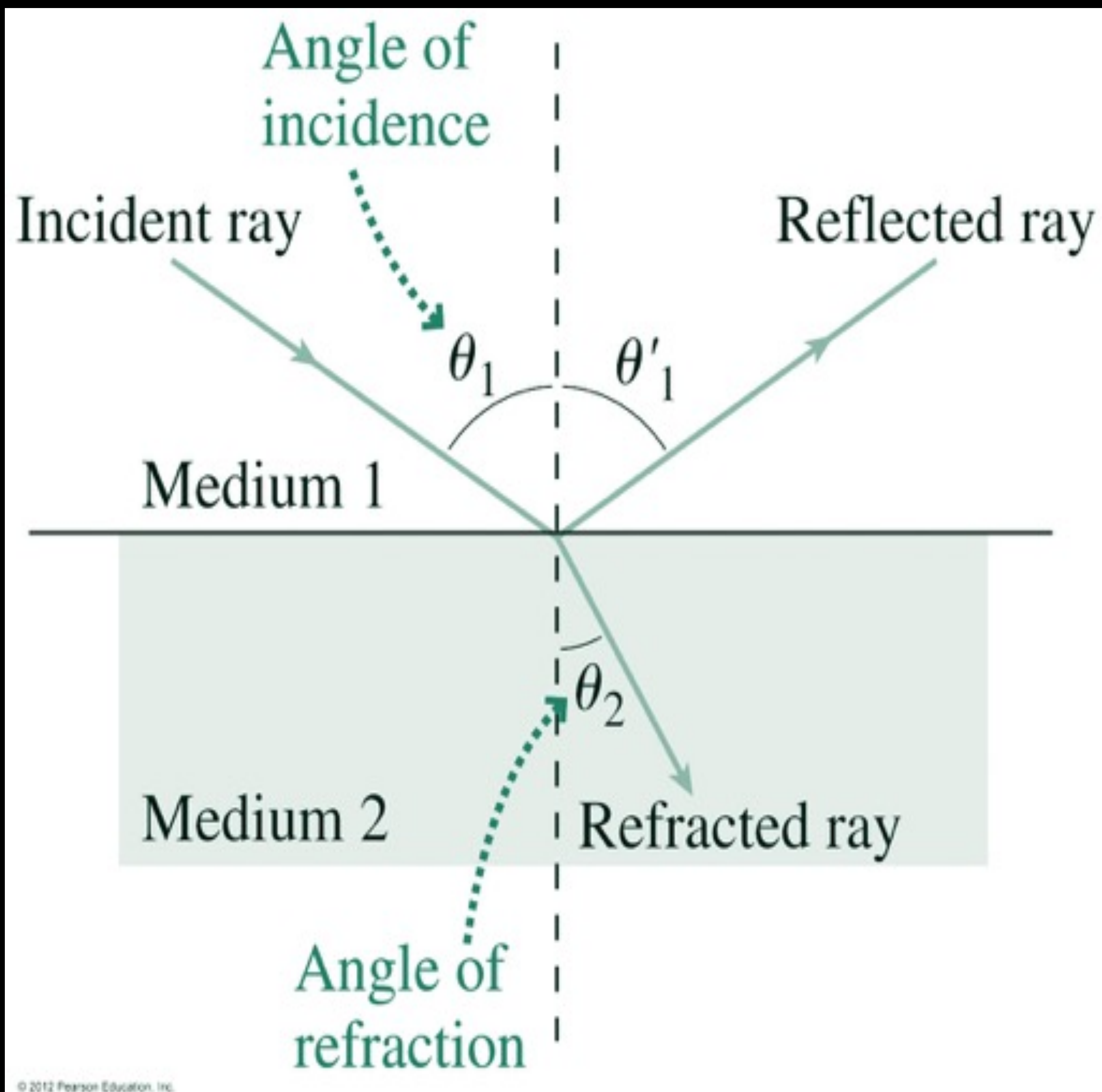
and since: $\lambda_1 = \frac{c}{f}\frac{1}{n_1}$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Snell's law

Refraction

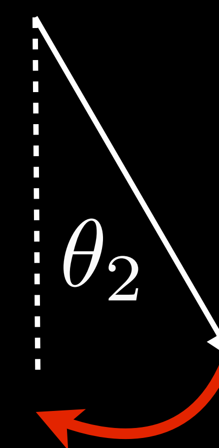
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



When going from low↓ n ...

... to high↑ n

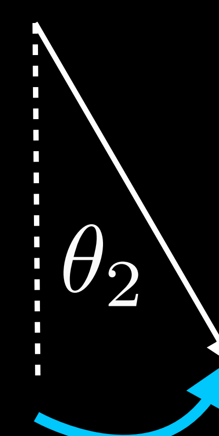
the ray bends towards to the normal



When going from high↑ n ...

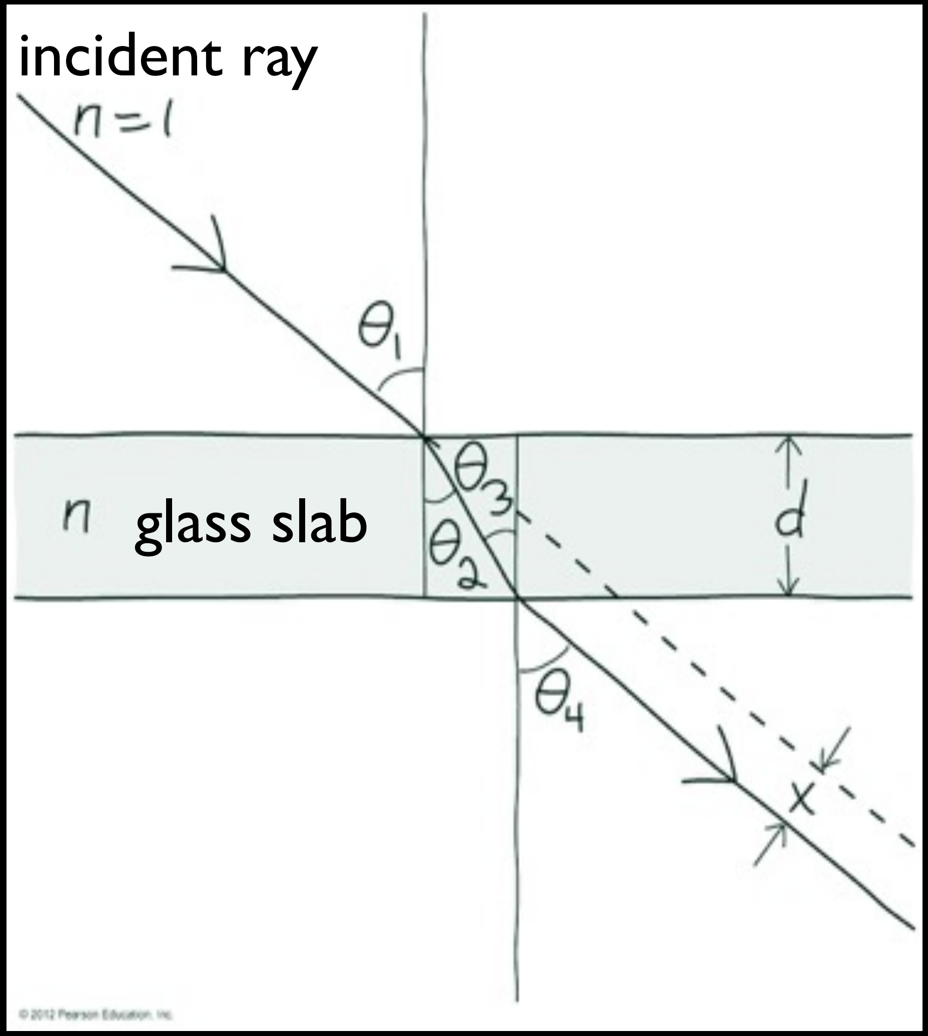
... to low↓ n

the ray bends away to the normal



Refraction

Show the ray exiting the glass slab is parallel to the incident ray.



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$ Snell's law

Refraction occurs:

entering block: low n to high n

$$1 \sin\theta_1 = n \sin\theta_2 \longrightarrow \sin\theta_2 = \sin\theta_1 / n$$

exiting block: high n to low n

$$n \sin\theta_3 = 1 \sin\theta_4 \longrightarrow \sin\theta_4 = n \sin\theta_3$$

slab faces parallel: $\theta_2 = \theta_3 \longrightarrow \sin\theta_4 = n\left(\frac{\sin\theta_1}{n}\right) = \sin\theta_1$

rays are parallel!

