Essential Physics I

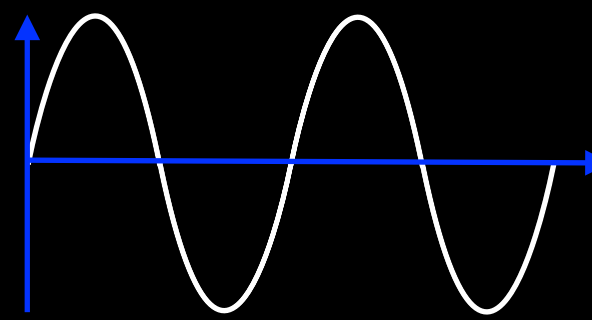
英語で物理学の

エッセンス I

Lecture 10: 23-06-16

Last lecture: review

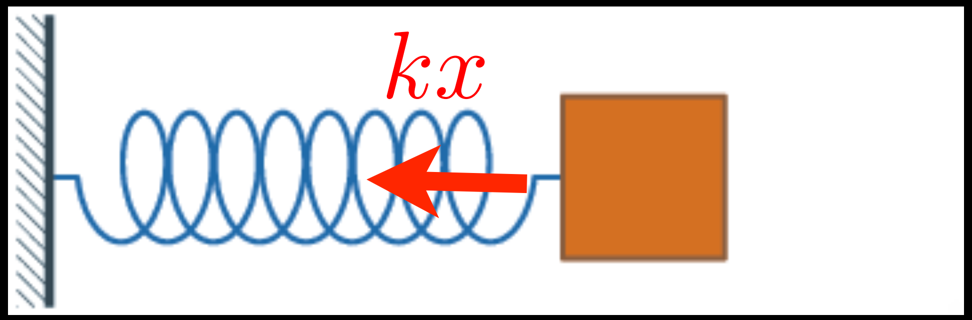
Oscillation: periodic motion where a force returns the system to equilibrium



Simple harmonic motion (SHM): force is proportional to displacement: $F \propto \Delta x$

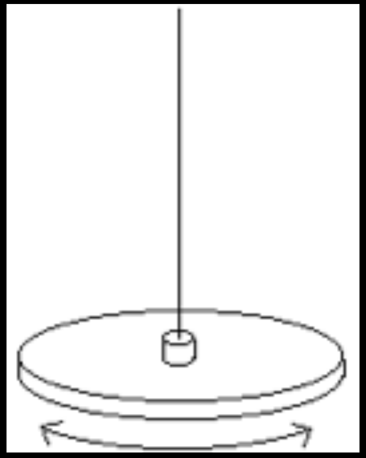
(1) Spring: $m\frac{d^2x}{dt^2} = -kx$

$$x(t) = A\cos(\omega t + \phi) \qquad w = \sqrt{\frac{k}{m}}$$



(2) Torsional pendulum: $I\frac{d^2\theta}{dt^2} = -\kappa\theta$

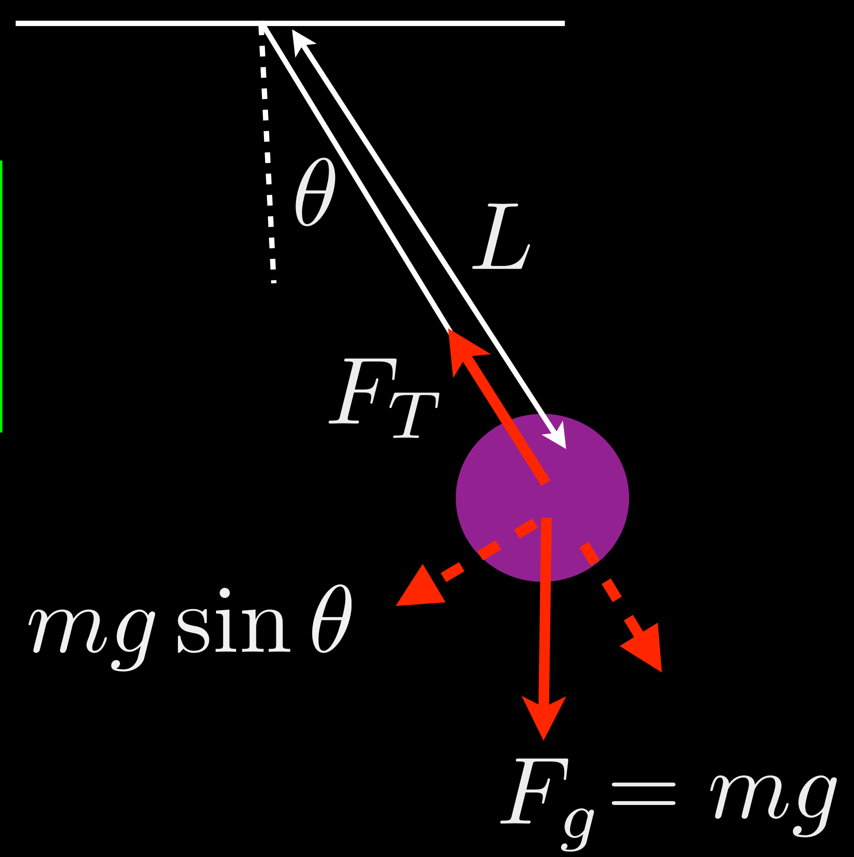
$$\theta(t) = A\cos\omega t\,, \qquad \omega = \sqrt{\frac{\kappa}{I}}$$



Last lecture: Simple Harmonic Motion

(3) Simple pendulum

string mass ≈ 0
ball's volume ≈ 0 (point mass)



$F_{\text{net}} = mg\sin\theta$

$$I\frac{d^2\theta}{dt^2} = -\tau = -LF_{\text{net}}$$

$$= -mgL\sin\theta$$

Force not directly proportional to θ ⟶ not SHM

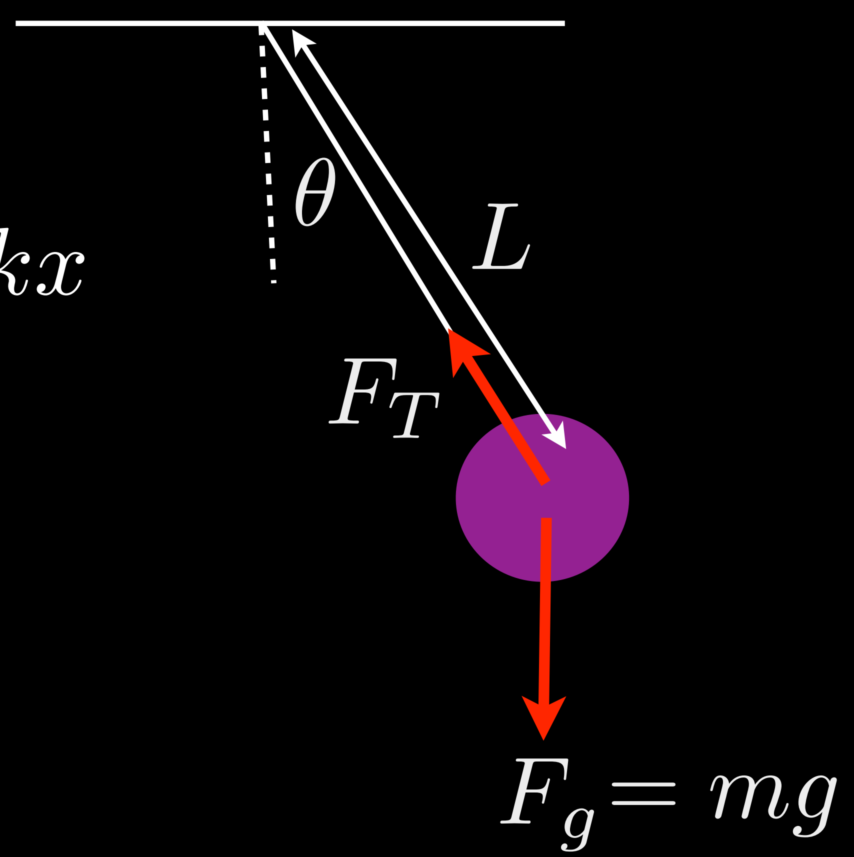
But for small oscillations $\sin\theta \approx \theta$

$$I\frac{d^2\theta}{dt^2} = -mgL\theta$$

Last lecture: Simple Harmonic Motion

(3) Simple pendulum

$$I\frac{d^2\theta}{dt^2} = -mgL\theta \quad \longleftrightarrow \quad m\frac{d^2x}{dt^2} = -kx$$



$$\boxed{\omega = \sqrt{\frac{mgL}{I}}}$$

Simple pendulum, point mass: $I = mL^2$

$$\omega = \sqrt{\frac{mgL}{mL^2}} = \sqrt{\frac{g}{L}}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}}$$

Last lecture: Simple Harmonic Motion

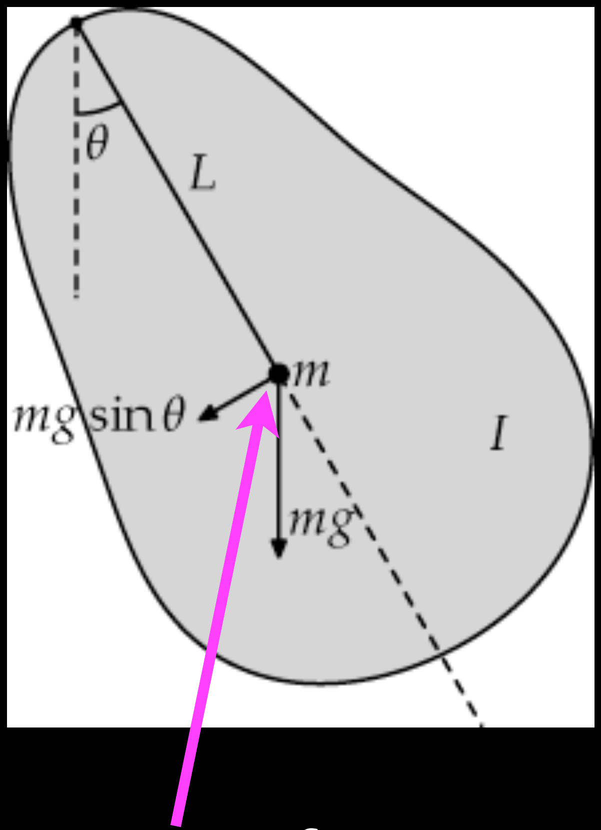
(4) Physical Pendulum

Not a point mass   e.g. a leg, punching bag etc

$$\omega = \sqrt{\frac{mgL}{I}}$$ is still true

but $I \neq mL^2$

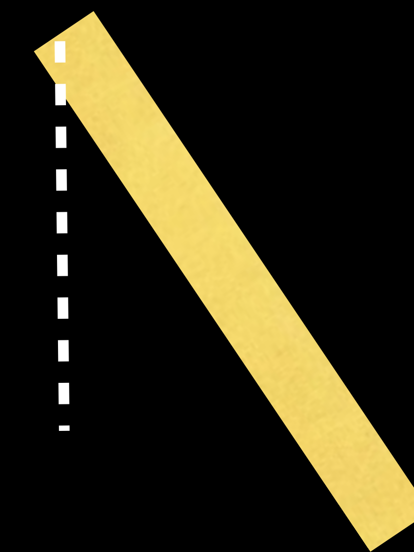
and L = distance to centre of gravity



centre of gravity

Simple Harmonic Motion

A meter stick is suspended from one end and set swinging. Find the period of the resulting oscillations.



(a) 2.3s

(b) 3.2s

(c) 1.6s

(d) 4.1s

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgL}}$$

centre of gravity for stick: $L = L_0/2 = 50\,\mathrm{m}$

$$T = 2\pi\sqrt{\frac{mL_0^2/3}{mgL_0/2}} = 2\pi\sqrt{\frac{2L_0}{3g}}$$

$$= 2\pi\sqrt{\frac{2(1.0\,\mathrm{m})}{3(9.81\,\mathrm{m/s^2})}} = 1.6\,\mathrm{s}$$

$$I = \frac{mR^2}{3}$$

Simple Harmonic Motion

Energy and SHM

Potential energy for a spring: $U = \frac{1}{2}kx^2 \quad = \frac{1}{2}k(A\cos\omega t)^2$

$$= \frac{1}{2}kA^2\cos^2\omega t$$

Kinetic energy for a spring: $K = \frac{1}{2}mv^2 \quad = \frac{1}{2}m(-\omega A\sin\omega t)^2$

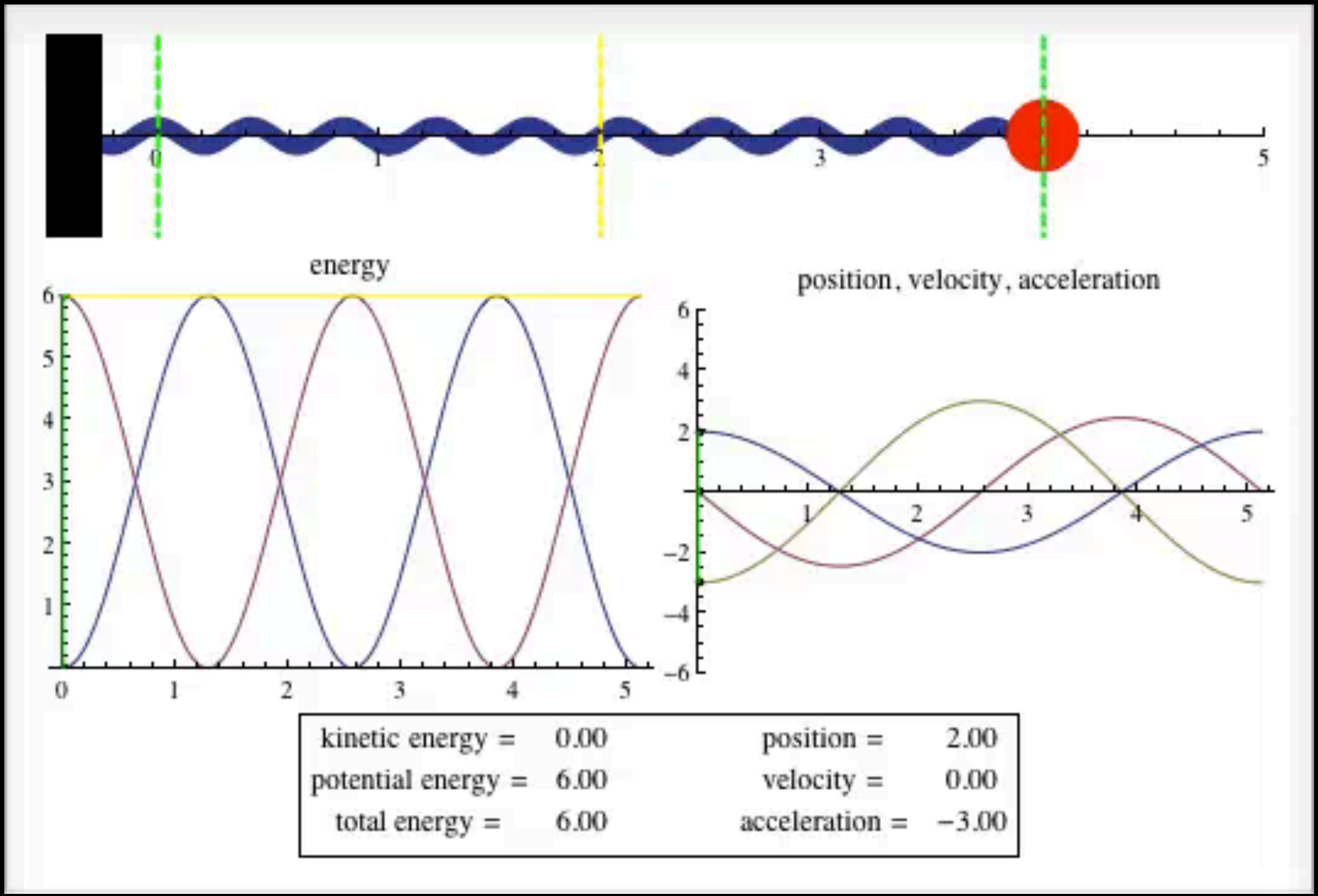
$$= \frac{1}{2}m\omega^2A^2\sin^2\omega t$$

$$= \frac{1}{2}kA^2\sin^2\omega t$$

Total energy: $E = U + K = \frac{1}{2}kA^2\cos^2\omega t + \frac{1}{2}kA^2\sin^2\omega t = \boxed{\frac{1}{2}kA^2}$

Simple Harmonic Motion

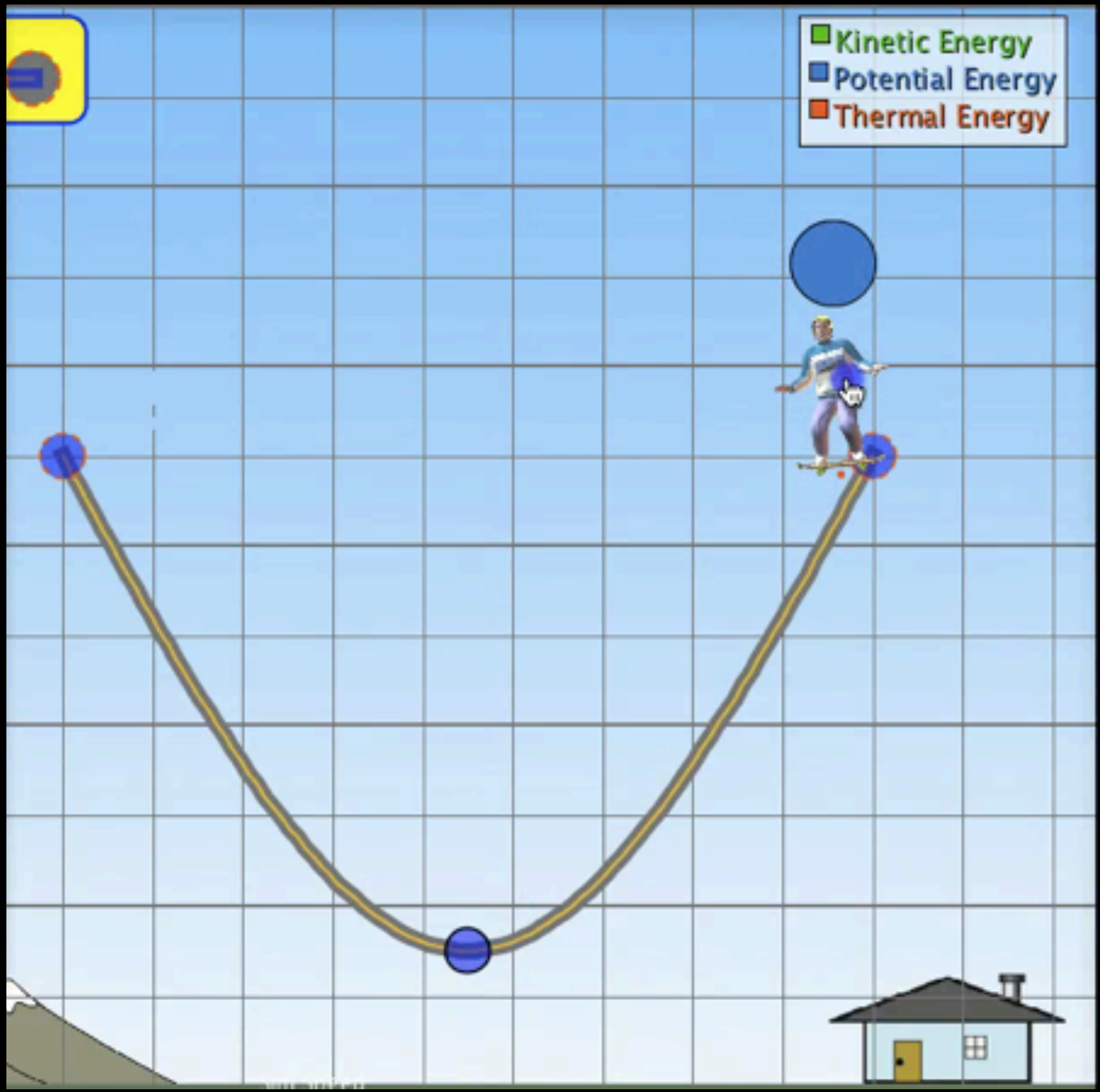
Energy and SHM



Simple Harmonic Motion

In real oscillating systems, air resistance & friction dissipate (remove) the oscillation energy.

We saw this before for the skateboarder



The energy loss causes the amplitude, A, to decrease.

The motion is said to be damped.

