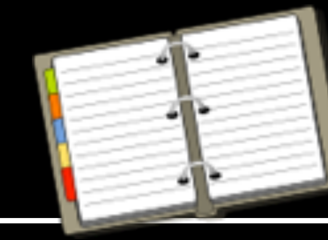
Essential Physics I

英語で物理学の

エッセンス I

Lecture 11: 27-06-16

Last lecture: review



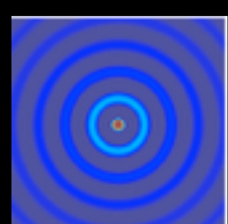
Wave is a moving oscillation. It carries energy but not matter.

$$v = \frac{\lambda}{T} = \lambda f = \sqrt{\frac{F}{\mu}}$$

String

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

If oscillation is SHM: $y(x,t) = A\cos(kx \pm \omega t)$

$I = \dfrac{P}{A}$  $= \dfrac{P}{4\pi r^2}$

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

Standing waves:

2 fixed

$L = \dfrac{m\lambda}{2}$ m = 1, 2, 3, ...

1 fixed,
1 open

$L = \dfrac{m\lambda}{4}$ m = 1, 3, 5 ....

Last lecture: review

Travelling wave:

$$y(x,t) = 4.0\cos(\underbrace{15}_{k}x - \underbrace{30}_{\omega}t)$$

What is the wave speed?

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

(a) 4 m/s

(b) 2 m/s

(c) 0.5 m/s

(d) 120 m/s

(e) 60 m/s

$$v = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{30}{15} = 2\,\mathrm{m/s}$$

Last lecture: review

A boat bobs up and down on a water wave.

It moves a vertical distance of 2 m in 1s.

A wave crest moves a horizontal distance of 10 m in 2 s.

What is the magnitude of the wave speed?

(a) 2.0 m/s

(b) 5.0 m/s

(c) 7.5 m/s

(d) 10.0 m/s

Last lecture: review

A boat bobs up and down on a water wave.

It moves a vertical distance of 2 m in 1s.

A wave crest moves a horizontal distance of 10 m in 2 s.

What is the magnitude of the wave speed?

Matter (boat) speed   Wave speed

(a) 2.0 m/s

(b) 5.0 m/s

(c) 7.5 m/s

(d) 10.0 m/s

A wave moves energy, but not matter.

$$v = \frac{\Delta x}{\Delta t} = \frac{10\,\mathrm{m}}{2\,\mathrm{s}} = 5.0\,\mathrm{m/s}$$

Slinky Experiment

What happens when the top of the slinky is released?

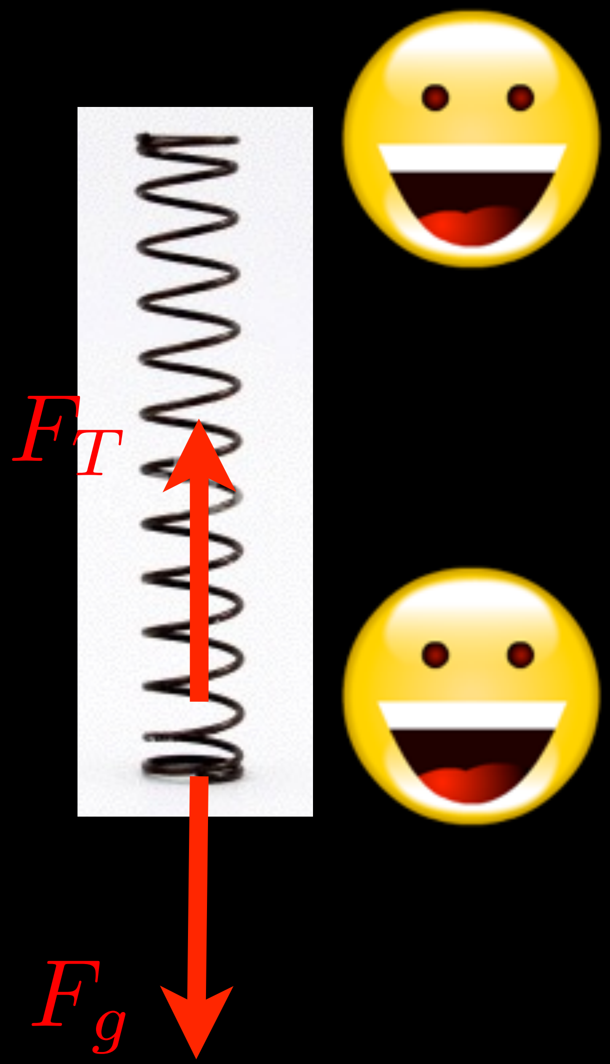
(a) the bottom drops first

(b) the top drops first

(c) top and bottom drop together

(d) top and bottom approach the centre

Slinky Experiment

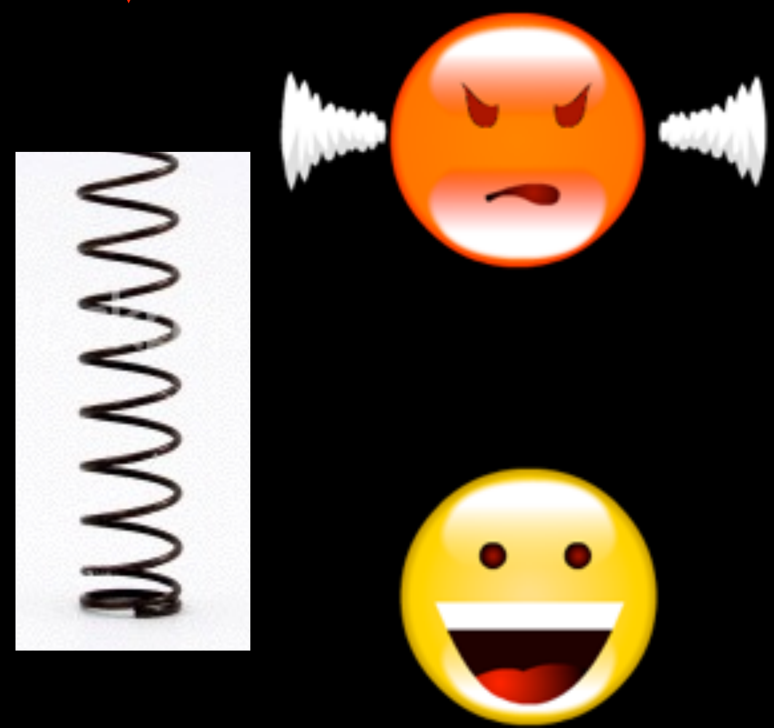


At the start, the spring is being held.

At the top, the hand's contact force and gravity balance.

At the bottom, gravity and tension balance.

The top is released.

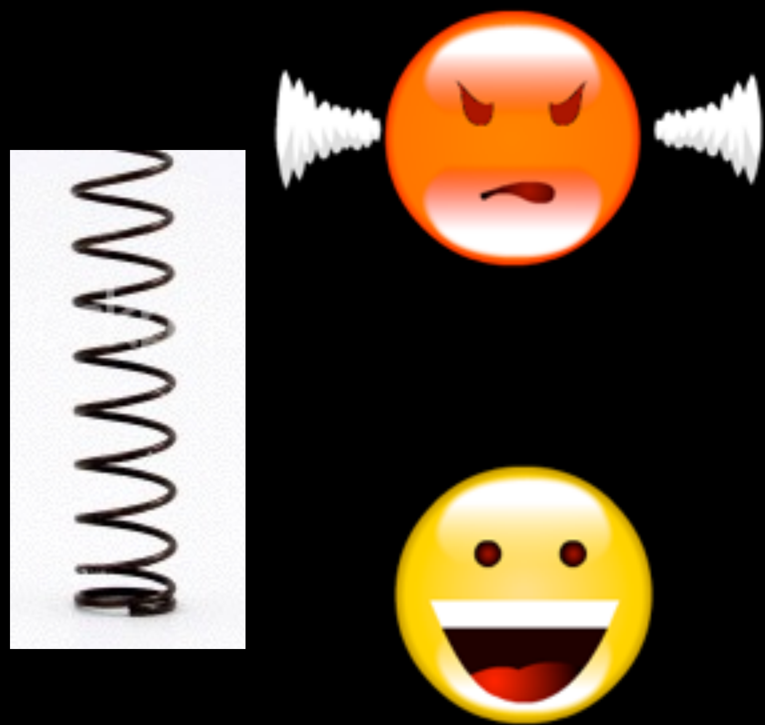


The top knows it has lost its upwards force and starts to fall.

But the bottom does not yet know the upwards force has gone.

The bottom stays still.

Slinky Experiment



The information about the lost upwards force travels down the spring in a wave.

When the wave reaches the bottom, the bottom knows it has lost the upwards force.

The bottom falls.

Fluids

What is a fluid?

A fluid is something that takes the shape of its container.

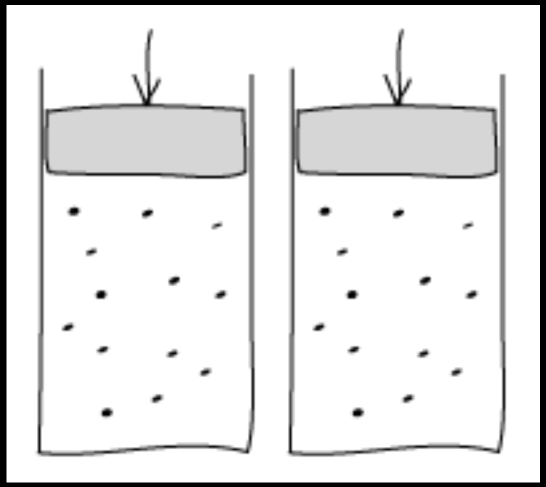
liquids are fluids

gases are fluids

solids are not fluids

What is a fluid?

liquid molecules are close together.

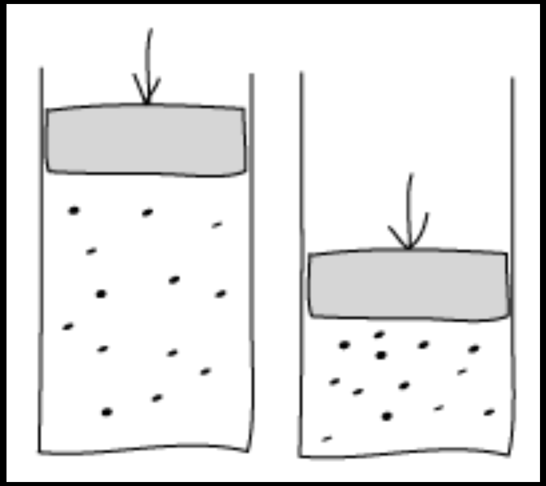


Difficult to push closer.

Liquids are incompressible.

Density is constant: $\rho = \frac{m}{V}$

gas molecules are far apart.