Refraction

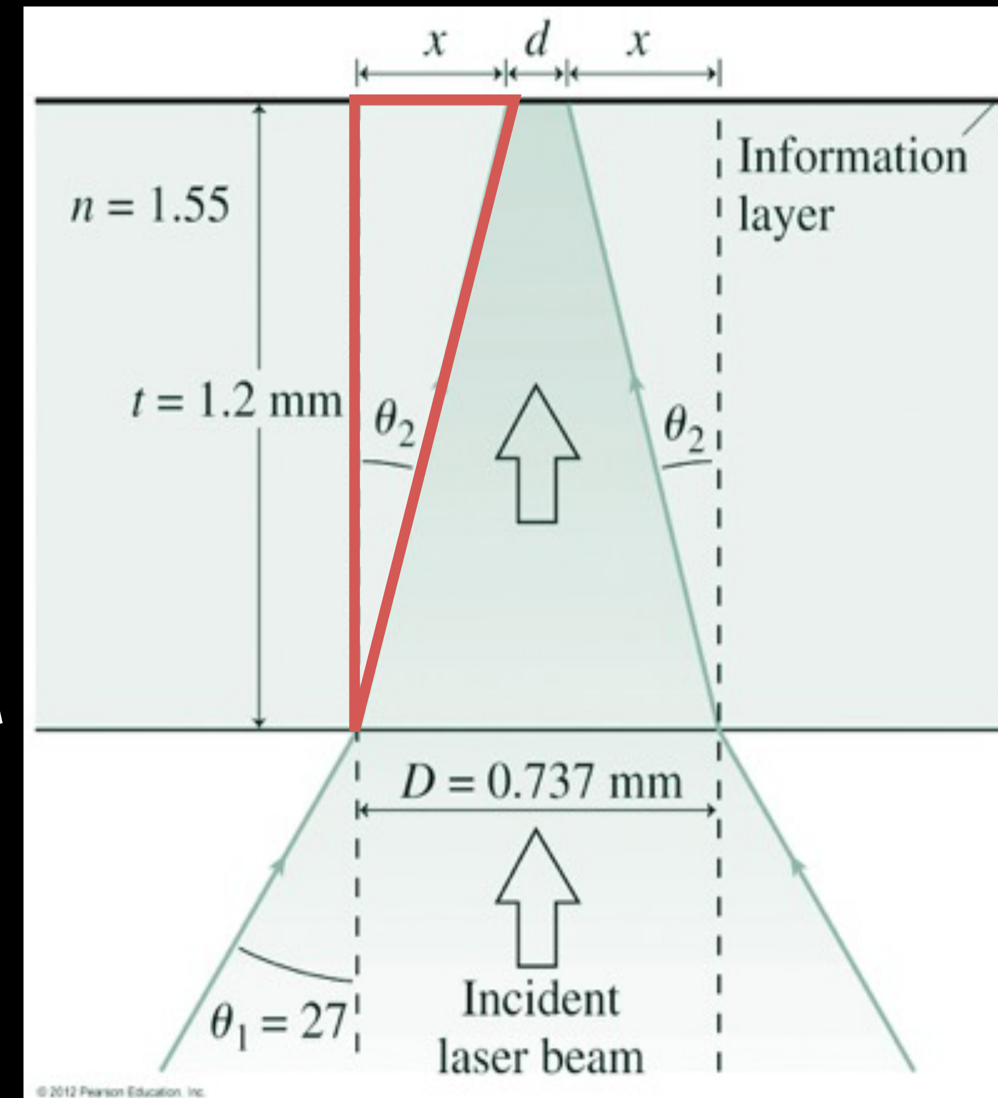
Laser ‘reads’ a CD

Entres CD 0.737 mm wide

Refracted to width d mm

What is d?

CD thickness



Snell’s law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\theta_2 = \sin^{-1}(n_1 \sin\theta_1 / n_2) = 17.03^\circ$$

$$d = D - 2x \;= D - 2t\tan\theta_2$$

$$= 1.80 \mu m$$

Refraction allows a larger laser to be used for CD players

Refraction

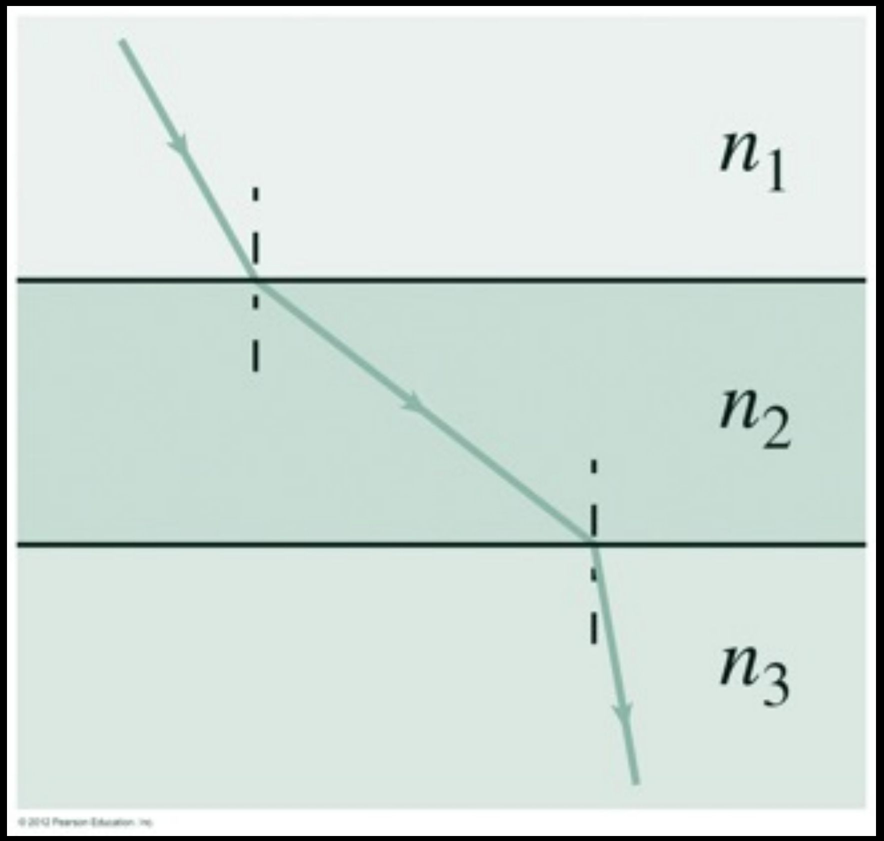
Rank the refractive indices.

(A) $n_1 > n_2 > n_3$

(B) $n_3 > n_1 > n_2$

(C) $n_3 > n_2 > n_1$

(D) $n_2 > n_1 > n_3$



Refraction

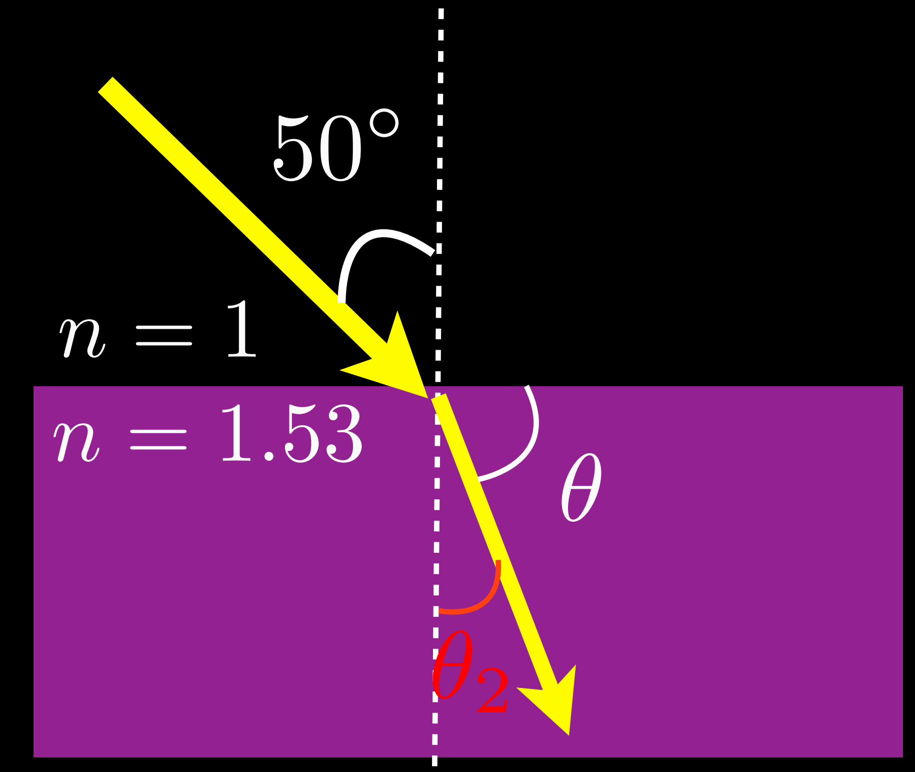
What is θ ?

(A) 20°

(B) 30°

(C) 50°

(D) 60°



Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$

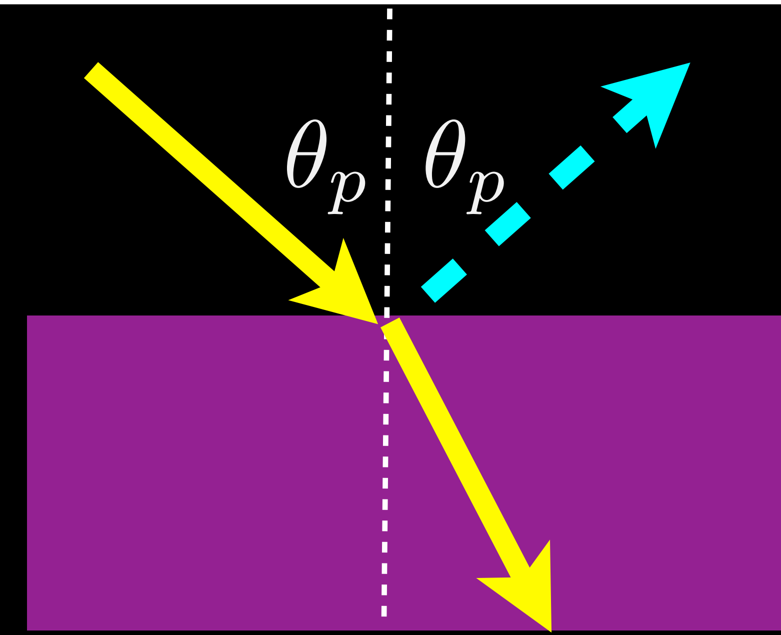
$$\sin 50^\circ = 1.53 \sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 50^\circ}{1.53}\right) = 30^\circ$$

$$\theta = 90 - \theta_2 = 60^\circ$$

Brewster Angle

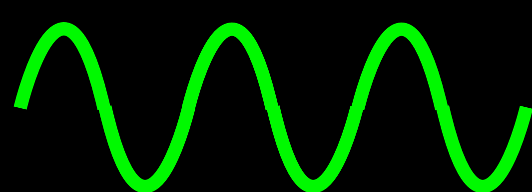
There is a special angle, θ_p, where no reflection occurs.



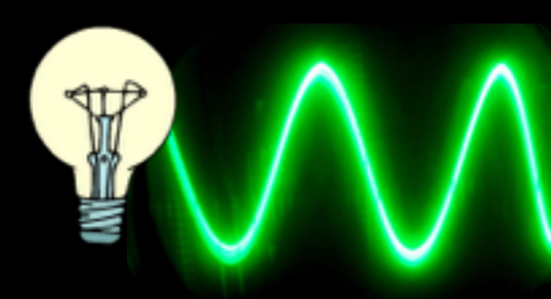
Why?