Simple Harmonic Motion

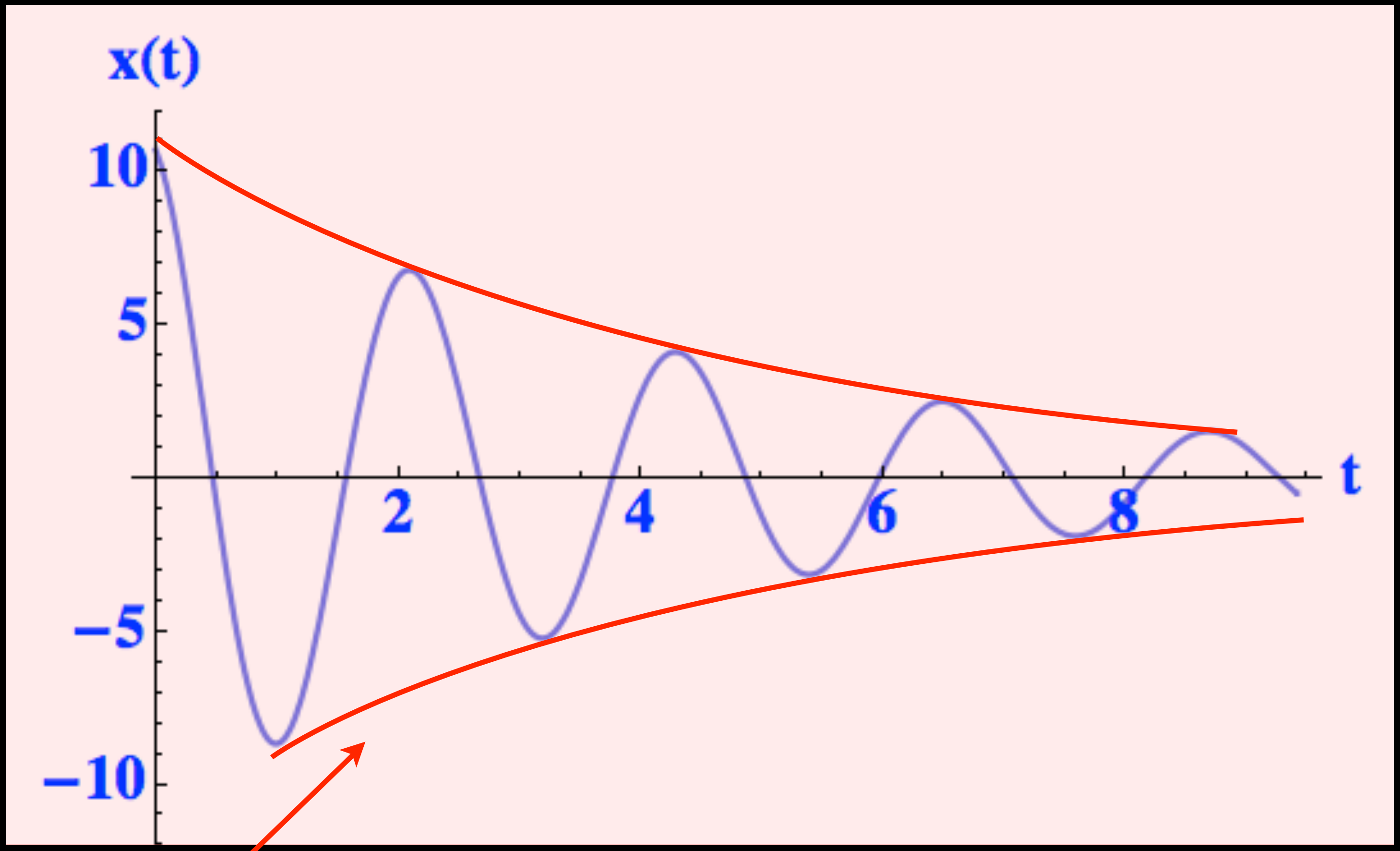
If only a little energy is lost each oscillation, the system behaves the same as for the undamped case - but with a gradual loss in A.

Often, the damping force is proportional to v:

$$F_d = -bv = -b\frac{dx}{dt}$$

Therefore:

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$



Solution: $x(t) = Ae^{-bt/2m}\cos(\omega t + \phi)$

Simple Harmonic Motion

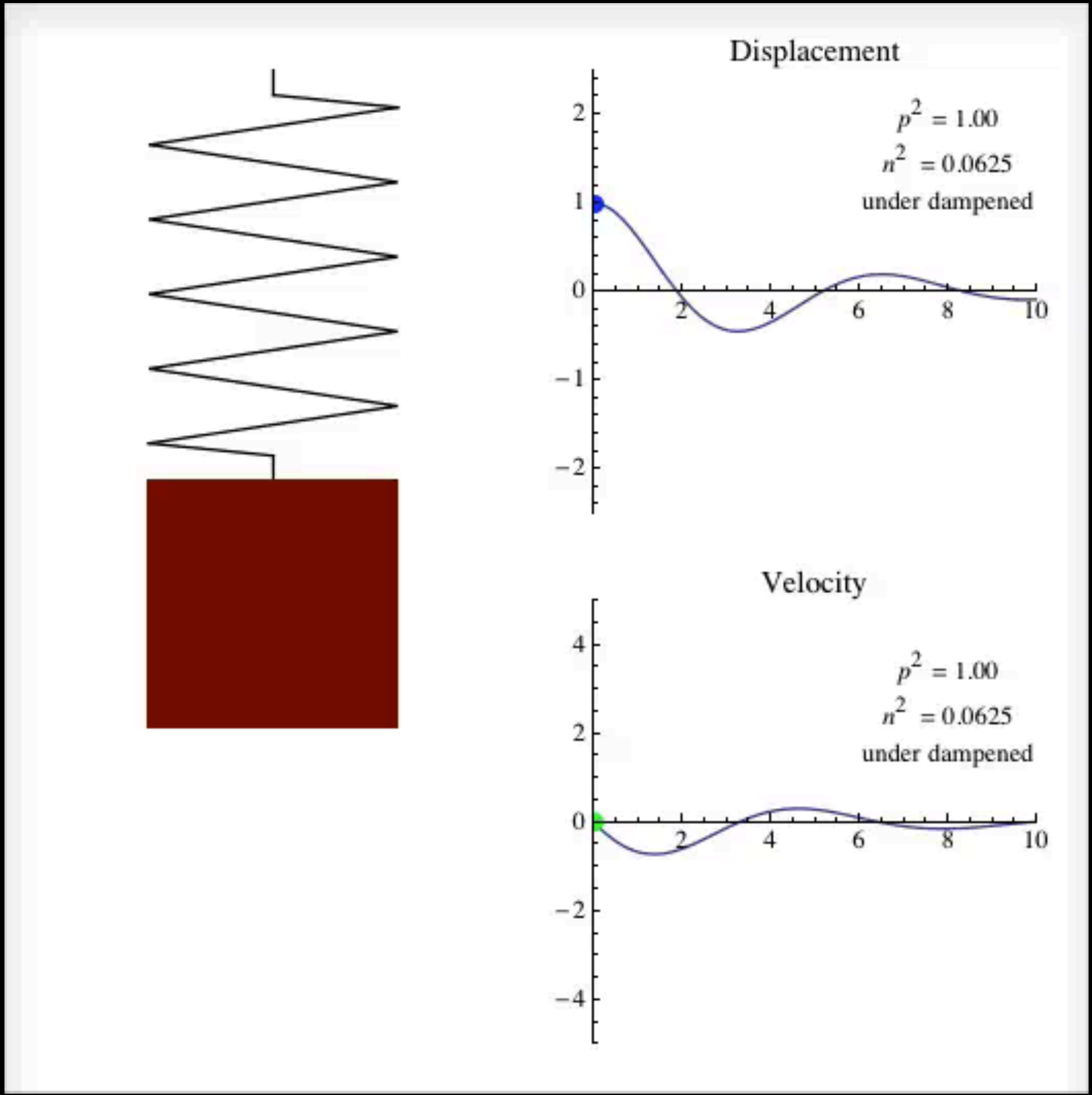
For weak damping,
ω is unchanged:

$$\omega = \sqrt{k/m}$$

For stronger damping, the motion slows.

ω becomes lower.

If there is oscillation, the system is underdamped.

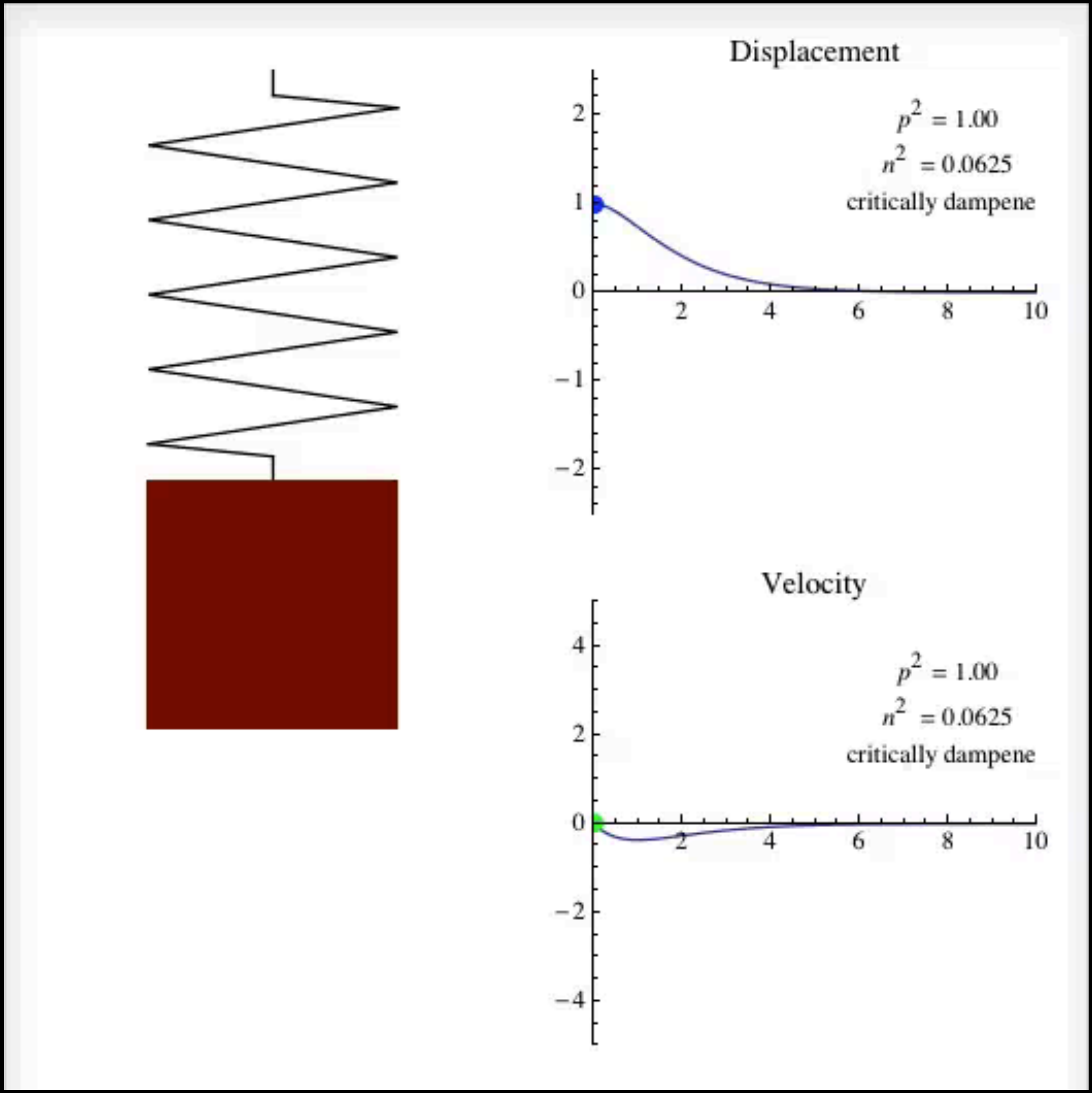


Simple Harmonic Motion

If the effect of the damping force equals that of the spring force

the system is **critically damped**.

It returns to equilibrium without oscillation.

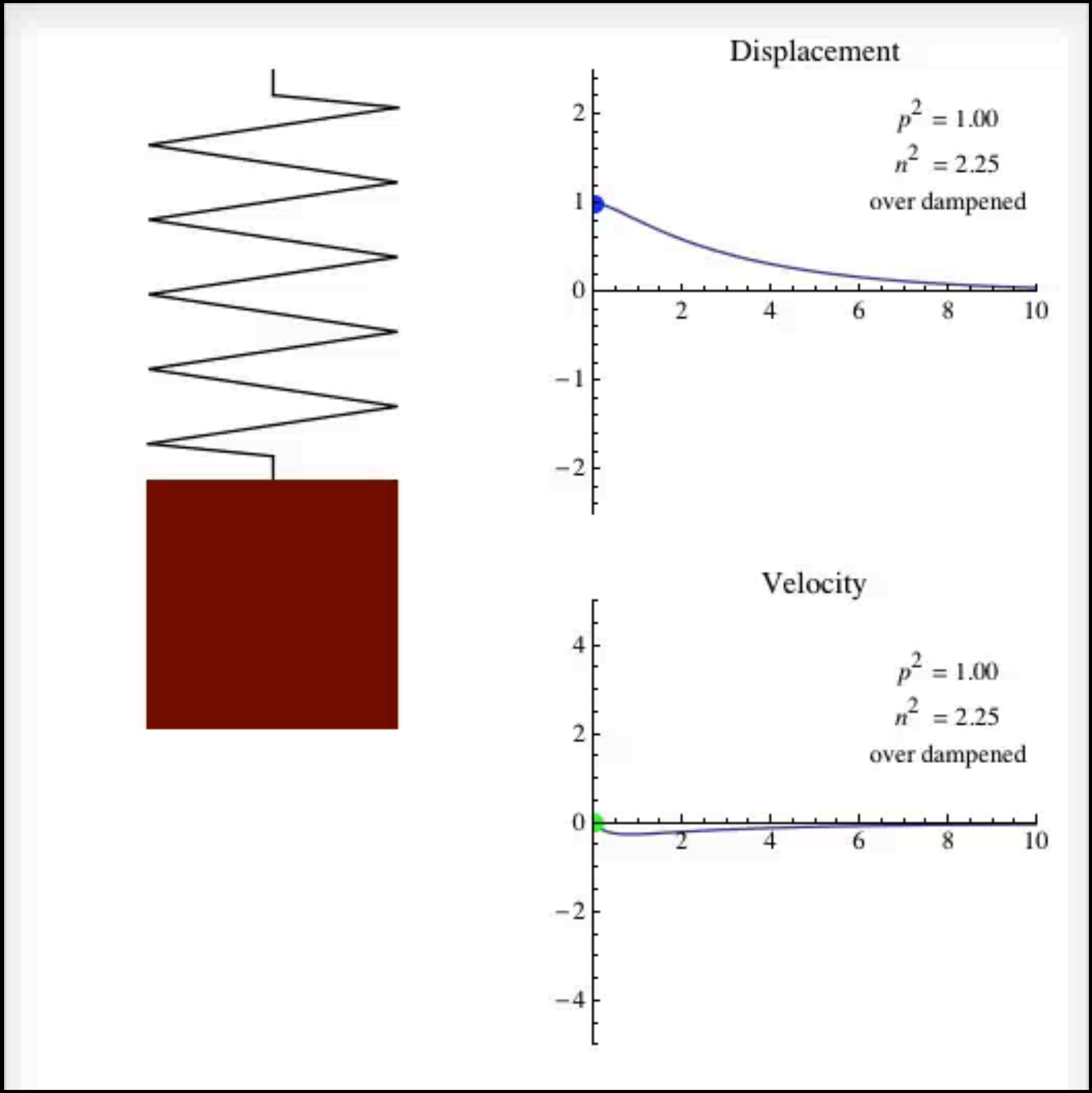


Simple Harmonic Motion

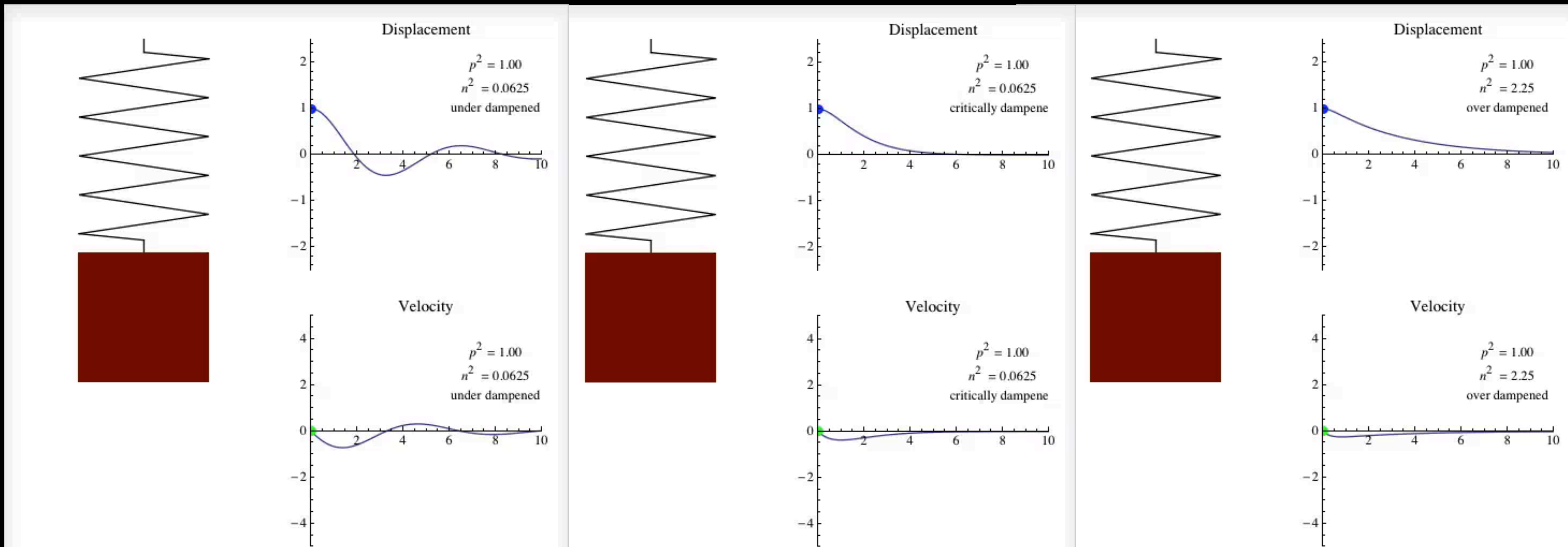
If the damping is stronger

the system is overdamped.

Damping dominates and return to equilibrium is slower.



Simple Harmonic Motion



underdamped

$$-bv << -kx$$

critically damped

$$-bv \simeq -kx$$

overdamped

$$-bv >> -kx$$

There are many real damped oscillators.

e.g. car shock absorbers damp oscillations to produce a quick return to equilibrium after a bump.