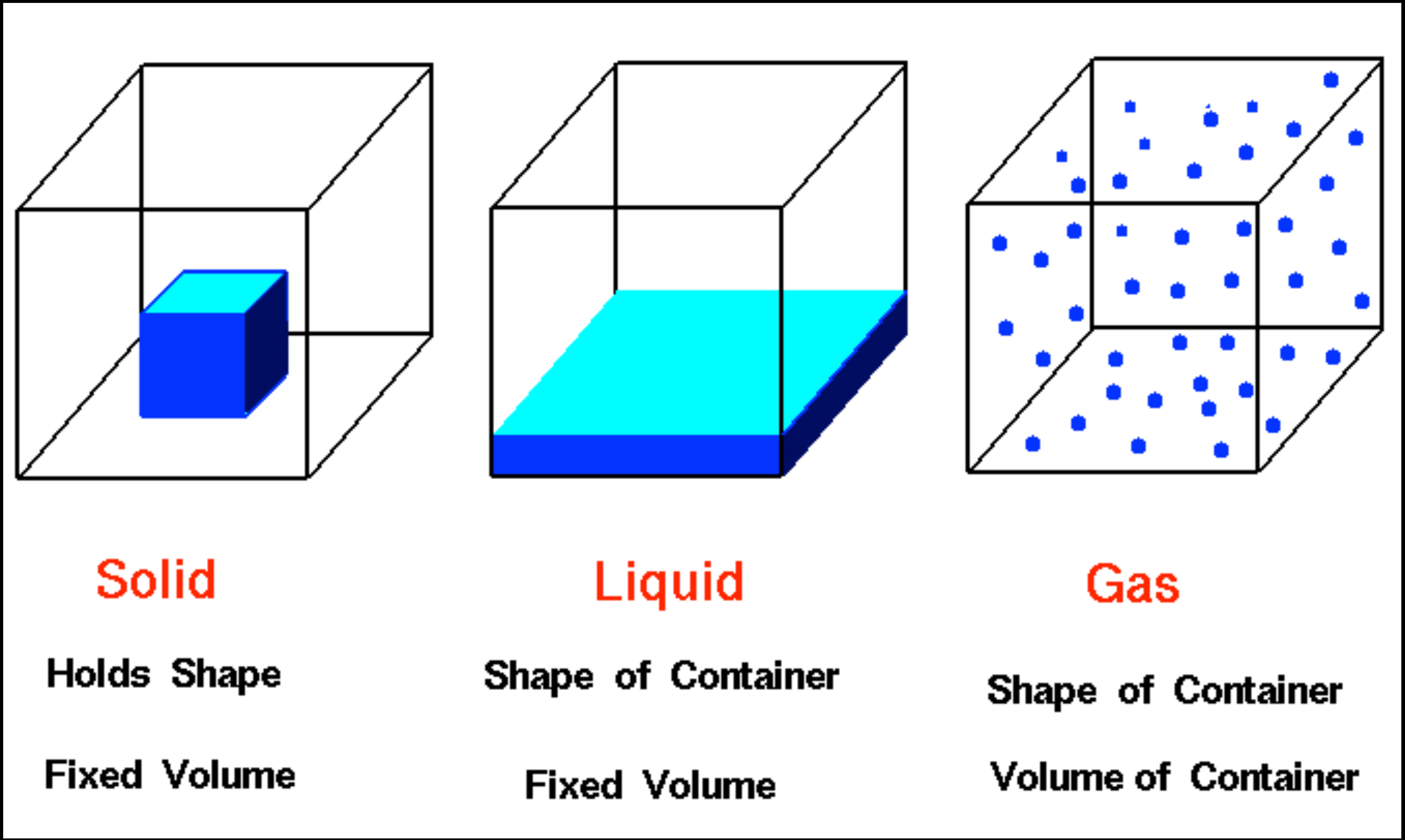


Easy to push closer.

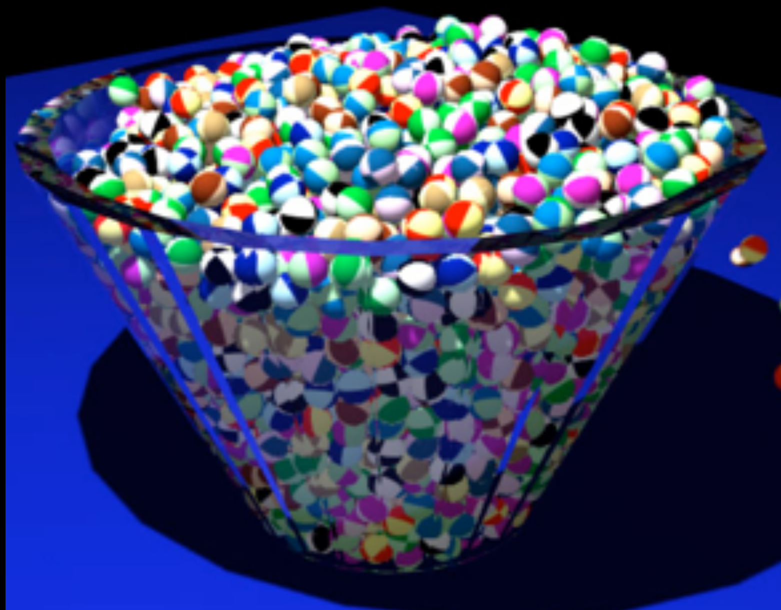
Gases are compressible.

Density changes.

What is a fluid?

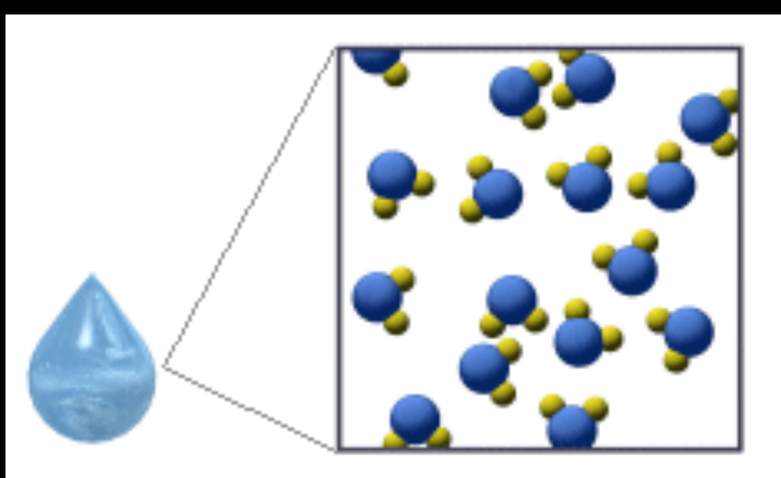


Fluid properties



we could use the laws of mechanics to calculate fluid motion....

....and apply Newton's laws to each molecule in the fluid.



But a drop of water contains 1,000,000,000,000,000,000,000 molecules.

So it would take the fastest computer many times the age of the Universe to calculate the motion!

Fluid properties

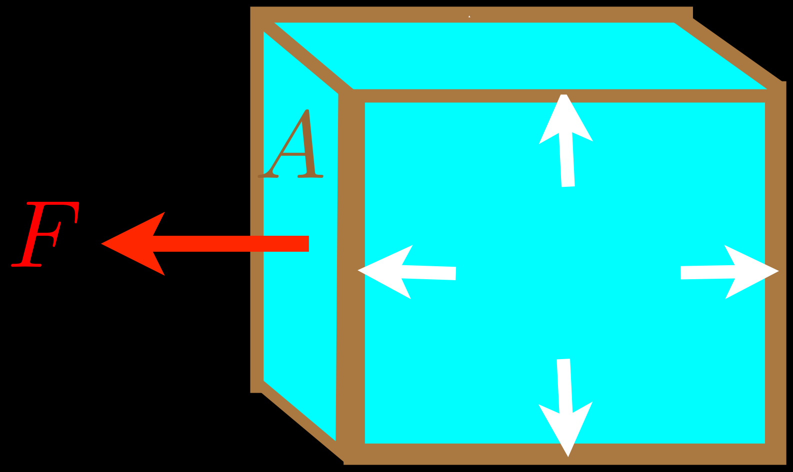
Consider fluid as continuous, rather than made from discrete (separate) particles.

Macroscopic (large-scale) properties:

Density: $\boxed{\rho = \frac{m}{V}}$

Pressure: $\boxed{P = \frac{F}{A}}$ $[\mathrm{N/m^2}] = [\mathrm{Pa}]$ pascal

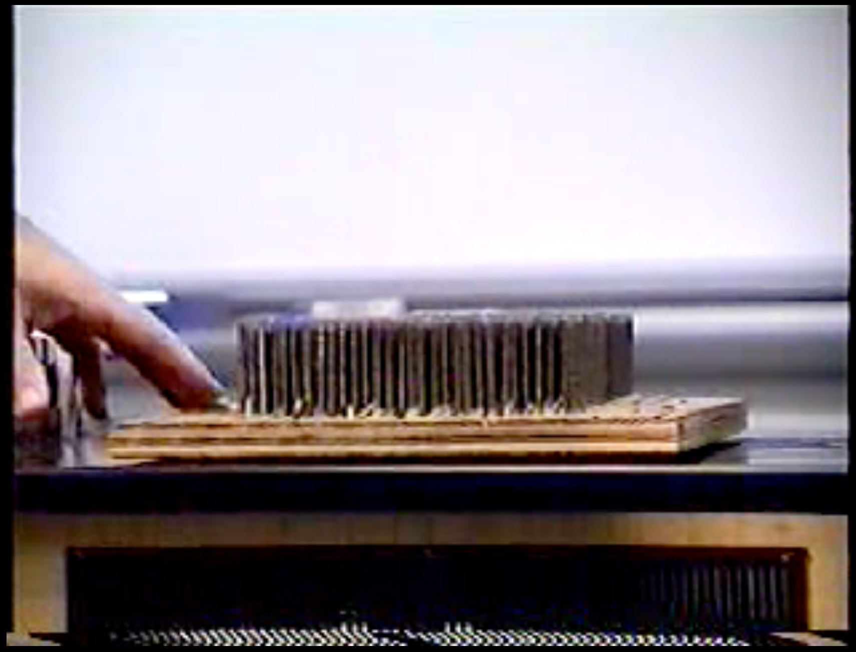
Pressure is a scalar (non-vector).
It applies in all directions.



Fluid properties

Pressure: $$\boxed{P = \frac{F}{A} \quad [\mathrm{N/m^2}]}$$

If the force is applied across a large area, the pressure is small:



If the pressure is distributed over many nails, it is not enough to pop the balloon.

If just one nail is used, the pressure is high and the balloon pops.

Hydrostatic equilibrium

If the fluid is at rest ($v = 0$), $\bar{F}_{\rm net} = 0$

= hydrostatic equilibrium

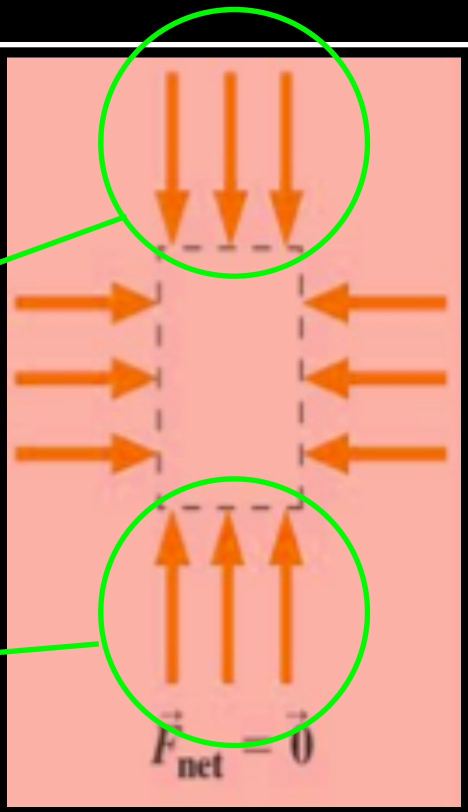
Since: $P = \dfrac{F}{A}$,

$$\bar{F}_{\rm net} = \bar{F}_1 + \bar{F}_2 = A(P_1 - P_2) = A\Delta P$$

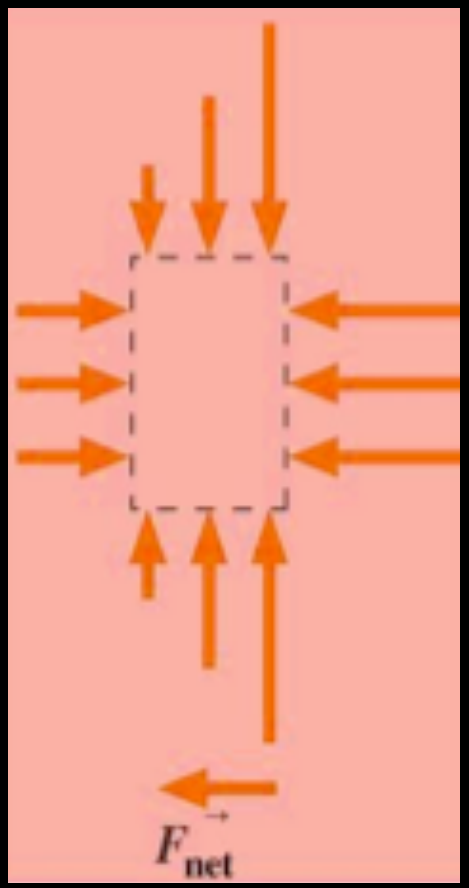
if $\bar{F}_{\rm net} = 0$, $P = \text{constant}$

Without external forces, hydrostatic equilibrium needs constant pressure

A pressure difference gives a force.