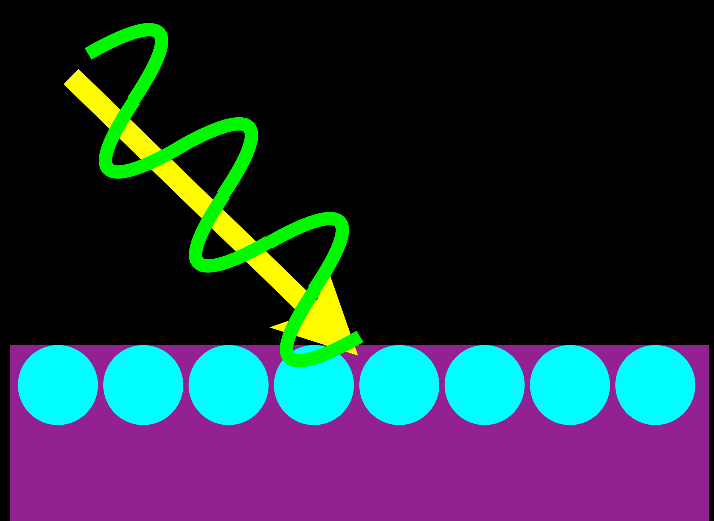
What is REALLY happening in reflection & refraction?

Light is wave...



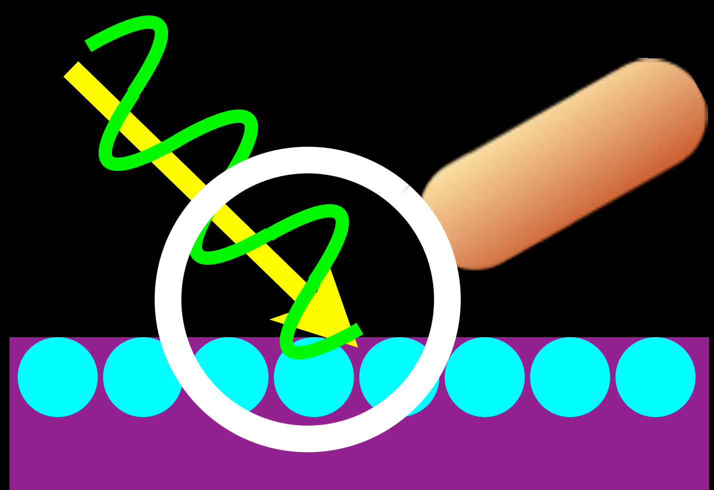
... an electric (and magnetic) wave

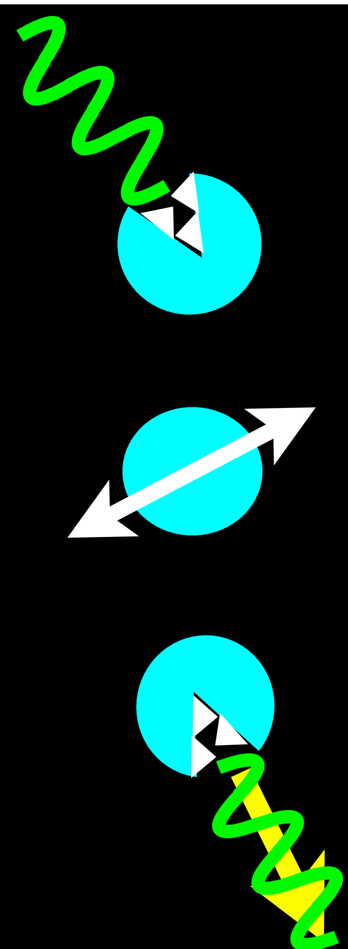




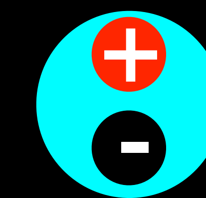
Reflection & refraction are interactions between the wave's electric field and atoms.

Brewster Angle



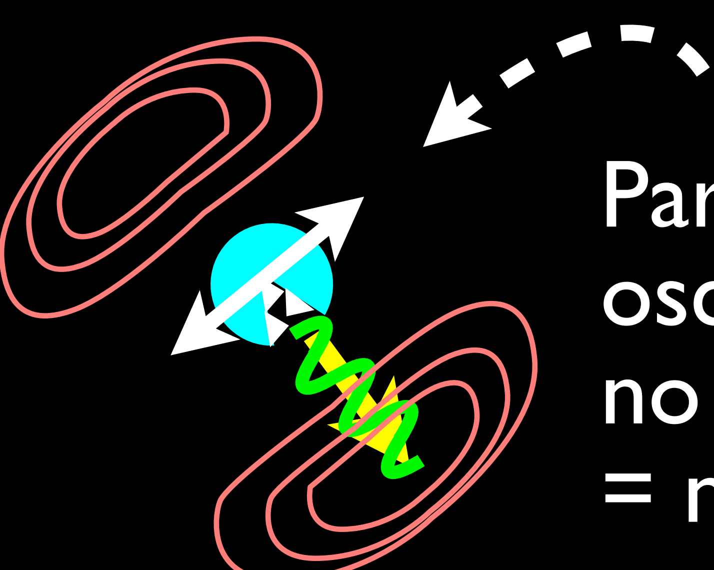


wave is absorbed by atom

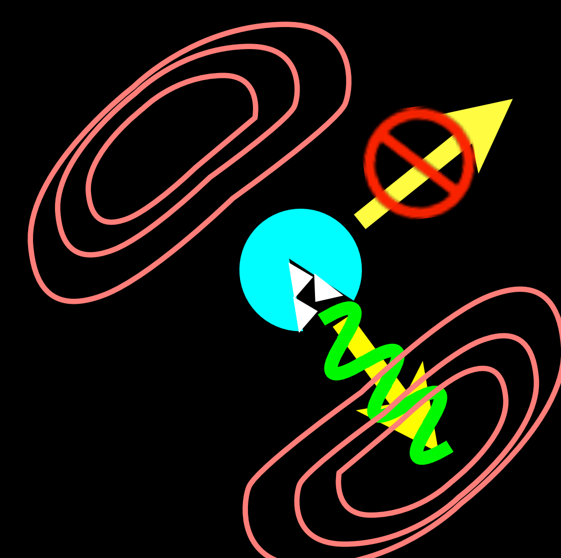
(actually a 'dipole' )

atom oscillates

oscillation produces refracted wave



Parallel to oscillation, there is no electric field = no wave



If reflected ray is in this direction, no reflection occurs!

Brewster Angle

Brewster angle, θ_p

occurs when: $\theta_p + \theta_2 = 90^\circ$

$$\theta_2 = 90^\circ - \theta_p$$

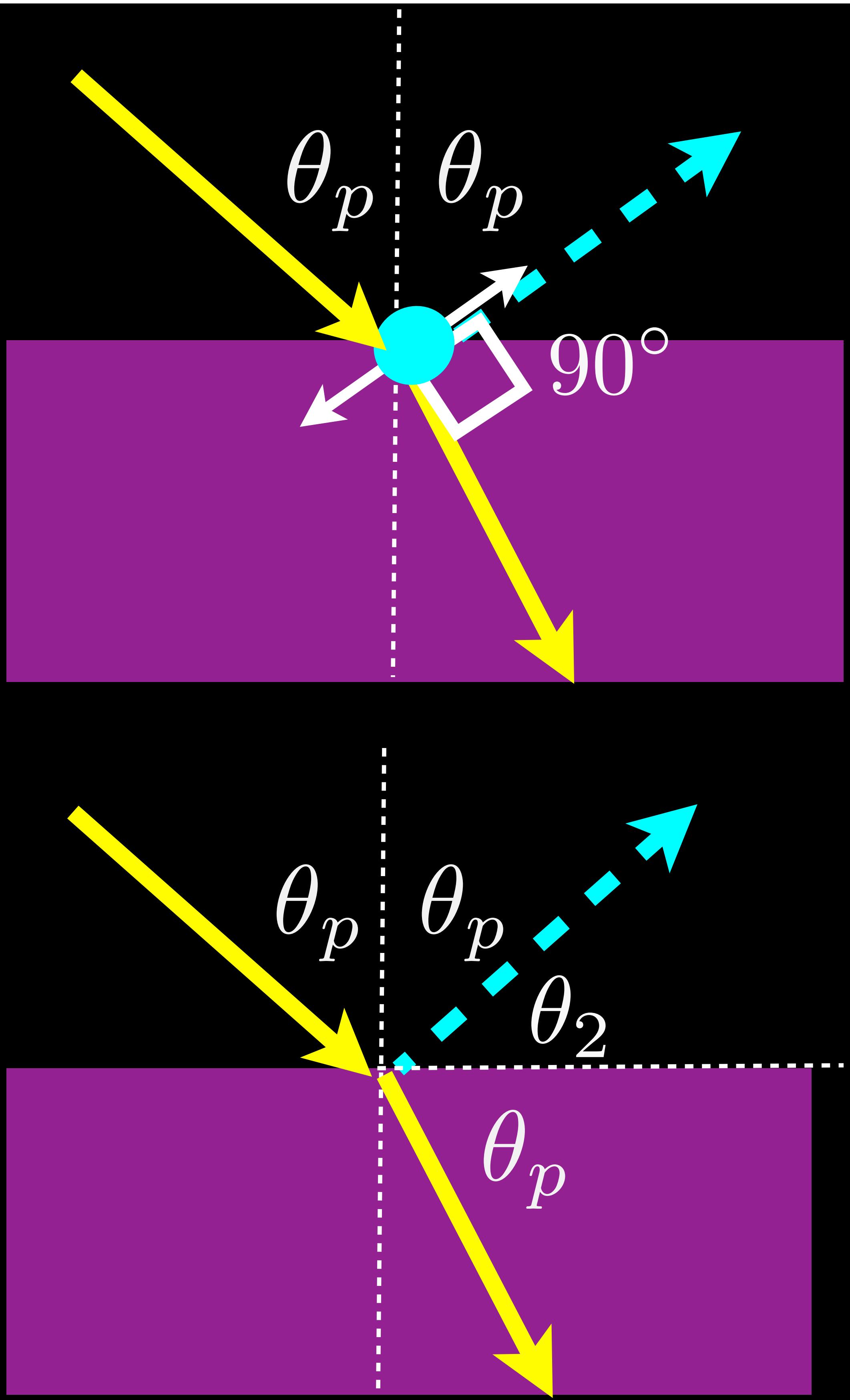
$$\sin\theta_2 = \sin(90^\circ - \theta_p)$$

$$= \cos\theta_p$$

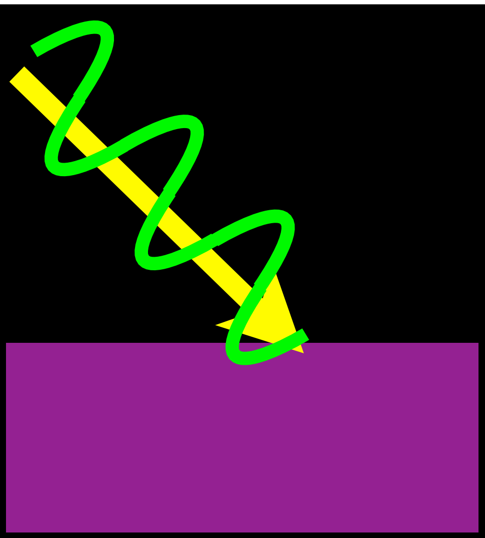
Snell's law: $\sin\theta_2 = \dfrac{n_1}{n_2}\sin\theta_p$