Simple Harmonic Motion

Oscillations can also be driven

e.g. pushing a child on a swing

Driving force: $F_d = F_0 \cos \omega_d t$

driving frequency

Then:

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_0 \cos \omega_d t$$

damping driving

Solution: $x = A\cos(\omega_d t + \phi)$

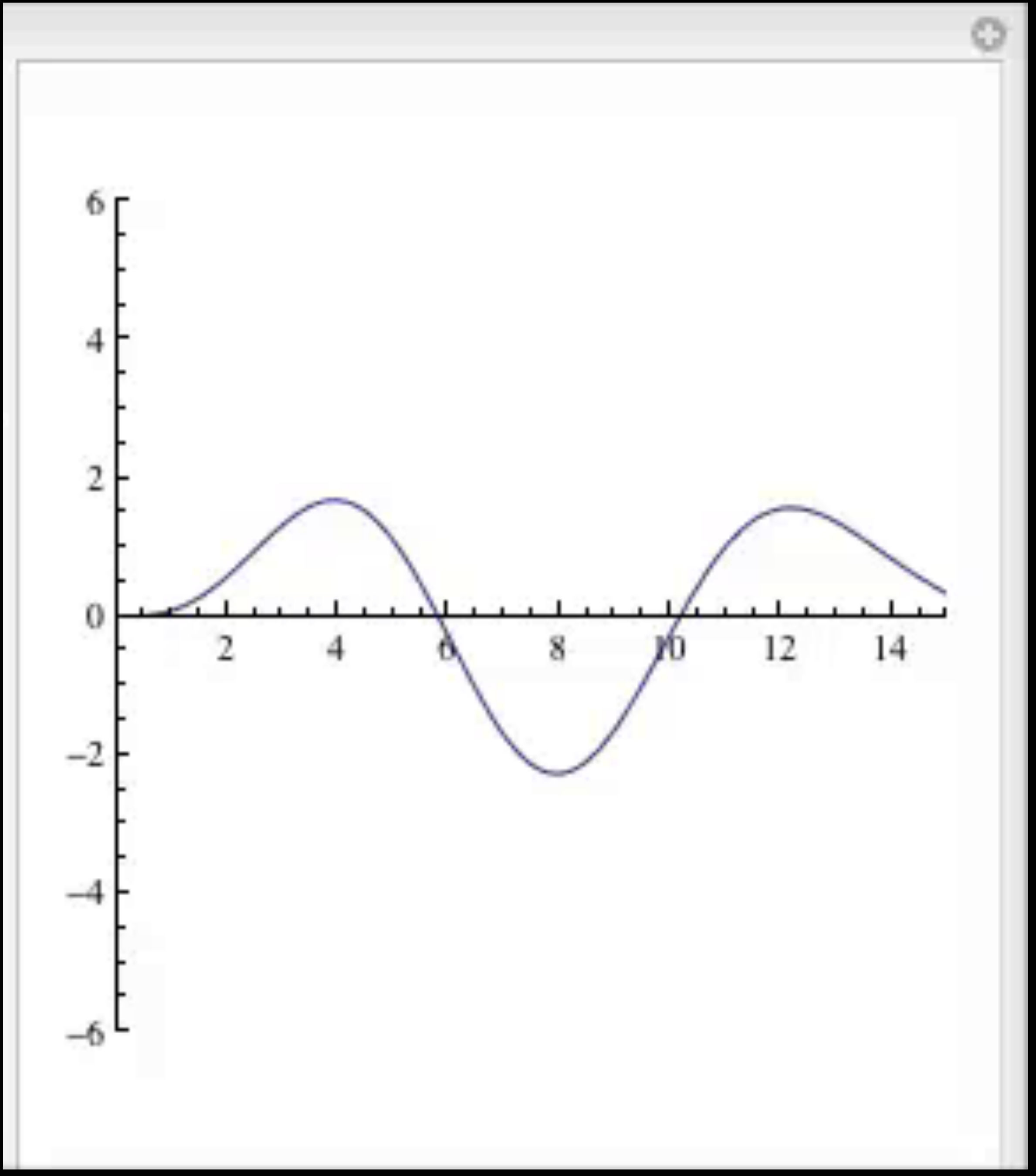
but: $$A(\omega) = \frac{F_0}{m\sqrt{(\omega_d^2 - \omega_0^2)^2 + b^2\omega_d^2/m^2}}$$

Simple Harmonic Motion

$$A(\omega) = \frac{F_0}{m\sqrt{(\omega_d^2 - \omega_0^2)^2 + b^2\omega_d^2/m^2}}$$

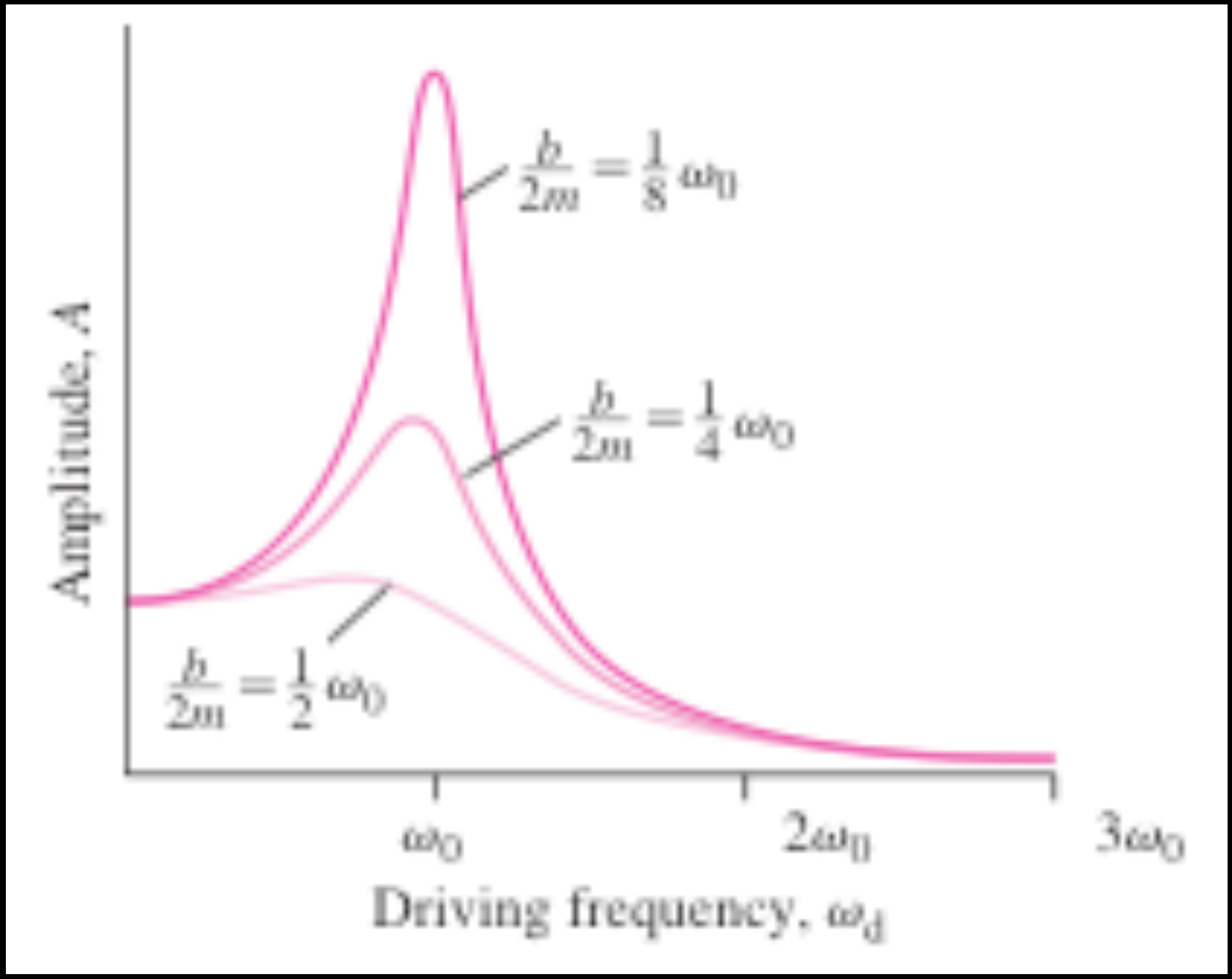
How does A change with driving force?

As driving force increases, A increases.... then decreases....



Simple Harmonic Motion

If the system is underdamped, A has a maximum at a some driving frequency.



At this frequency, the amplitude gets very large.

This is known as resonance.

Driving forces may be accidental, e.g. wind or earthquakes.

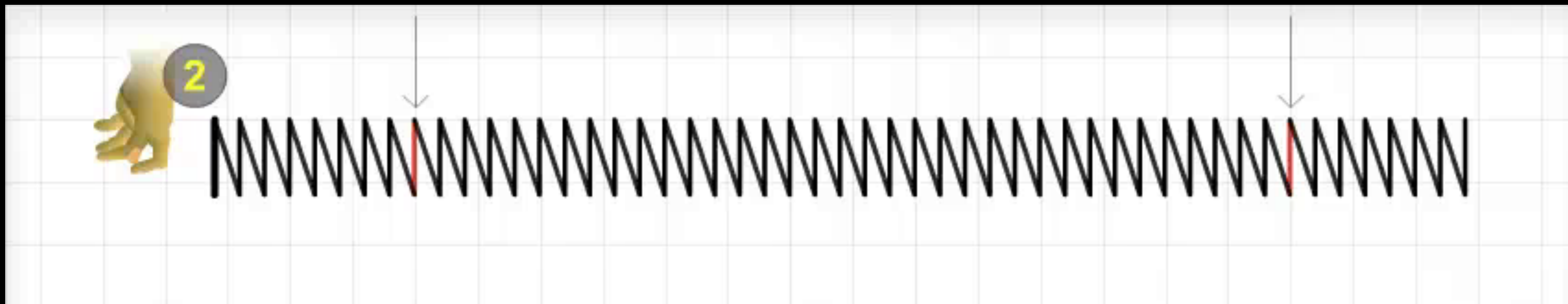
It is very important for engineers to know when resonance will occur.....

Simple Harmonic Motion

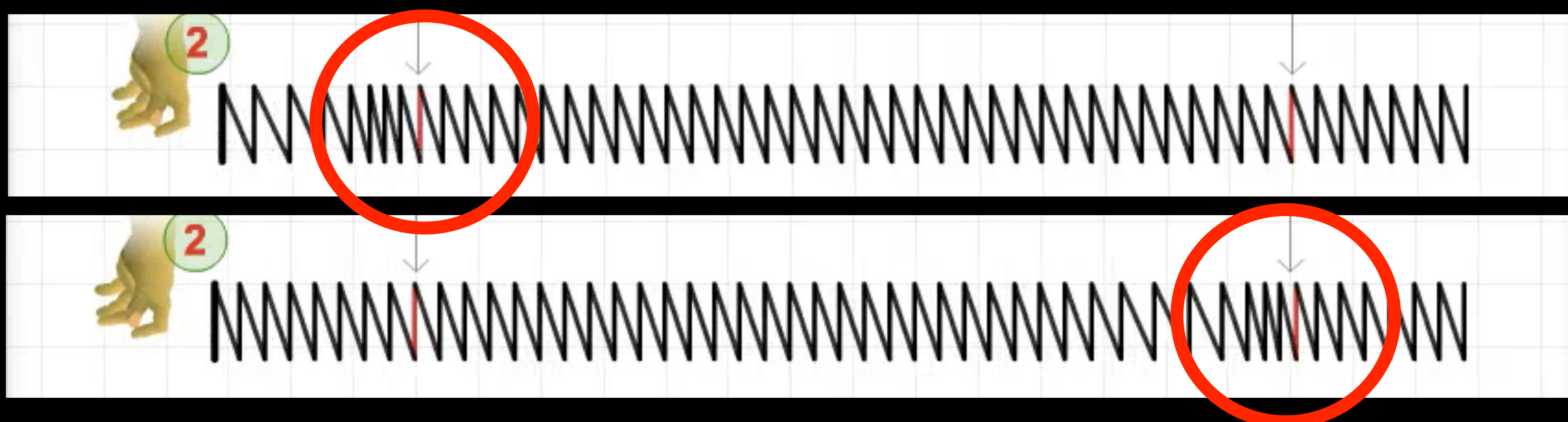
1940 collapse of the Tacoma Narrows Bridge

Waves

What is a wave?



This is a wave.



A small disturbance moves along the spring.

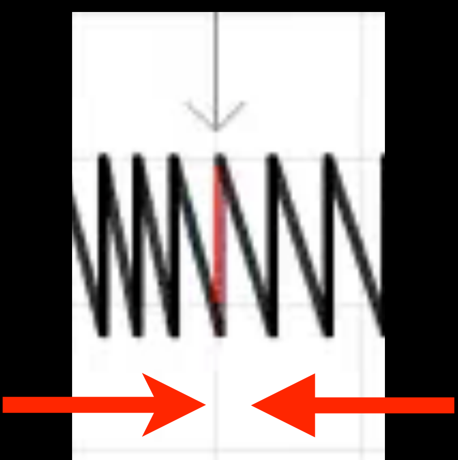
Each part of the spring makes a small oscillation,

then returns to equilibrium

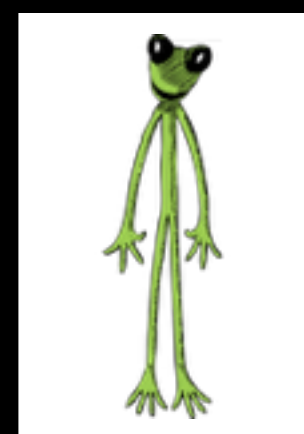
What is a wave?

Equilibrium

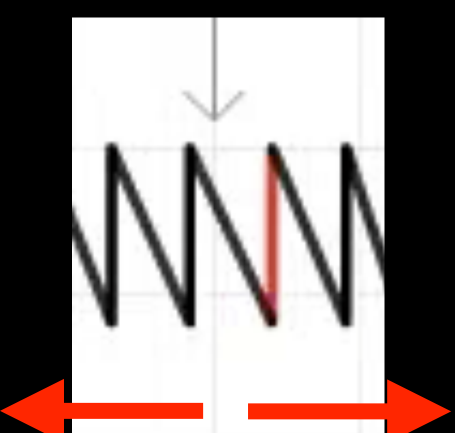
The spring makes a small oscillation as the wave passes.



Compression

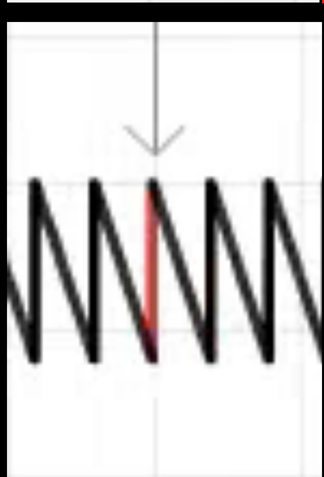


It then returns to its normal state.



Expansion

The wave carries the energy of the oscillation onto the next part of the spring.



Equilibrium

A wave moves energy, but not matter.

What is a wave?

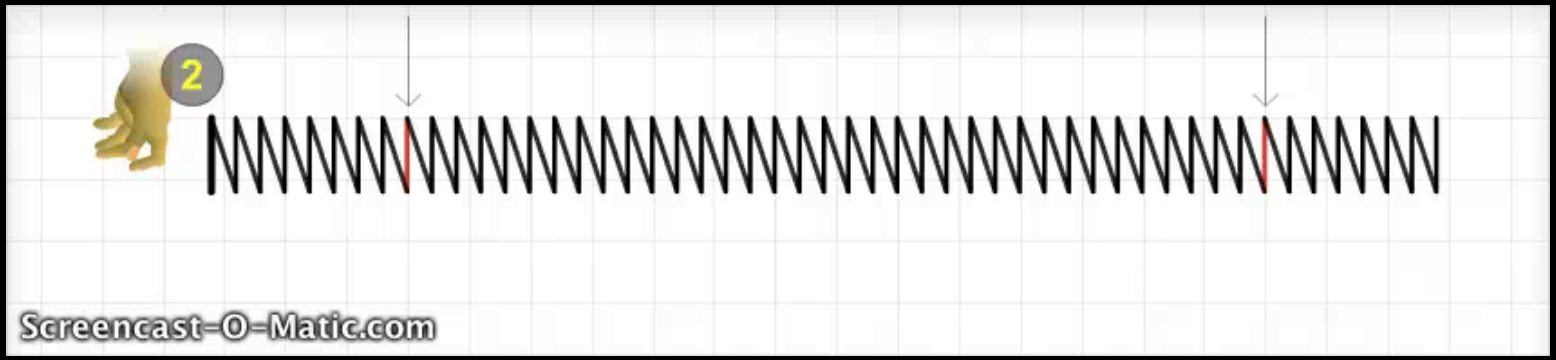
True or false?

In order for Ryoma to hear Taka, air molecules must move from Ryoma's lips to Taka's ear?

(A) True

(B) False

A wave moves energy, but not matter.



Types of wave

Two types of waves:

this lecture

mechanical

disturbance in a medium,

e.g. water, air, a spring, earth, violin string....

next semester

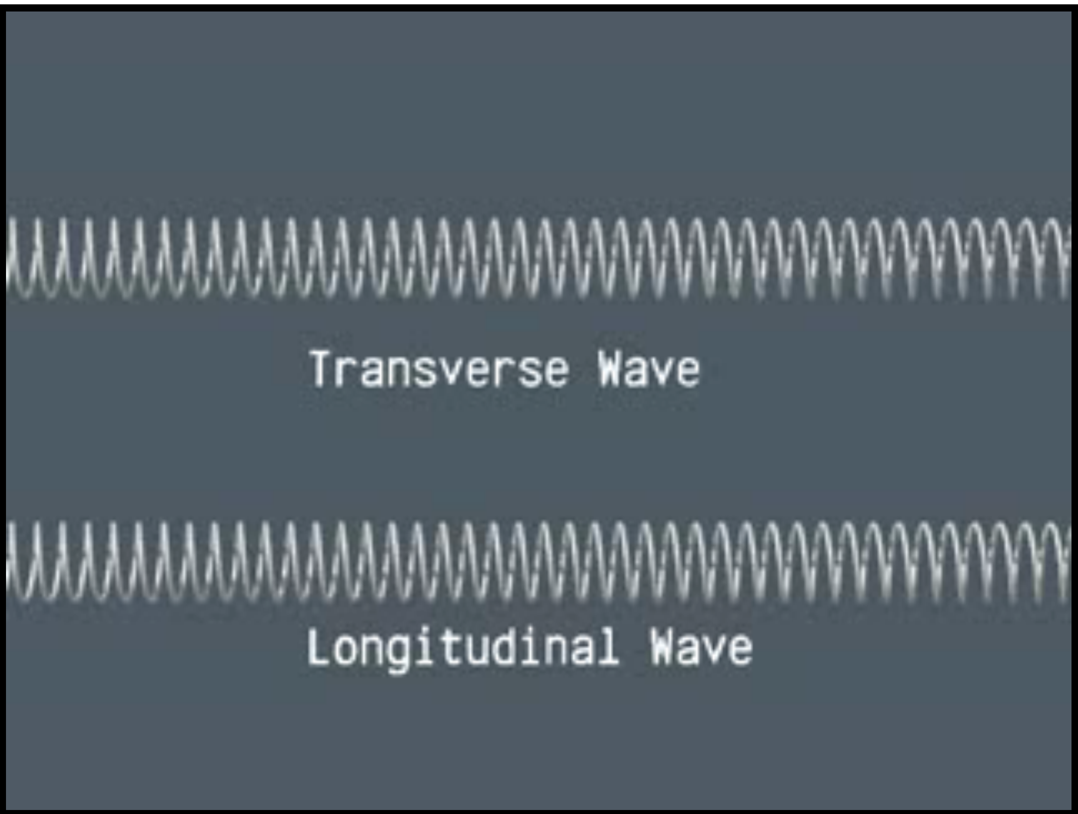
electromagnetic

no medium

Types of wave

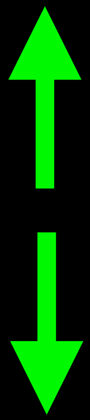
mechanical

Two types of waves:



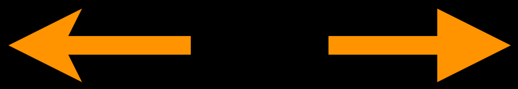
Transverse — oscillation perpendicular to wave motion

e.g. water



Longitudinal — oscillation parallel to wave motion

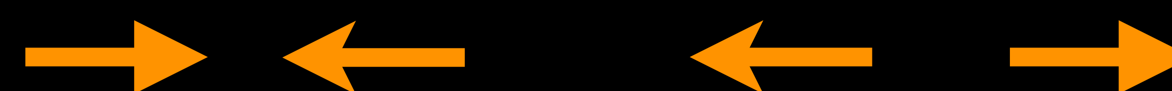
e.g. sound, water



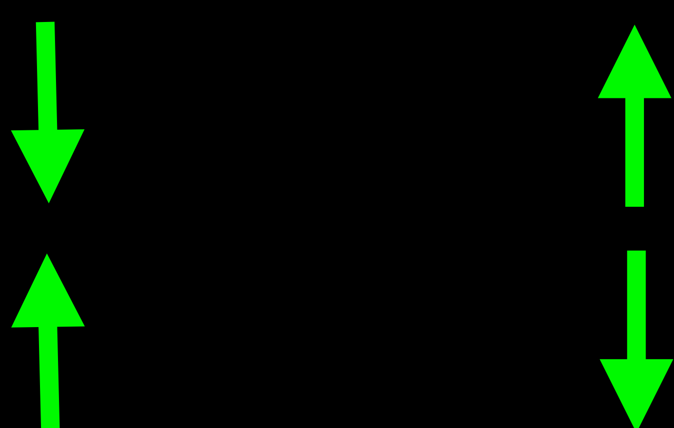
Types of wave

transverse wave

longitudinal wave

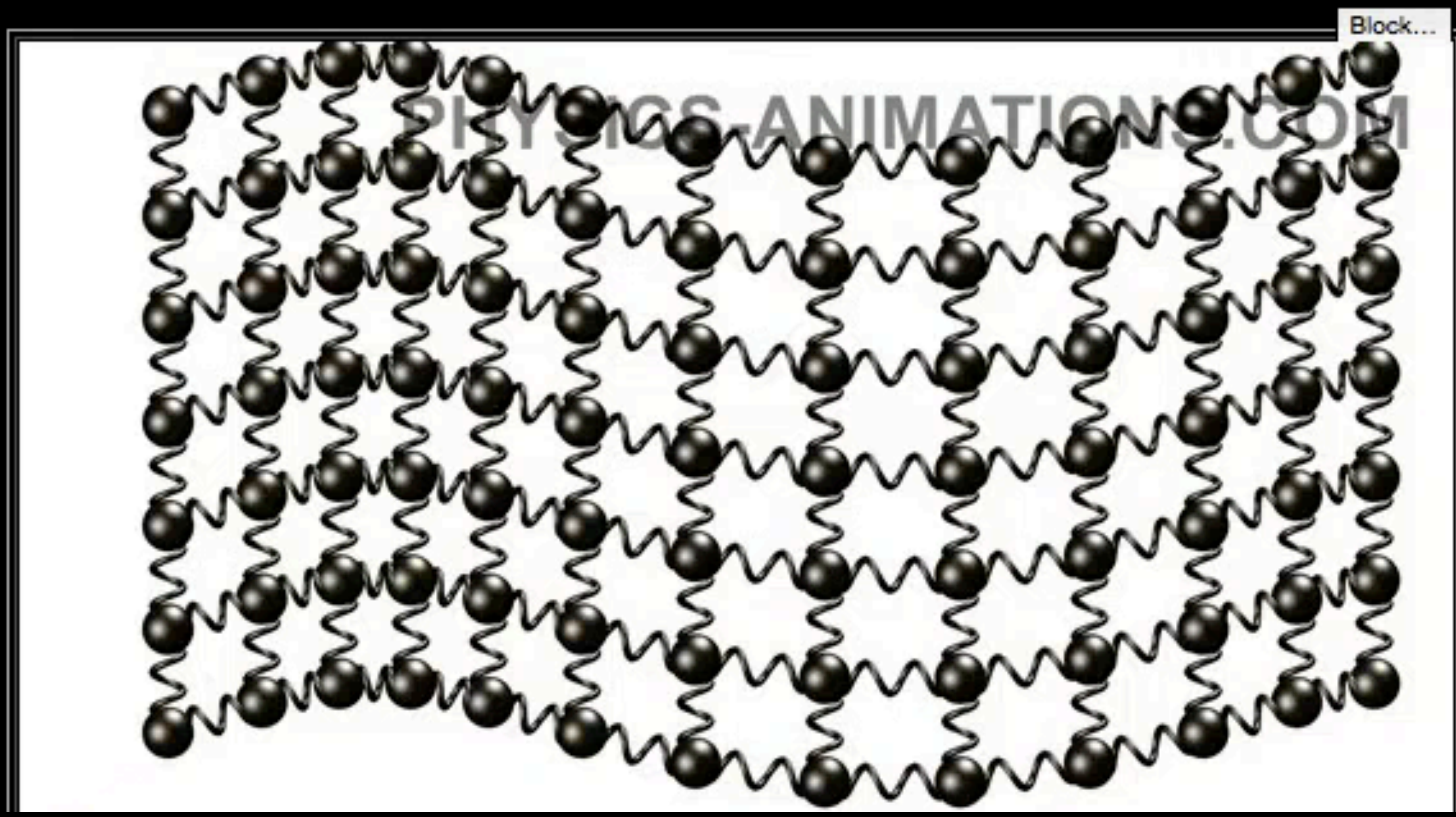


compression expansion

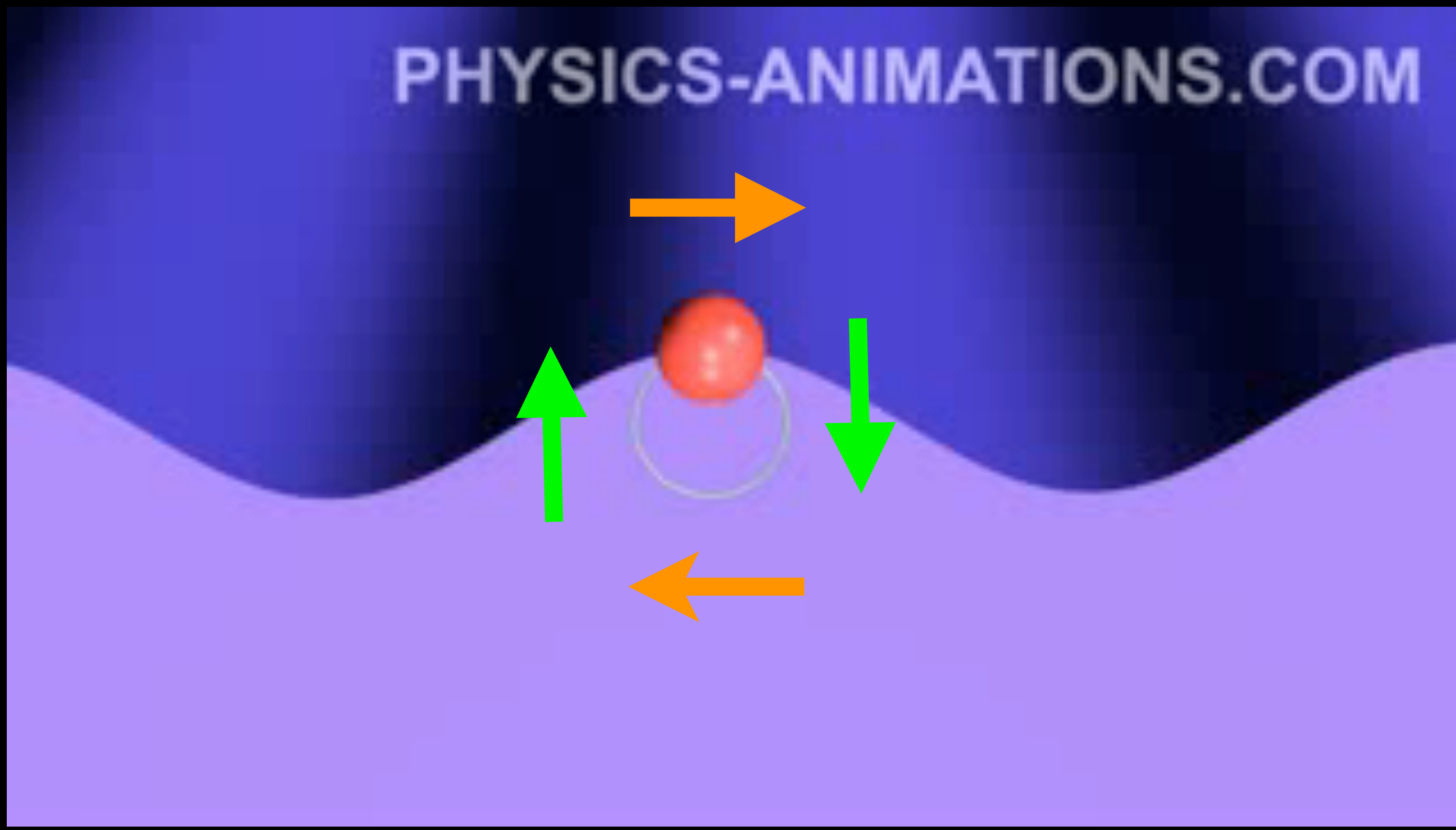


compression expansion

Types of wave



A wave with longitudinal and transverse components



e.g. a water wave

Types of wave

A single disturbance is a pulse

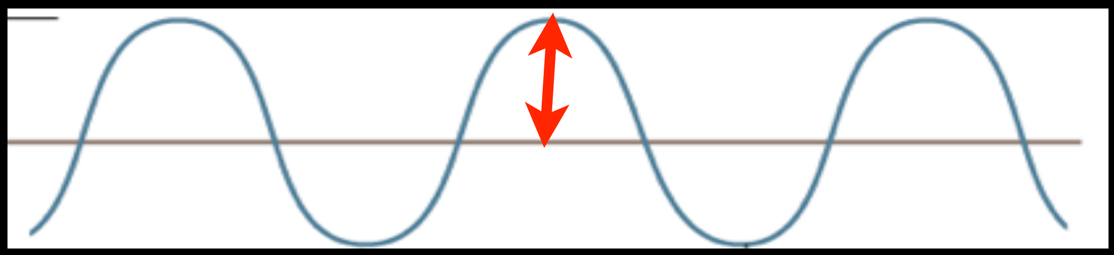
Ongoing disturbances are a continuous wave

In-between is a wave train

Wave properties

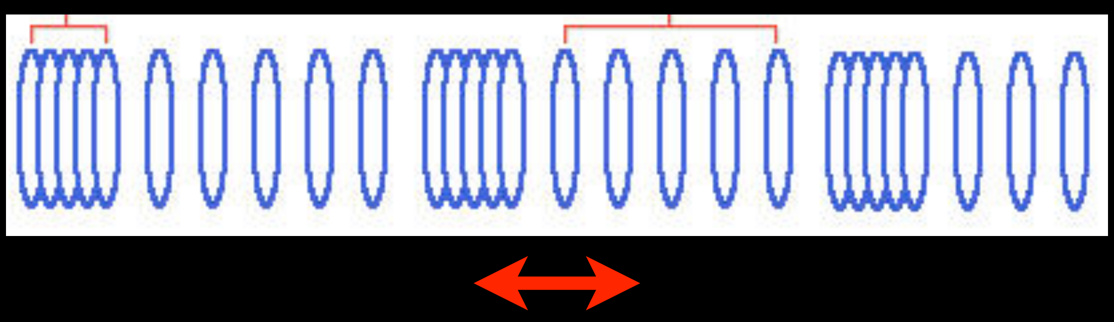
Amplitude: maximum value of the disturbance

e.g. water wave



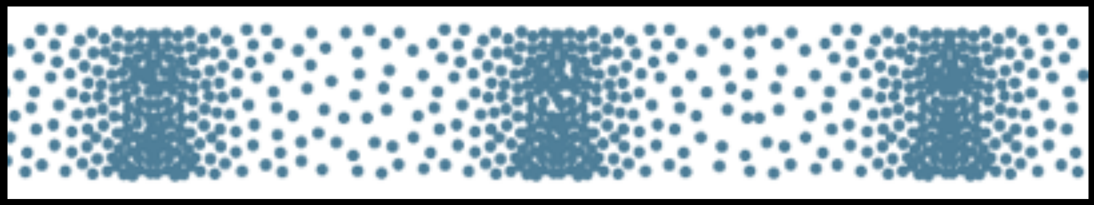
maximum height above water level

e.g. spring



maximum displacement

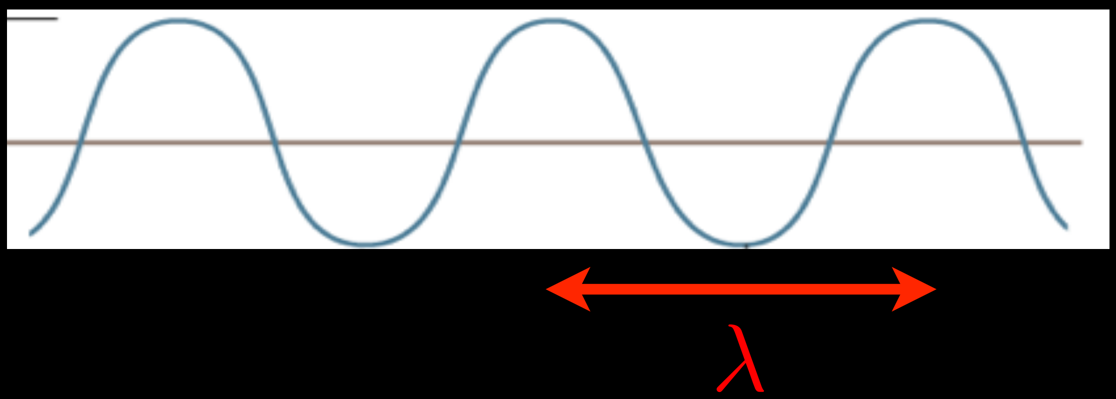
e.g. sound



maximum change in air pressure

Wave properties

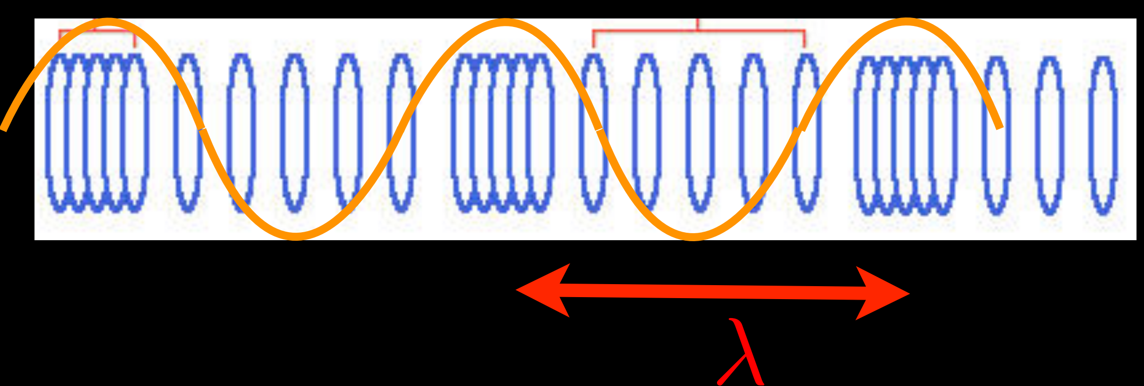
e.g. water wave



wavelength: λ

distance over which the wave pattern repeats

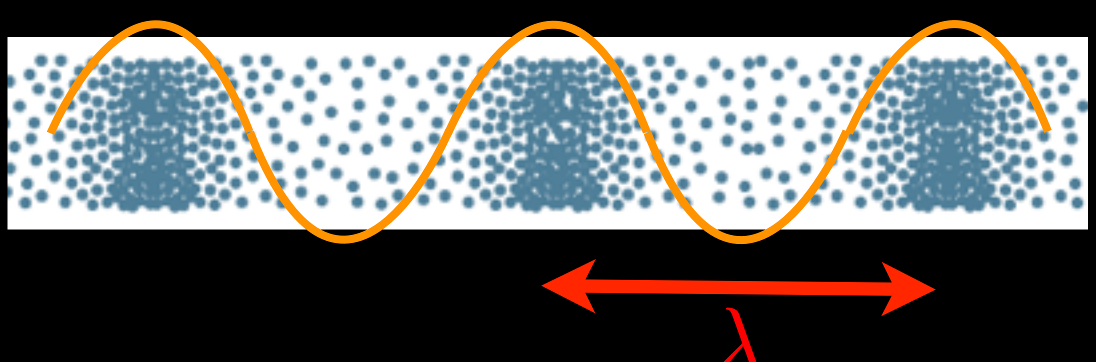
e.g. spring



period: T

time for one oscillation

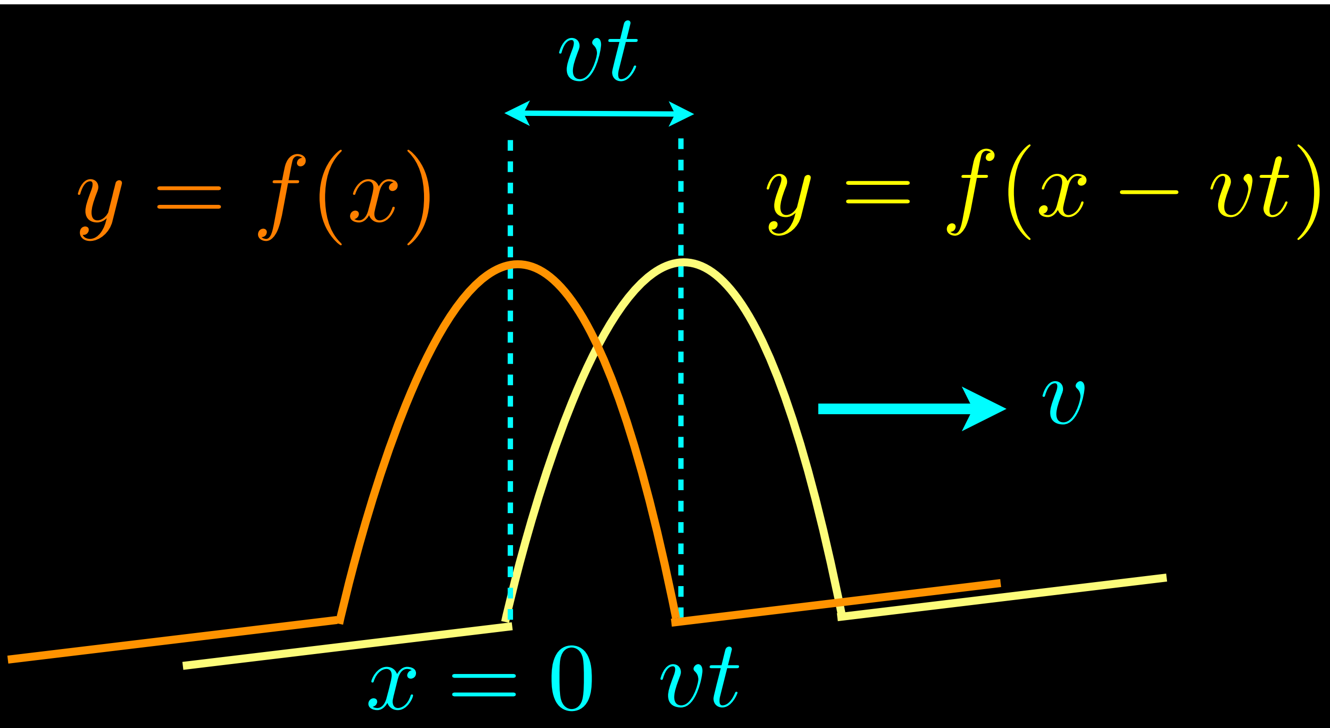
e.g. sound



frequency: f

number of oscillations / time: $f = \frac{1}{T}$

Wave properties



wave speed: v

speed of disturbance along the medium:

$$v = \frac{\lambda}{T} = \lambda f$$

If disturbance is simple harmonic oscillation:

$$y(x=0,t) = A\cos(\omega t) \longrightarrow y(x,t) = A\cos(kx \pm \omega t)$$

when $t = T$: $\omega T = 2\pi \longrightarrow \omega = \frac{2\pi}{T}$

when $x = \lambda$: $k\lambda = 2\pi \longrightarrow k = \frac{2\pi}{\lambda}$ wave number

$$v = \frac{\lambda}{T} = \frac{2\pi/k}{2\pi/\omega} = \frac{\omega}{k}$$

Wave properties

Quiz

A seismograph (measures earthquake) located at 1200 km from an earthquake detects seismic waves 5.0 minutes after the quake.

The seismograph oscillates in step (same time) with the waves, at 3.1 Hz.

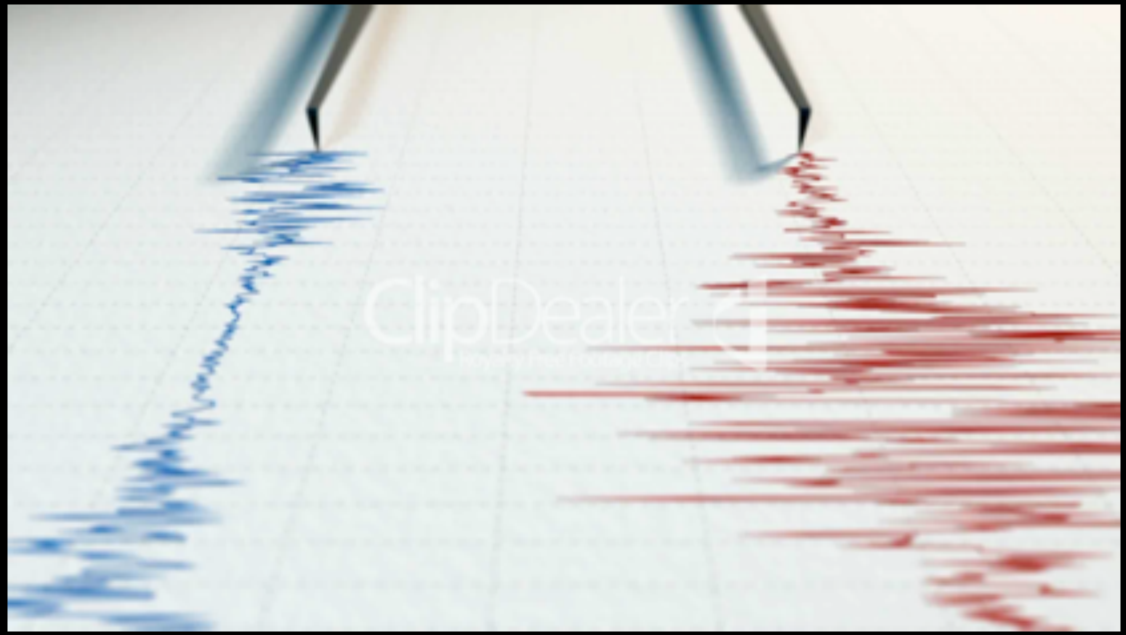
What is the wavelength?