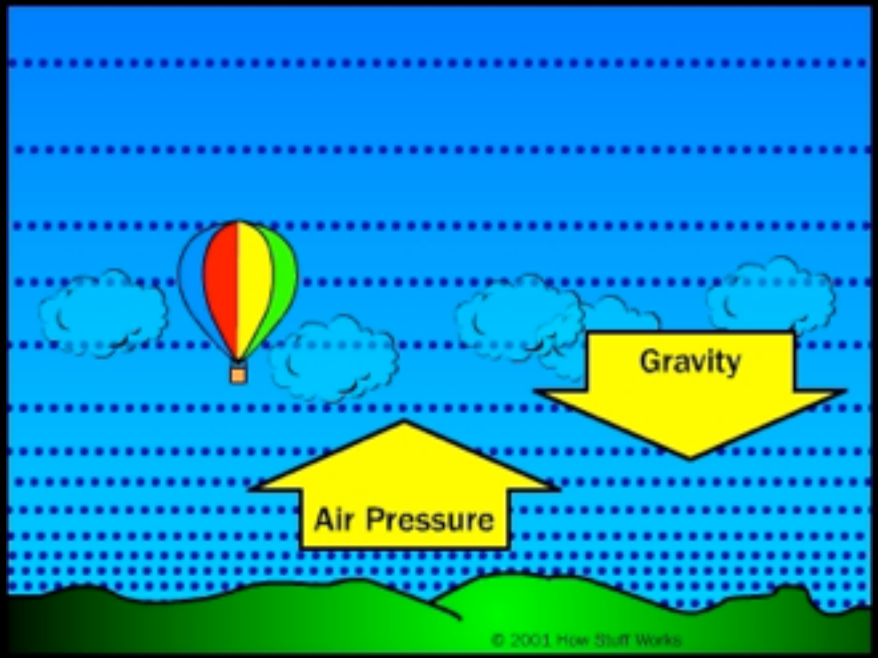
constant P



increasing P

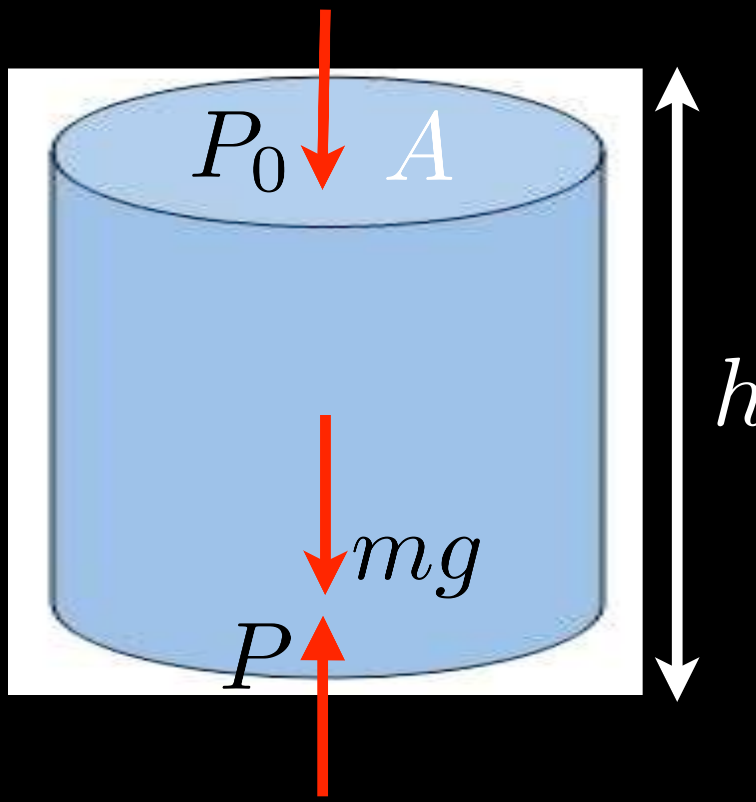
Hydrostatic equilibrium

With gravity, the pressure force balances the gravitational force.



Since the pressure force comes from pressure difference (ΔP), it increases with depth.

Consider forces on a column of fluid:



$$F_P = F_{P0} + F_g \longrightarrow PA = P_0 A + mg$$

since: $m = \rho V = \rho A \Delta h$

$$PA - P_0 A = \rho A \Delta h g \longrightarrow \boxed{P = P_0 + \rho g \Delta h}$$

for liquid (constant ρ)

More generally:

$$\frac{\Delta P}{\Delta h} = \rho g \longrightarrow \boxed{\frac{dP}{dh} = \rho g}$$

Hydrostatic equilibrium

Hydrostatic equilibrium

Through which hole will the water come out fastest?

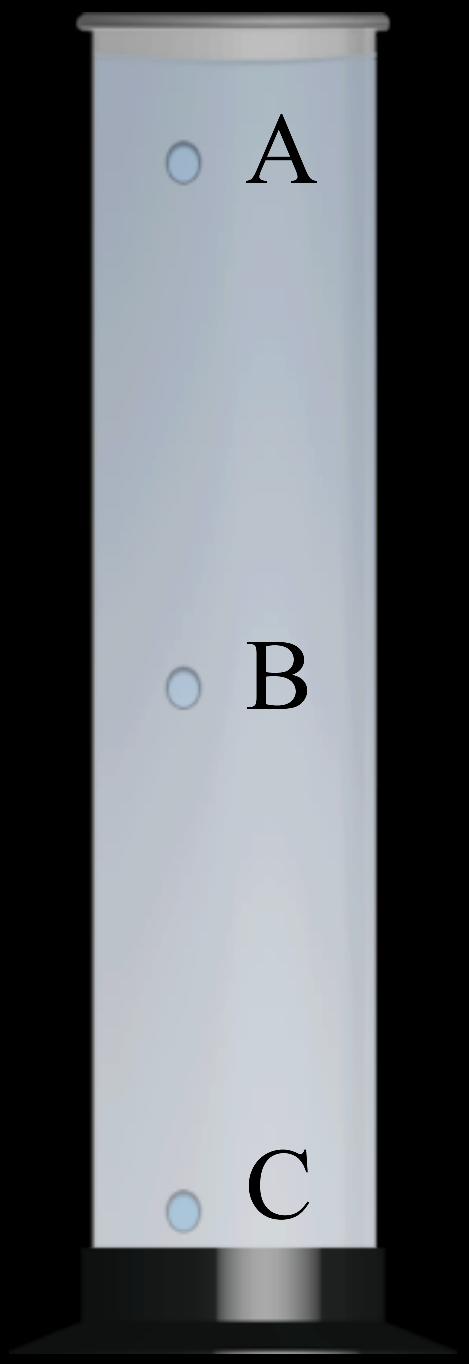
(a)

(b)

(c) $\longrightarrow$ $P = P_0 + \rho g \Delta h$

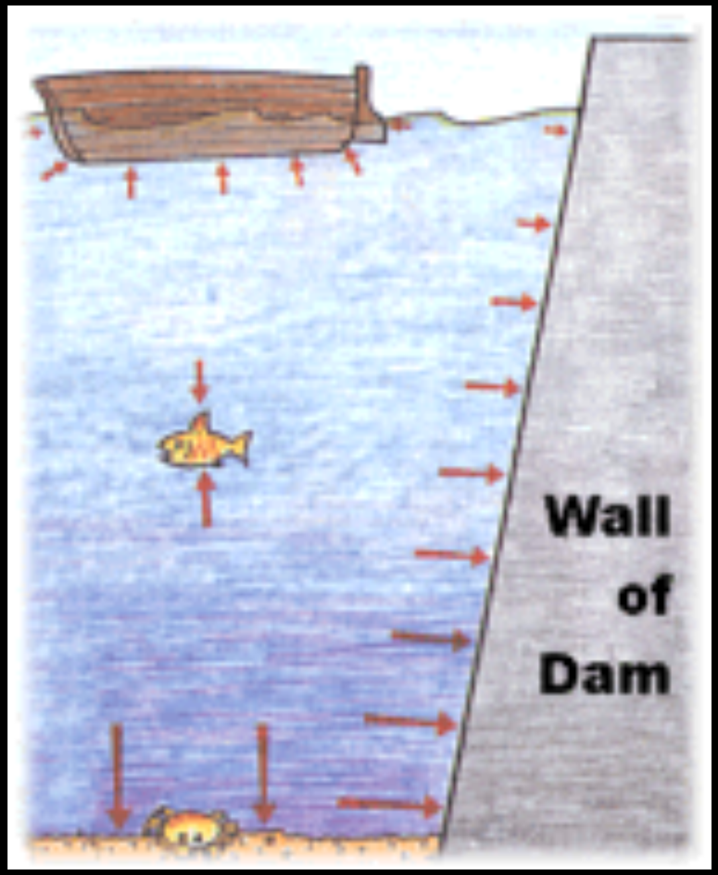
$$P = \frac{F}{A}$$

P is higher at (C), therefore force is greater and velocity is higher.



Hydrostatic equilibrium

Quiz



Two dams are identical in size and shape and the water levels at both are the same.

1 dam holds back a lake containing 2,000,000 m^3 of water while the other holds back a 4,000,000 m^3 lake.

Which statement is correct?

(1) The dam with the larger lake has twice the average force on it.

(2) The dam with the smaller lake has twice the average force on it.

(3) The dam with the larger lake has a slightly larger average force.

(4) None of the above $\longrightarrow$ $P = P_0 + \rho g \Delta h \longrightarrow$ if h the same, P the same

Hydrostatic equilibrium

Find the pressure at a depth of 10 m below the surface of a lake if the pressure at the surface is 1 atm (atmosphere).