Therefore: $$\tan\theta_p = \frac{n_2}{n_1}$$

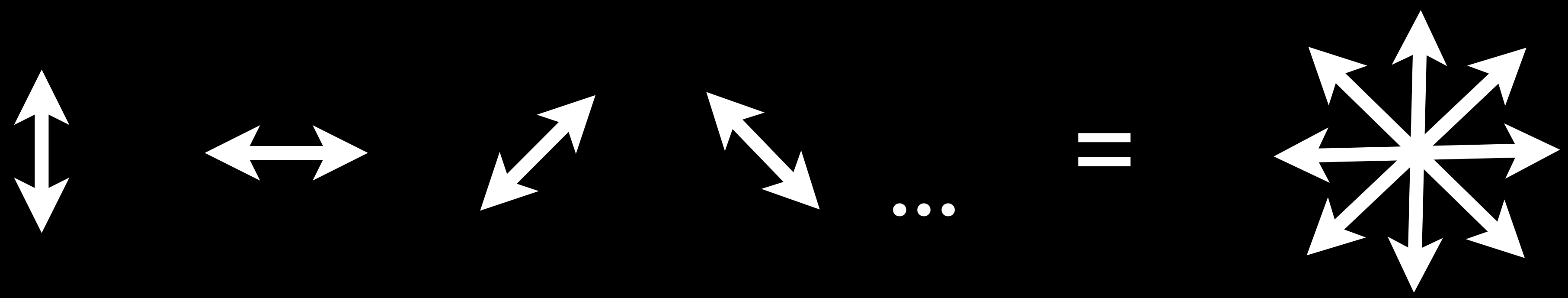
Air/glass interface: $\theta_p = 56^\circ$

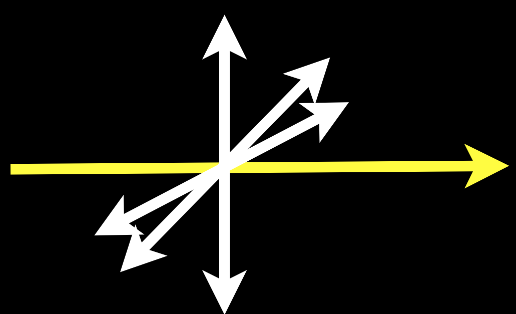


Brewster Angle = Polarising Angle



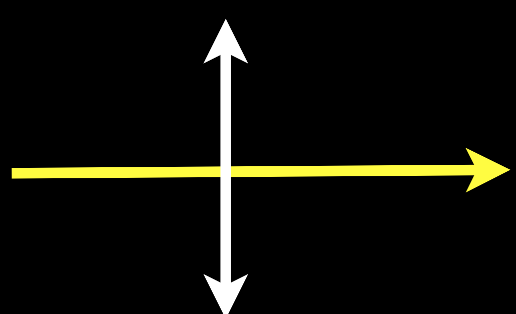
Normally, light is made from waves that oscillate in different directions:



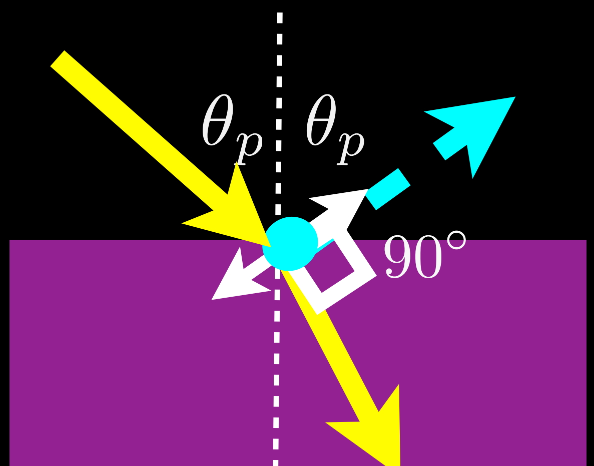


Same direction of motion,

different oscillation directions



If light only oscillates in one direction, it is polarised



Non-polarised light at the Brewster angle reflects polarised light:

only light component with oscillations perpendicular to atom oscillation.

Brewster Angle

Quiz

Find the refractive index of a material with a Brewster (polarising) angle in air of 62°.

(A) 1.5

(B) 1.0

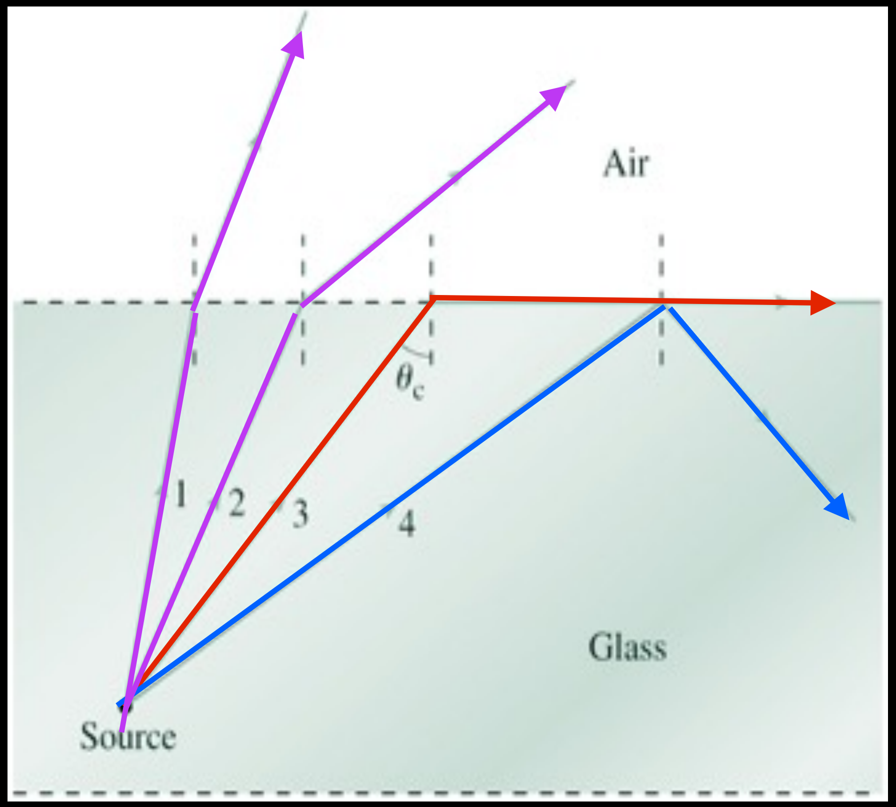
(C) 1.9

(D) 1.7

$$n_2 = n_1 \tan\theta_p = (1.0)\tan(62^\circ) = 1.9$$

Total Internal Reflection

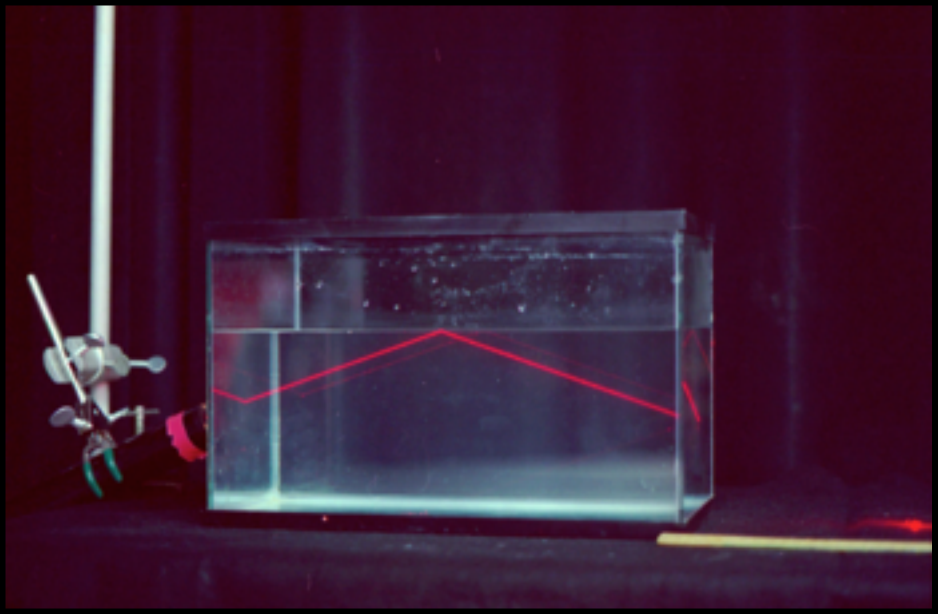
Light moving from medium with high n to low n is bent away from the normal.



If the angle of refraction $> 90^\circ$, total internal reflection occurs.

Light cannot escape the glass.