(a) 1.3 km

(b) 77.4 km

(c) 0.08 km

(d) 0.001 km



Wave properties

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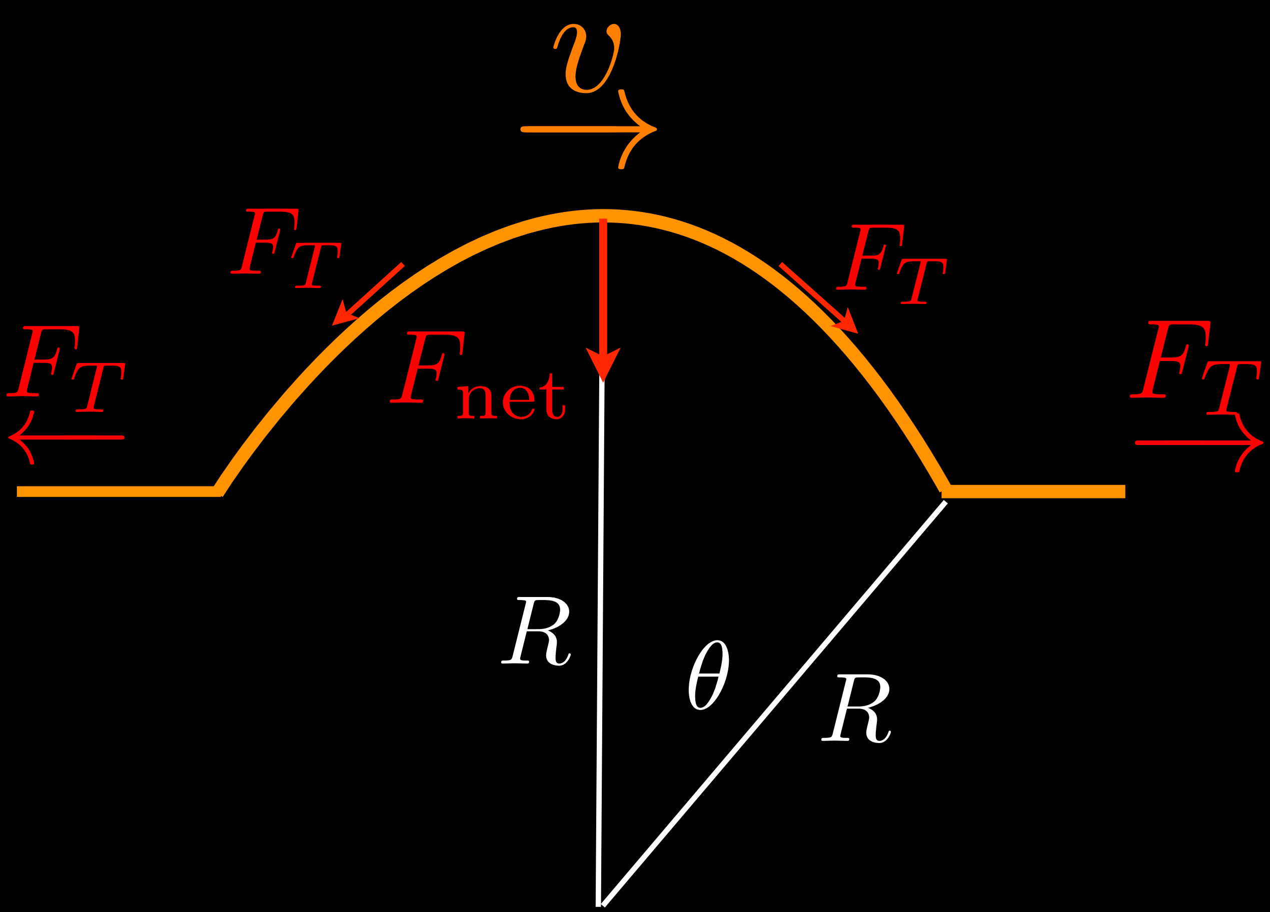
$$\lambda = \frac{v}{f}$$

$$= \frac{d}{tf} = \frac{1.2 \times 10^6}{(3 \times 10^2\,\mathrm{s})(3.1\,\mathrm{Hz})}$$

$$= 1.3 \times 10^3\,\mathrm{m} = 1.3\,\mathrm{km}$$

Wave on a string



<u>Velocity</u>

mass / unit length: μ

Assume perturbation is small:

(1) tension constant

(2) $\sin\theta \simeq \theta$

$$F_{\text{net}} = 2F_T \sin\theta = \frac{mv^2}{R} = \frac{2\theta R\mu v^2}{R} = 2\theta\mu v^2 \longrightarrow v = \sqrt{\frac{F}{\mu}}$$

circular motion

$$m = \mu\Delta s = 2\theta R\mu$$

Wave on a string

Quiz

Which curve best represents the variation of wave velocity with tension in a vibrating string?

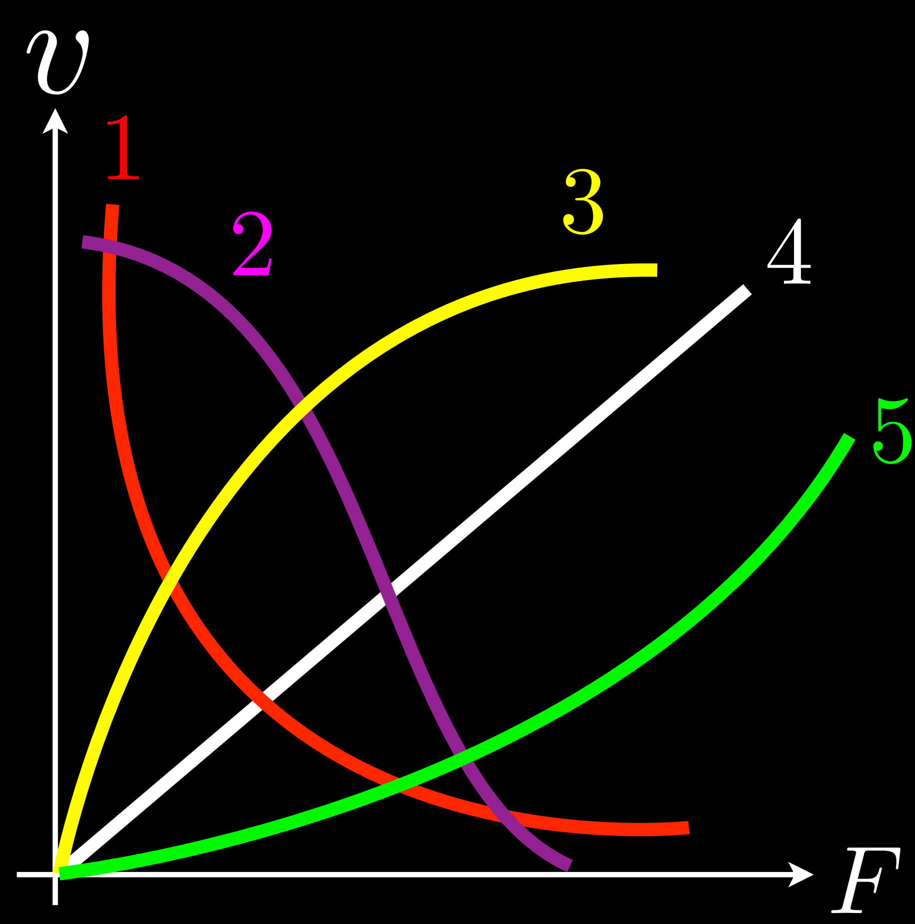
(a) 1

(b) 2

(c) 3

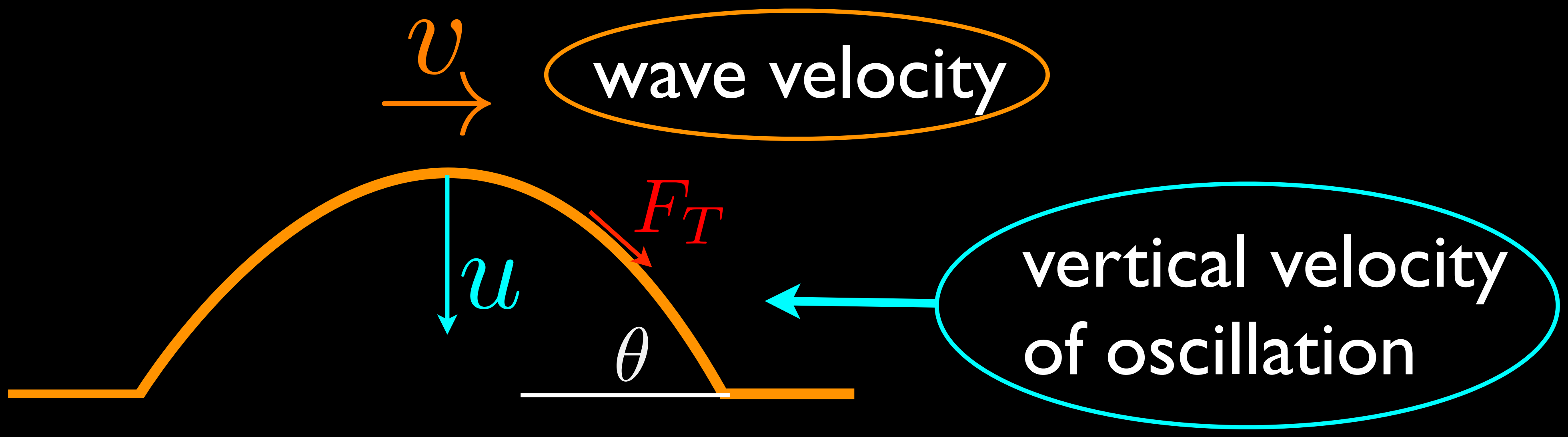
(d) 4

(e) 5



$$v = \sqrt{\frac{F}{\mu}}$$

Wave on a string



Power force x velocity = ?

$$u = \frac{dy}{dt} = A\omega \sin(kx - \omega t)$$

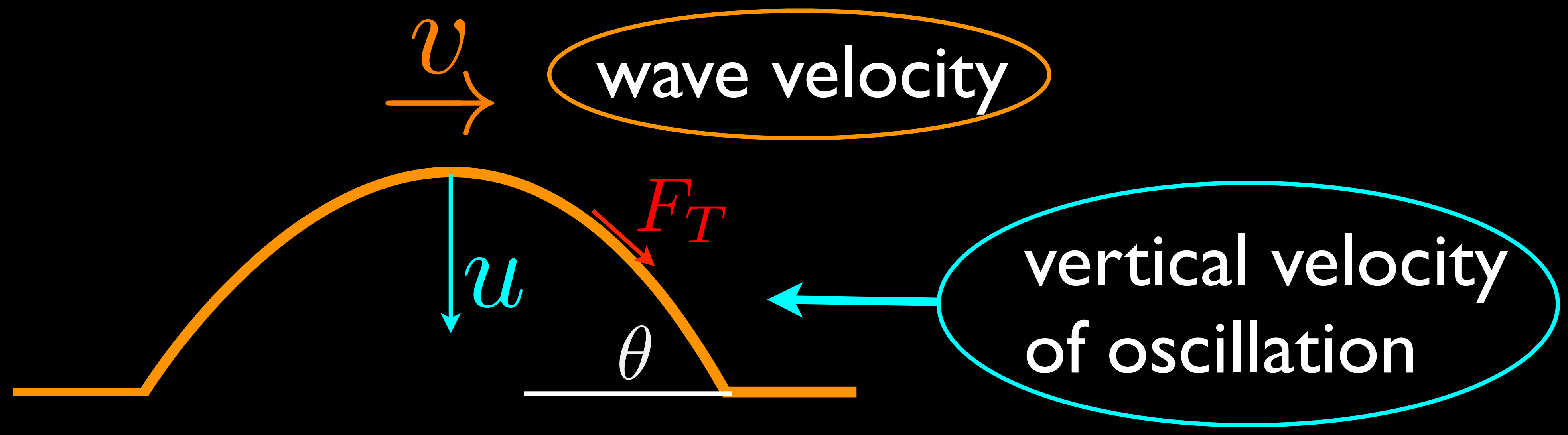
$$\tan\theta \simeq \theta = \frac{dy}{dx} = -kA\sin(kx - \omega t)$$

$$P = -F\sin\theta u \simeq -F\theta u = F\omega k A^2 \sin^2(kx - \omega t)$$

average: $\frac{1}{2}$

$$\bar{P} = \frac{1}{2} F\omega k A^2$$

Wave on a string



Power

force x velocity

$$\bar{P} = \frac{1}{2} F \omega k A^2$$

Since: $v = \sqrt{\frac{F}{\mu}} \longrightarrow F = v^2 \mu$

and: $v = \frac{\omega}{k}$

So: $F = \frac{\omega}{k} v \mu \longrightarrow$

$$\bar{P} = \frac{1}{2} F \omega k A^2 = \frac{1}{2} \mu \omega^2 A^2 v$$

Wave on a string

The equation for particle displacement in a medium where there is a simple harmonic progressive wave is:

$$y(x,t) = (2/\pi)\sin\pi(x - 4t)$$

units are SI.

For a particle at x = 10 m when t = 2s, the particle speed is:

(a) 0

(b) 2 m/s

(c) $4/\pi$ m/s

(d) 4 m/s

(e) 8 m/s

Waves move energy, not matter:

Velocity of particle $\neq$ wave velocity, v

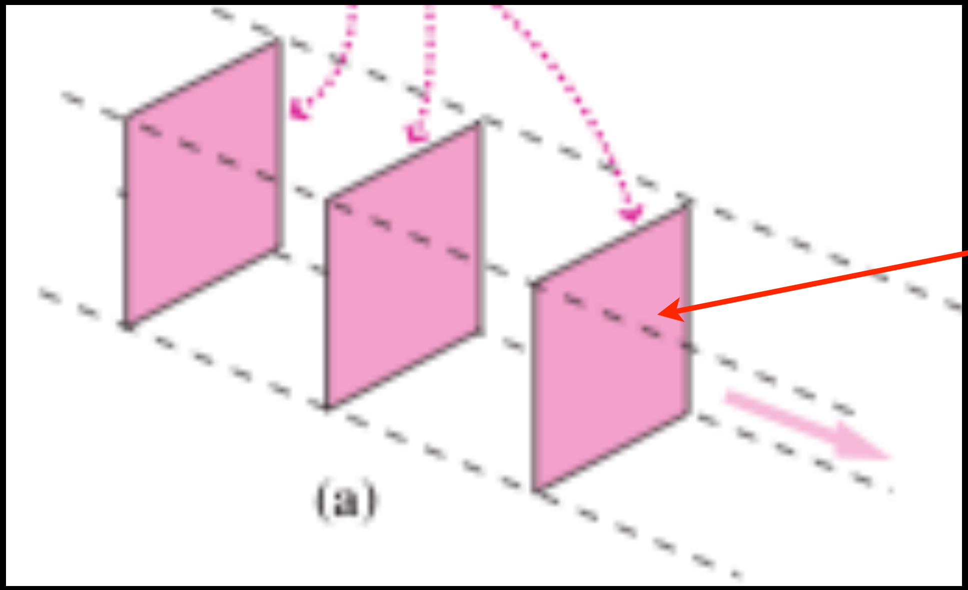
$=$ vertical velocity, u

$$u = \frac{dy}{dt} = \frac{2}{\pi}(\cos\pi(x - 4t))(-4\pi)$$

$$= \frac{2}{\pi}(-4\pi)\cos(2\pi) = -8\,\mathrm{m/s}$$

Wave on a string

<u>Intensity</u> $I = \frac{P}{A} \quad \mathrm{W/m^2}$

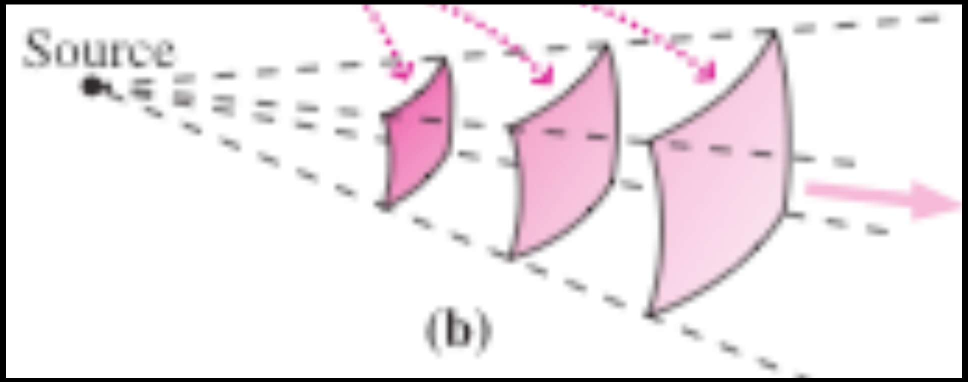


plane wave

A = Area perpendicular to wave velocity

Plane wave: area constant with distance

Spherical wave: $I = \frac{P}{A} = \frac{P}{4\pi r^2}$



spherical wave

Wave energy spread out over larger and larger area.

Intensity decreases further from the source.