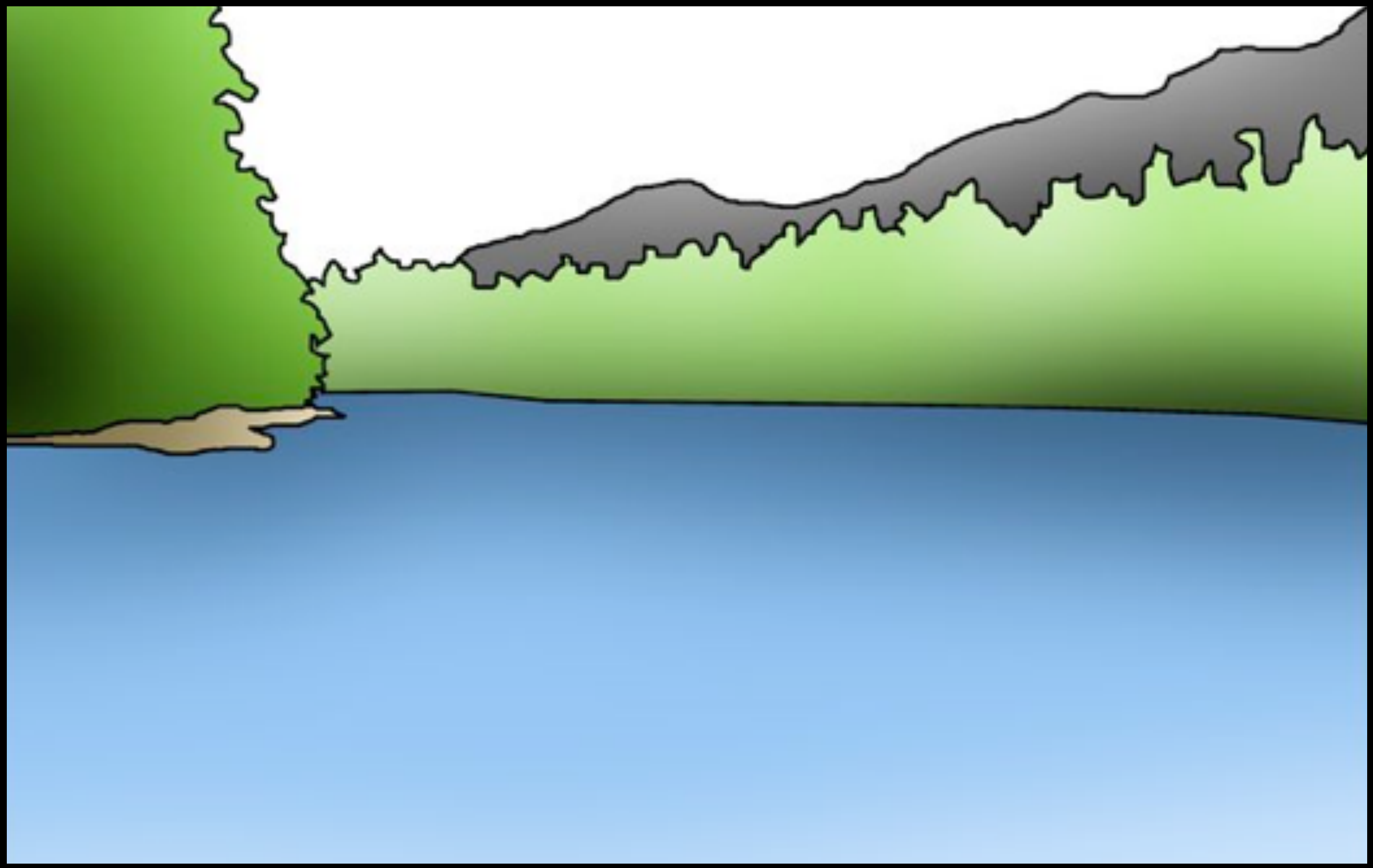
$$1\text{atm} = 101\text{kPa}$$
$$\rho = 10^3\text{kg/m}^3$$

(a) 1 atm

(b) 199,000 atm

(c) 1.97 atm

(d) 199 atm



Hydrostatic equilibrium

Find the pressure at a depth of 10 m below the surface of a lake if the pressure at the surface is 1 atm (atmosphere).

$$\boxed{\begin{aligned} 1\mathrm{atm} &= 101\mathrm{kPa} \\ \rho &= 10^3\mathrm{kg/m^3} \end{aligned}}$$

(a) 1 atm

(b) 199,000 atm

(c) 1.97 atm

(d) 199 atm

$$\begin{aligned} P &= P_0 + \rho g \Delta h \\ &= 101\,\mathrm{kPa} + (10^3\,\mathrm{kg/m^3})(9.81\,\mathrm{m/s^2})(10\,\mathrm{m}) \\ &= 199\,\mathrm{kPa} = 1.97\,\mathrm{atm} \end{aligned}$$

Pascal's Law

Since: $P = P_0 + \rho g \Delta h$

An increase in pressure here

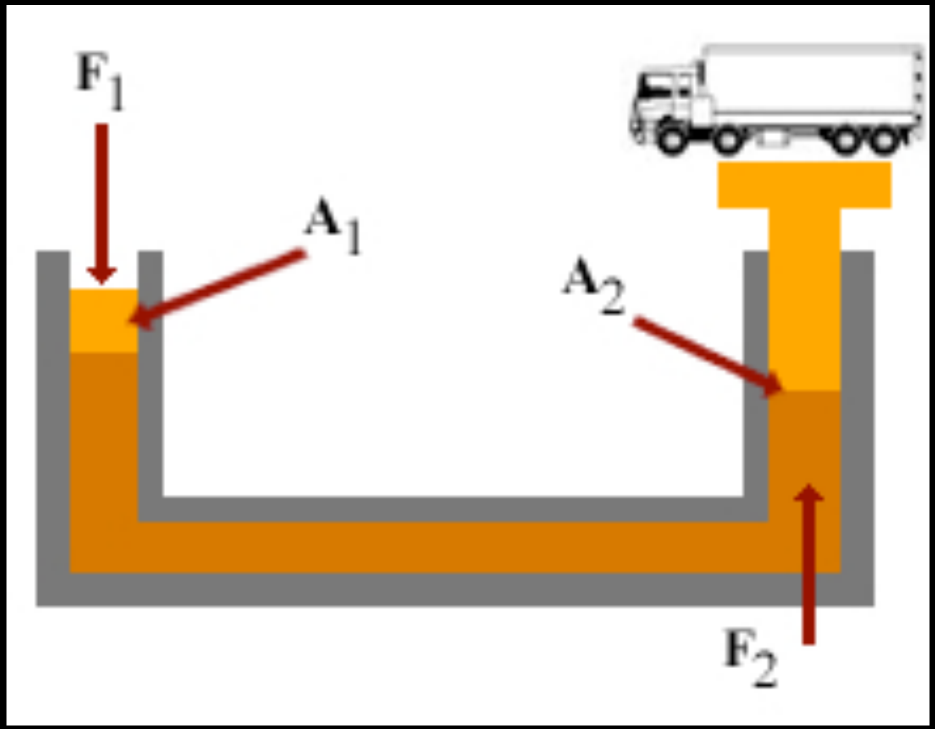
Gives the same increase in pressure everywhere in the fluid

Pascal's law:

A pressure increase anywhere is felt everywhere in the fluid.

<u>Application: hydraulic lift</u>

small force F_1, gives pressure $P = \dfrac{F_1}{A_1}$



This pressure is felt at the right-hand end to give: $F_2 = A_2 P$

The area is larger, so the force is bigger.

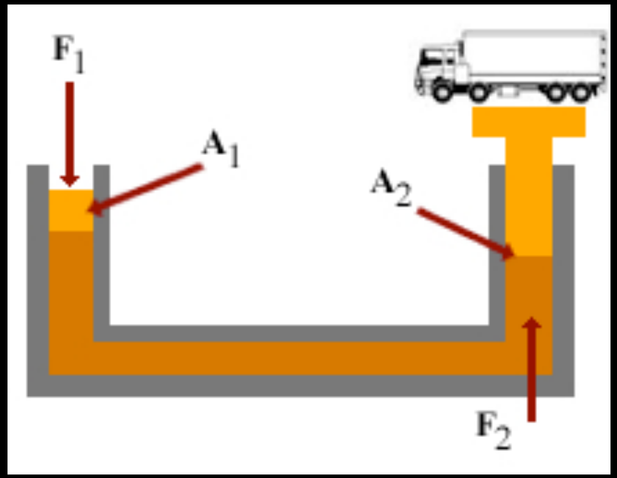
Pascal's Law

Quiz

The large piston in a hydraulic lift has a radius of 20 cm. What force must be applied to the small piston of radius 2 cm to raise a car of mass 1500 kg?

$$F_{\mathrm{car}} = mg = (1500\,\mathrm{kg})(9.81\,\mathrm{m/s^2})$$

$$= 1.47 \times 10^4\ \mathrm{N}$$



(a) 14.7 N

(b) 1470 N

(c) 147 N

(d) 15 N

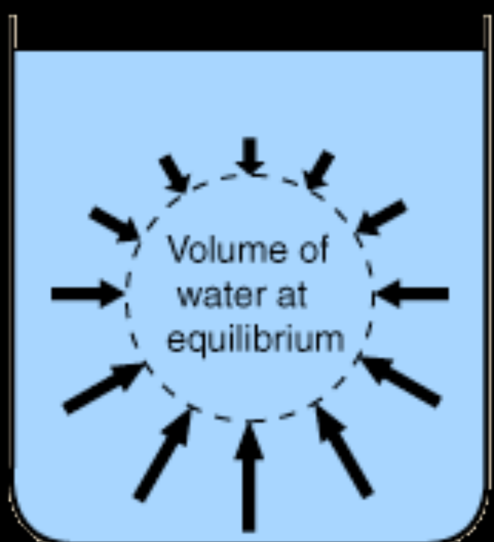
$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \longrightarrow F_1 = \frac{A_1}{A_2} F_2 = \frac{\pi r_1^2}{\pi r_2^2} mg$$

$$= \frac{2\,\mathrm{cm}^2}{20\,\mathrm{cm}^2} \times 1.47 \times 10^4\ \mathrm{N}$$

$$= 147\,\mathrm{N} \quad (\sim 331\,\mathrm{lb})$$

Archimedes' Principal

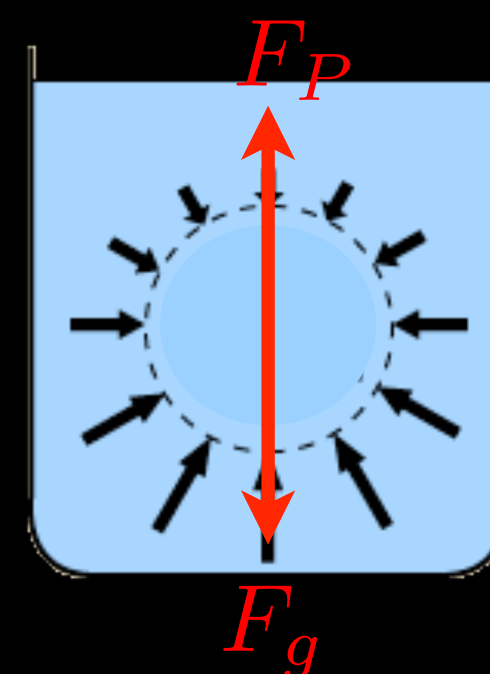
Float or sink?

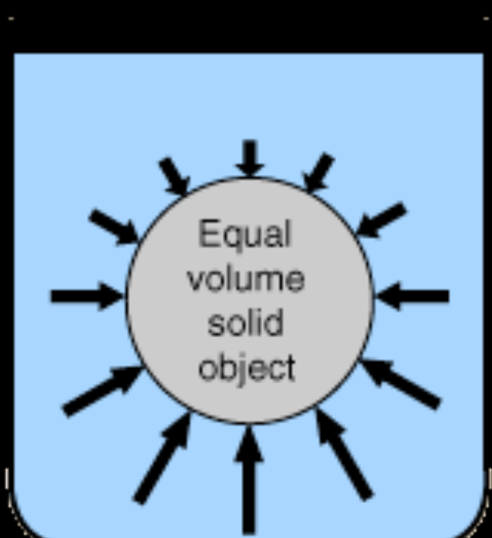


Pressure (P) on a volume of fluid.