The incident ray's critical angle is when the angle of refraction $= 90^\circ$



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 \sin\theta_c = n_2 \sin 90^\circ \Rightarrow \sin\theta_c = \frac{n_2}{n1}$$

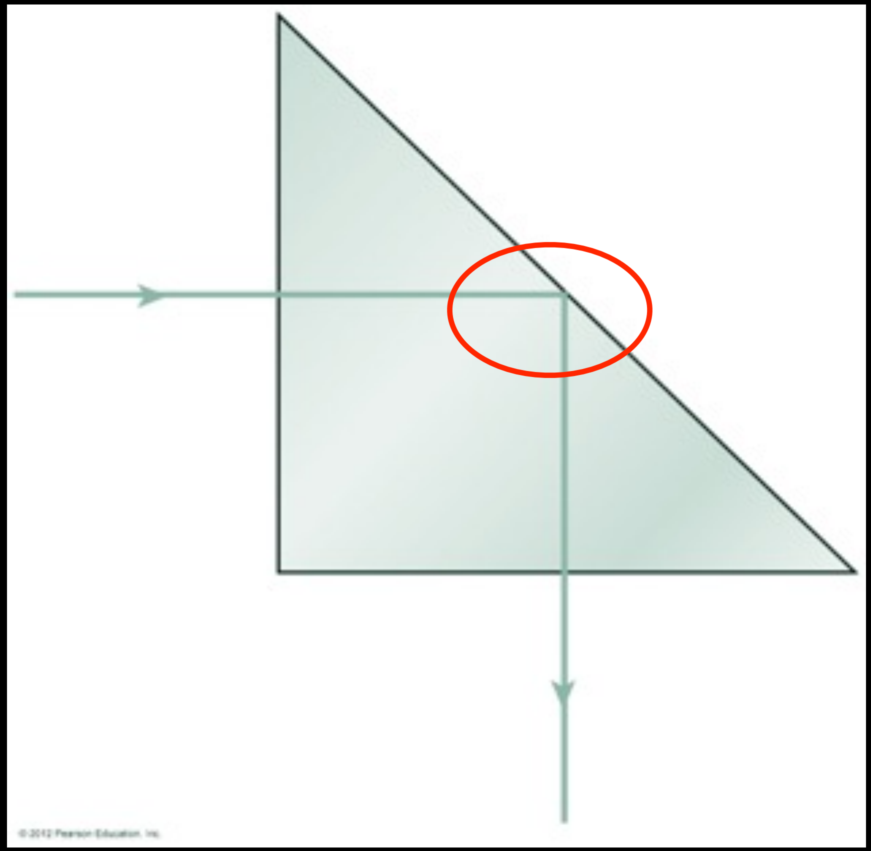
Total Internal Reflection

The glass prism has *n* = 1.5 and is surrounded by air (*n* = 1).

What would happen to the incident light ray if the prism were immersed in water (*n* =1.333)?

(A) Most would exit into the water through the diagonal face and some would be reflected.

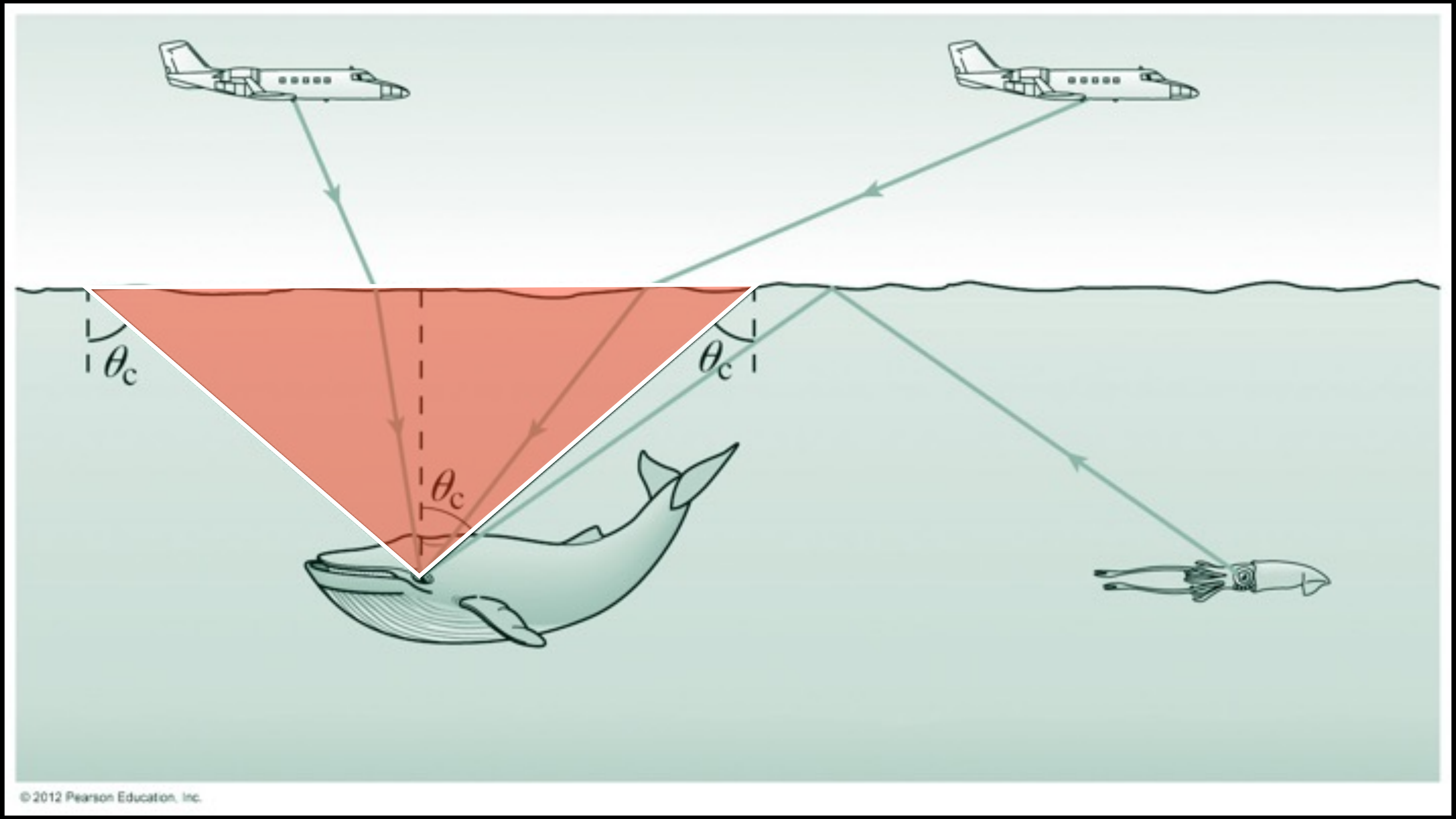
(B) Most would be reflected and some would exit into the water through the diagonal face.



$$\sin\theta_c = \frac{n_2}{n1}$$ increase

Total Internal Reflection

A whale's view



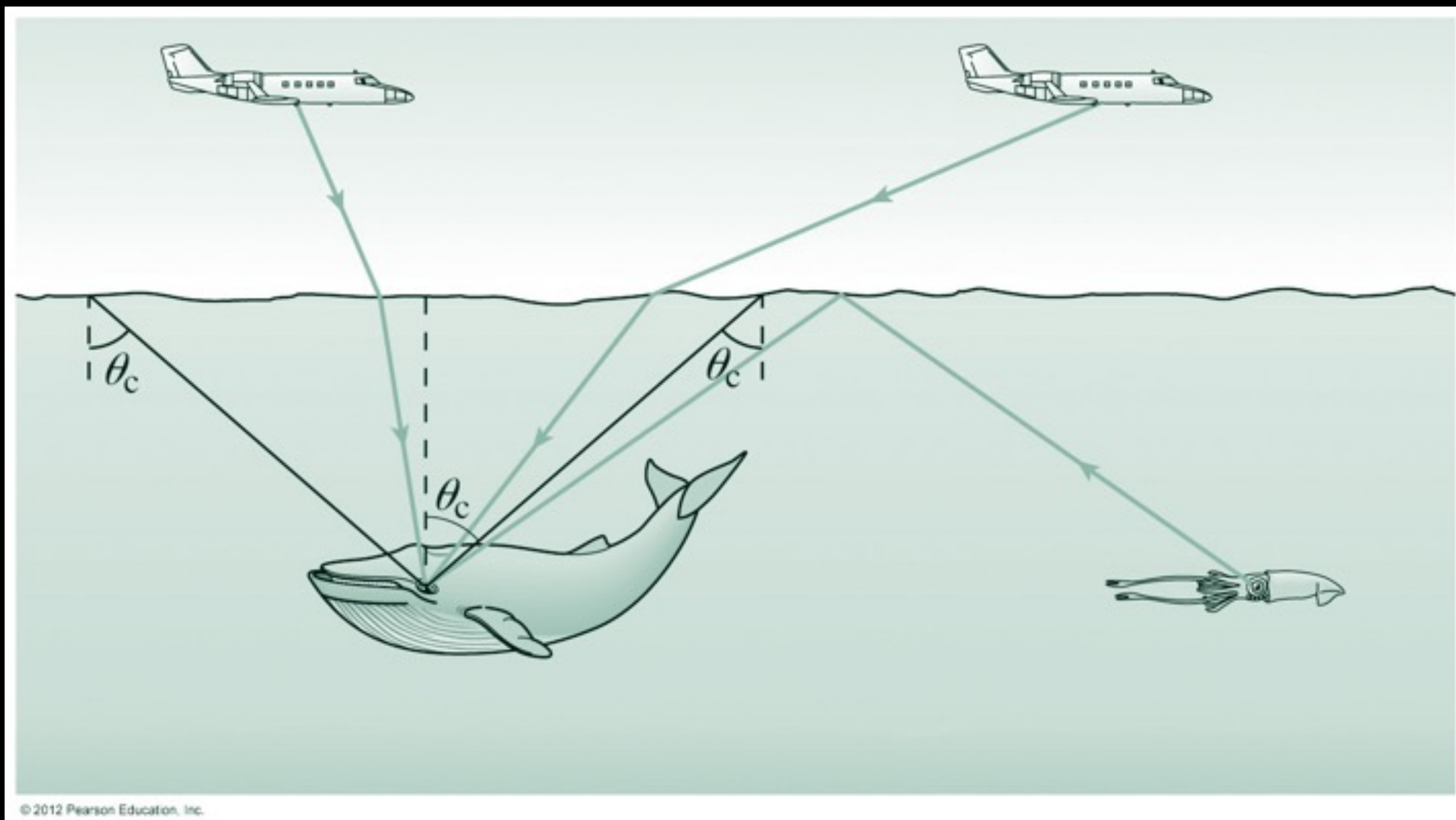
Light rays in the red region are from objects above the water.

Light rays outside red region are from objects in the water.