Wave on a string

Quiz

Two waveforms of the same frequency are moving to the right with velocity, v. The power P_A transmitted by wave A is equal to:

(a) $2P_B/3$

(b) $9P_B/4$

(c) $\sqrt{2}P_B/\sqrt{3}$

(d) $4P_B/9$

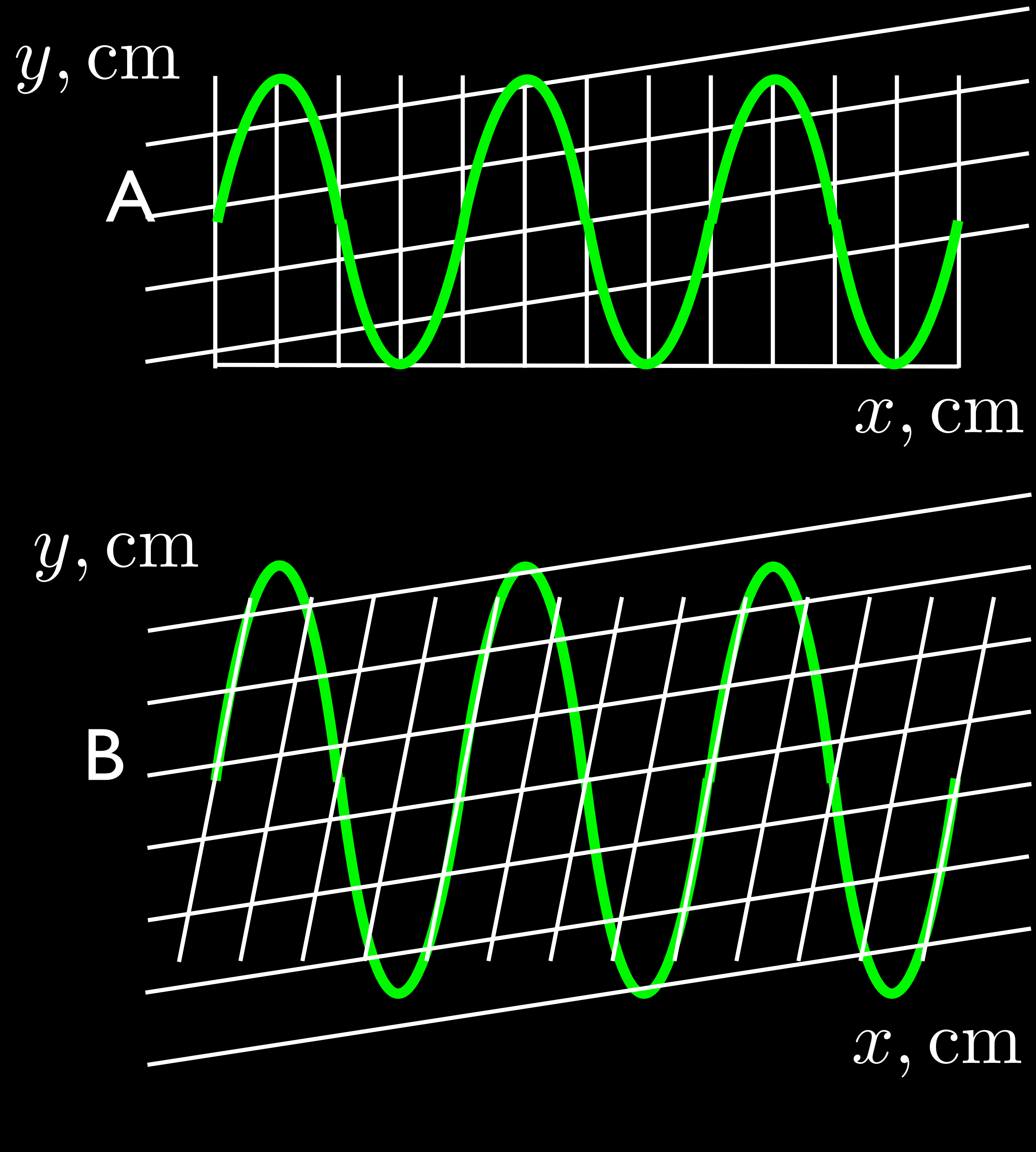
(e) P_B

$$\bar{P} = \frac{1}{2}F\omega k A^2$$

=

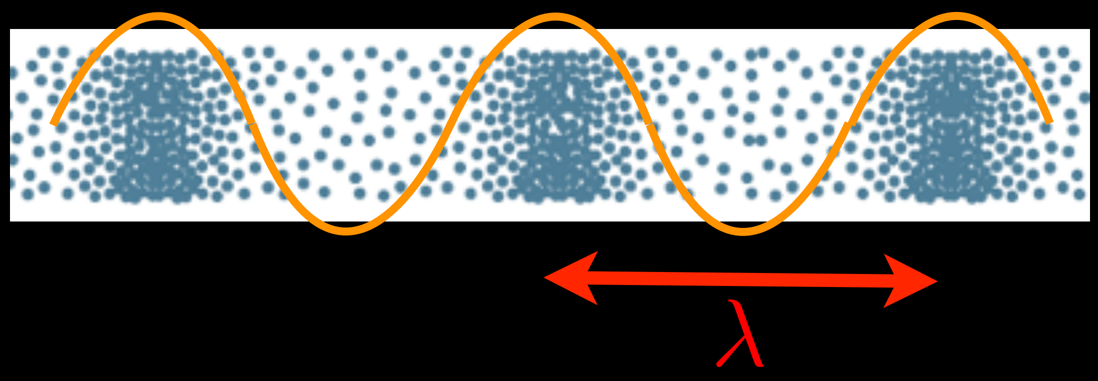
$$v = \sqrt{\frac{F}{\mu}}$$

$$\frac{P_A}{P_B} = \frac{A_A^2}{A_B^2} = \frac{4}{9}$$



Sound waves

sound wave



Longitudinal wave

Travels through solid, liquid & gases

In air, disturbance is a change in pressure and density.

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Air: $\gamma = \frac{7}{5}$

Helium: $\gamma = \frac{5}{3}$

Sound intensity measured in decibels

$$\beta = 10 \log \left(\frac{I}{I_0} \right) \quad \mathrm{dB}$$

$$I_0 = 10^{-12} \mathrm{W/m^2}$$

Sound waves

A loud speaker is adjusted so that it produces a sound 2 x the intensity of its original sound.

What is the change in the sound level, beta?

(a) 2 dB

(b) 30 dB

(c) 20 dB

(d) 3 dB

Sound waves

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(a) 2 dB

(b) 30 dB

(c) 20 dB

(d) 3 dB

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

$$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_0}\right) - 10\log\left(\frac{I_1}{I_0}\right)$$

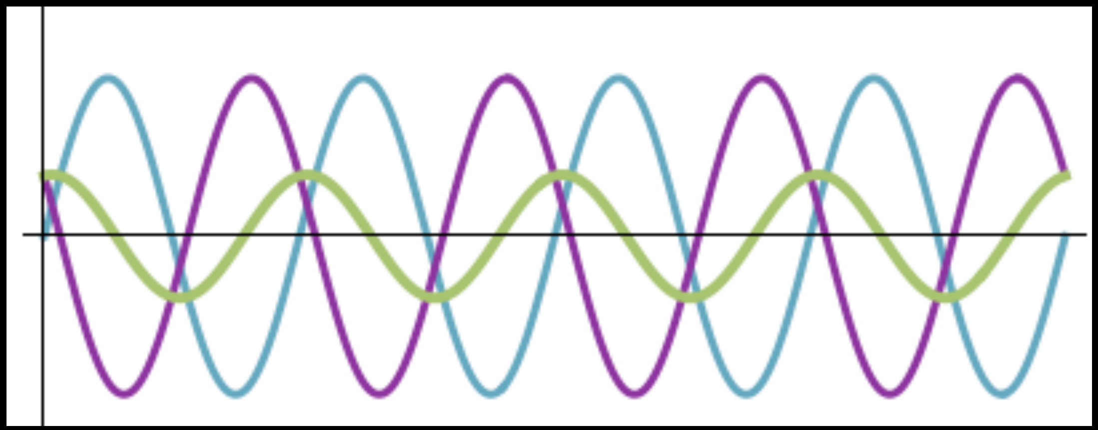
$$= 10\log I_2 - 10\log I_0 - 10\log I_1 + 10\log I_0$$

$$= 10\log I_2 - 10\log I_1$$

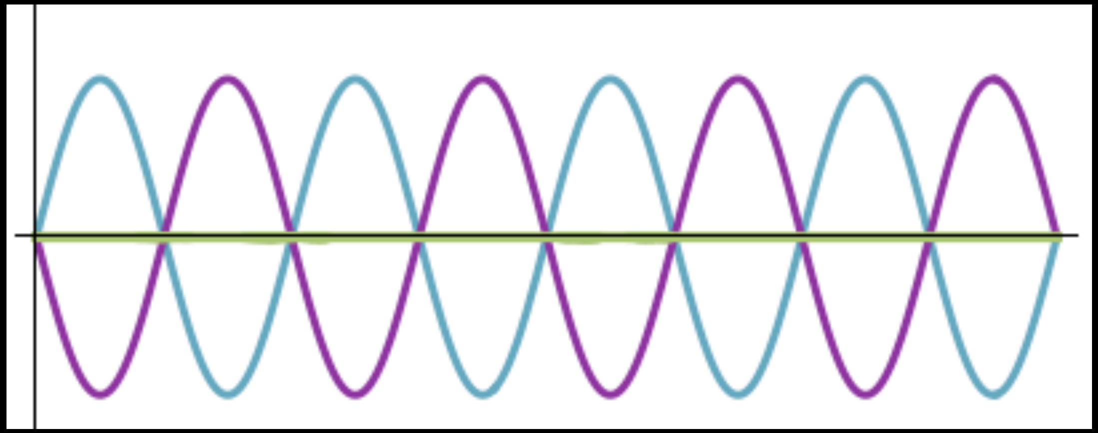
$$= 10\log\left(\frac{I_2}{I_1}\right) \quad = 10\log(2) = 3\,\mathrm{dB}$$

Interference

superposition principal: most waves can be added

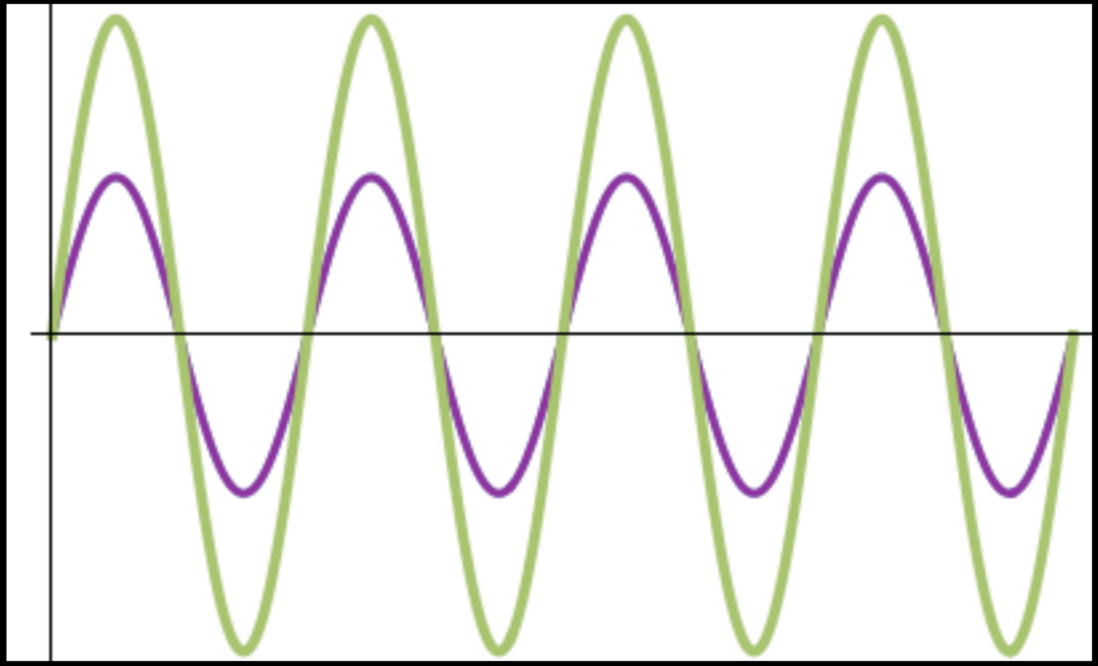


wave I + wave 2 = resulting wave



wave I + wave 2 cancel

Destructive interference

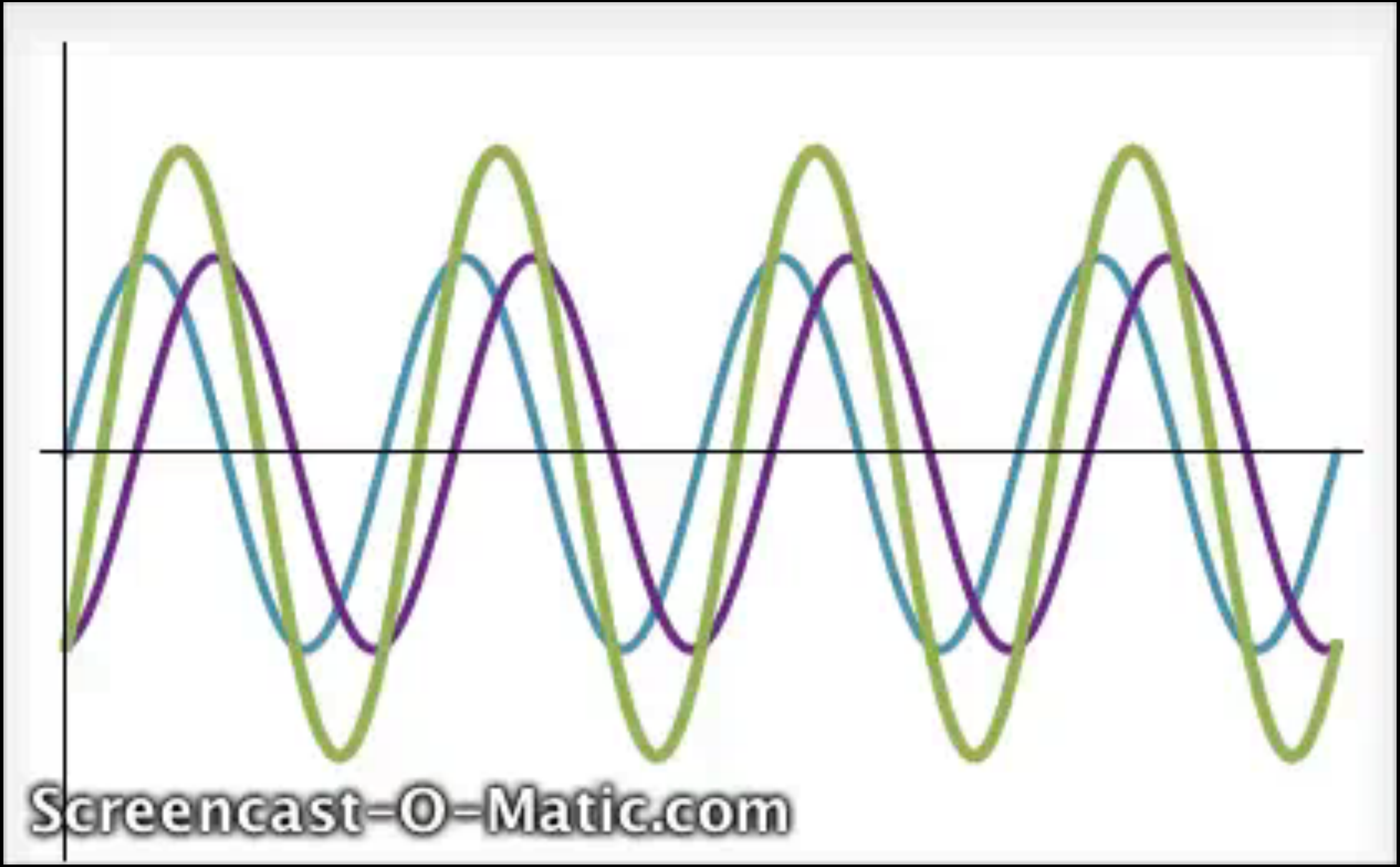


waves coincide

Constructive interference

Interference

superposition principal: most waves can be added



Interference

In graph A, 2 waves a shown at time t.

Which curve in B represents the wave from their superposition?

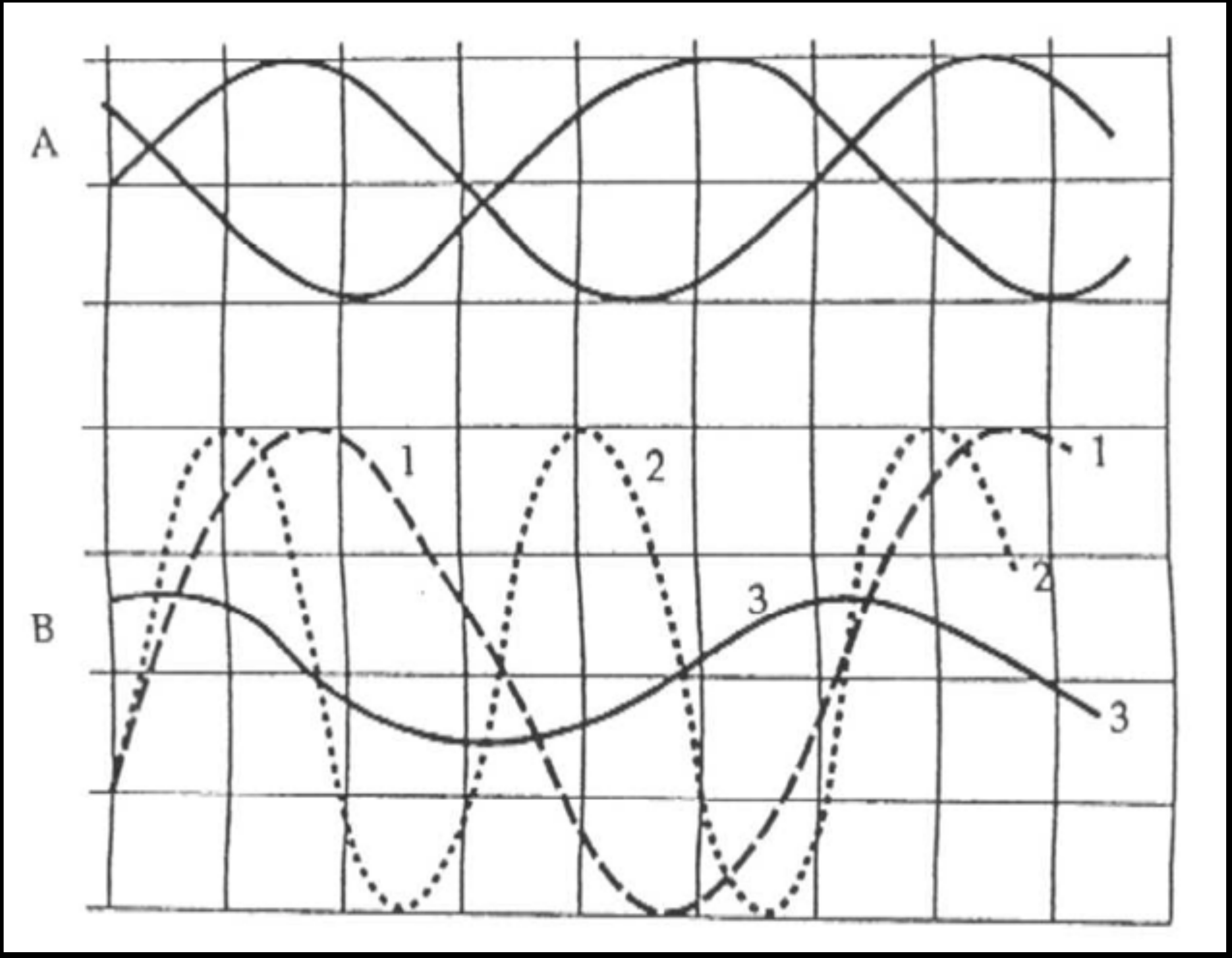
(a) 1

(b) 2

(c) 3

(d) The resultant is 0 for all x

(e) none

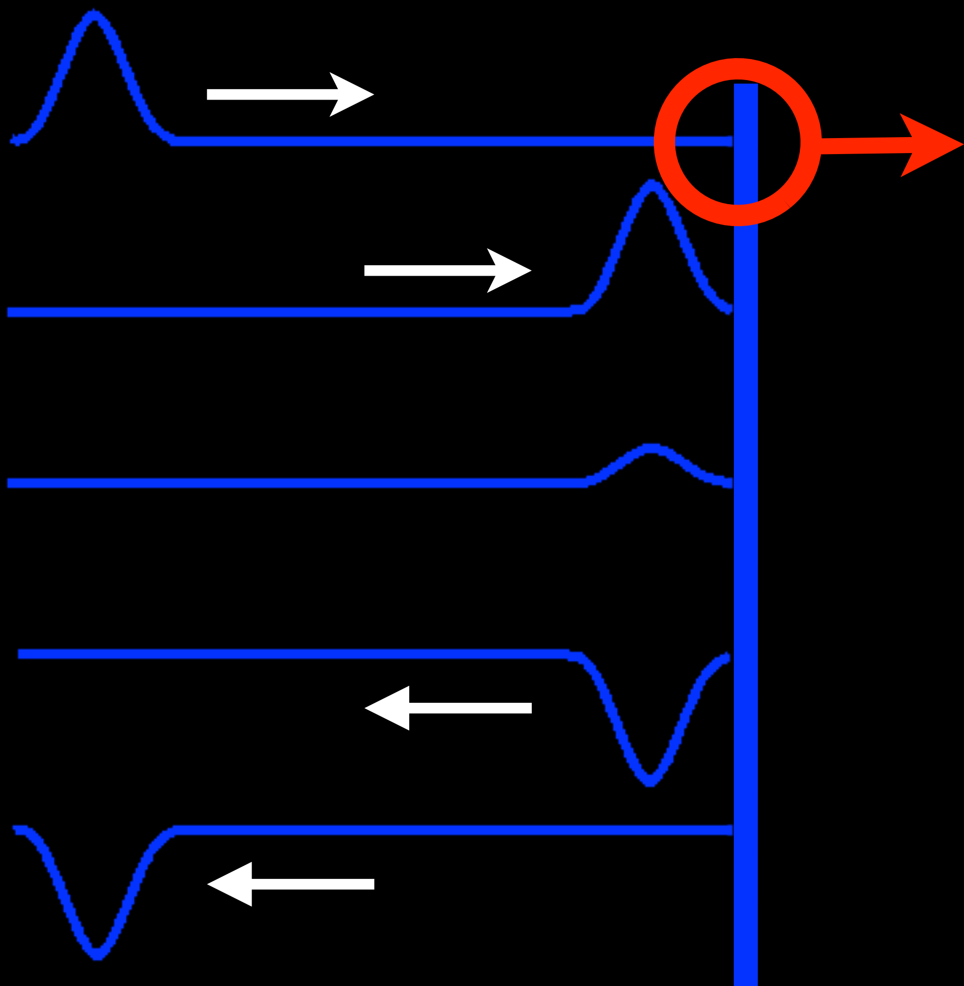


Reflection

When a wave hits something it cannot travel through, it must reflect

.... otherwise, where would the energy go?