Pressure and gravity balance.

$$F_P = F_g = mg$$





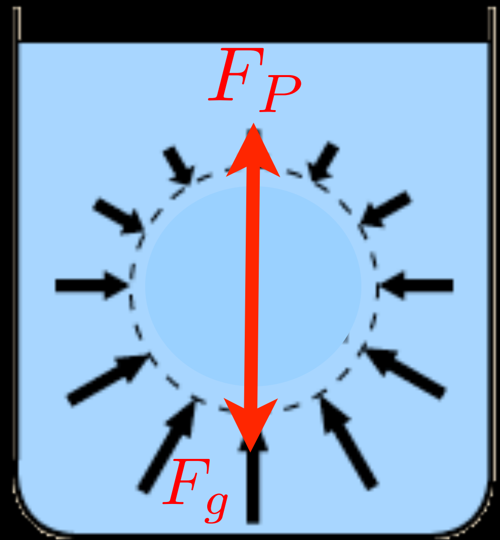
If we replace that volume with a solid object, the remaining fluid is the same.

Therefore, P (and F_P) is the same.

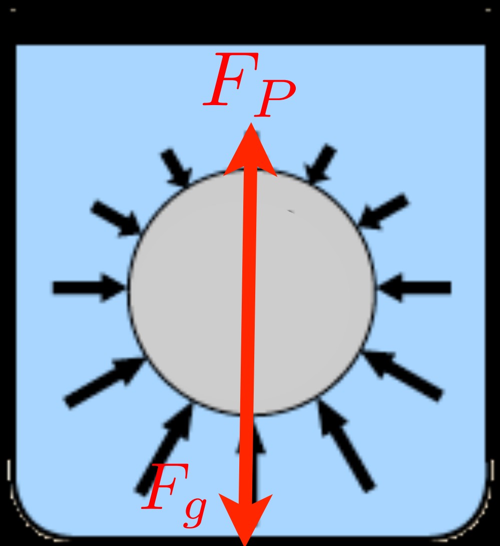
The pressure force on the object is the buoyancy force.

It is equal to the weight of fluid displaced (removed).

Archimedes' Principal



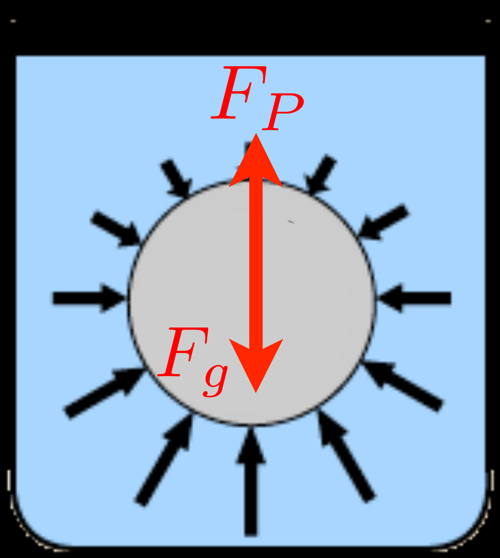
$$F_P = F_{g,f} = m_{(\text{fluid})}g$$



If the object is heavier than the fluid, its gravitational force will be bigger than the buoyancy (P) force.

$$F_P < F_{g,o} = m_{(\text{object})}g$$

Object will sink.



If the object is lighter than the fluid, its gravitational force will be smaller than the buoyancy (P) force.

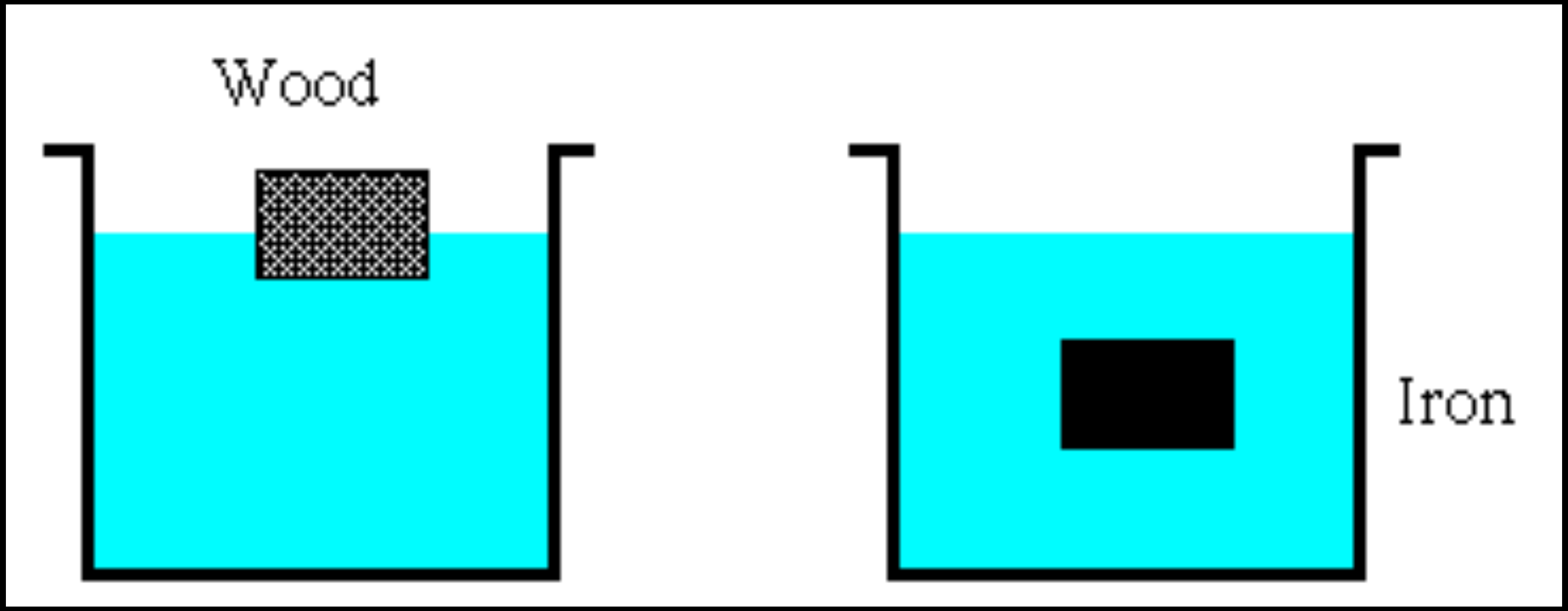
$$F_P > F_{g,o} = m_{(\text{object})}g$$

Object will float.

Archimedes' Principal

Quiz

The wood and iron have equal volumes. The wood floats and the iron sinks. Which has the great buoyant force?



(a) wood

(b) iron

Iron displaces the greater weight of water.

(c) same

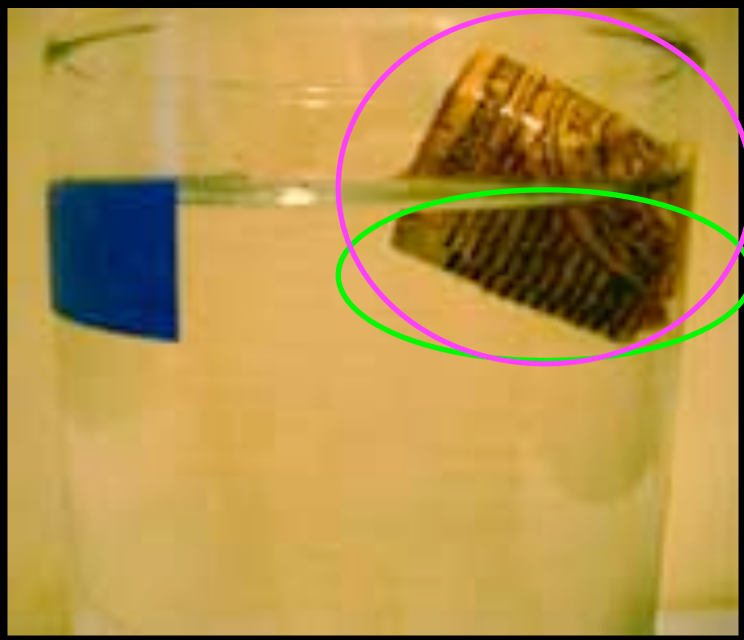
Archimedes' Principal

Example

A cork has a density of $200\mathrm{kg/m^3}$. Find the fraction of the volume of the cork that is submerged when the cork floats in water.

Buoyant force = weight of water displaced

$$= m_W g = (\rho_W V')g$$



Gravitational force: $\rho_c g V$

To float: Gravitational force = Buoyant force

$$\frac{V'}{V} = ?$$

$$\rho_c g V = \rho_W g V'$$

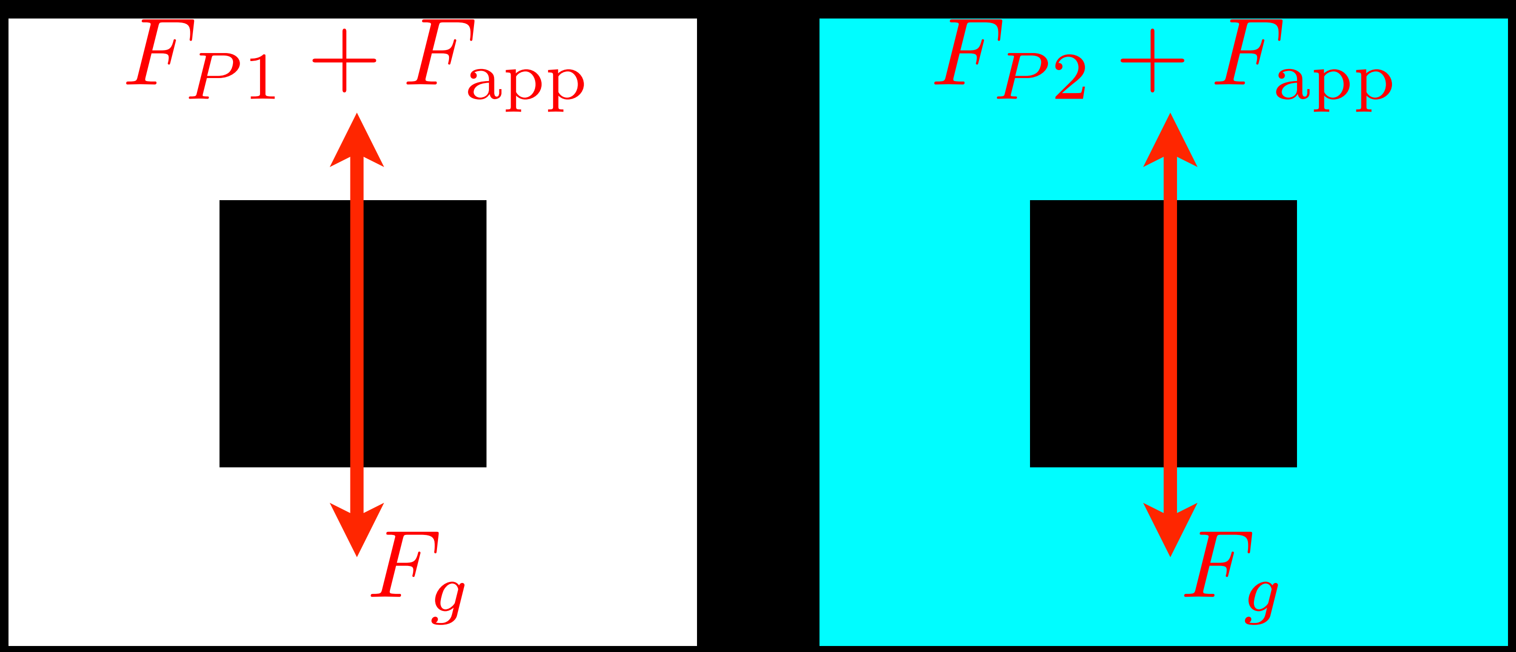
$$\frac{V'}{V} = \frac{\rho_c}{\rho_W} = \frac{200\mathrm{kg/m^3}}{1000\mathrm{kg/m^3}} = \frac{1}{5}$$

Archimedes' Principal

Quiz

On land, the most massive concrete block you can carry is 25 kg.