Total Internal Reflection

Example

A whale's view

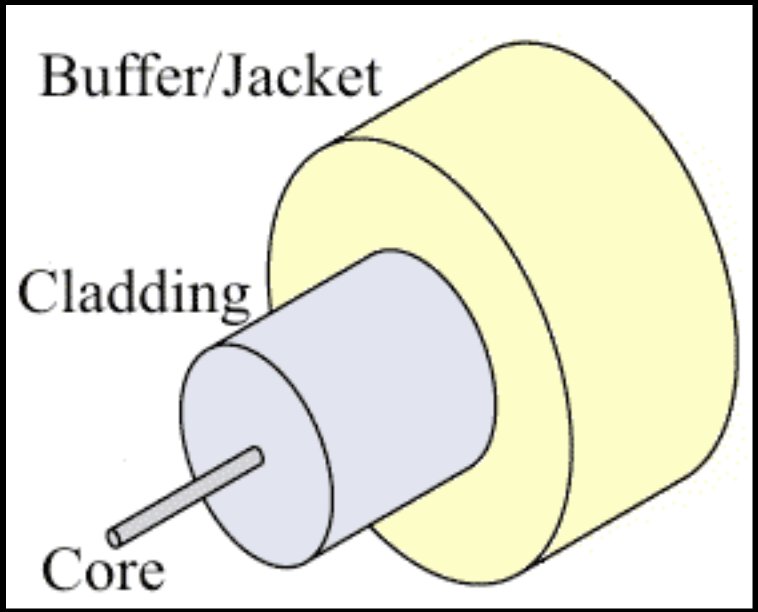


What is θ, the half-angle of the cone in which the whale sees above the water?

$$\theta_c = \sin^{-1}(1/1.333) = 48.6°$$

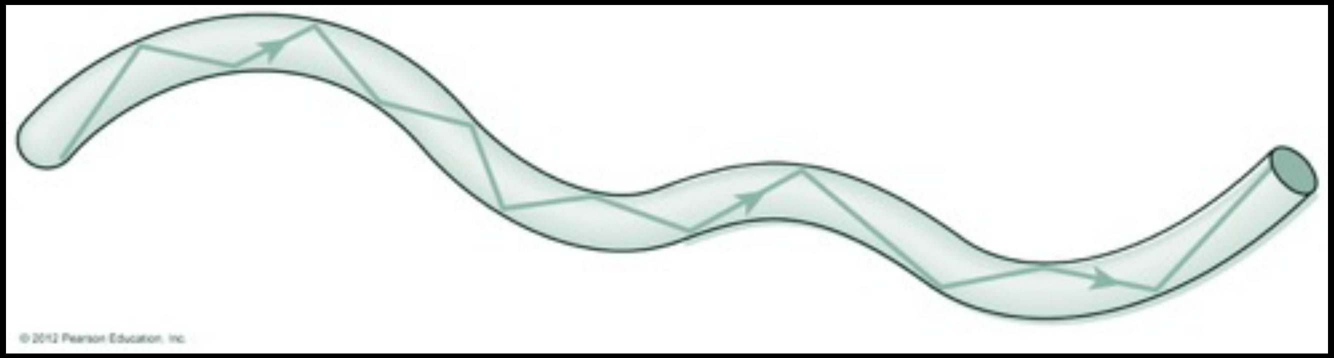
Total Internal Reflection

Information for the internet, telephones and television is carried in optical fibres.



Cable has glass core inside 'cladding'.

The cladding has a lower refractive index, n



Total internal reflection at core/cladding interface.

Lighter and more durable than copper wire.

Two wires on right carry same rate of information!

optical fibre

copper wire

Total Internal Reflection

What is the critical angle for light in a glass with n = 1.52 when the glass is immersed in water (n = 1.333).

(A) 61.3°

(B) 41.1°

(C) 80.9°

(D) none

$$\theta_c = \sin^{-1}(n_2/n_1)$$

$$= \sin^{-1}(1.333/1.52) = 61.3^\circ$$

Total Internal Reflection

What is the critical angle for light in a glass with n = 1.52 when the glass is immersed in benzene (n = 1.501).

(A) $61.3°$

(B) $41.1°$

(C) $80.9°$

(D) none

$$\theta_c = \sin^{-1}(n_2/n_1)$$

$$= \sin^{-1}(1.501/1.52) = 80.9°$$

Total Internal Reflection

What is the critical angle for light in a glass with n = 1.52 when the glass is immersed in diidomethane (n = 1.738).

(A) $61.3°$

(B) $41.1°$

(C) $80.9°$

(D) none

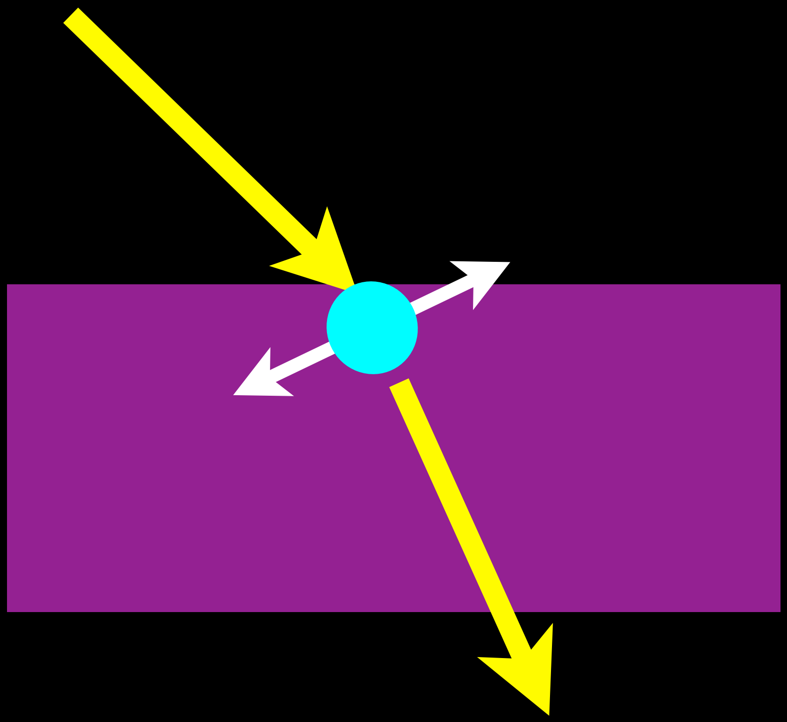
$$n_1(= 1.52) > n_2(= 1.738)$$

$$\sin^{-1}(1.738/1.52)$$ no solution!

no total internal reflection for light moving in glass.

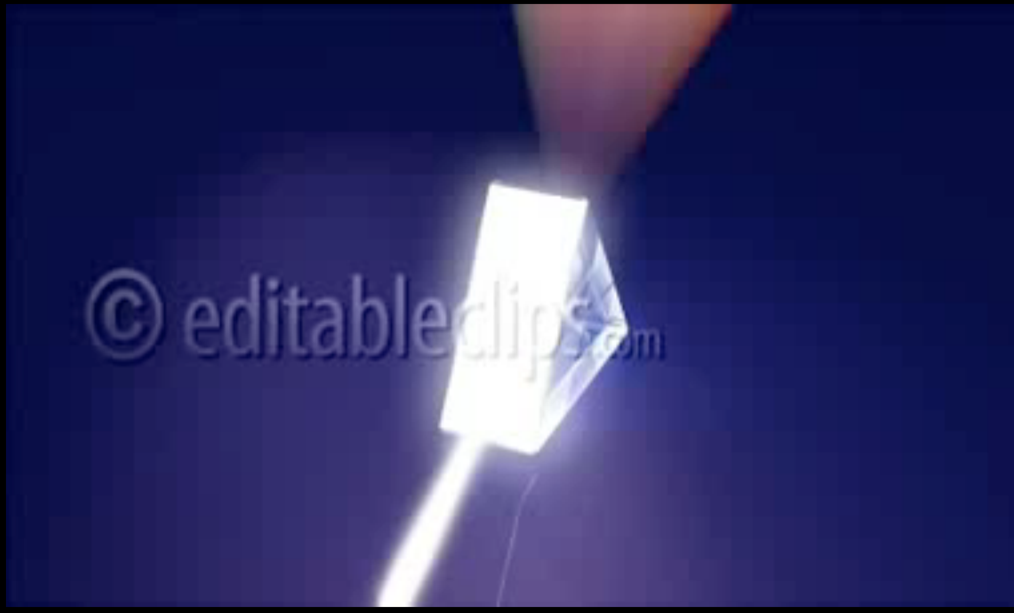
Dispersion

Refraction is the interaction between the light wave and the atoms.



It depends on the frequency of the wave

The refractive index, n, depends on wave frequency.



Different frequencies refract through different angles.

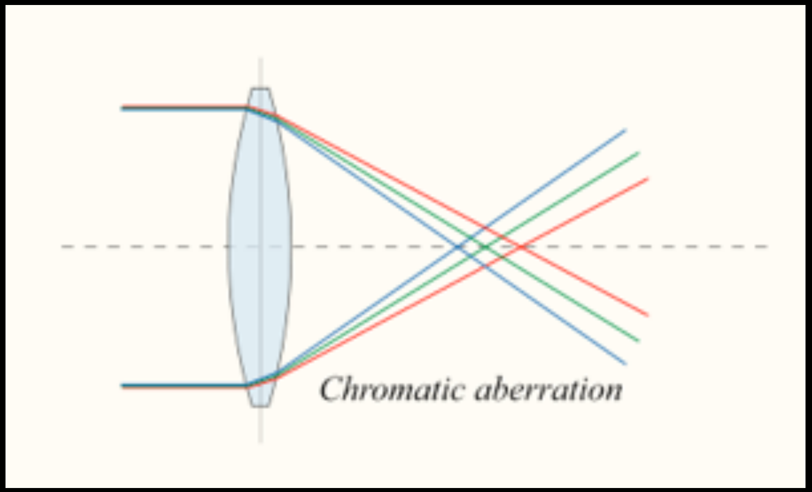
This is dispersion.