Reflection

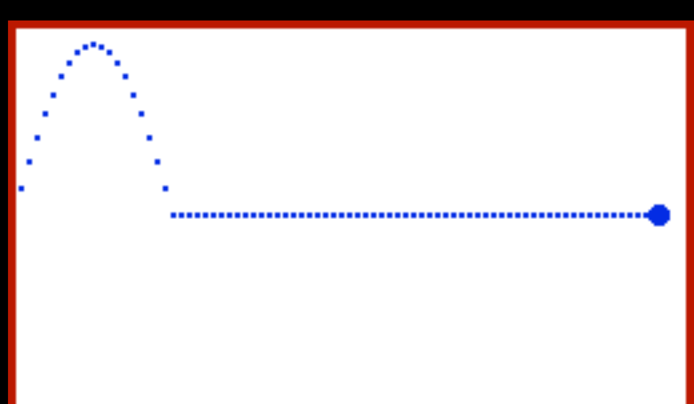


fixed end

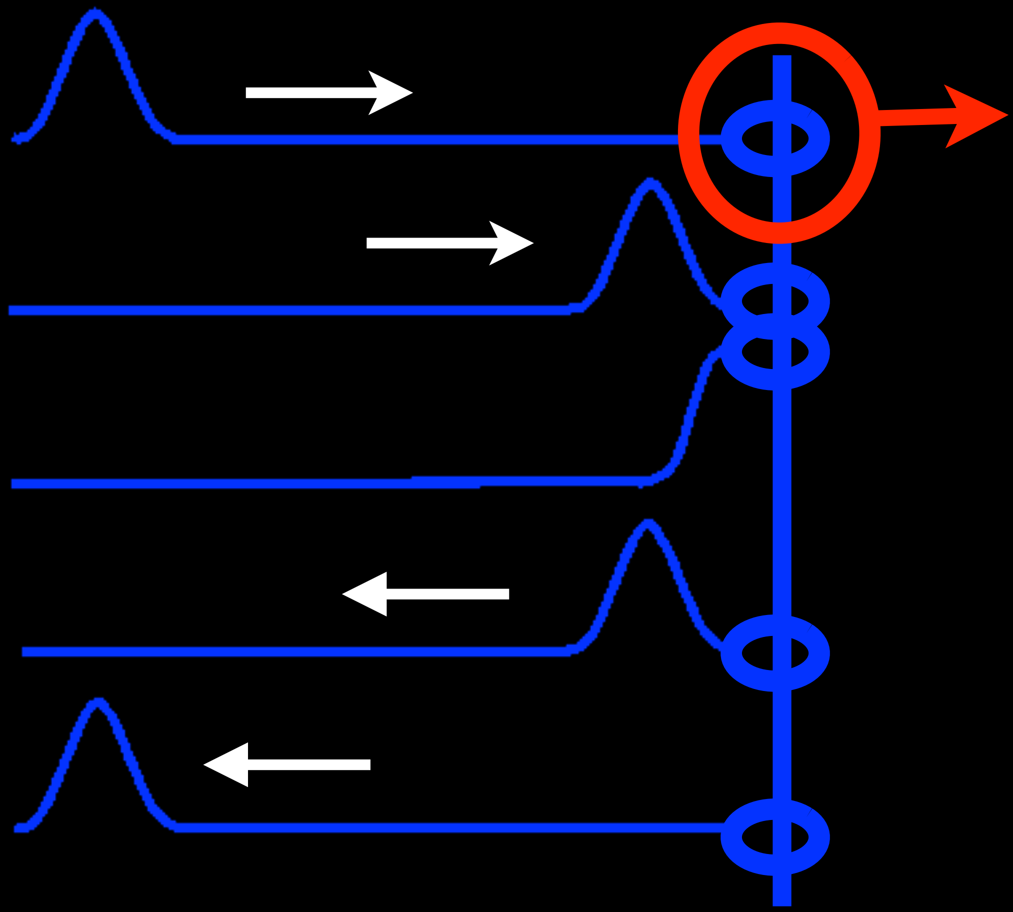
Wave height must be 0 at end.

Incident (in-coming) and reflected (out-going) wave must interfere destructively.

Reflected wave is inverted.



Reflection

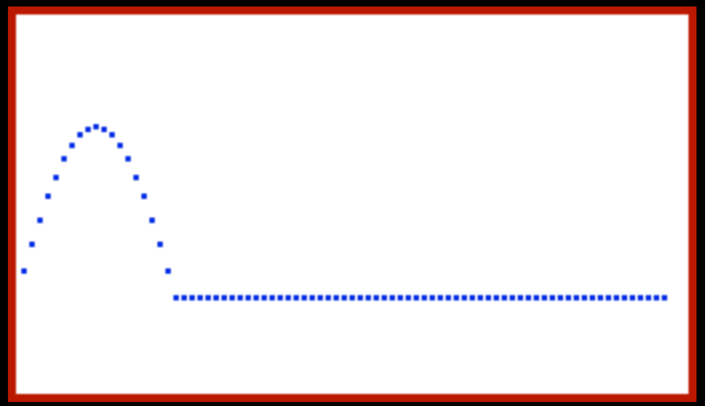


loose end: ring free to move on pole

Wave pulse pushes ring up.

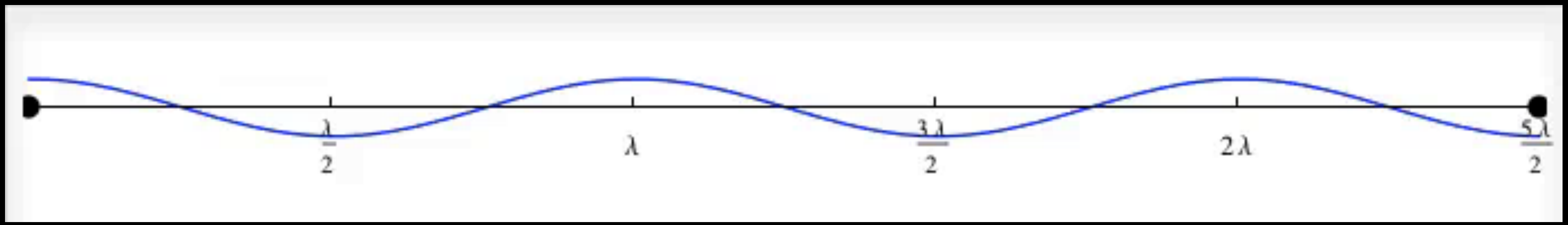
Wave height is at maximum at end.

Reflected wave not inverted.

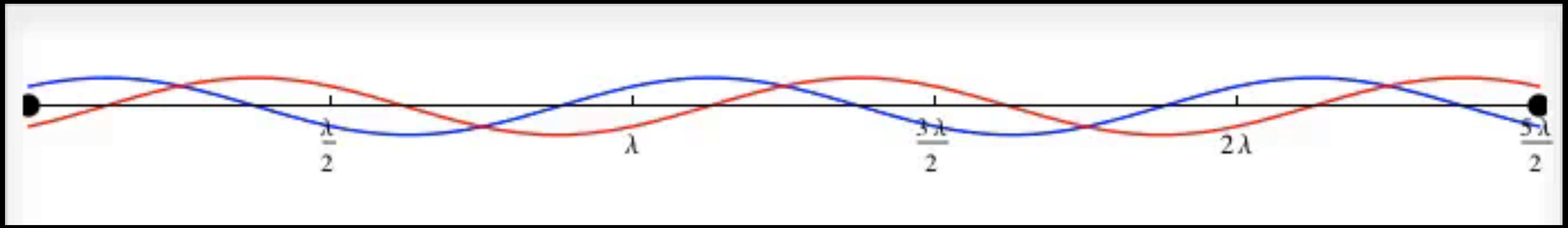


Standing waves

String with 2 fixed ends

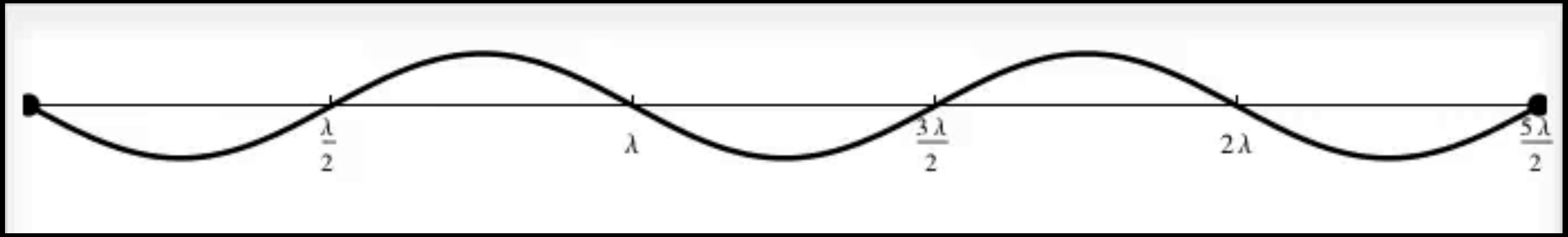


in-coming wave



in-coming wave
& reflected wave

If exact number of half-wavelengths between fixed ends:



superposition

Standing wave

Standing waves

Mathematically:

$$y(x,t) = y_1 + y_2 \quad = A[\cos(kx - \omega t) - \cos(kx + \omega t)]$$

in-coming

reflected

since: $$\cos\alpha - \cos\beta = -2\sin\left[\frac{1}{2}(\alpha+\beta)\right]\sin\left[\frac{1}{2}(\alpha-\beta)\right]$$

Therefore: $$y(x,t) = 2A\sin kx \sin\omega t$$

$$\alpha = kx - \omega t$$
$$\beta = kx + \omega t$$

amplitude depends on position

simple harmonic motion

Standing waves

Mathematically:

$$y(x,t) = y_1 + y_2 \quad = A[\cos(kx - \omega t) - \cos(kx + \omega t)]$$

in-coming

reflected

since: $$\cos\alpha - \cos\beta = -2\sin\left[\frac{1}{2}(\alpha+\beta)\right]\sin\left[\frac{1}{2}(\alpha-\beta)\right]$$

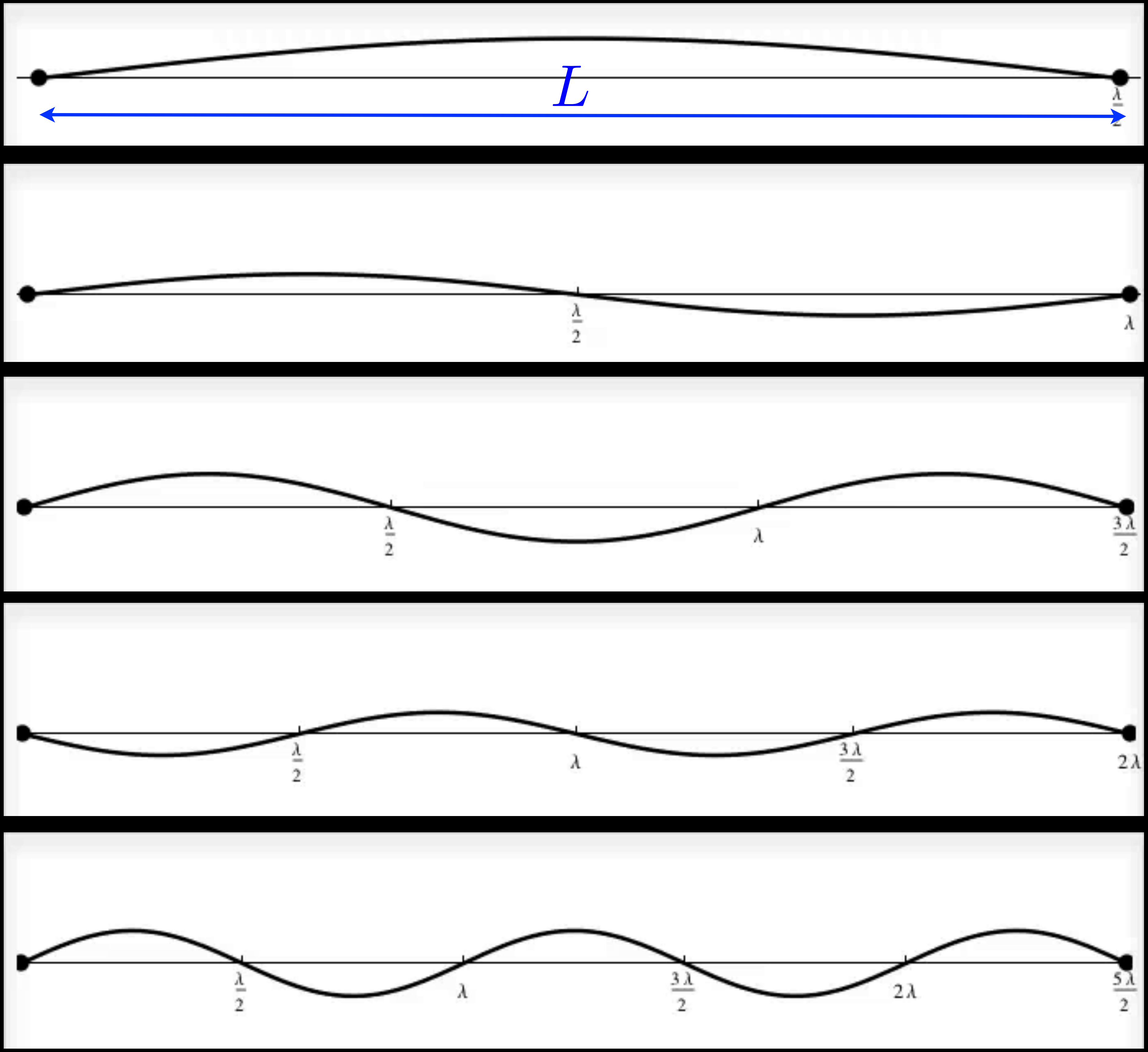
Therefore: $$y(x,t) = 2A\sin kx \sin\omega t$$

Amplitude 0 at x = 0, x = L:

$$\sin m\pi = 0$$

$$kL = m\pi \longrightarrow L = \frac{m\lambda}{2}$$ m = 1, 2, 3, ...

Standing waves



$$L = \frac{m\lambda}{2}$$

m = 1

m = 2

m = 3

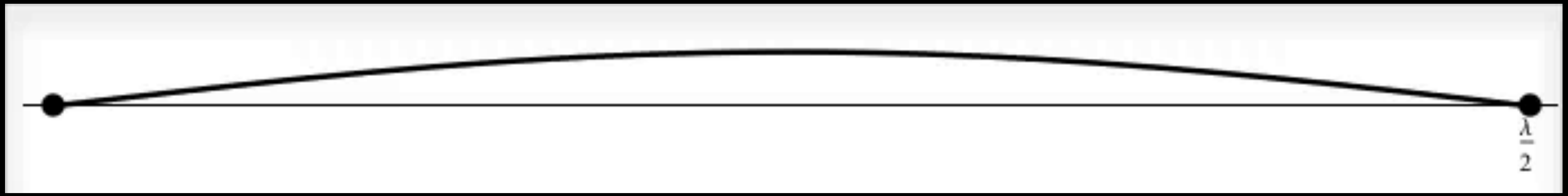
m = 4

m = 5

Standing waves

The wavelengths of standing waves are called harmonics or modes

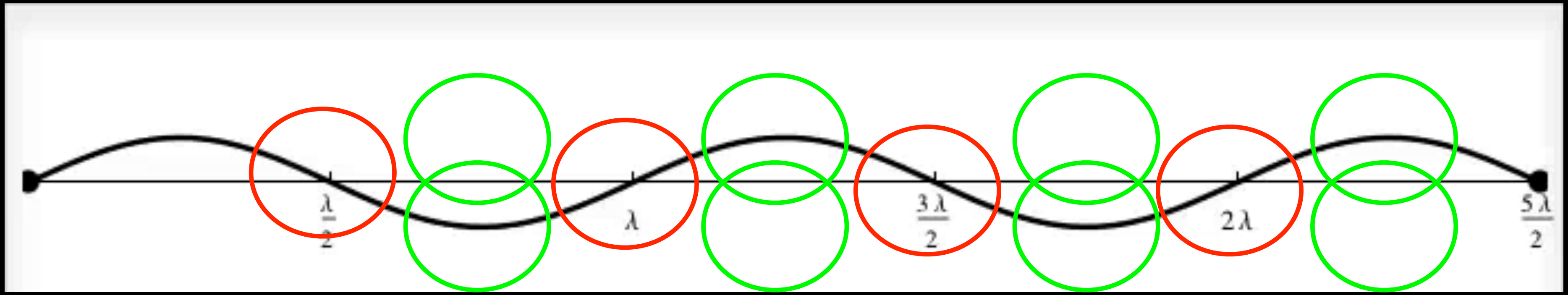
$$L = \frac{m\lambda}{2}$$ m is the mode number



m = 1

m = 1 is the fundamental mode, the longest possible standing wave

m > 1 are overtones



nodes do not move

anti-nodes oscillate between maximum and minimum

Standing waves

A standing wave is shown below.

If the period of the wave is T, the shortest time it takes for the wave to go from the solid curve to the dashed curve is

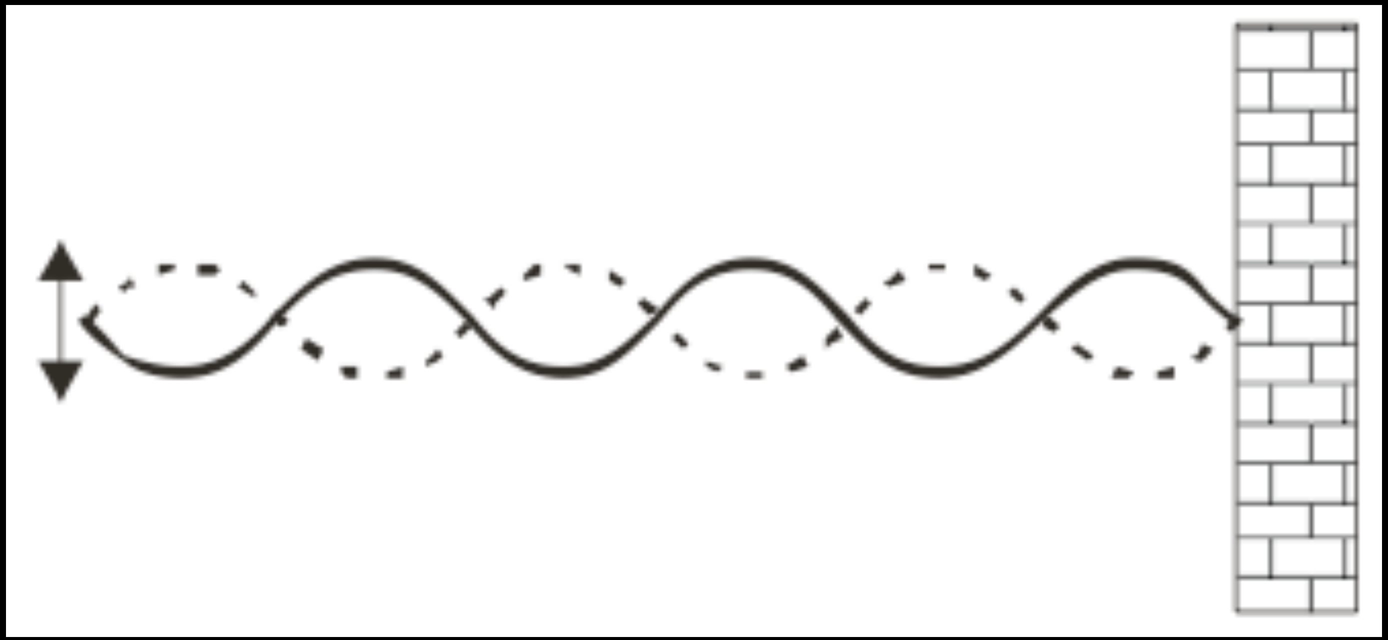
(a) T/4

(b) T/3

(c) T/2

(d) 3T/4

(e) none



Standing waves

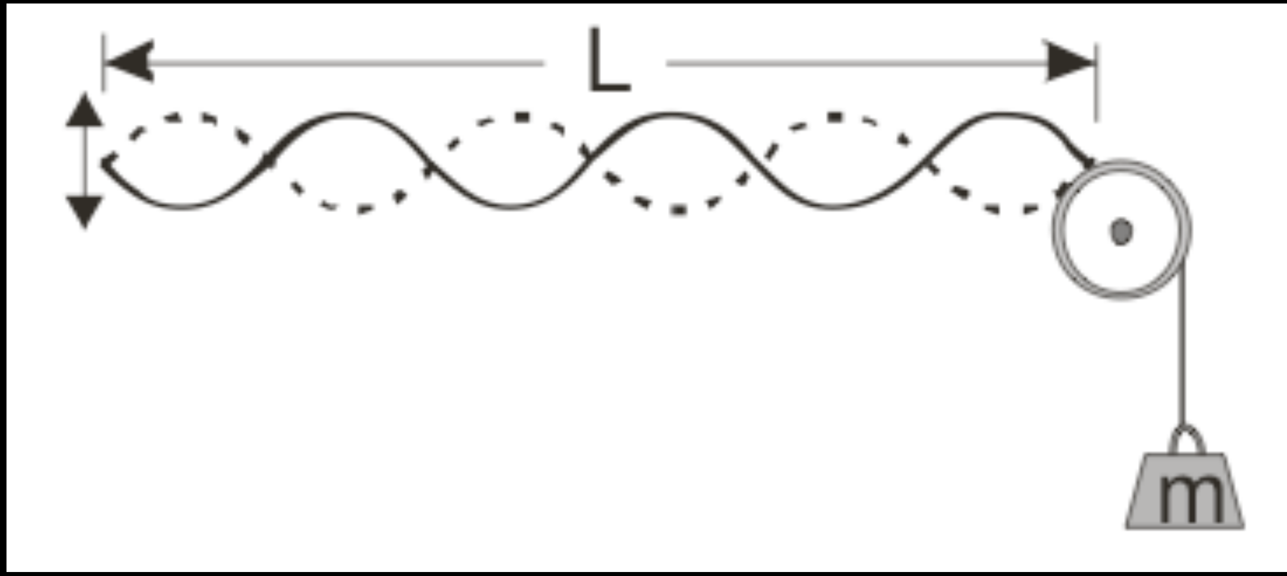
Quiz

A string of linear density μ and length L is under a constant tension T = mg. One end of the string is attached to a tunable harmonic oscillator. A resonant standing wave is observed ....