If concrete's density is 2200kg/m^3, how massive a block could you carry underwater?



$\rho_W = 1000\text{kg/m}^3$

(a) 55 kg

(b) 266 kg

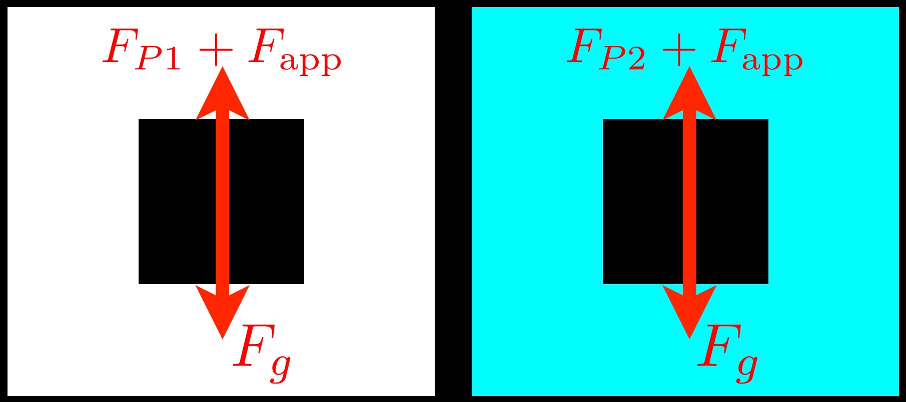
(c) 46 kg

(d) 100 kg

Archimedes' Principal

On land, the most massive concrete block you can carry is 25 kg.

If concrete's density is $2200 \mathrm{kg/m^3}$, how massive a block could you carry underwater?



In water: $F_{P2} + F_{\mathrm{app}} - m_c g = 0$

$$F_{\mathrm{app}} = m_c g - F_{P2}$$

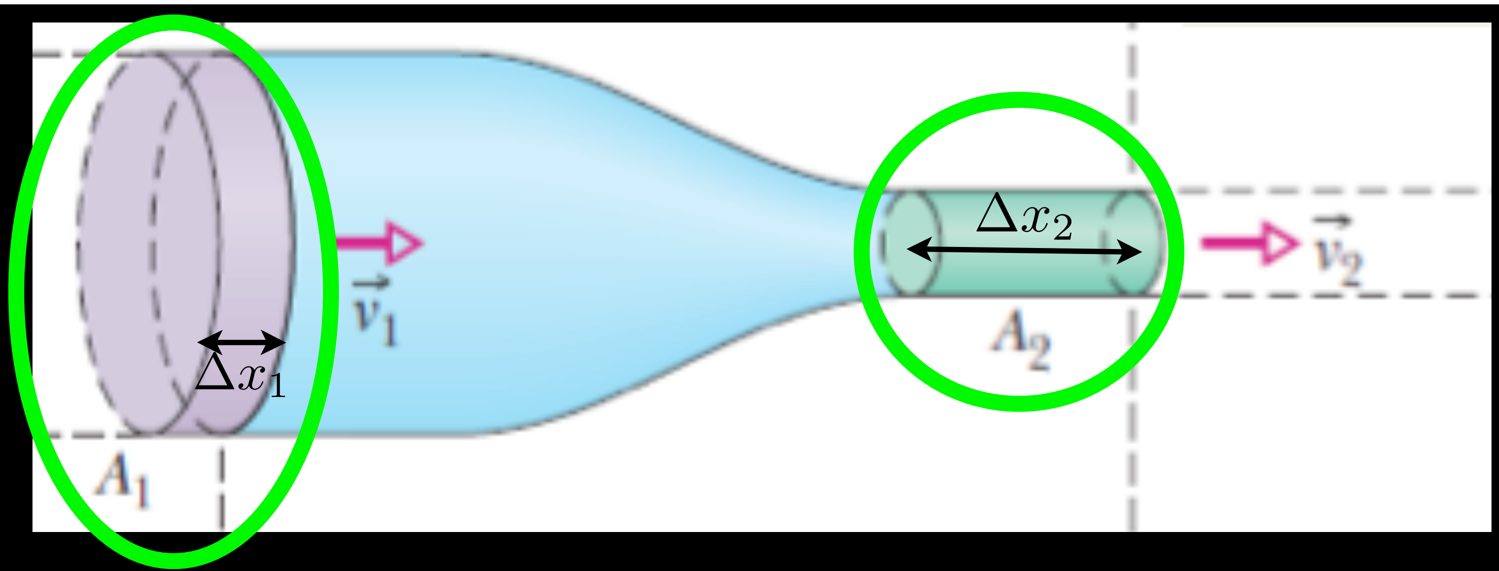
Max $F_{\mathrm{app}} = 25\,\mathrm{kg} \times g$

Buoyancy force $F_{P2} = \rho_W g V_c = \rho_W g \dfrac{m_c}{\rho_c}$

$$(25\,\mathrm{kg})g = m_c g - \rho_W g \frac{m_W}{\rho_c} = m_c g \left(1 - \frac{\rho_W}{\rho_c}\right)$$

$$m_c = 25 \left(\frac{\rho_c}{\rho_c - \rho_W}\right) = 46\,\mathrm{kg}$$

Conservation of Mass



Mass of fluid entering in time Δt = Mass of fluid exiting in time Δt

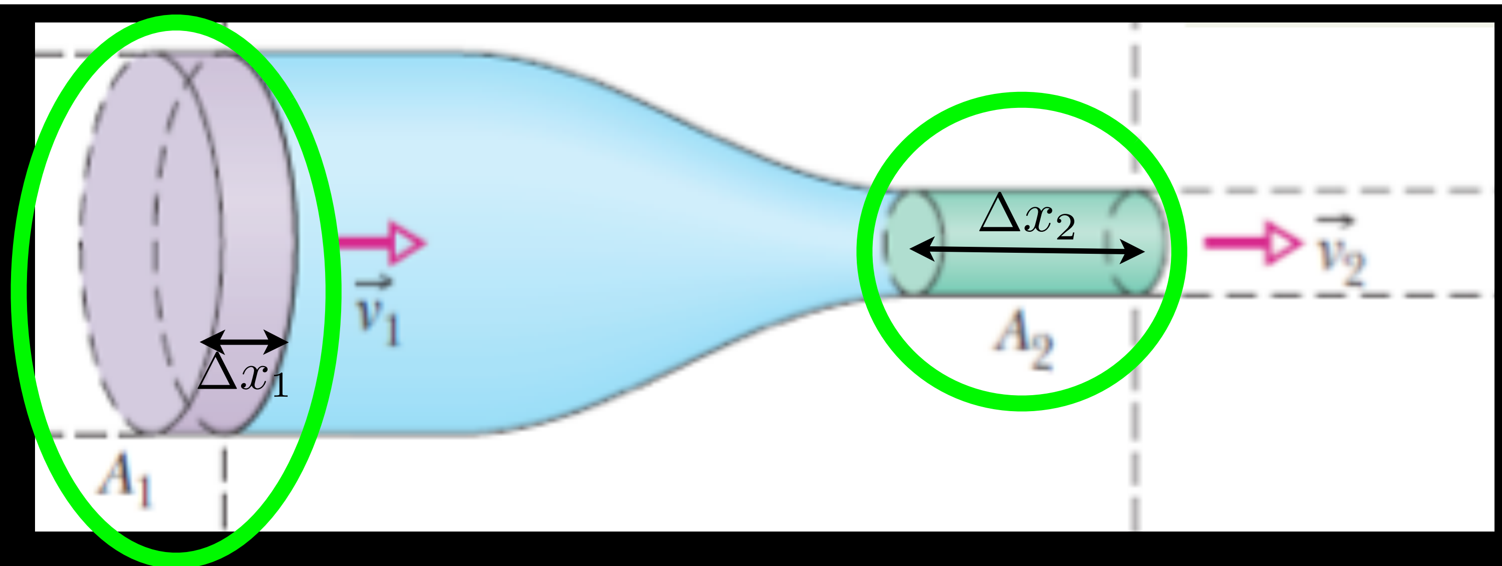
$$m = \rho_1 V_1 \qquad m = \rho_2 V_2$$

$$= \rho_1 A_1 \Delta x_1 \qquad = \rho_2 A_2 \Delta x_2$$

$$= \rho_1 A_1 v_1 \Delta t \qquad = \rho_2 A_2 v_2 \Delta t$$

=

Conservation of Mass



Mass of fluid entering in time Δt = Mass of fluid exiting in time Δt

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$$= \rho_1 A_1 \Delta x_1 \qquad = \rho_2 A_2 \Delta x_2$$

$$= \rho_1 A_1 v_1 \Delta t \qquad = \rho_2 A_2 v_2 \Delta t$$

$\boxed{\rho A v = \text{ constant along flow}}$ mass flow rate

for liquid (constant ρ)

$\boxed{A v = \text{ constant along flow}}$ volume flow rate

Conservation of Mass

Quiz

Water flows through a pipe that has a constriction in the middle as shown. How does the speed of the water in the constriction compare to the speed of the water in the rest of the pipe?

(A) It is bigger

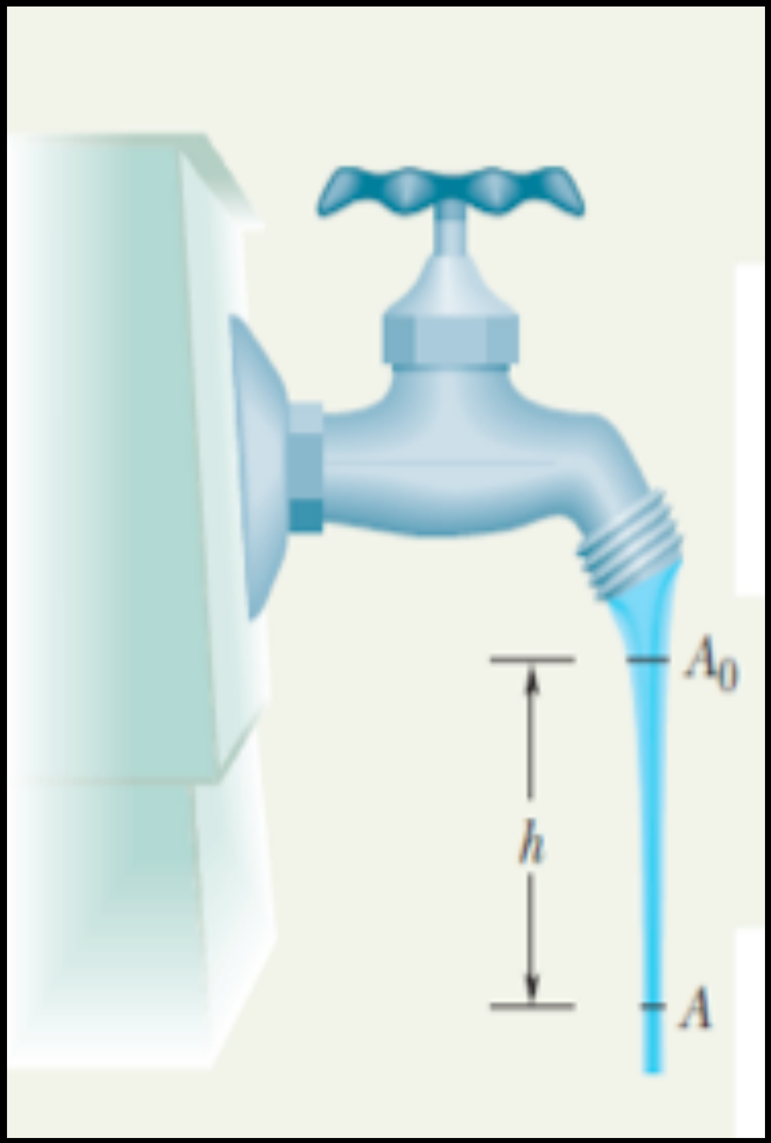
(B) It is smaller

(C) It is the same

$Av =$ constant along flow

Conservation of Mass

Quiz



A stream of water falls from a tap.