Dispersion

Dispersion can be bad:

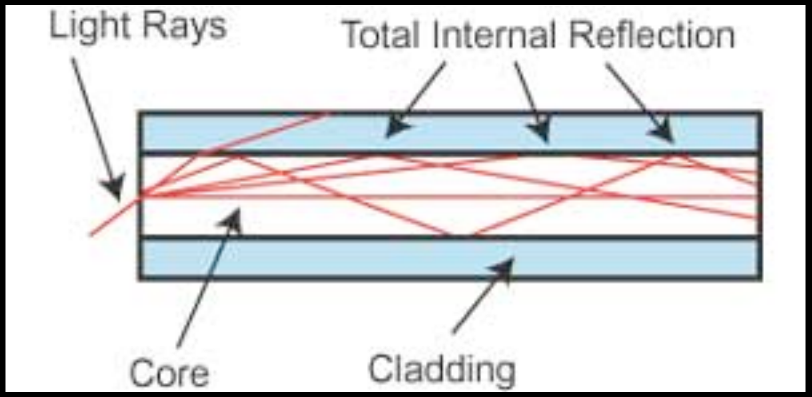
Lenses focus colours as different places.

“Chromatic aberration”



Dispersion in optical fibres.

Rays take different paths by reflecting at different angles.

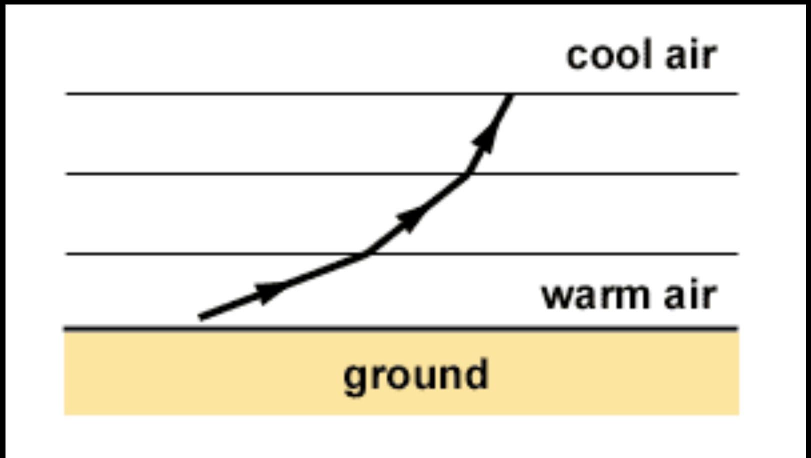


Dispersion can be good:

Measuring dispersion in the atmosphere allows GPS devices to correct for atmospheric conditions.

Mirages

Mirages occur when the refractive index, n changes with position.

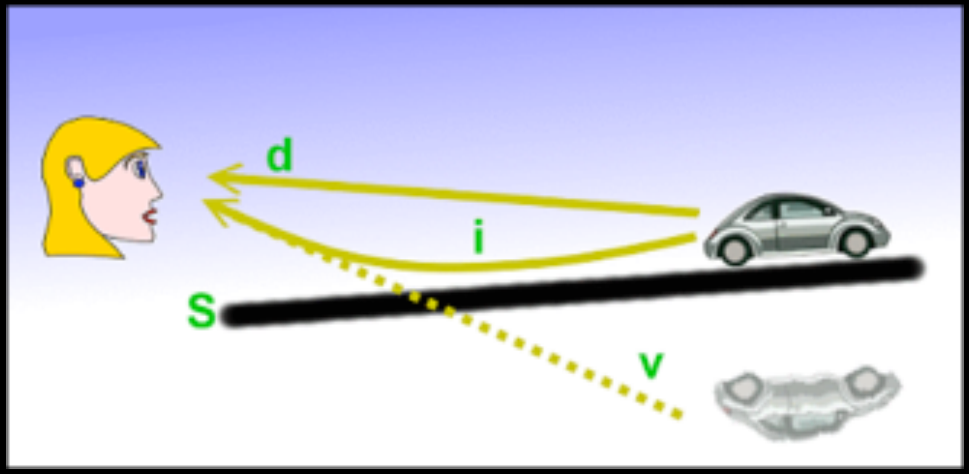


The ray travels in a curved path.

e.g. A temperature difference produces a density difference.

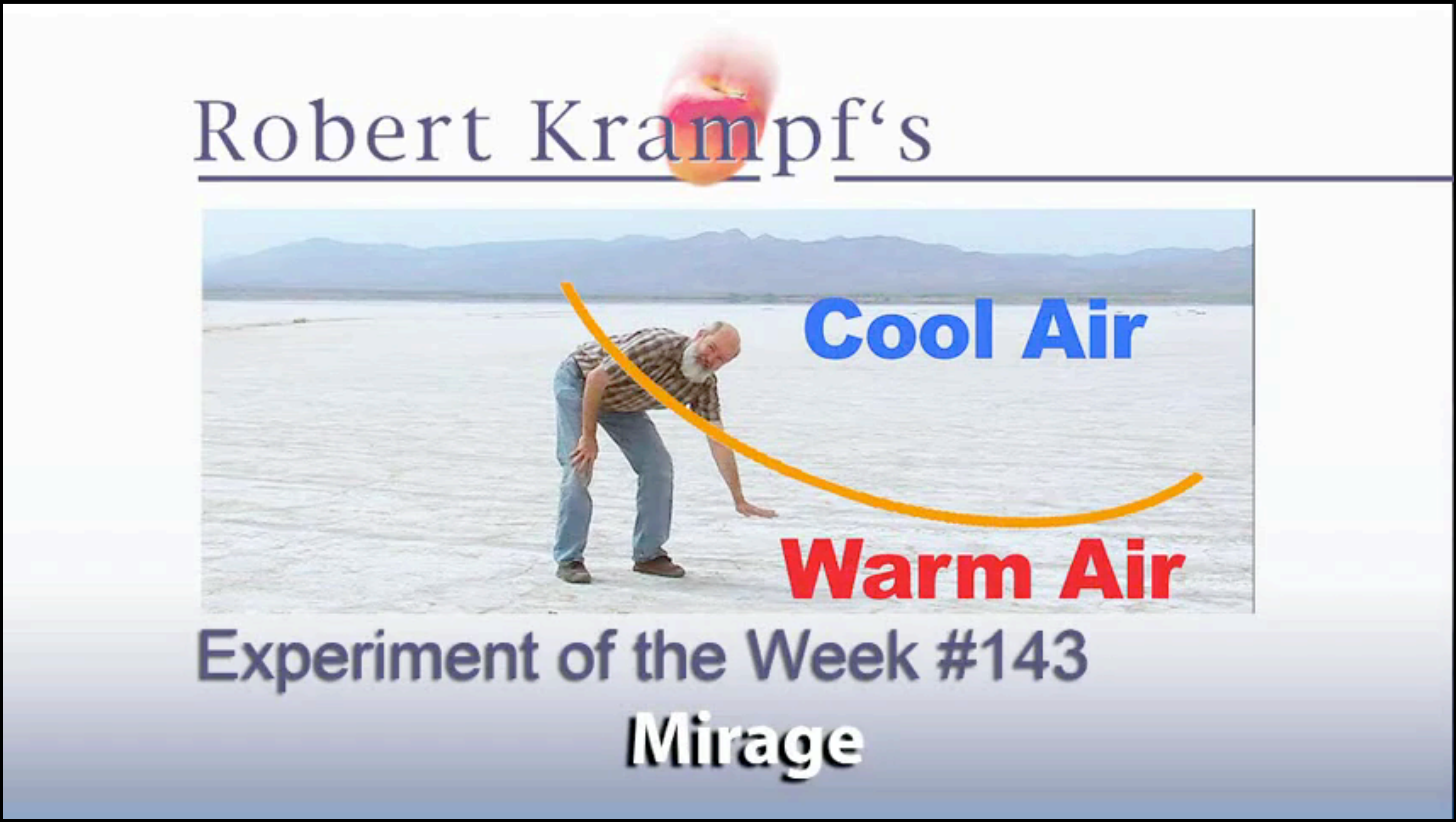
Causes n to vary.

"Water" on the road on a hot day



Brain assumes ray, i, is straight
"see" a reflection of the car, v

Mirages

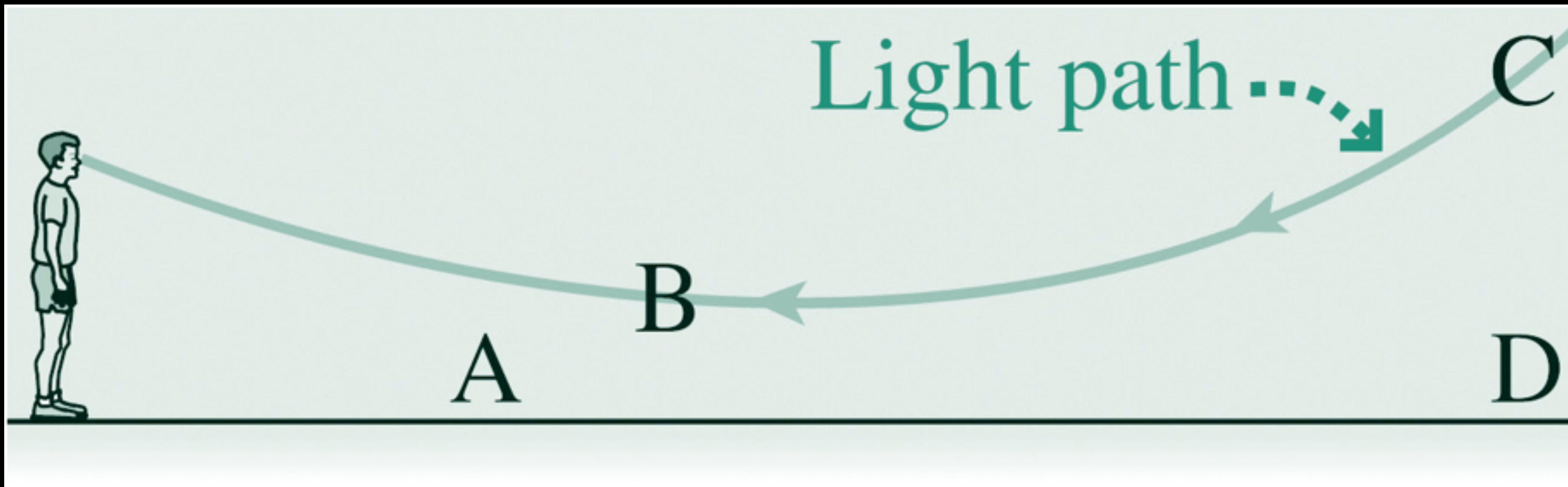


Robert Krampf's
Cool Air
Warm Air
Experiment of the Week #143
Mirage

Mirages

Quiz

Light path over a hot road.