(a) at any frequency

(b) when $f = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}$

when n = 1, 2, 3...



(c) when $f = \frac{n}{L}\sqrt{\frac{mg}{\mu}}$ when n = 1, 2, 3...

(d) when $f = \frac{nv_s}{2L}$ when n = 1, 2, 3... v_s is speed of sound

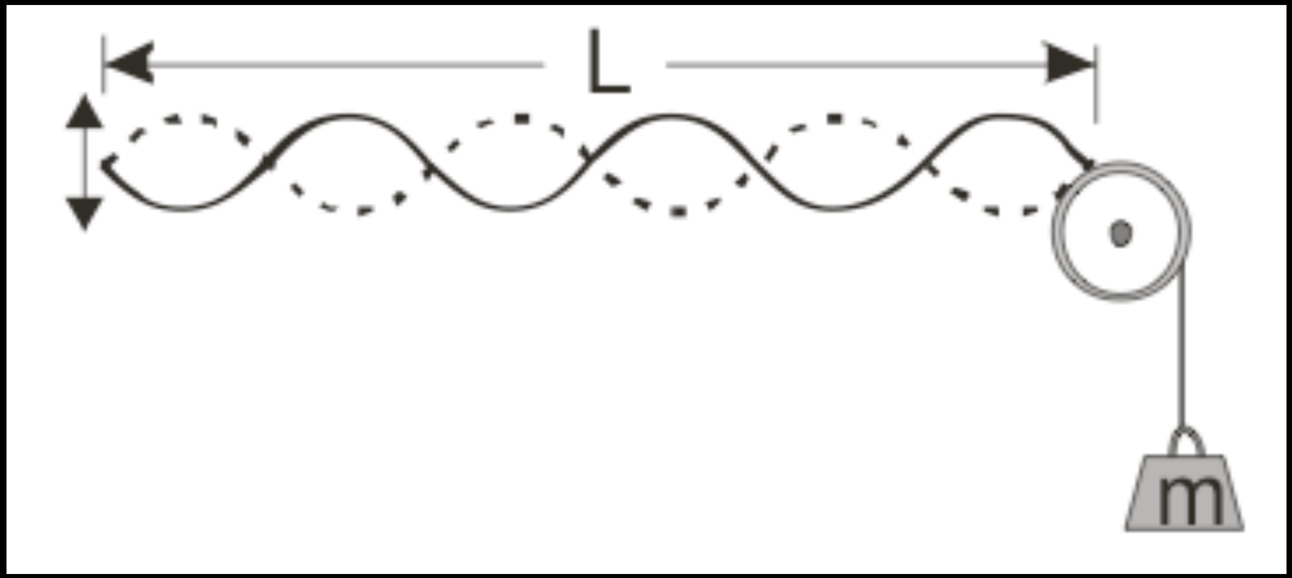
Standing waves

Quiz

A string of linear density μ and length L is under a constant tension T = mg. One end of the string is attached to a tunable harmonic oscillator. A resonant standing wave is observed ....

$$v = \sqrt{\frac{F_T}{\mu}} = \lambda f$$

$$= \frac{2L}{n} f$$



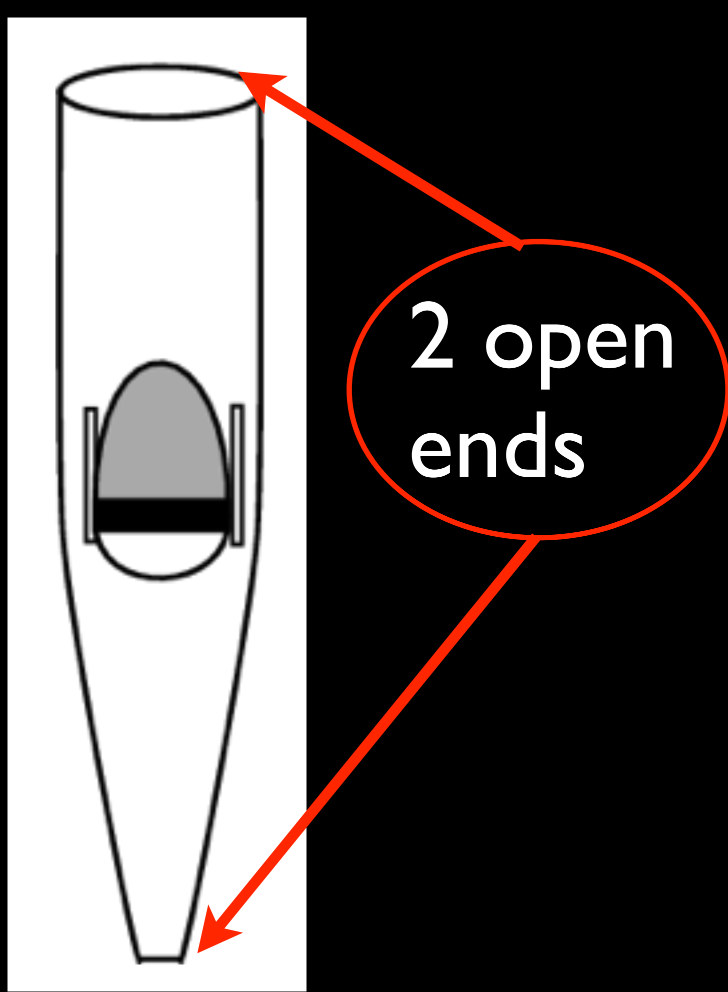
$$f = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}$$

Standing waves

Musical instruments

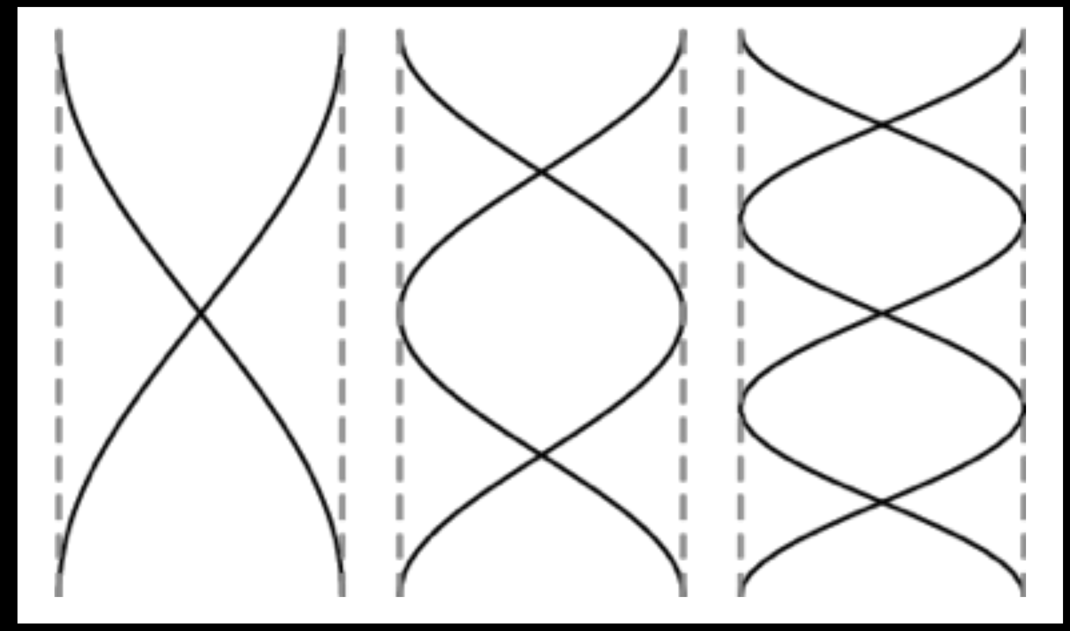
Stringed instruments (e.g. violin, piano, guitar) have standing waves like those just described (2 fixed ends).

Wind instruments (e.g. organ, bassoon, flute) make standing waves in air columns, which have open (not fixed) ends.



Open ends are fixed by pressure

They are anti-nodes (maximum amplitude)



Standing waves

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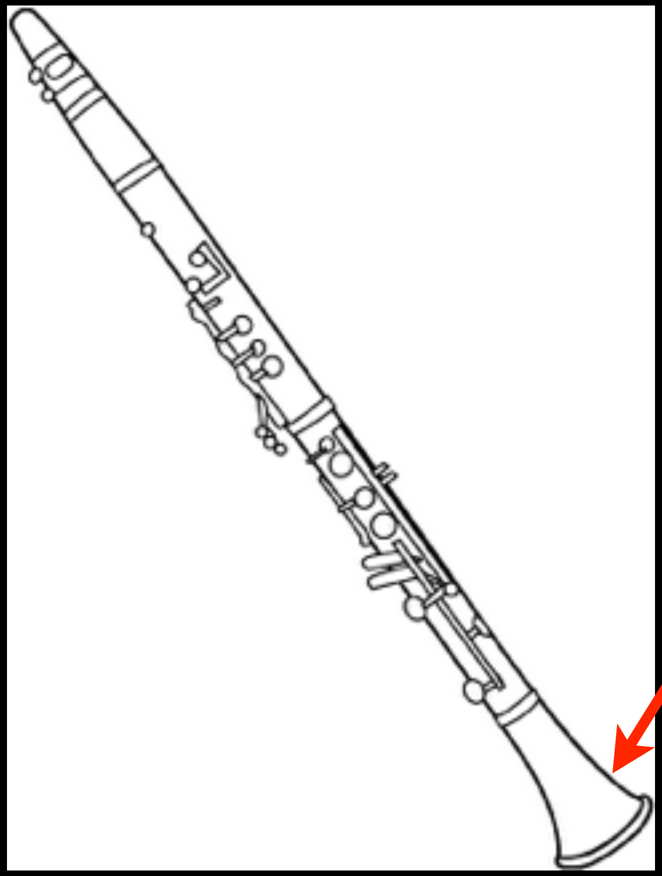
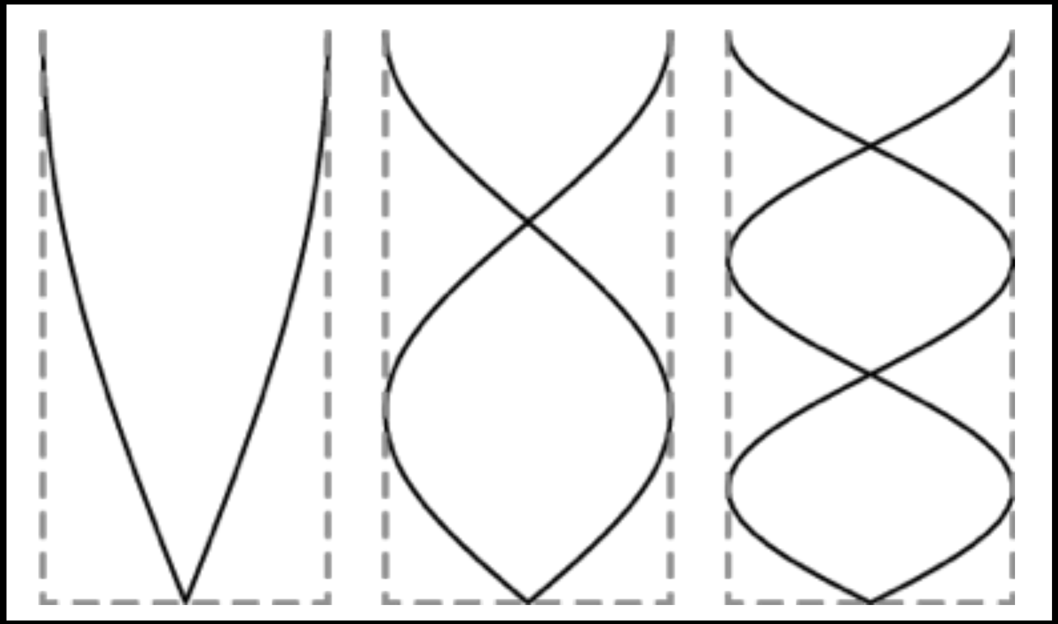


1 open end

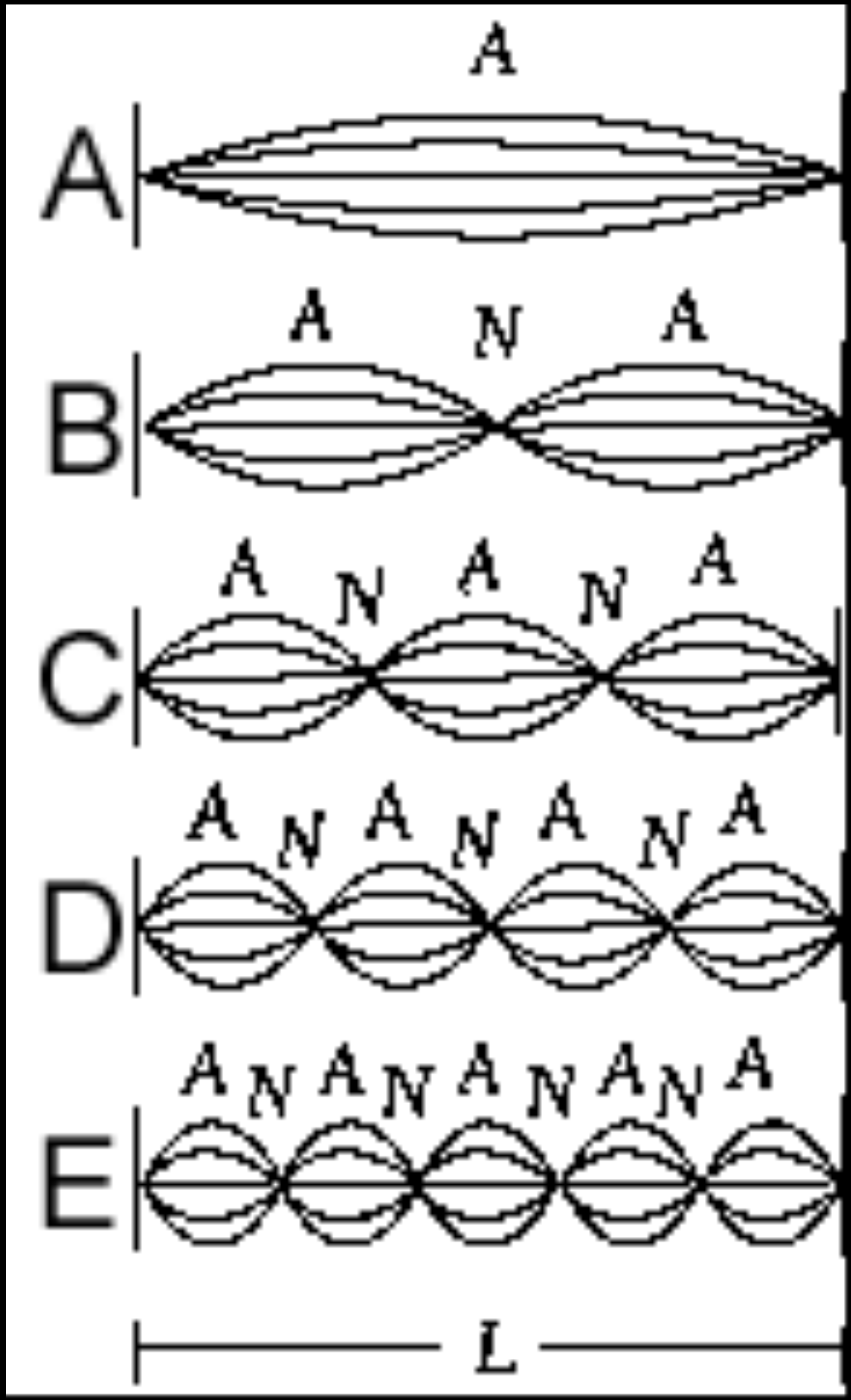
odd integer number of quarter-wavelengths

$$L = \frac{m\lambda}{4} \quad m = 1, 3, 5 \ldots.$$



Standing waves

The figure represents a string of length L, fixed at both ends, vibrating in several harmonics.



Which string shows the 3rd harmonic?

(a) A

(b) B

(c) C

(d) D

(e) E

$$\longrightarrow L = \frac{m\lambda}{2}$$ m = 1, 2, 3, ...

Standing waves

Quiz

The figure shows a standing wave in a pipe that is closed at one end. The frequency associated with this wave pattern is called the ....

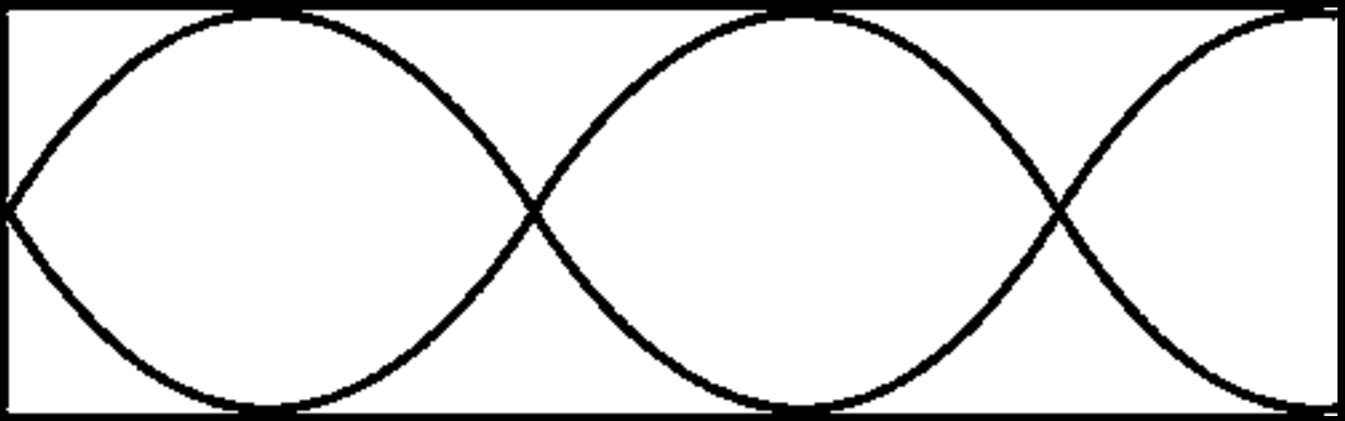
(a) 1st harmonic

(b) 2nd harmonic

(c) 3rd harmonic

(d) 4th harmonic

(e) 5th harmonic



$$L = \frac{m\lambda}{4}$$ m = 1, 3, 5 ....

Standing waves

Quiz

Of the sound sources shown, that which is vibrating with its first harmonic is the ...

(a) whistle

(b) organ pipe

(c) string

(d) rod

(e) spring