Its cross-sectional area changes from

$A_0 = 1.2\text{cm}^2$ to $A_1 = 0.35\text{cm}^2$

The 2 levels are separated by h = 45mm.

What is the initial velocity and volume flow rate?

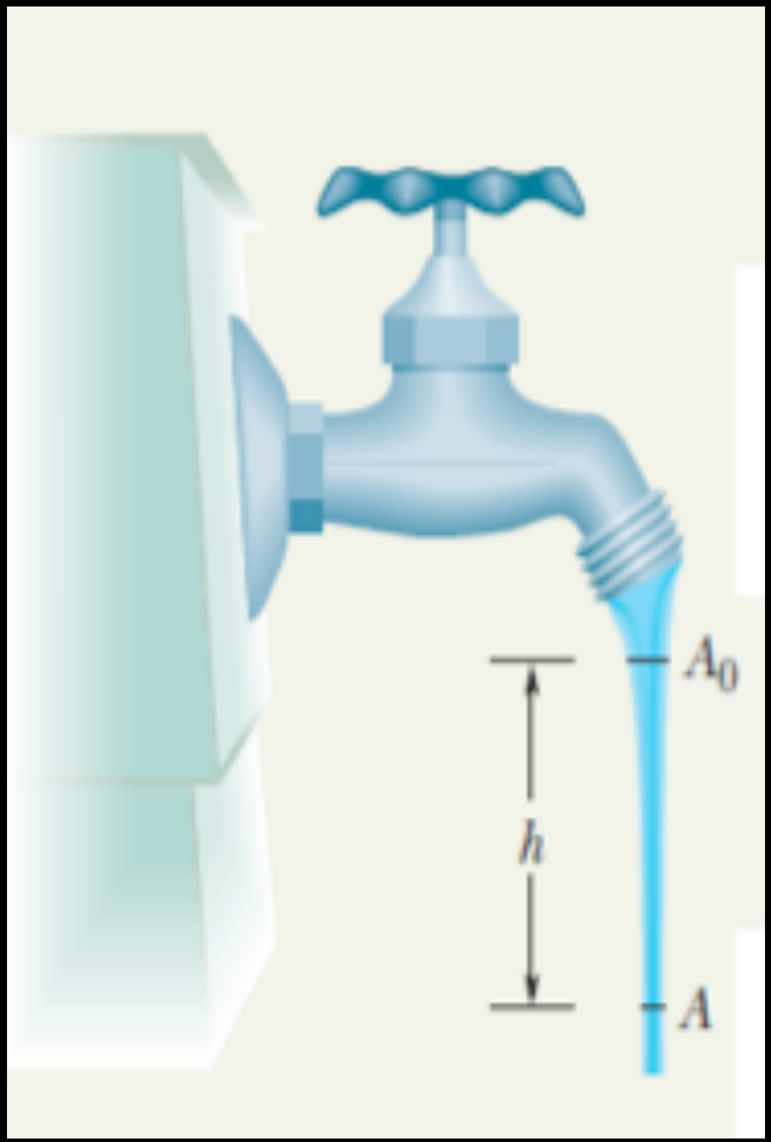
(Hint: use constant acceleration equations)

(a) $v_0 = 31.1\text{cm/s}$ volume flow rate: $37\text{cm}^3/\text{s}$

(b) $v_0 = 9.1\text{cm/s}$ volume flow rate: $10.9\text{cm}^3/\text{s}$

(c) $v_0 = 28.6\text{cm/s}$ volume flow rate: $34\text{cm}^3/\text{s}$

Conservation of Mass



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(Hint: use constant acceleration equations)

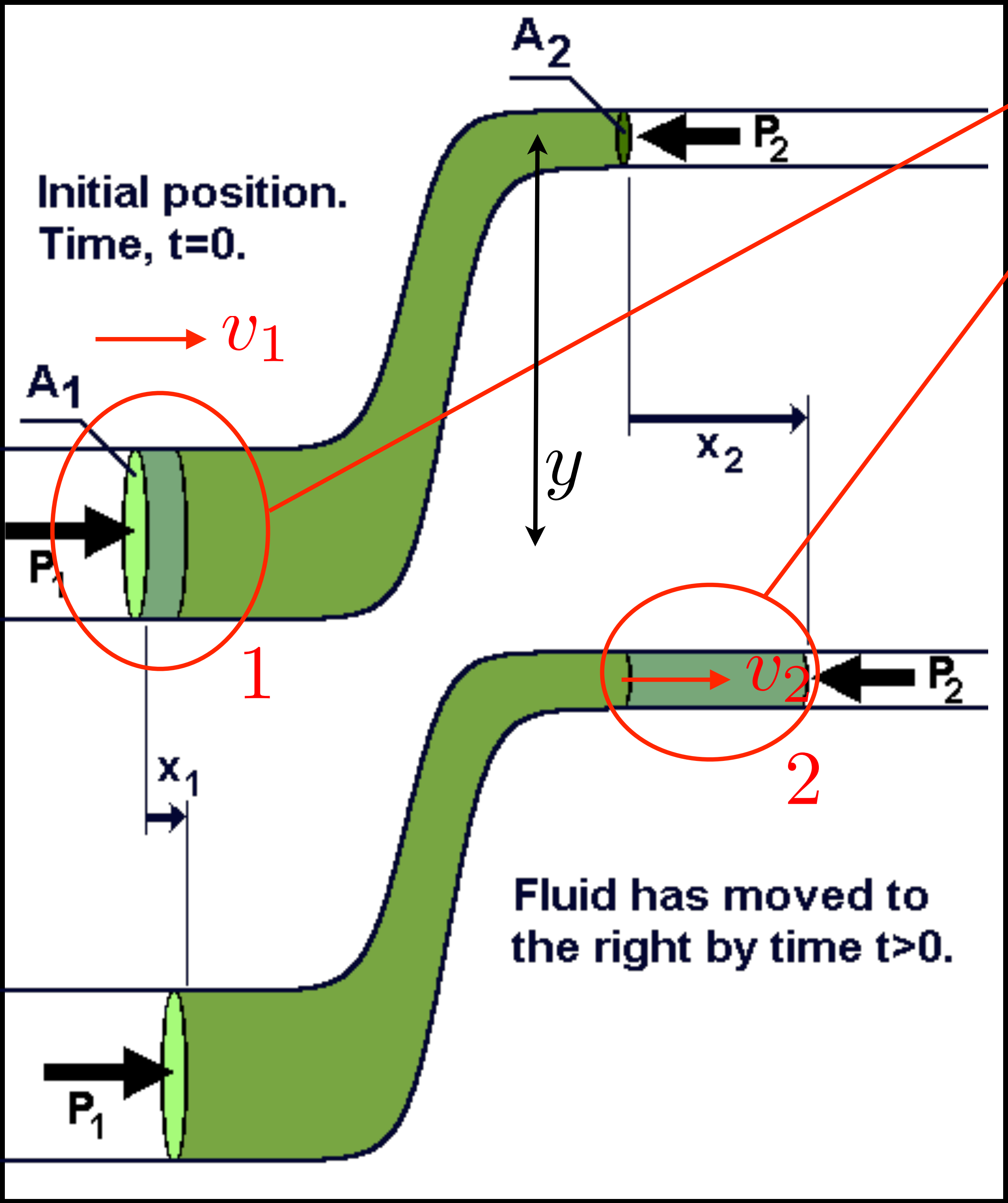
Conservation of mass: $A_0 v_0 = Av \quad \rightarrow v_0^2 = A^2 v^2 / A_0^2$

$$v^2 = v_0^2 + 2gh$$

$$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}} = 28.6\,\mathrm{cm/s}$$

Volume flow rate: $A_0 v_0 = 34\,\mathrm{cm}^3/\mathrm{s}$

Conservation of Energy



Fluid moves along pipe

$$\Delta K = \frac{1}{2}m(v_2^2 - v_1^2)$$

= work done

Fluid to the left exerts pressure force at (1) as fluid moves x_1

$$W_1 = F_1 x_1 \; = P_1 A_1 x_1$$

Fluid to the right exerts opposite pressure force at (2) as fluid moves x_2

$$W_2 = -F_2 x_2 = -P_2 A_2 x_2$$

Work done against gravity: $W_g = -\rho V g(y_2 - y_1)$

Conservation of Energy

$$\Delta K = W_1 + W_2 + W_g$$

$$\frac{1}{2}m(v_2^2 - v_1^2) = P_1 \underbrace{A_1 x_1}_{V} - P_2 \underbrace{A_2 x_2}_{V} - \rho V g(y_2 - y_1)$$

For incompressible fluids: $A_1 x_1 = A_2 x_2 = V$

$$P_1 - P_2 - \rho g(y_2 - y_1) = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$\boxed{P + \rho g y + \frac{1}{2}\rho v^2 = \text{ constant}}$$

Bernoulli's equation

Conservation of Energy

Quiz

Two empty pop cans are placed about ¼” apart on a frictionless surface. If you blow air between the cans, what happens?

A) The cans move toward each other.

B) The cans move apart.

C) The cans don’t move at all.

velocity increases, so pressure decreases between the cans.

The higher outside pressure pushes them towards each other.

$$P + \rho g y + \frac{1}{2}\rho v^2 = \text{ constant}$$

Conservation of Energy

A liquid with density $\rho = 791\mathrm{kg/m^3}$ flows through a horizontal pipe that narrows from $A_1 = 1.2 \times 10^{-3}\mathrm{m^2}$ to $A_2 = A_1/2$.

The pressure difference between wide and narrow sections is 4120 Pa.