Is the air's refractive index....

(a) increase from left to right

(b) increase from right to left

(c) increase upwards

(d) increase downwards

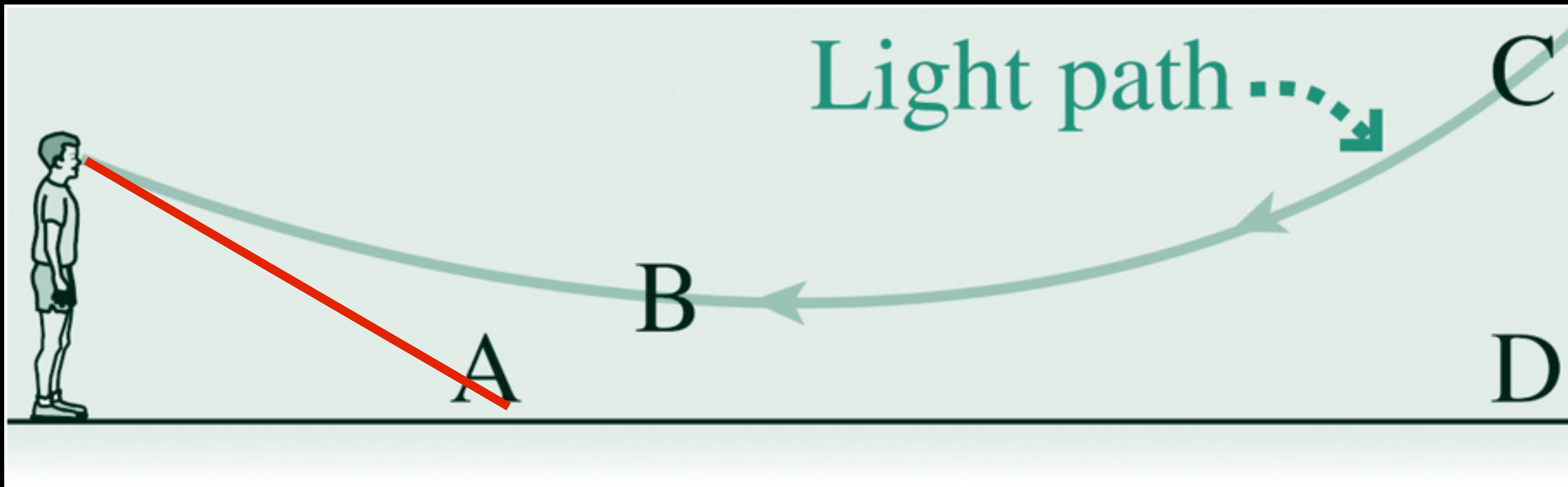
Light is bending away from the normal

n is higher at the top.

Mirages

Quiz

Light path over a hot road.



The person sees a mirage that looks like water.

It appears to be at:

(a) point A

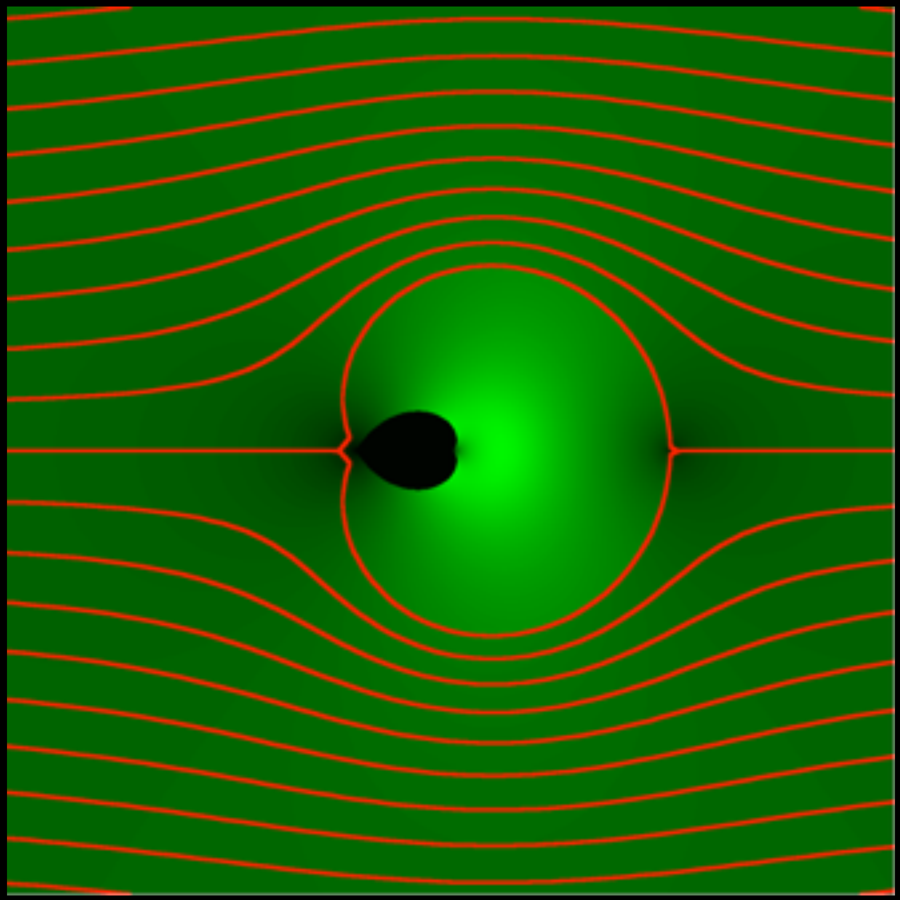
(b) point B

(c) point C

(d) point D

Mirages

Mirages work by bending light....



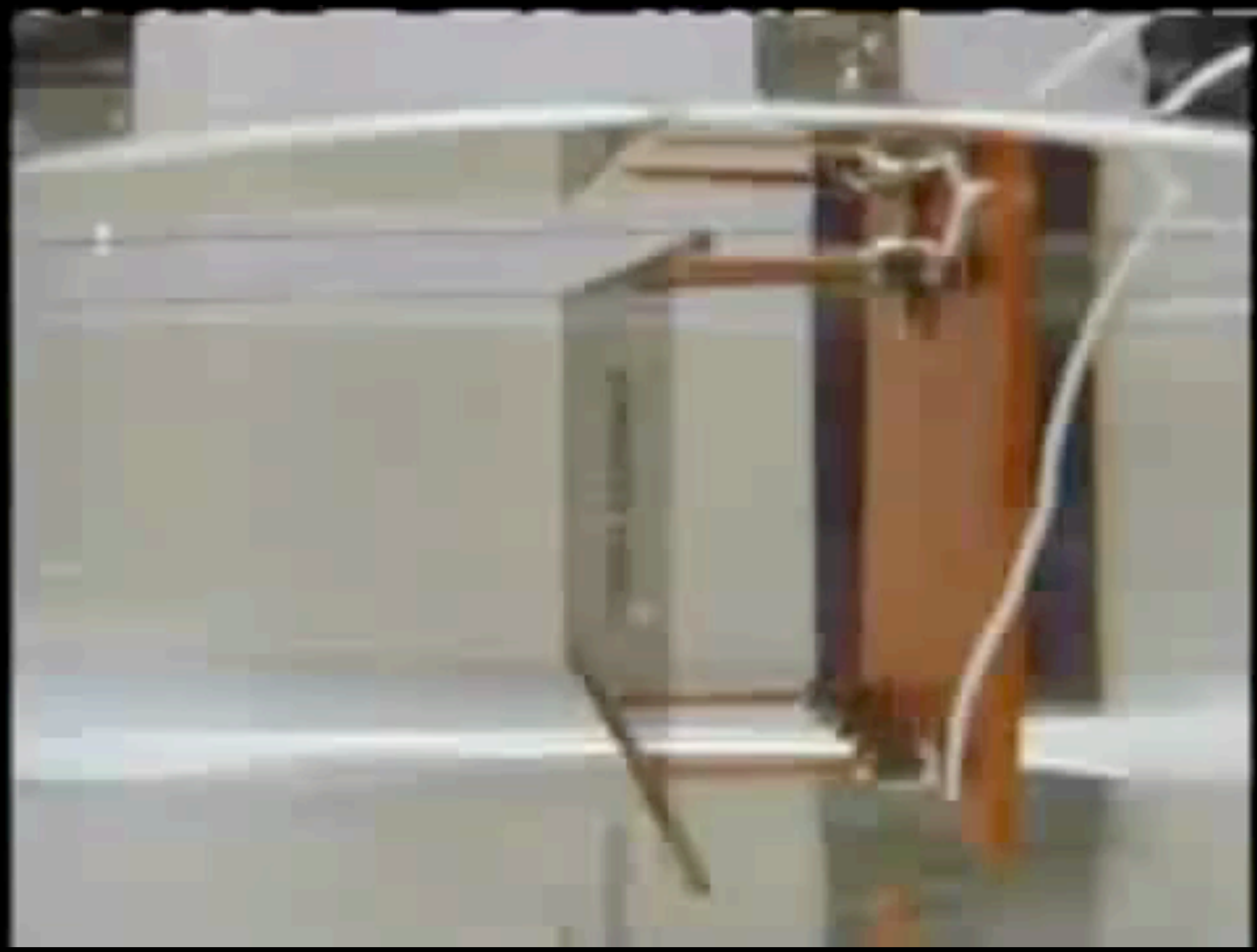
Research from St Andrew University, UK

What if we bent light completely around an object?

Could we make it invisible?

Yes! ..... in theory.

Mirages



Mirages

What has been done in the laboratory?

(A) An object has been destroyed with light

(B) An object has been made invisible

(C) An object has been made to absorb light

(D) An object has been made to look like water

Mirages

How was this done?

(A) The material becomes so hot it melts

(B) The material becomes so hot light is reflected from it

(C) The material becomes so hot light passes through it

(D) The material becomes so hot light is bent

Mirages

How does the army truck become invisible to infrared camera?

(A) It becomes hot and bends light

(B) It becomes hot and light passes through the truck

(C) It matches the temperature of the air exactly

(D) It reflects the light.