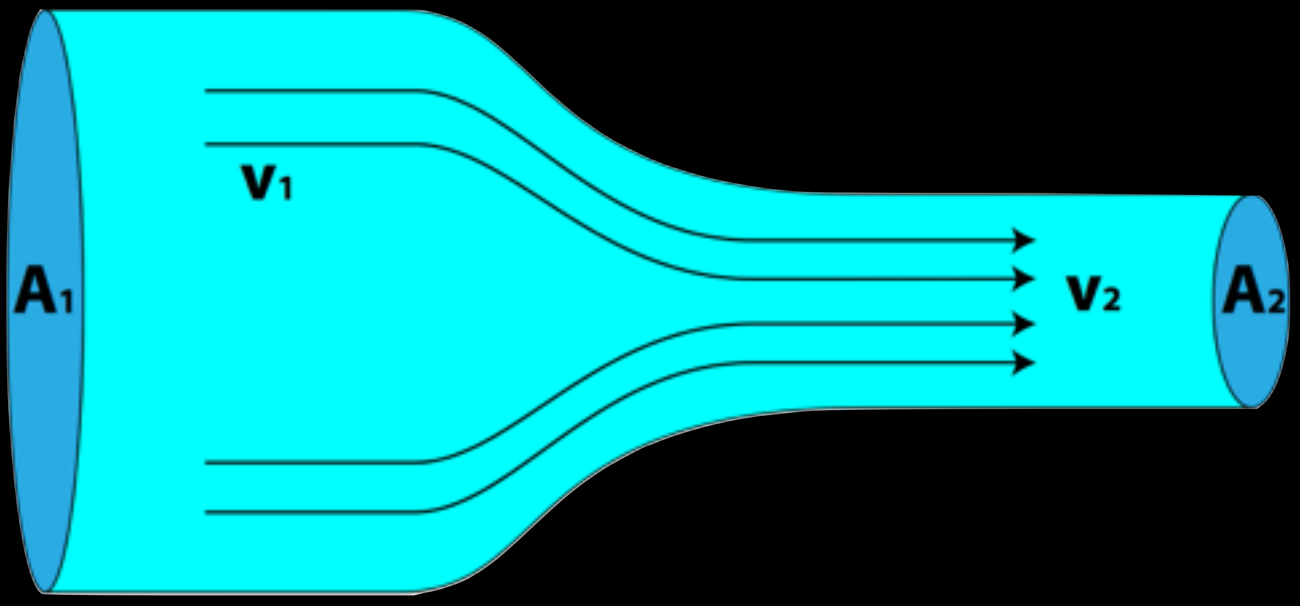
What is the volume flow rate Av?

(a) $4.48 \times 10^{-3}\mathrm{m^3/s}$

(b) $2.24 \times 10^{-3}\mathrm{m^3/s}$

(c) $3.03 \times 10^{-6}\mathrm{m^3/s}$

(d) $6.06 \times 10^{-6}\mathrm{m^3/s}$



Conservation of Energy

Quiz

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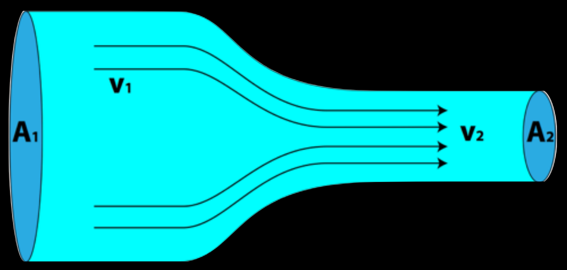


$$A_1 v_1 = A_2 v_2 = R_v \quad \text{(volume flow rate)}$$

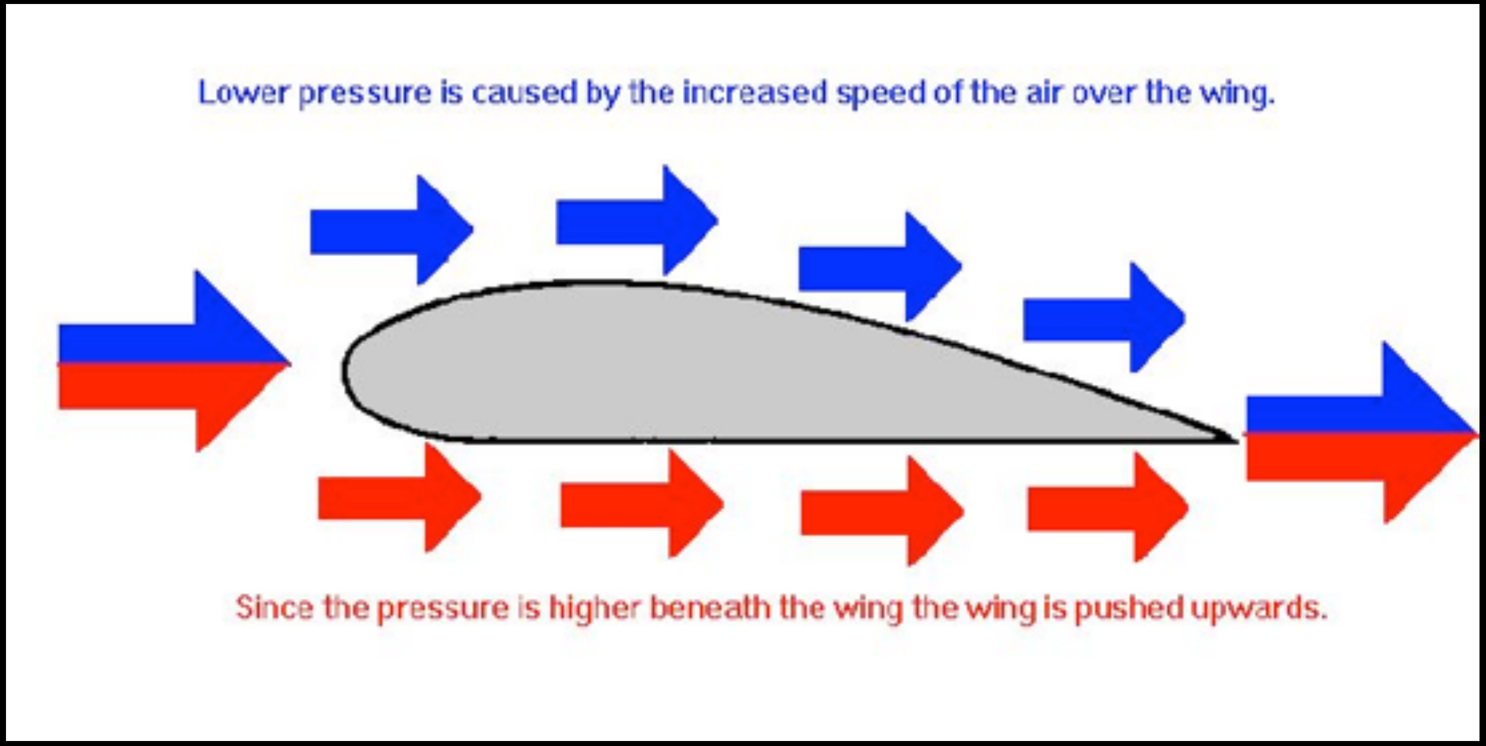
Bernoulli: $P_1 + \rho g y + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y + \frac{1}{2}\rho v_2^2$

$$v_1 = \frac{R_v}{A_1} \qquad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \qquad v_2 = \frac{R_v}{A_2} = \frac{2R_v}{A_1}$$

$$R_v = A_1\sqrt{\frac{2(P_1 - P_2)}{3\rho}} = 1.2 \times 10^{-3}\sqrt{\frac{(2)(4120\,\mathrm{Pa})}{(3)(791\,\mathrm{kg/m^2})}} = 2.24 \times 10^{-3}\,\mathrm{m^3/s}$$

Lift and curve

How do aeroplanes fly?



Aeroplane wing:

The distance travelled above the curved wing is larger than below it.

From conservation of mass, the air flow must take the same time to go over and under the wing.

Therefore, air above the wing moves faster.

From Bernoulli's equation: $P + \rho g y + \frac{1}{2}\rho v^2 =$ constant

A higher velocity gives a lower pressure above the wing.

This pressure difference gives an upwards force.

Lift and curve

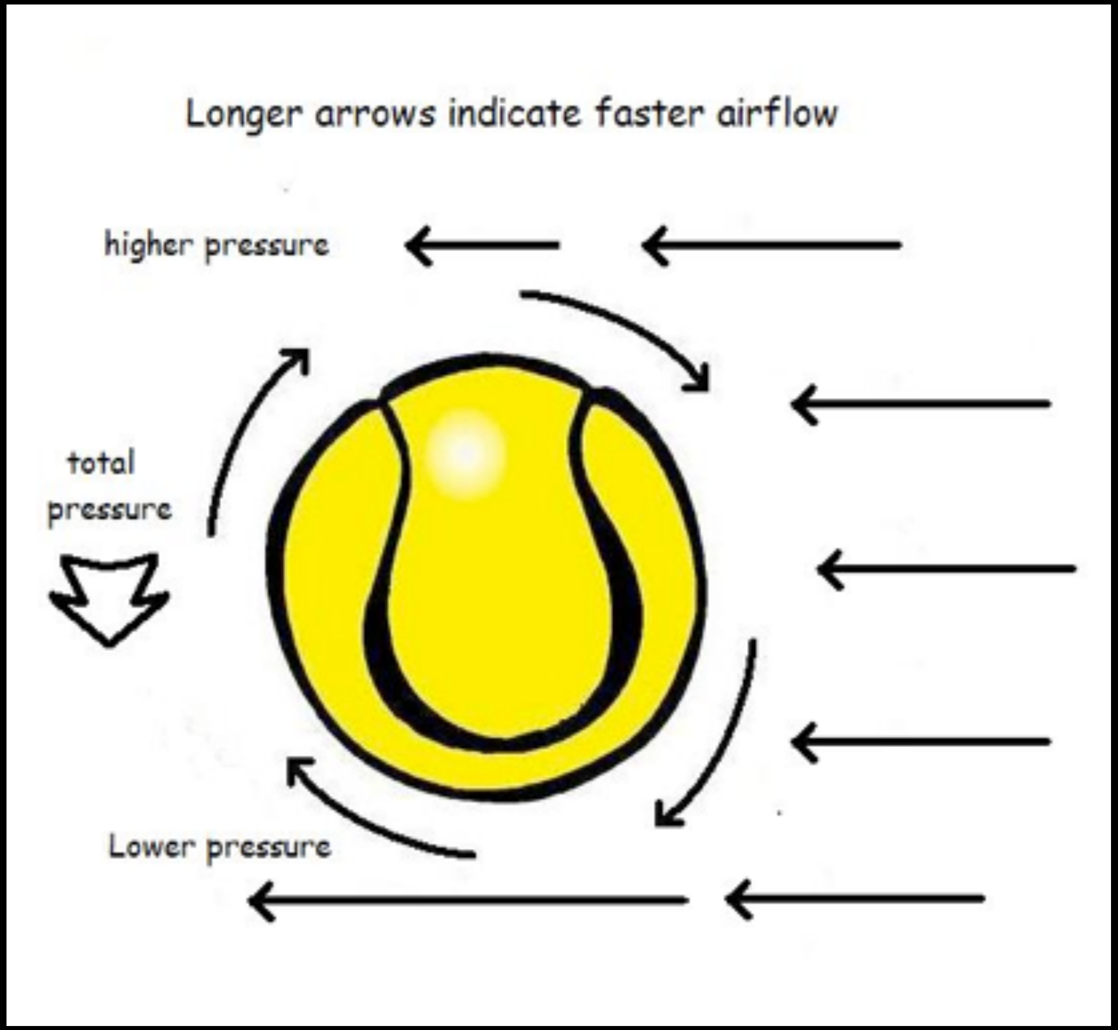
Similarly for a curving ball:

Bottom of ball, the spin and air velocity are in the same direction.

The ball drags that air, making it faster.

Velocity is higher and so pressure is lower.

Pressure difference gives downwards force.



Lecture 11 : Summary

- Fluid properties: pressure, density, flow velocity

- Archimedes' Principal: buoyancy force from pressure is equal to the weight of the displaced fluid by an object.

  Objects less dense that fluid will float
  Object more dense will sink

- Continuity Equation: conservation of matter

  $\rho A v =$ constant along flow

- Bernoulli's Equation: conservation of energy

  $P + \rho g y + \frac{1}{2}\rho v^2 =$ constant

  (relate flow speed and pressure)